Some Notes on Rates in Qweak Monte Carlo

July 17, 2013

1 Introduction

When presenting the results of a Geant4 simulation for Qweak, there are a number of things that are of general interest and ought be included as output by default. This list includes the following:

1. The *effective "solid angle"*, for checking overall normalization and comparing acceptance of various reactions.

2. The *scattering rate*, for comparing with rates measured in event mode running.

3. The *photoelectron spectrum*, for comparing with the ADC spectra measured during event mode running.

4. The *detector integral response*, for comparing relative signal sizes measured during integral running.

The basic equation, for linking the simulation with data, applies to total scattering rate detected,

$$\mathcal{R} = \mathscr{L} \int_x \sigma(x)\epsilon(x), \tag{1}$$

where $\mathcal{R}$ is the scattering rate in the detector, σ is the cross section, ϵ is the effective solid angle of the detector, $\mathscr{L}$ is the luminosity and x represents the various degrees of freedom over which the integral is done. The Monte Carlo technique itself is used to implicitly do the integral over dimensions such as the scattering position in the target and various sizes and types of radiative effects. This is done by sampling and weighting the various processes so as to correspond to the physical situation. In addition, there is an explicit dependence on the kinematics of the reaction itself, $\phi, \cos\theta, E'$. For the general case of scattering rates from inclusive inelastic scattering, the dependence of the cross section and the acceptance on the kinematics is shown as

$$\mathcal{R} = \mathscr{L} \int_{\phi,\cos\theta,E'} \frac{d\sigma}{d\phi d\cos\theta dE'} \epsilon(\phi, \cos\theta, E') d\phi d\cos\theta dE', \tag{2}$$

or, written in differential form,

$$\mathcal{R}(\phi, \cos\theta, E') = \mathscr{L}\frac{d\sigma}{d\phi d\cos\theta dE'}\epsilon(\phi, \cos\theta, E') \tag{3}$$

The specific case of elastic scattering is similar but does not have a separate dependence on E' since energy and momentum conservation dictate a specific E' for any θ.

There is an additional piece in party violation experiments, where instead of counting scattered particles, the signal in the detector is integrated. Here we are instead interested in the signal size in the detector, $\mathcal{S}$, where each event deposits a different number of photoelectrons, $\mathcal{P}$, which can depend on the kinematics.

$$\mathcal{S}(\phi, \cos\theta, E') = \mathscr{L}\frac{d\sigma}{d\phi d\cos\theta dE'}\epsilon(\phi, \cos\theta, E')\mathcal{P}(\phi, \cos\theta, E') \tag{4}$$

2 Effective Solid Angle

The effective solid angle ϵ, is a measure of the phase space over which the detectors are able to accept scattered particles. It is measured in units of solid angle (a fraction of 4π) for elastic scattering. For inelastic scattering, the equivalent quantity is in units of both solid angle *and* energy of the scattered particle, so ϵ could be referred to as the effective phase space but is typically still referred to as a solid angle. Here

$$\epsilon = \frac{N_{det}}{N_{gen}}\mathcal{G}_V, \tag{5}$$

where N_{det} is the number of detected events, N_{gen} is the number of generated events, and $\mathcal{G}_V = \Delta\phi\Delta\cos\theta\Delta E'$ is the generation volume - the kinematic ranges over which initial events are generated.

The total success rate, A, is the fraction of successfully "detected" simulation events, given by

$$A = \frac{N_{det}}{N_{gen}}. \tag{6}$$

If the simulation limits are chosen sufficiently wide to cover the available phase space, then increasing the generation ranges, $\mathcal{G}_V$, will decrease the success rate such that the effective solid angle is unchanged.

For some purposes it is useful to generalize the success rate into an acceptance function, as a function of some parameter y, such as scattering position in the target. In that case, the generating information for all events in the simulation will need to be saved regardless of whether they hit the detector. The histogram $A(y)$ is obtained by first making the individual histograms $N_{det}(y)$ and $N_{gen}(y)$ and then taking the ratio bin-by-bin.

$$A(y) = \frac{N_{det}(y)}{N_{gen}(y)} \tag{7}$$

Unlike the total success rate, an acceptance function is a general property of the apparatus.

3 Scattering Rate

Comparing rates for data and simulation requires agreement in the treatment of the analysis for both. In particular, it is important to correctly take the trigger threshold into account and to obtain equivalent independent normalization.

