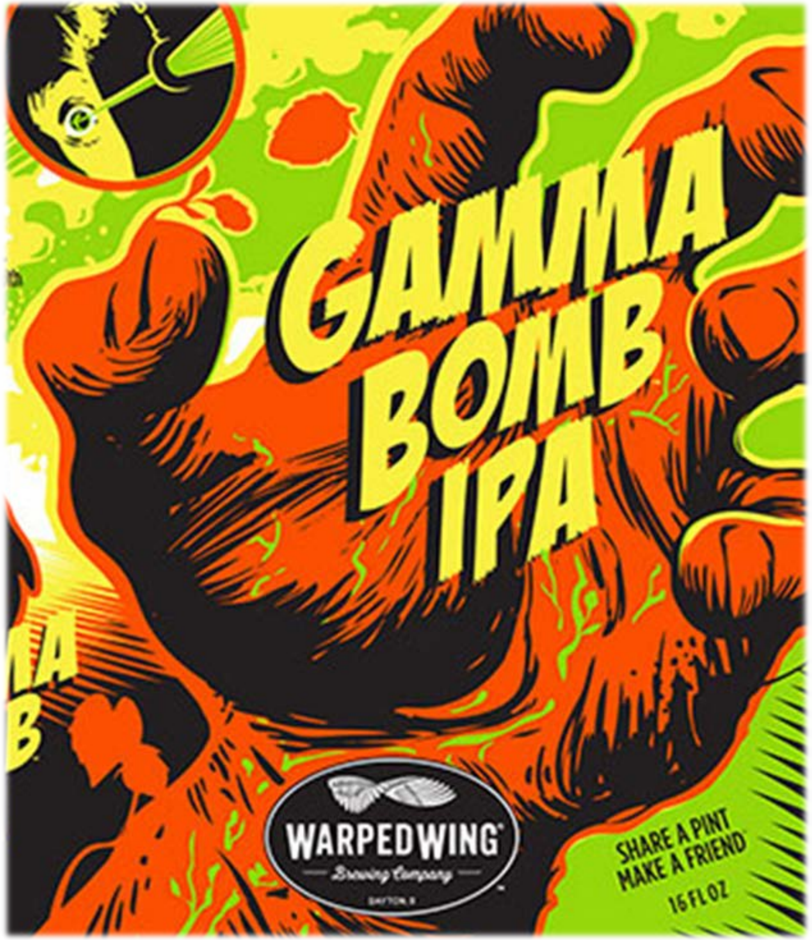
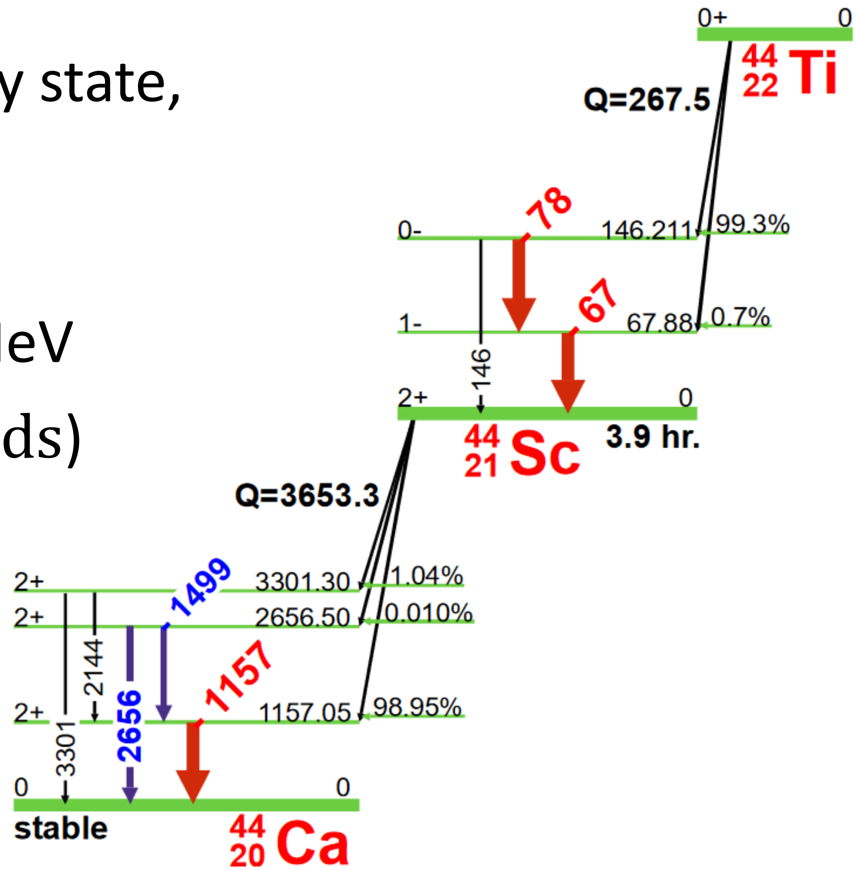
Lecture 9: γ *Decay*

- Basics
- Energetics
- Transition rates
- Angular correlations
- γ strength functions



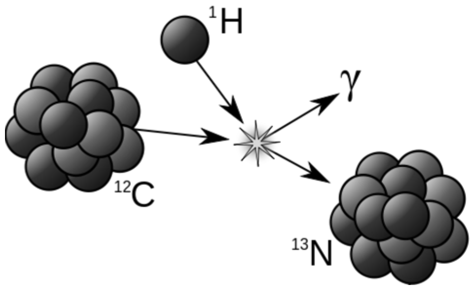
γ decay basics

- γ decay is a de-excitation from an excited state to a lower energy state, preceded by some decay or reaction
- *[Just to be clear]* Z & A are unchanged
- γ ray energies can span anywhere from several keV to several MeV
- γ decay lifetimes are typically extremely short ($\tau \lesssim \mathrm{femtoseconds}$)
  [with the exception of isomeric states]



INEL γ Spectrum Catalog

γ decay energetics



- γ decay can be used to probe excited state energies, but is $E_\gamma = E_{xs}$?
- For a decay from a higher-lying excited state to a lower-energy one,
  $M_{higher}c^2 = M_{lower}c^2 + E_\gamma + KE_{recoil}$
- $M_{higher}c^2 - M_{lower}c^2 \equiv E_{xs} = E_\gamma + KE_{recoil}$
- $KE_{recoil} = \frac{p_{recoil}^2}{2M_{recoil}}$
- Conservation of momentum dictates $p_\gamma = p_{recoil}$, so $KE_{recoil} = \frac{p_\gamma^2}{2M_{recoil}}$
- Recall that for a massless particle, $E = pc$, so $KE_{recoil} = \frac{E_\gamma^2}{2M_{recoil}c^2}$
- Consider $E_\gamma = 2\mathrm{MeV}, \mathrm{A} = 50$: $KE_{recoil} = \frac{4\mathrm{MeV}^2}{2 \cdot 50 \cdot 931.5\mathrm{MeV}} \approx 43\mathrm{eV}$
- For, a $5\mathrm{MeV}$ γ from ^{3}He *(e.g. populated by d+p)*, $KE_{recoil} \approx 4.5\mathrm{keV}$

Do we have to worry about EM interactions within the nucleus?

•How does the photon wavelength compare to the nuclear size?

•$E_\gamma = h\nu = \frac{hc}{\lambda} = \frac{2\pi\hbar c}{\lambda} \approx \frac{2\pi(197 MeV\cdot fm)}{\lambda}$

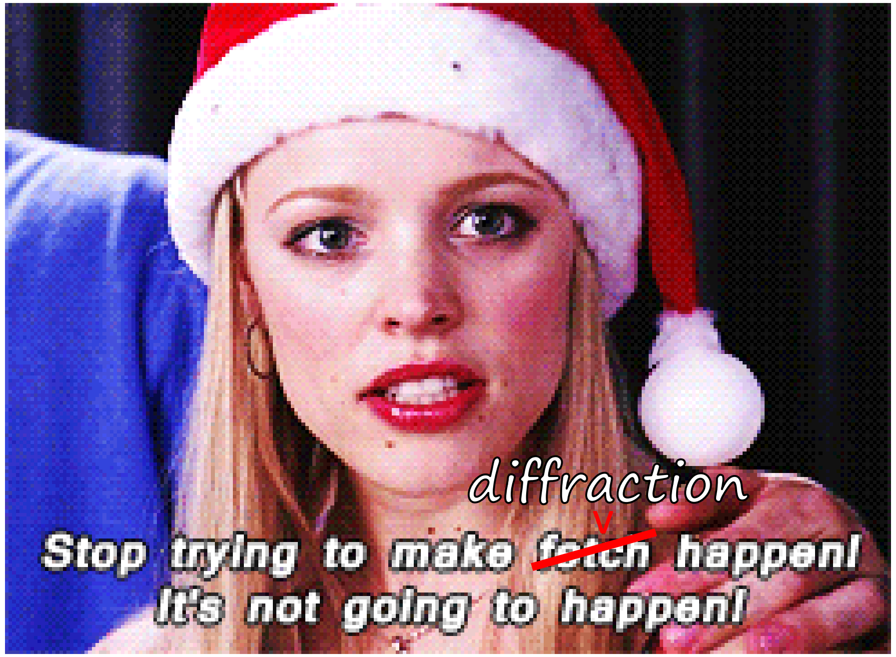
•$\lambda \approx \frac{2\pi(197 MeV\cdot fm)}{E_\gamma}$ …for $E_\gamma = 10\text{MeV}$: $\lambda \approx 124 fm$

