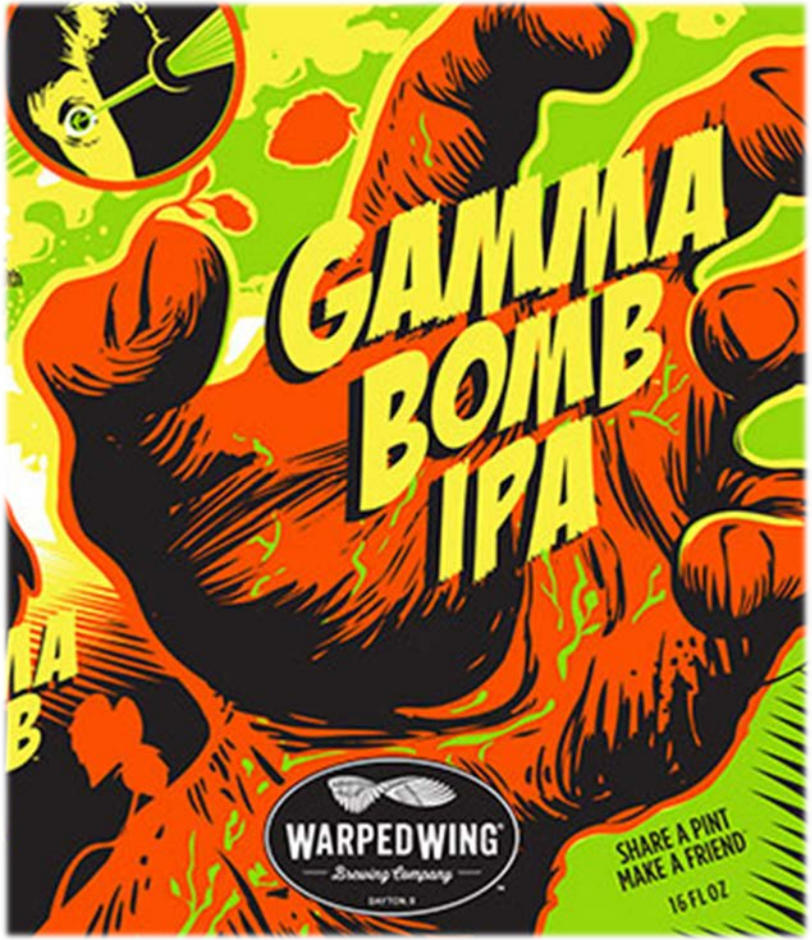
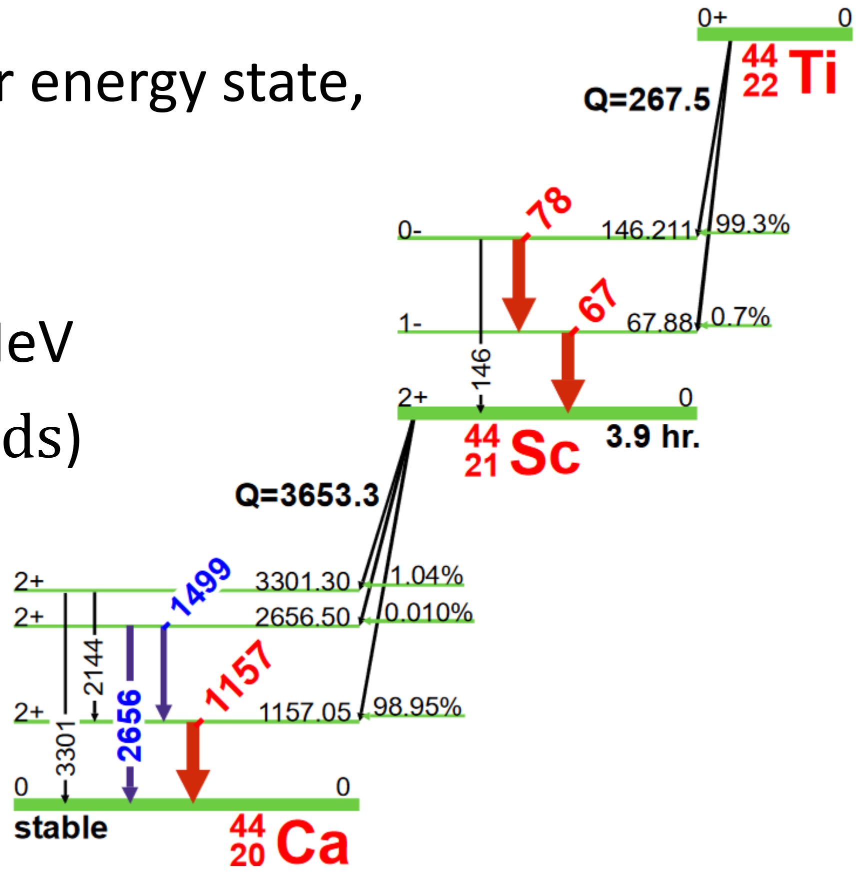
Lecture 9: γ *Decay*