3.1 Trigger Threshold

Discussion of how to analyse the data to extract scattering rates is beyond the scope of this document. The current best estimate for the hardware trigger threshold used during data taking is 1.5 photoelectrons (PEs) for each photomultiplier tube, in coincidence. This number is not based on a rigorous analysis and will hopefully be made more precise. The simulation produces events with an integer number of PEs, so in principle, a rigorous treatment of would require additional software cuts for the data corresponding to a cut on the integer number of PEs in the simulation. However, particularly in the absence of such data analysis, it is important to determine the sensitivity of any rates obtained from simulation on the trigger threshold level by repeating the analysis with 3 different PE cuts: $\mathcal{P} > 0$, $\mathcal{P} > 1$ and $\mathcal{P} > 2$.

3.2 Luminosity

The Luminosity, $\mathscr{L}$, is the proportionality factor between cross section σ and the number of interactions per second,

$$\mathcal{R} = \mathscr{L}\sigma. \tag{8}$$

This is equal to the product of the flux of particles in the beam, N_b (dimensions of 1/time), and the areal number of scatterers in the target, N_t (dimensions of 1/area),

$$\mathscr{L} = N_b N_t \tag{9}$$

The number of particles in the beam is given by the definition of the Ampere,

$$N_b = 6.241 \times 10^{12} \text{ Hz } \mu\text{A}^{-1}. \tag{10}$$

The number of scatterers in the target is given by

$$N_t = \frac{\rho_A N_A}{M_A}, \tag{11}$$

where N_A is Avagadro's Number, M_A is the average mass of the target nuclei in atomic mass units, and ρ_A is the areal density. The areal density can be decomposed as $\rho_A = l \cdot \rho_V$ where l is the target thickness along the beam direction and ρ_V is the usual volume density, which is appropriate for the cryogenic target. For the solid targets it is more precise to measure ρ_A directly from other methods.

For example, the Qweak hydrogen target has the luminosity calculated as follows.

$$\begin{aligned}
\mathscr{L} &= N_b \frac{\rho_A N_A}{M_A} \\
&= 6.24 \times 10^{12} \text{ Hz } \mu\text{A}^{-1} \cdot \frac{34.4\text{cm} \cdot 0.0713\text{g cm}^{-3} \cdot 6.02 \times 10^{23}\text{mol}^{-1}}{1.007\text{g mol}^{-1}} \\
&= 9.14 \times 10^{36} \text{ Hz } \mu\text{A}^{-1}\text{cm}^{-2} \\
&= 9.14 \times 10^{6} \text{ Hz } \mu\text{A}^{-1}\mu\text{barn}^{-1}, \qquad (12)
\end{aligned}$$

where we have used the definition of the the barn, 1 μbarn $= 10^{-30}$ cm^2.

3.3 Practical

In practice, the following rules should be observed when working with rates:

1. Analysis based on the number of successful events needs a cut $\mathcal{P} > 0$ to be meaningful. For this reason, all histograms should have the cut $\mathcal{P} > 0$ in order to make the statistics box, assumed to show the number of successful events N_{det}, correct. In addition, further analysis to determine the sensitivity to a trigger threshold should be done if any comparison with data will be made.

2. Histograms should be filled with the weight $\sigma \cdot \mathscr{L}\mathcal{G}_V/N_{gen}$ (where $\mathscr{L}\mathcal{G}_V/N_{gen}$ is a number for the simulation as a whole and σ is obtained event-by-event) and the y-axis labeled "*rate (Hz μA^{-1})*".

4 Photoelectron Spectrum

The challenge here is to do an "absolute comparison" of the PE spectrum from simulation with the ADC spectrum from event mode data taking. Such a comparison should use an independent normalization of the PEs per ADC channel. These have been determined during LED calibration running and are available in the log book. In addition, both spectra should be filled in units of Hz μA^{-1} for independent normalization. The previous section describes how to determine these rates from the simulation.

5 Detector Integral Response

In the integrating mode there is a potential coupling between the signal level in the detector and the kinematics of the scattering, such that $\langle \sigma S \rangle \neq \langle \sigma \rangle \langle S \rangle$. Histograms should be filled with the weight $\mathcal{P}\sigma \cdot \mathscr{L}\mathcal{G}_V/N_{gen}$ (where $\mathscr{L}\mathcal{G}_V/N_{gen}$ is a number for the simulation as a whole and $\mathcal{P}\sigma$ is obtained event-by-event) and the y-axis labeled "*Current mode yield (PEs μA^{-1})*". In this case it is not clear that it will be possible to independently normalize the data into units of

PE rate. Nevertheless, the full weighting is still necessary for simulation analysis so that simulations with different generation limits, number of thrown events and various targets can be compared in a meaningful way. When plotting the detector integral response, since the weight includes PEs, the $\mathcal{P} = 0$ events don't contribute to the spectrum—however they would still contribute to the number of events in the histogram, which appears in the stats box, so the cut $\mathcal{P} > 0$ should still be applied.