•For a large nucleus, $A = 200$, the diameter $D \approx 2 \cdot 1.2 fm \cdot A^{1/3} \approx 14 fm$

•So, even for an extreme case, $\lambda \gg D$

•For antennas and diffraction, one needs $\lambda \lesssim D$

i.e. it's not going to happen



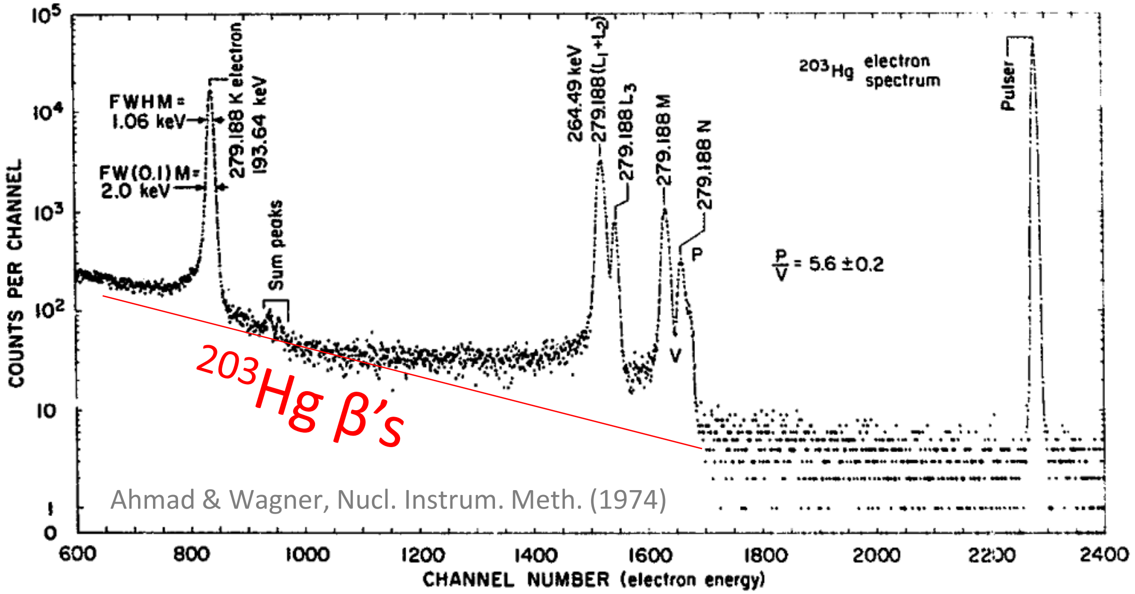
γ decay types

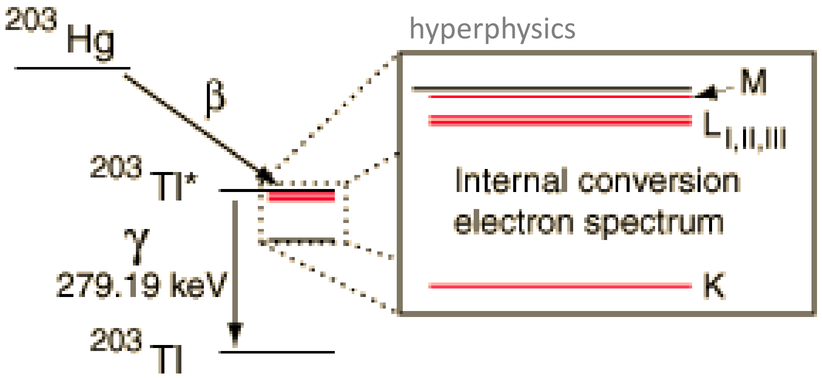
- Parity and angular momentum are conserved during γ decay
- Photons carry some integer angular momentum with a minimum $l = 1$, where l is referred to by the multipole 2^l
  - $l = 1: dipole, l = 2\ quadrupole, \cdots$
- A photon's parity depends not only on l, but also on the decay type
- A photon decay corresponds to shift in the nucleus's charge and matter distribution
  - Shift in the charge distribution = change in electric field = *Electric*
  - Shift in the current distribution [i.e. orbitals of protons]= change in magnetic field = *Magnetic*
- The selection rules corresponding to a particular decay type are:

Radiation Type	Name	$l = \Delta I$	$\Delta\pi$
E1	Electric dipole	1	Yes
M1	Magnetic dipole	1	No
E2	Electric quadrupole	2	No
M2	Magnetic quadrupole	2	Yes
E3	Electric octupole	3	Yes
M3	Magnetic octupole	3	No
E4	Electric hexadecapole	4	No
M4	Magnetic hexadecapole	4	Yes

How does $0^+ \rightarrow 0^+$ happen? *Internal conversion*

- Since $l_{min} = 1$ for a photon, de-excitation by photon emission isn't possible
- Instead the process of internal conversion can happen, whereby a nucleus interacts electromagnetically with an orbital electron and de-excites by ejecting that orbital electron
- This process operates in competition with γ decay for any transition, not just $0^+ \rightarrow 0^+$
- The energy of the emitted electron is: $E_{IC} = E_{xs} - E_{BE,e^-}$, where E_{xs} is the decay transition energy, and E_{BE,e^-} is the electron binding energy





A similar, but different phenomenon is Internal Pair Conversion, where a photon emitted by the nucleus with $E_\gamma > 2m_ec^2$ interacts with the coulomb field of the nucleus to create an e^+–e^- pair. See e.g. A. Wuosmaa et al. PRC(R) 1998.

γ decay constant

- As with β decay, the decay constant is described by the expectation value of a small perturbation multiplied with the final state density, i.e. by Fermi's Golden Rule
  - $\lambda = \frac{2\pi}{\hbar}\left|\langle\Psi_{final}|H'|\Psi_{initial}\rangle\right|^2\rho(E)$
- However, here $\rho(E)$ is the product of the density of nuclear states and density of electromagnetic states available in the system created by the transition and Ψ_{final} is a product of the wavefunction of the final nucleus and the outgoing electromagnetic wave
- Without proof*, it turns out that lots of gymnastics with E&M will result in:
  - $\lambda\left(l_\gamma, J_i, \pi \to J_f, \pi\right) = \frac{8\pi(l_\gamma+1)}{l_\gamma\left[(2l_\gamma+1)!!\right]^2}\frac{\left({}^{E_\gamma}/_{\hbar c}\right)^{2l_\gamma+1}}{\hbar}B(l_\gamma, J_i, \pi \to J_f, \pi)$,

  *for a proof, see Appendix 25 of Quantum Mechanics for Engineers (L. van Dommelen)

  where $n!!$ is the double-factorial of n (product of all odd or even integers from n to 2 or 1), and $B(l_\gamma, J_i, \pi \to J_f, \pi)$ is the "reduced transition probability", which is the square of the modulus of the expectation value of the transition operator (for El or Ml)
- These B are extremely nasty to deal with…so lucky for us, back in the mists of time, Weisskopf derived general expressions for different transitions types, assuming the decay was due to a single nucleon undergoing a transition

Weisskopf (a.k.a. single-particle) estimates for λ

- $\lambda(l_\gamma, J_i, \pi \to J_f, \pi) = \frac{8\pi(l_\gamma+1)}{l_\gamma[(2l_\gamma+1)!!]^2} \frac{\left(E_\gamma/\hbar c\right)^{2l_\gamma+1}}{\hbar} B(l_\gamma, J_i, \pi \to J_f, \pi),$
- Reduced transition probabilities assuming the initial to final state transition is due to a single nucleon re-orienting itself within a nucleus of uniform density with $R = r_0 A^{1/3}$ are:
  - $B_{s.p.}(E, l_\gamma) = \frac{1}{4\pi}\left[\frac{3}{l_\gamma+3}\right]^2 r_0^{2l_\gamma} A^{2l_\gamma/3} e^2 (fm)^{2l_\gamma}$