- Basics
- Energetics
- Transition rates
- Angular correlations
- γ strength functions



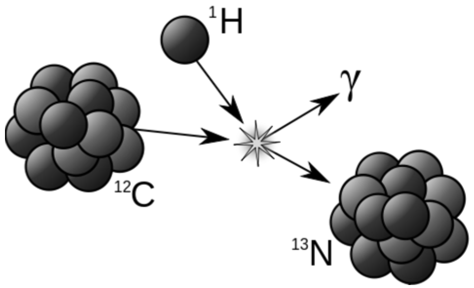
γ decay basics

- γ decay is a de-excitation from an excited bound state to a lower energy state, preceded by some decay or reaction
- *[Just to be clear]* Z & A are unchanged
- γ ray energies can span anywhere from several keV to several MeV
- γ decay lifetimes are typically extremely short ($\tau \lesssim \mathrm{femtoseconds}$)
  [with the exception of isomeric states]



INEL γ Spectrum Catalog

γ decay energetics

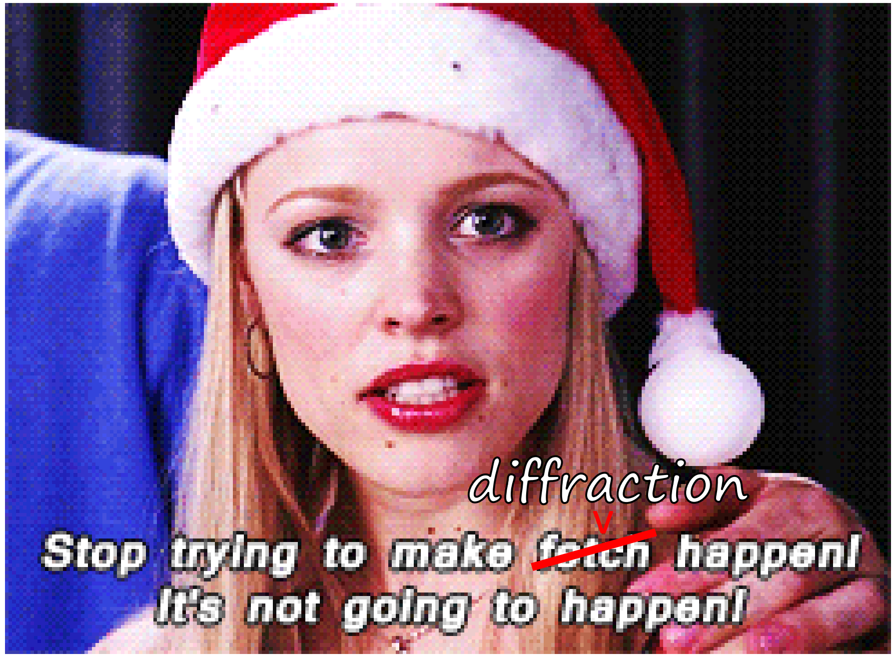


- γ decay can be used to probe excited state energies, but is $E_\gamma = E_{xs}$?
- For a decay from a higher-lying excited state to a lower-energy one,
  $M_{higher}c^2 = M_{lower}c^2 + E_\gamma + KE_{recoil}$
- $M_{higher}c^2 - M_{lower}c^2 \equiv E_{xs} = E_\gamma + KE_{recoil}$
- $KE_{recoil} = \frac{p_{recoil}^2}{2M_{recoil}}$
- Conservation of momentum dictates $p_\gamma = p_{recoil}$, so $KE_{recoil} = \frac{p_\gamma^2}{2M_{recoil}}$
- Recall that for a massless particle, $E = pc$, so $KE_{recoil} = \frac{E_\gamma^2}{2M_{recoil}c^2}$
- Consider $E_\gamma = 2\text{MeV}, \text{A} = 50$: $KE_{recoil} = \frac{4\text{MeV}^2}{2 \cdot 50 \cdot 931.5\text{MeV}} \approx 43\text{eV}$
- For, a 5MeV γ from ^{3}He *(e.g. populated by d+p)*, $KE_{recoil} \approx 4.5\text{keV}$

Do we have to worry about EM interactions within the nucleus?