    The units for B change with l_γ!
  - $B_{s.p.}(M, l_\gamma) = \frac{10}{\pi}\left[\frac{3}{l_\gamma+3}\right]^2 r_0^{(2l_\gamma-2)/2} \mu_n^2 (fm)^{2l_\gamma-2}$, where the nuclear magneton $\mu_n = \frac{e\hbar}{2m_p c}$
- Note: There is a steep dependency of λ on l, so only one multipole of a decay type will matter
- The equations above are still a huge pain to work with and more noble souls have worked-out the decay constant for various situations.
- Using Q in MeV, λ_γ in s^{-1} for a nucleus with mass number A is given by:

L. van Dommelen, Quantum Mechanics for Engineers (2012)

$$\boxed{\lambda^{E\ell} = C_{E\ell} A^{2\ell/3} Q^{2\ell+1} \qquad \lambda^{M\ell} = C_{M\ell} A^{(2\ell-2)/3} Q^{2\ell+1}}$$

ℓ :	1	2	3	4	5
$C_{E\ell}$:	$1.0\ 10^{14}$	$7.3\ 10^{7}$	34	$1.1\ 10^{-5}$	$2.4\ 10^{-12}$
$C_{M\ell}$:	$3.1\ 10^{13}$	$2.2\ 10^{7}$	10	$3.3\ 10^{-6}$	$7.4\ 10^{-13}$

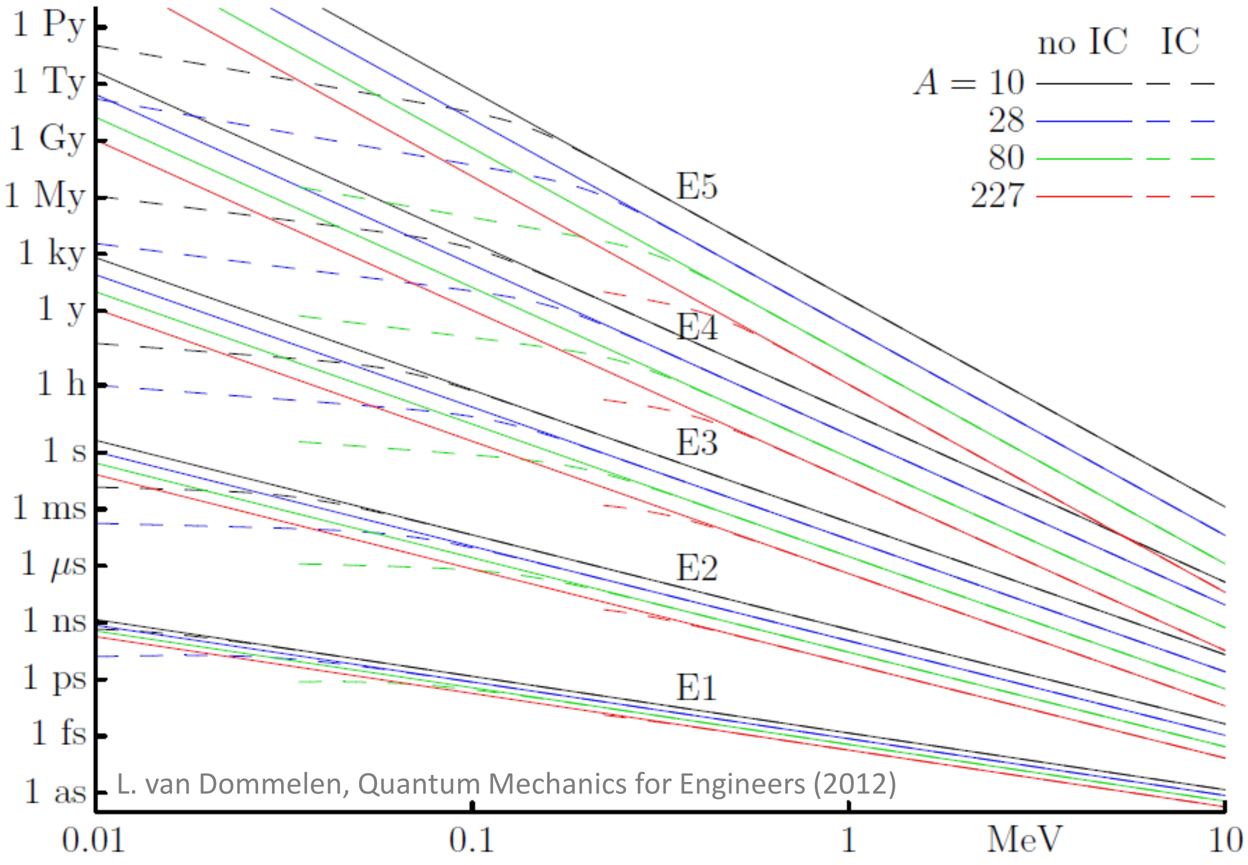
- Weisskopf estimates are generally within a few orders of magnitude of the real answer, so γ decay constants are often quoted as the ratio to this estimate in "Weisskopf Units" [w.u.]

Weisskopf (a.k.a. single-particle) estimates for Γ_γ

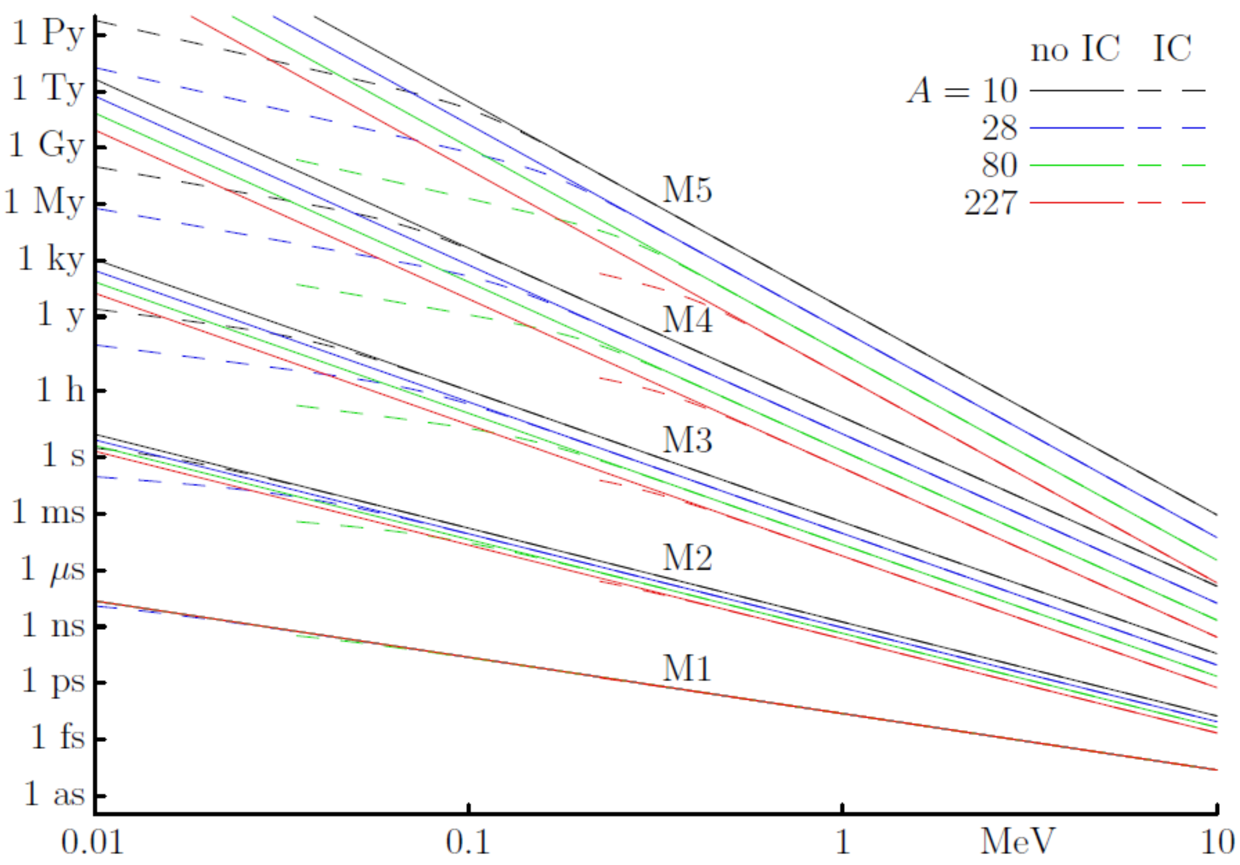
- For nuclear reactions, you may instead want the gamma-width
- The decay constant can be converted to the half-life, which can be converted to a partial width via the uncertainty principle (see Radioactive Decay lecture)
- To save you the effort, here are the Weisskopf widths for the transition types most commonly involved in astrophysical reaction rate calculations:
  (with Γ_γ in units of eV, E_γ in units of MeV, and A as the mass number):
  - E1: $\Gamma_\gamma(E_\gamma) = 6.8 \times 10^{-2} A^{2/3} E_\gamma^3$
  - M1: $\Gamma_\gamma(E_\gamma) = 2.1 \times 10^{-2} E_\gamma^3$
  - E2: $\Gamma_\gamma(E_\gamma) = 4.9 \times 10^{-6} A^{4/3} E_\gamma^5$
  - M2: $\Gamma_\gamma(E_\gamma) = 1.5 \times 10^{-8} A^{2/3} E_\gamma^5$
- Note that these are upper limits. Actual E1 widths are often x10^{-4} lower.
  The best thing to do is use data from nearby nuclides and the same transition type to determine what a reasonable reduction factor would be for the Weisskopf estimate
  - You can find experimental data with the “Nuclear Levels and Gamma Search” under the NuDat portion of the NNDC webpage

Weisskopf (a.k.a. single-particle) estimates for $t_{½}$

$t_{½}(E_\gamma)$ *for **Electric** Transitions (from Weisskopf)*



$t_{½}(E_\gamma)$ *for **Magnetic** Transitions (from Moszkowski)*

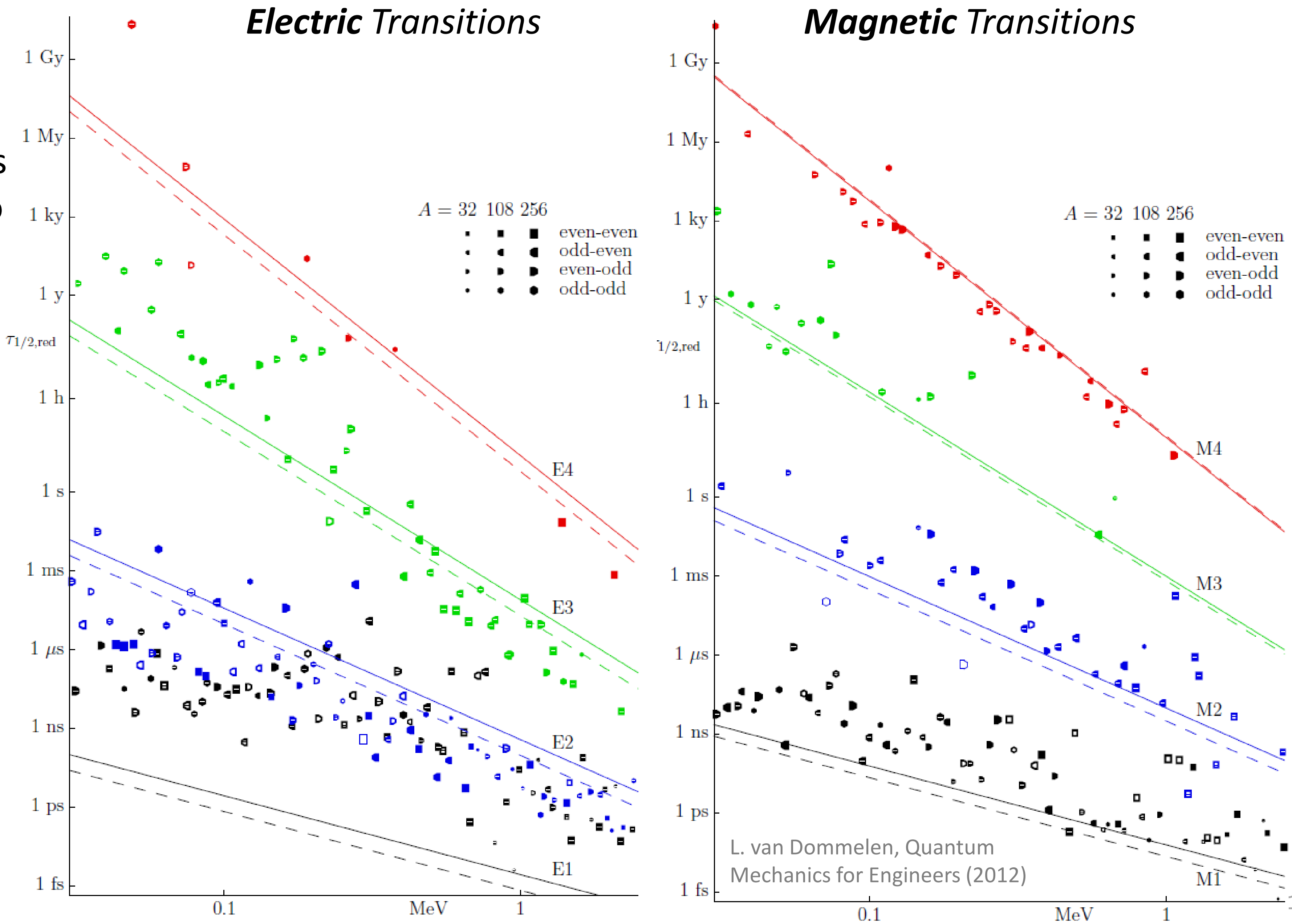


Note that E transitions of a given multipole and E_γ are ~100X faster than M transitions with the same E_γ,ℓ

Now we see how it is that low-energy high-spin states exist as isomeric states.

Weisskopf estimates for $t_{½}$ compared to data

- Single-particle estimates aren't too bad
- The general spread is explained away by the fact that transitions are often not going to be due to a single particle rearranging itself, but rather a collective effect
- $E2$ are thought to be faster because collective de-excitations *(rotation, vibration)* will favor $\Delta J = 2, \Delta\pi = no$
- However, other lower-order multipoles are thought to be suppressed because shape-changes will generally be complex and, in general, poorly described by smoothly varying Legendre polynomials (low l) used for the EM interaction