- How does the photon wavelength compare to the nuclear size?
- $E_\gamma = h\nu = \frac{hc}{\lambda} = \frac{2\pi\hbar c}{\lambda} \approx \frac{2\pi(197 MeV \cdot fm)}{\lambda}$
- $\lambda \approx \frac{2\pi(197 MeV \cdot fm)}{E_\gamma}$ …for $E_\gamma = 10\text{MeV}$: $\lambda \approx 124 fm$
- For a large nucleus, $A = 200$, the diameter $D \approx 2 \cdot 1.2 fm \cdot A^{1/3} \approx 14 fm$
- So, even for an extreme case, $\lambda \gg D$
- For antennas and diffraction, one needs $\lambda \lesssim D$

i.e. it's not going to happen



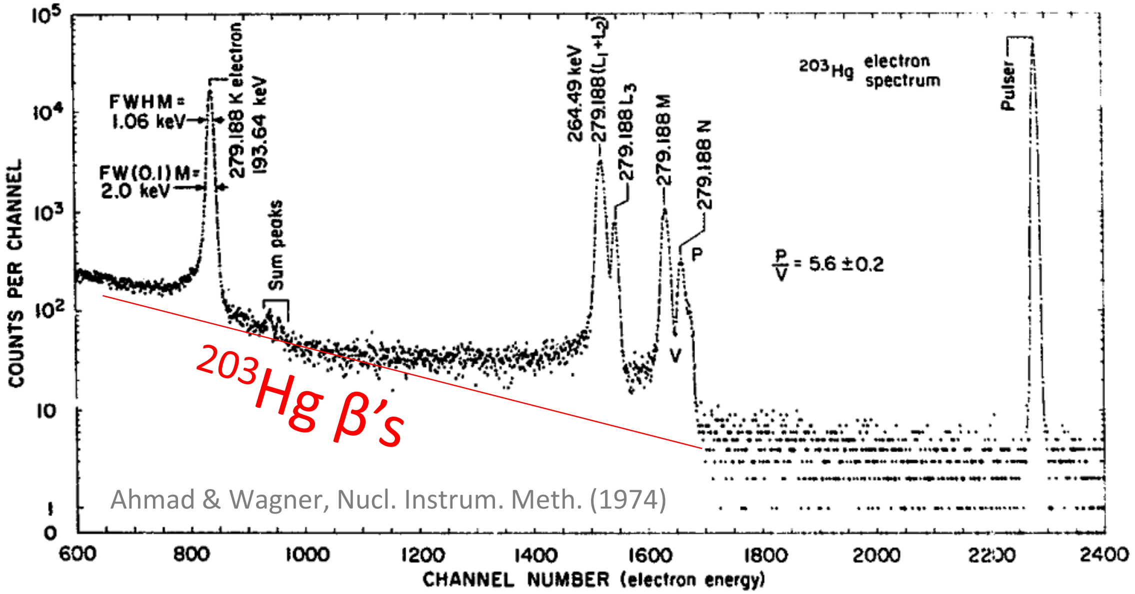
γ decay types

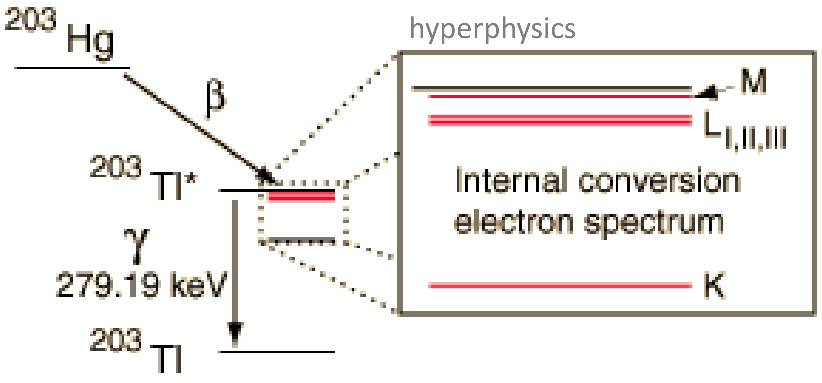
- Parity and angular momentum are conserved during γ decay
- Photons carry some integer angular momentum with a minimum $l = 1$, where l is referred to by the multipole 2^l
  - $l = 1: dipole, l = 2\ quadrupole, \cdots$
- A photon's parity depends not only on l, but also on the decay type
- A photon decay corresponds to shift in the nucleus's charge and matter distribution
  - Shift in the charge distribution = change in electric field = *Electric*
  - Shift in the current distribution [i.e. orbitals of protons]= change in magnetic field = *Magnetic*
- The selection rules corresponding to a particular decay type are:

Radiation Type	Name	$l = \Delta I$	$\Delta\pi$
E1	Electric dipole	1	Yes
M1	Magnetic dipole	1	No
E2	Electric quadrupole	2	No
M2	Magnetic quadrupole	2	Yes
E3	Electric octupole	3	Yes
M3	Magnetic octupole	3	No
E4	Electric hexadecapole	4	No
M4	Magnetic hexadecapole	4	Yes

How does $0^+ \rightarrow 0^+$ happen? *Internal conversion*

- Since $l_{min} = 1$ for a photon, de-excitation by photon emission isn't possible
- Instead the process of internal conversion can happen,
  whereby a nucleus interacts electromagnetically with an orbital electron and de-excites by ejecting that orbital electron
- This process operates in competition with γ decay for any transition, not just $0^+ \rightarrow 0^+$