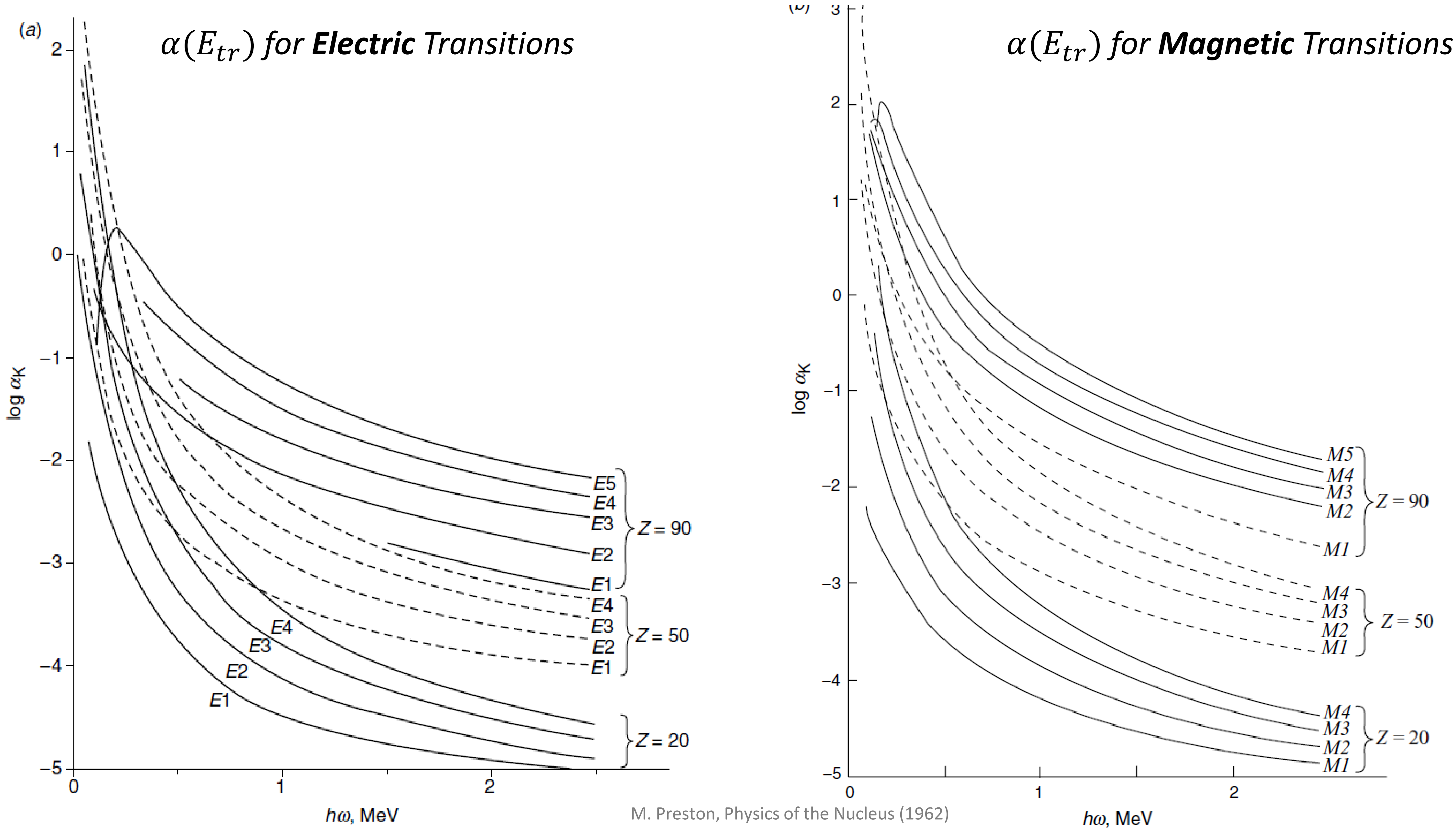


Internal conversion coefficient, α

- Don’t forget about our old friend, internal conversion, which competes with γ decay
- Competition between the two is described by the internal conversion coefficient $\alpha = \frac{\lambda_{IC}}{\lambda_\gamma}$,
  so $\lambda = \lambda_{IC} + \lambda_\gamma = \lambda_\gamma(1+\alpha)$
- α depends on the density of electrons near the nucleus, and so some friendly atomic physicists have done the dirty work of calculating the following approximate formulas:
  - $\alpha(El) = \frac{Z^3}{n^3}\left(\frac{l}{l+1}\right)\alpha_{f.s.}^4\left(\frac{2m_ec^2}{Q}\right)^{l+5/2}$; $\alpha(Ml) = \frac{Z^3}{n^3}\alpha_{f.s.}^4\left(\frac{2m_ec^2}{Q}\right)^{l+3/2}$,
    where $\alpha_{f.s.} \approx \frac{1}{137}$, Q is the transition Q-value,
    and n is the principal quantum number of the orbital electron being ejected

  These rely on the Born approximation, so Z<<137 ought to apply
- The atomic orbitals $K, L, M, N, O, \cdots$ correspond to $n = 1, 2, 3, 4, 5, \cdots$
- Clearly this process is favored for high-Z nuclei, …but also for $Q < 1.022 MeV$ $l = 0$ transitions
- For $0^+ \rightarrow 0^+$ transitions, $\lambda_{E0} = 3.8 \cdot Z^3 A^{4/3} Q^{1/2}$, with Q in MeV and λ in s^{-1}

Internal conversion coefficient, α



M. Preston, Physics of the Nucleus (1962)

λ_{E0} predictions compared to data

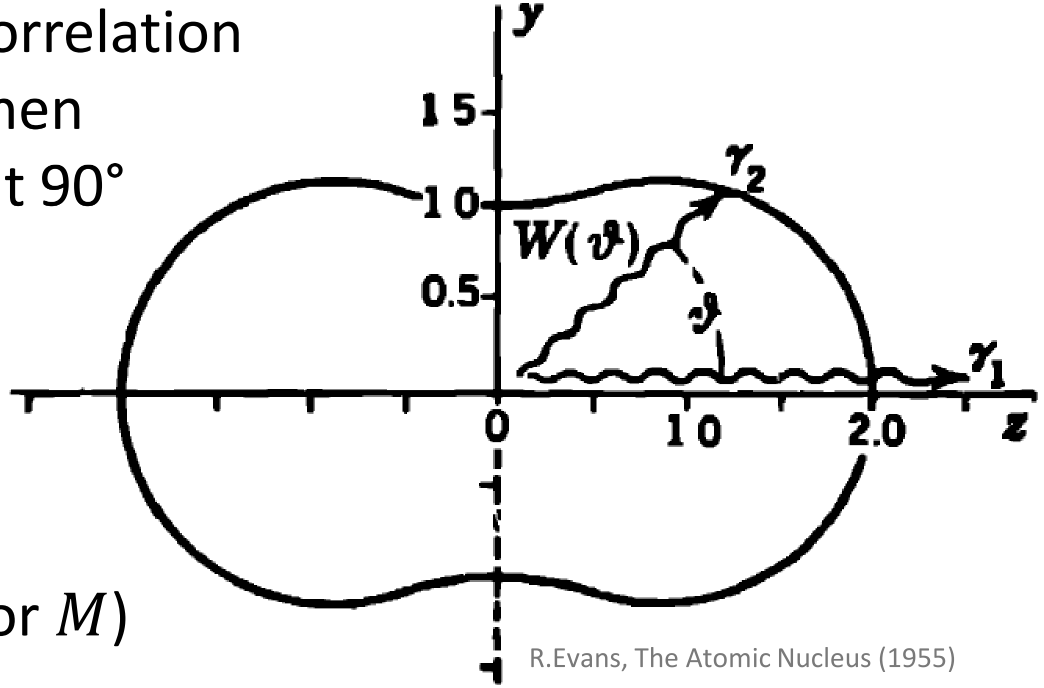
Nucleus	Q	$\tau_{1/2}$ μs	$\tau_{1/2,\mathrm{red}}$ μs	I.C. Theory μs
$^{184}_{80}\mathrm{Hg}$	0.375	0.00062	0.80	0.72
$^{72}_{36}\mathrm{Kr}$	0.671	0.0263	0.88	0.54
$^{72}_{32}\mathrm{Ge}$	0.691	0.4442	10.	0.53
$^{98}_{42}\mathrm{Mo}$	0.735	0.0218	1.8	0.51
$^{192}_{82}\mathrm{Pb}$	0.769	0.00075	1.1	0.50
$^{98}_{40}\mathrm{Zr}$	0.854	0.064	4.4	0.47
$^{194}_{82}\mathrm{Pb}$	0.931	0.0011	1.6	0.45
$^{96}_{40}\mathrm{Zr}$	1.582	0.0380	2.6	0.35
$^{90}_{40}\mathrm{Zr}$	1.761	0.0613	3.8	0.33
$^{68}_{28}\mathrm{Ni}$	1.770	0.2760	4.0	0.33
$^{40}_{20}\mathrm{Ca}$	3.353	0.00216	0.0057	0.24
$^{32}_{16}\mathrm{S}$	3.778	0.00254	0.0025	0.23
$^{16}_{8}\mathrm{O}$	6.048	0.00007	0.000003	0.18

For these decays e^+–e^- pair creation is a significant decay mode

L. van Dommelen, Quantum Mechanics for Engineers (2012)

γ angular correlations

- Radioactive decay, including γ decay, is isotropic when nuclei are oriented at random, which is generally the case in a laboratory setting
- However, the γ angular distribution relative to a fixed axis on a given nucleus, e.g. the angular momentum projection of the state the nucleus originates from, is anisotropic
- Measuring one γ-ray essentially identifies the nuclear axis, so measuring the next γ-ray, which has an angular distribution relative to the angular momentum projection of the state populated by the first γ-ray, will result in an angular correlation
- We choose the original γ-ray to be at $\theta_{CM} = 0$, and then the relative intensity to the intensity of the 2nd γ-ray at 90° $W(\theta) = \frac{Intensity\ at\ \theta}{Intensity\ at\ \theta_{cm}=90°}$ is described by a sum of several Legendre polynomials, since after all the populated m and photon multipolarity l correspond to a given Legendre polynomial
- Note $W(\theta)$ does not depend on the γ-decay type (E or M)

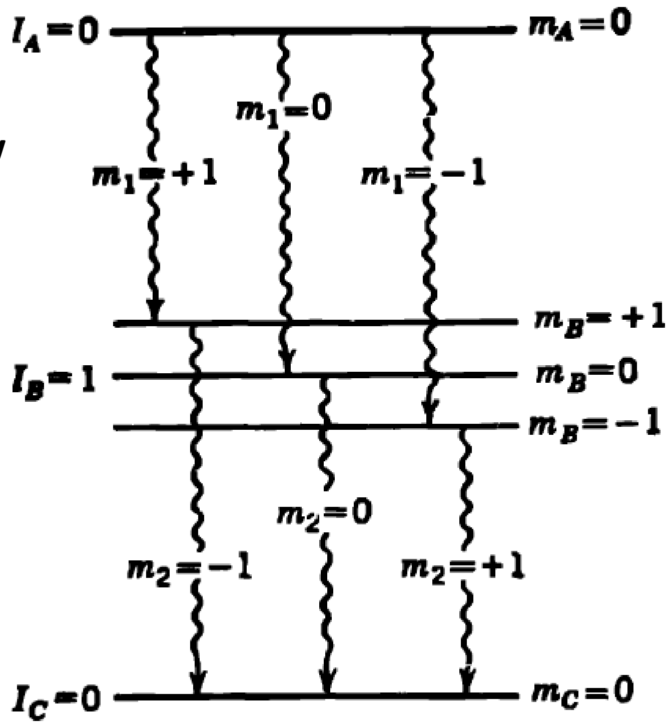


R.Evans, The Atomic Nucleus (1955)

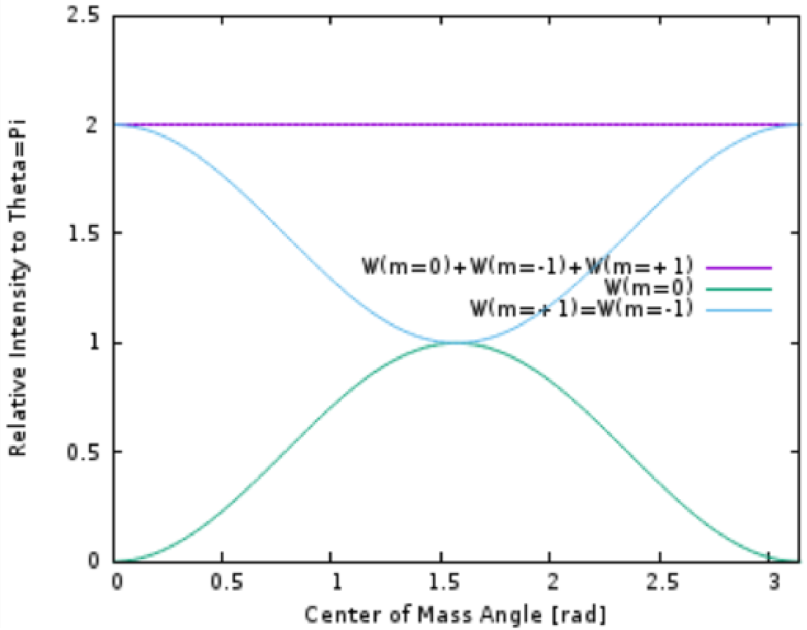
In all this and the following talk of W(θ), θ refers to the center of mass angle …however, as we showed earlier, the nuclear recoil is mostly negligible, so $\theta_{CM} \approx \theta_{lab}$.

γ angular correlations, dipole case

R.Evans, The Atomic Nucleus (1955)



- Let's restrict ourselves to dipole-dipole ($l_{\gamma_1}=1, l_{\gamma_2}=1$) radiation for now
- The first decay will populate the $m=-1,0,1$ magnetic sub-states with equal probability
- The angular distribution for these three radiations are described by
  - $W_{l=1,\boldsymbol{m=0}}(\theta) \propto \sin^2(\theta)$
  - $W_{l=1,\boldsymbol{m=+1}}(\theta) = W_{l=1,\boldsymbol{m=-1}}(\theta) \propto \ (1+\cos^2(\theta))$
- Their sum is an isotropic distribution
- However, for the second decay, we can gate on events where the 1st decay is located on our arbitrarily chosen $\theta=0$ axis
- In that case, $W_{l=1,\boldsymbol{m=0}}(0)=0$, so the second γ (which also has $l=1$) will have an anisotropic distribution
- Thus, any γ-γ coincidence displaying such an angular correlation is indicative of dipole-dipole radiation
- This is mighty handy, since, if we know the ground-state J, (e.g. $J=0$ for even-even nuclei) then we can build-up from there to get J for the preceding two levels



Arfken, Klema, & McGowan, Phys. Rev. Lett. (1952)

γ angular correlations, general case

- Generally speaking, $W(\theta)$ for any γ-γ coincidence is defined by a sum of Legendre polynomials:
  - $W(\theta) = \sum_{i=0}^{i=l} a_{2i}\, P_{2i}(\cos\theta)$
  - i.e. $W(\theta) = 1 + a_2\cos^2(\theta) + a_4\cos^4(\theta) + \cdots a_{2l}\cos^{2l}(\theta)$, where the normalization is such that $W(90°) = 1$
- The coefficients a_i are fit to data and the results are checked against the expected results for particular combinations of J_i, J_i, J_f, l_1, l_2
- For common cases, pre-tabulated values are available to compare to

R.Evans, The Atomic Nucleus (1955)

γ-γ cascade $I_A(l_1)I_B(l_2)I_C$	$W(\vartheta)\,d\Omega = (1 + a_2\cos^2\vartheta + a_4\cos^4\vartheta)\,d\Omega$	
	a_2	a_4
0(1)1(1)0	1	0
1(1)1(1)0	$-\frac{1}{3}$	0
1(2)1(1)0	$-\frac{1}{3}$	0
2(1)1(1)0	$+\frac{1}{13}$	0
3(2)1(1)0	$-\frac{3}{29}$	0
0(2)2(2)0	-3	$+4$
1(1)2(2)0	$-\frac{1}{3}$	0
2(1)2(2)0	$+\frac{3}{7}$	0
2(2)2(2)0	$-\frac{15}{13}$	$+\frac{16}{13}$
3(1)2(2)0	$-\frac{3}{29}$	0
4(2)2(2)0	$+\frac{1}{8}$	$+\frac{1}{24}$