- The energy of the emitted electron is: $E_{IC} = E_{xs} - E_{BE,e^-}$,
  where E_{xs} is the decay transition energy, and E_{BE,e^-} is the electron binding energy





A similar, but different phenomenon is Internal Pair Conversion, where a photon with $E_\gamma > 2m_ec^2$ interacts with the coulomb field of the nucleus to create an e^+–e^- pair. See e.g. A. Wuosmaa et al. Phys. Rev. C Rapid Comm. 1998

γ decay constant

- As with β decay, the decay constant is described by the expectation value of a small perturbation multiplied with the final state density, i.e. by Fermi's Golden Rule
  - $\lambda = \frac{2\pi}{\hbar}\left|\langle\Psi_{final}|H'|\Psi_{initial}\rangle\right|^2\rho(E)$
- However, here $\rho(E)$ is the product of the density of nuclear states and density of electromagnetic states available in the system created by the transition and Ψ_{final} is a product of the wavefunction of the final nucleus and the outgoing electromagnetic wave
- Without proof*, it turns out that lots of gymnastics with E&M will result in:
  - $\lambda(l_\gamma, J_i, \pi \rightarrow J_f, \pi) = \frac{8\pi(l_\gamma+1)}{l_\gamma[(2l_\gamma+1)!!]^2}\frac{\left({E_\gamma}/{\hbar c}\right)^{2l_\gamma+1}}{\hbar}B(l_\gamma, J_i, \pi \rightarrow J_f, \pi)$,

  where $n!!$ is the double-factorial of n (product of all odd or even integers from n to 2 or 1), and $B(l_\gamma, J_i, \pi \rightarrow J_f, \pi)$ is the "reduced transition probability", which is the square of the modulus of the expectation value of the transition operator (for El or Ml)

*for a proof, see Appendix 25 of Quantum Mechanics for Engineers (L. van Dommelen)

- These B are extremely nasty to deal with…so lucky for us, back in the mists of time, Weisskopf derived general expressions for different transitions types, assuming the decay was due to a single nucleon undergoing a transition