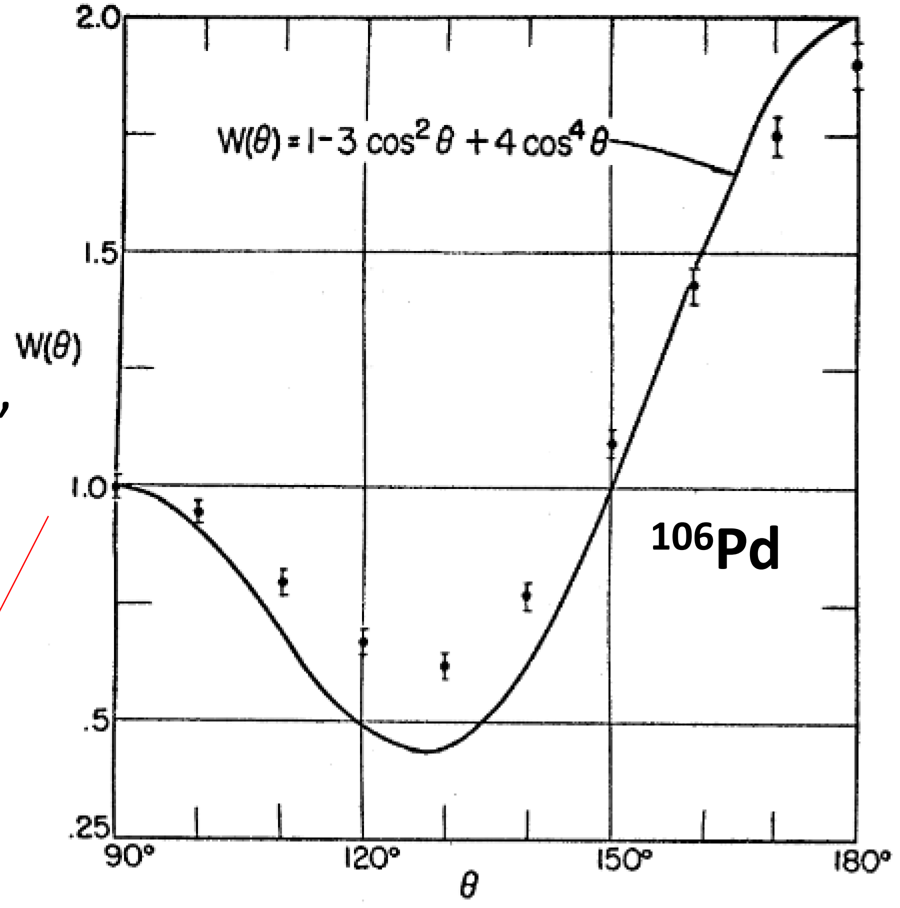
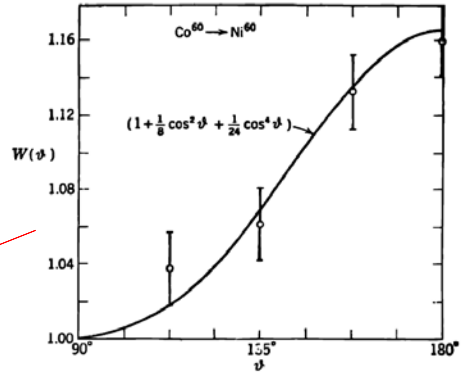
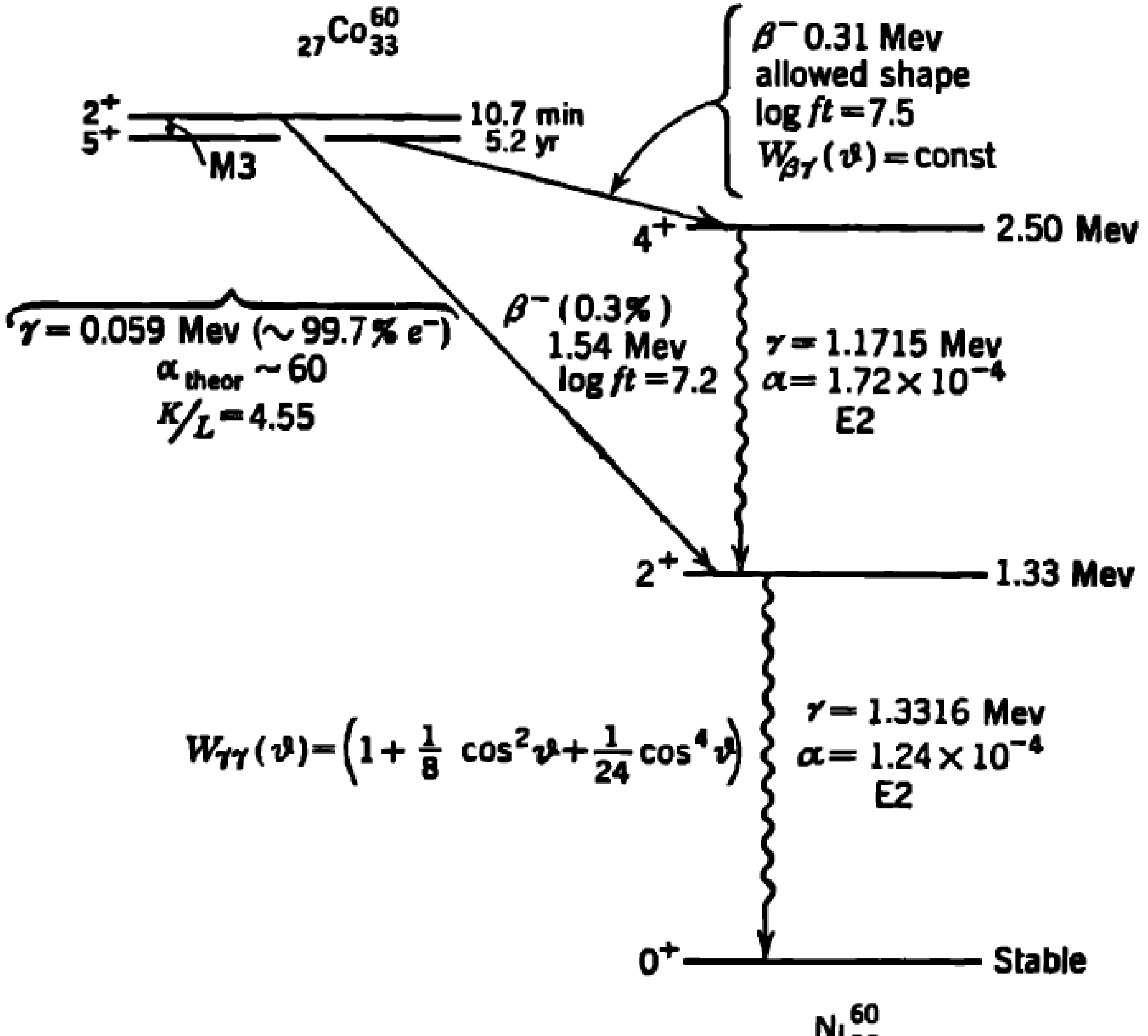


FIG. 1. Angular correlation of the 513- and 624-kev gamma-ray cascade in Pd^{106} (not corrected for finite solid angles).



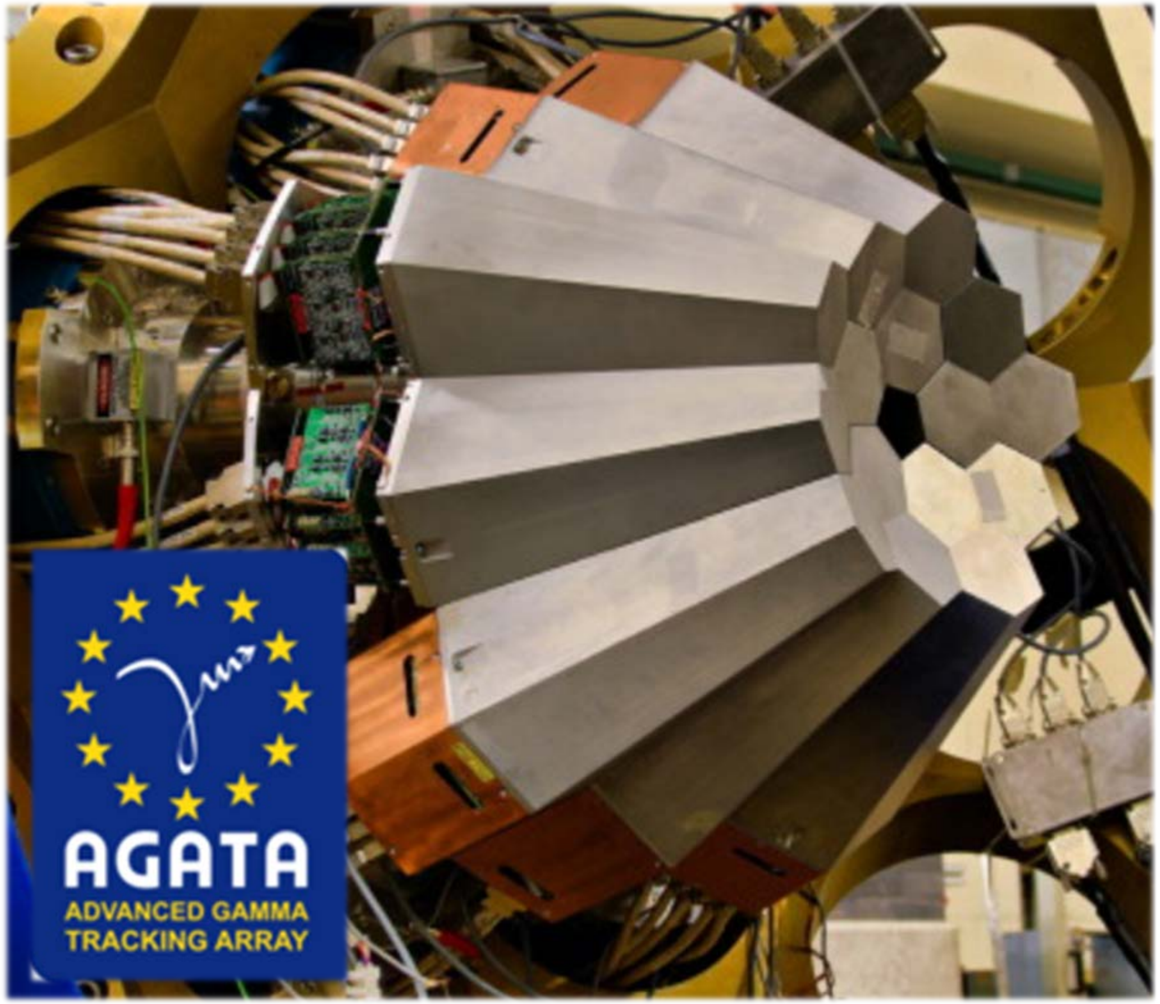
Spectroscopy with γ decay

- Particularly when combined with ft from β-decay, λ_γ and/or $W(\theta)$ provide powerful tools to determine or at least constrain J^π for nuclei



R.Evans, The Atomic Nucleus (1955)

Spectroscopy with γ decay: Modern Tools



γ strength function, γSF

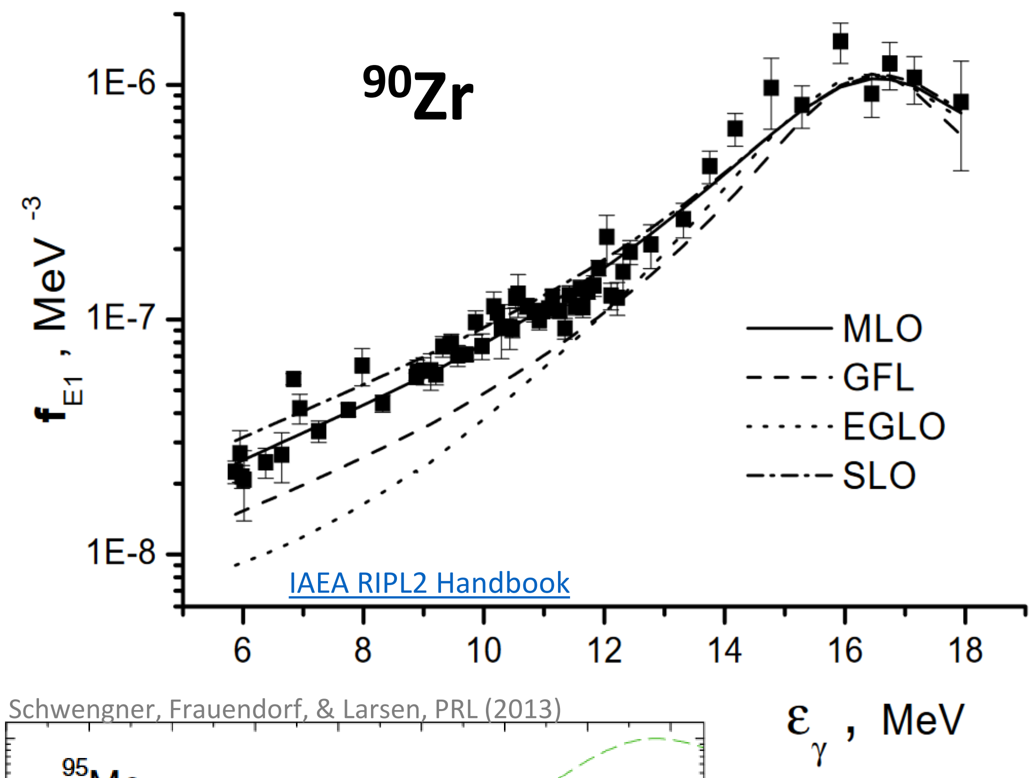
- Recall from a few lectures ago, back from when we were young and care free, that it's often in our best interest to consider the statistical properties of the nucleus
- For instance, a decay or capture reaction might populate a high excitation energy of a nucleus, up where there are a large number of nuclear levels
- The relevant question is then, *what is the probability of emitting a γ ray of a particular energy?*
- This is encapsulated in the Transmission Coefficient, in analogy to the transmission coefficient we saw for nucleons (α particles, specifically) earlier, which here is related to λ_γ
- The transmission coefficient for emitting a γ of energy E_γ and multipolarity Xl is:

$$T_{Xl}\left(E_\gamma\right) = 2\pi E_\gamma^{2l+1} f_{Xl}(E_\gamma)$$

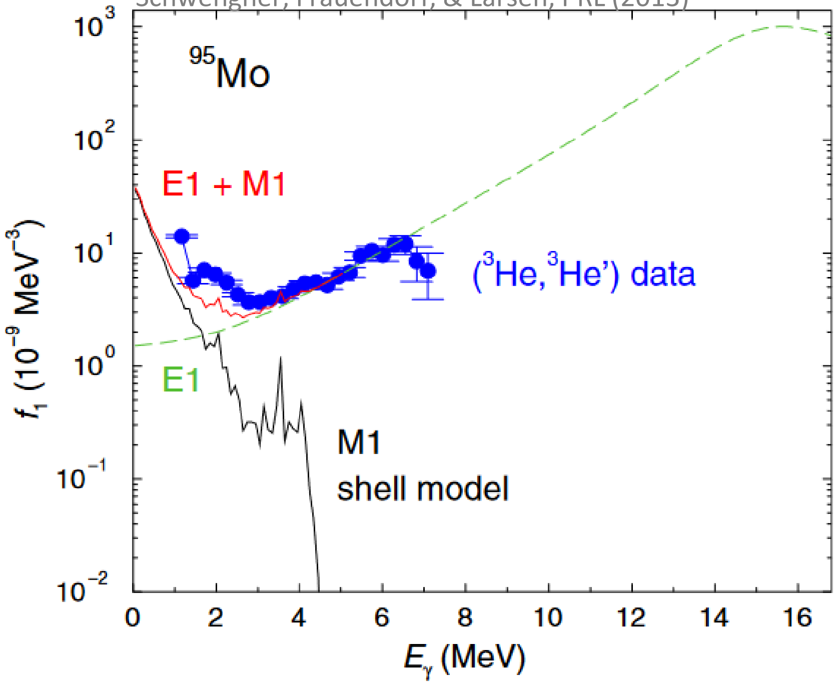
- Here you'll recognize the factor E_γ^{2l+1}, which is a scaling for the decay rate common to all λ_{Xl}
- $f_{Xl}(E_\gamma)$ is the "gamma strength function", γSF, which captures the E_γ probability distribution, as well as all the realistic features that make λ_{Xl} deviate from the single-particle model
- That $f_{Xl}(E_\gamma)$ is independent of excitation energy, indeed of the properties of the initial and final states for the decay, is known as the "Brink-Axel hypothesis"

γ strength function, γSF

- What functional form should we use for $f_{Xl}(E_\gamma)$?
- Just like our α transmission coefficients from earlier, T_γ describes the process of the γ going out of *or into* the nucleus
- So, we can make a guess at T_γ using photoabsorption cross section data, from which we're guided towards a Lorentzian shape
- Recent experimental and theoretical results show that M1 strength function has a special enhancement, called the "up-bend", at low E_γ



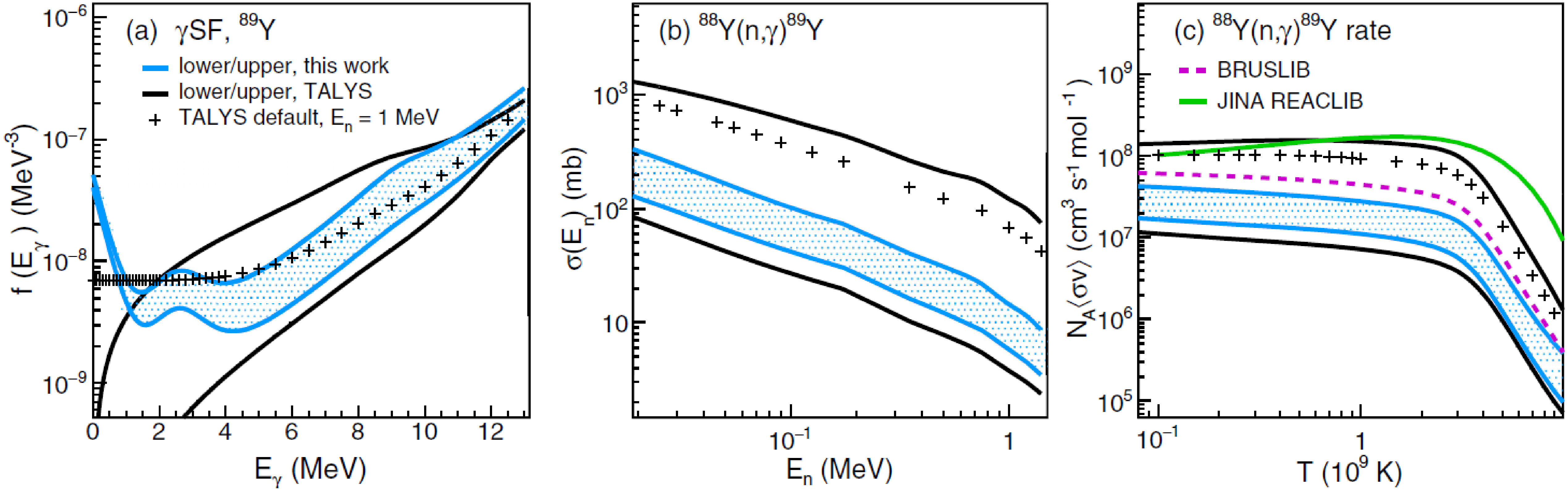
Schwengner, Frauendorf, & Larsen, PRL (2013)



γSF impact (selected examples)

(γ,α), (γ,p), (γ,n) for the p-process, (p,γ) for the rp-process, (n,γ) for the r- and s- processes

A.C. Larsen et al. Phys. Rev. C (2016)



Further Reading

- Chapter 9: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapter 7: Nuclear & Particle Physics (B.R. Martin)
- Chapter 14, Section : Quantum Mechanics for Engineers (L. van Dommelen)
- Chapter 4: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapter 6, Section 4: The Atomic Nucleus (R. Evans)
- Chapter 7: IAEA RIPL2 Handbook