Weisskopf (a.k.a. single-particle) estimates for λ

- $\lambda(l_\gamma, J_i, \pi \to J_f, \pi) = \frac{8\pi(l_\gamma+1)}{l_\gamma[(2l_\gamma+1)!!]^2} \frac{\left(E_\gamma/_{\hbar c}\right)^{2l_\gamma+1}}{\hbar} B(l_\gamma, J_i, \pi \to J_f, \pi),$
- Reduced transition probabilities assuming the initial to final state transition is due to a single nucleon re-orienting itself within a nucleus of uniform density with $R = r_0 A^{1/3}$ are:
  - $B_{s.p.}(E, l_\gamma) = \frac{1}{4\pi}\left[\frac{3}{l_\gamma+3}\right]^2 r_0^{2l_\gamma} A^{2l_\gamma/3} e^2 (fm)^{2l_\gamma}$
  - $B_{s.p.}(M, l_\gamma) = \frac{10}{\pi}\left[\frac{3}{l_\gamma+3}\right]^2 r_0^{(2l_\gamma-2)/2} \mu_n^2 (fm)^{2l_\gamma-2}$, where the nuclear magneton $\mu_n = \frac{e\hbar}{2m_p c}$

The units for B change with l_γ!

- Note: There is a steep dependency of λ on l, so only one multipole of a decay type will matter
- The equations above are still a huge pain to work with and more noble souls have worked-out the decay constant for various situations.
- Using Q in MeV, λ_γ in s^{-1} for a nucleus with mass number A is given by:

L. van Dommelen, Quantum Mechanics for Engineers (2012)

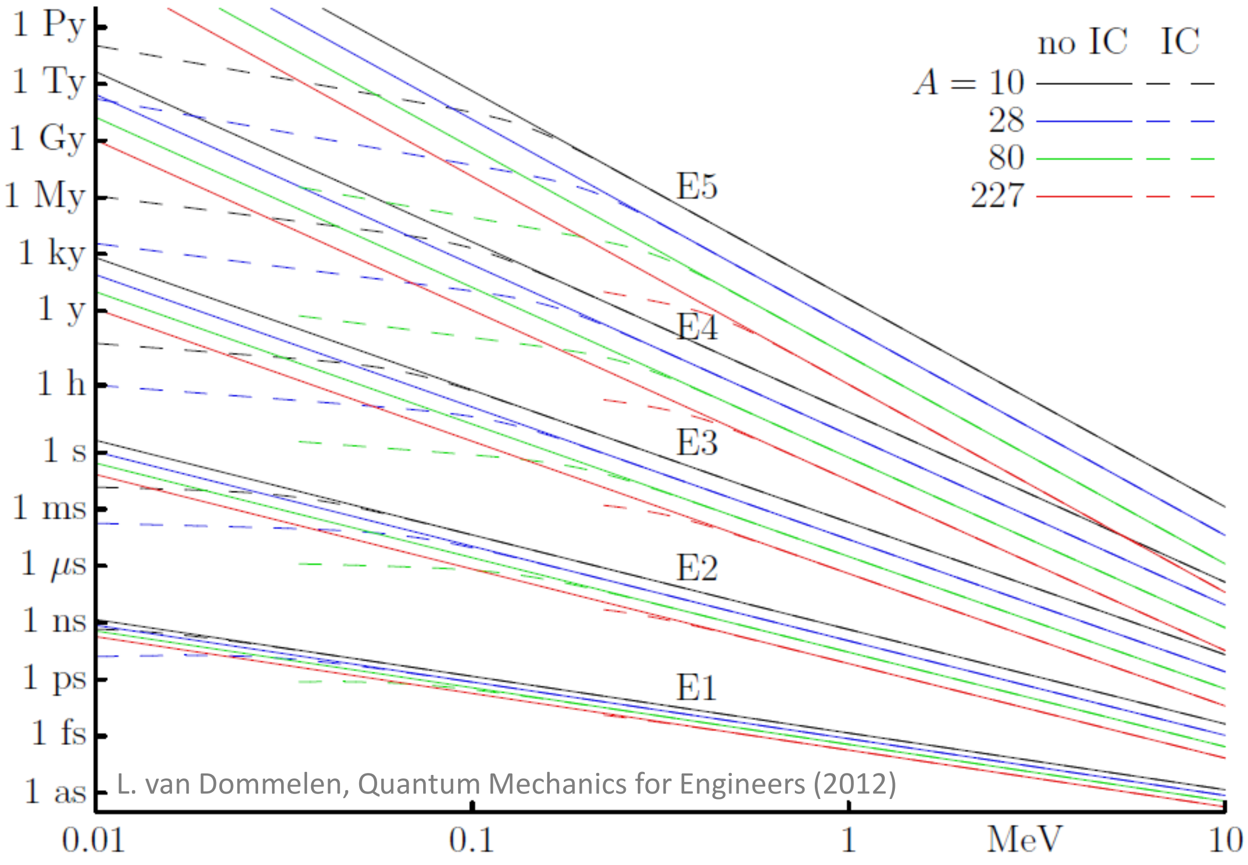
$$\lambda^{E\ell} = C_{E\ell} A^{2\ell/3} Q^{2\ell+1} \qquad \lambda^{M\ell} = C_{M\ell} A^{(2\ell-2)/3} Q^{2\ell+1}$$

ℓ :	1	2	3	4	5
$C_{E\ell}$:	$1.0\,10^{14}$	$7.3\,10^{7}$	34	$1.1\,10^{-5}$	$2.4\,10^{-12}$
$C_{M\ell}$:	$3.1\,10^{13}$	$2.2\,10^{7}$	10	$3.3\,10^{-6}$	$7.4\,10^{-13}$

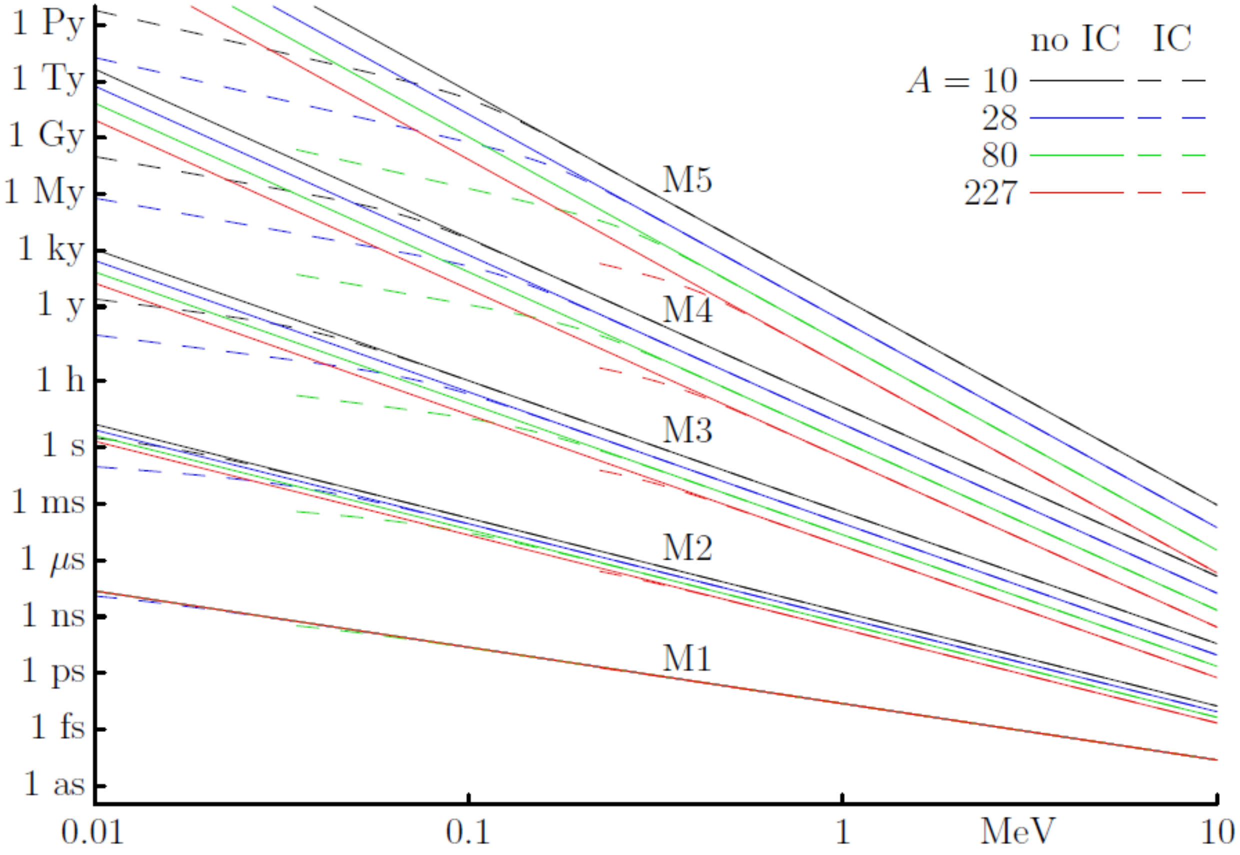
- Weisskopf estimates are generally within an order of magnitude of the real answer, so γ decay constants are often quoted as the ratio to this estimate in “Weisskopf Units” [w.u.]

Weisskopf (a.k.a. single-particle) estimates for $t_{½}$

$t_{½}(E_\gamma)$ *for* ***Electric*** *Transitions (from Weisskopf)*



$t_{½}(E_\gamma)$ *for* ***Magnetic*** *Transitions (from Moszkowski)*

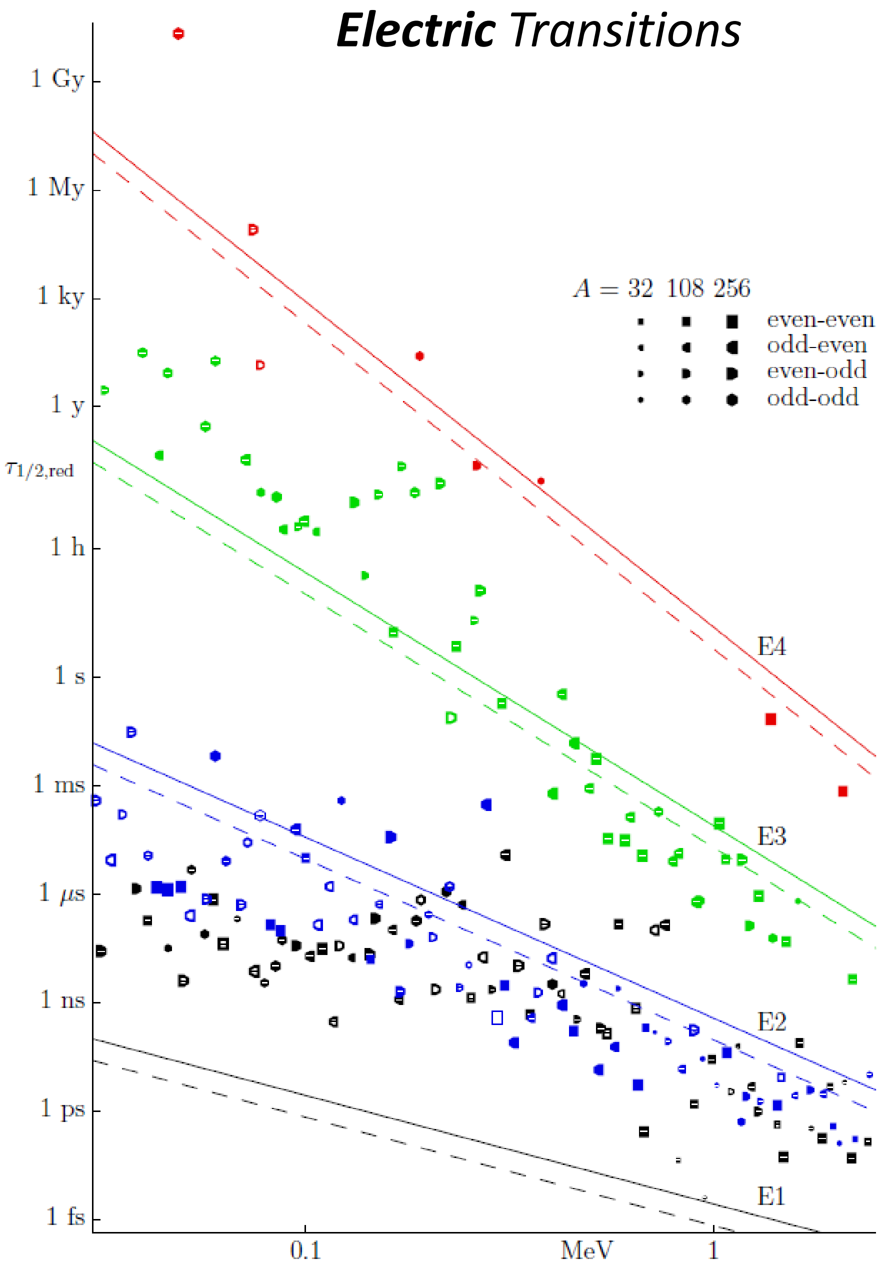


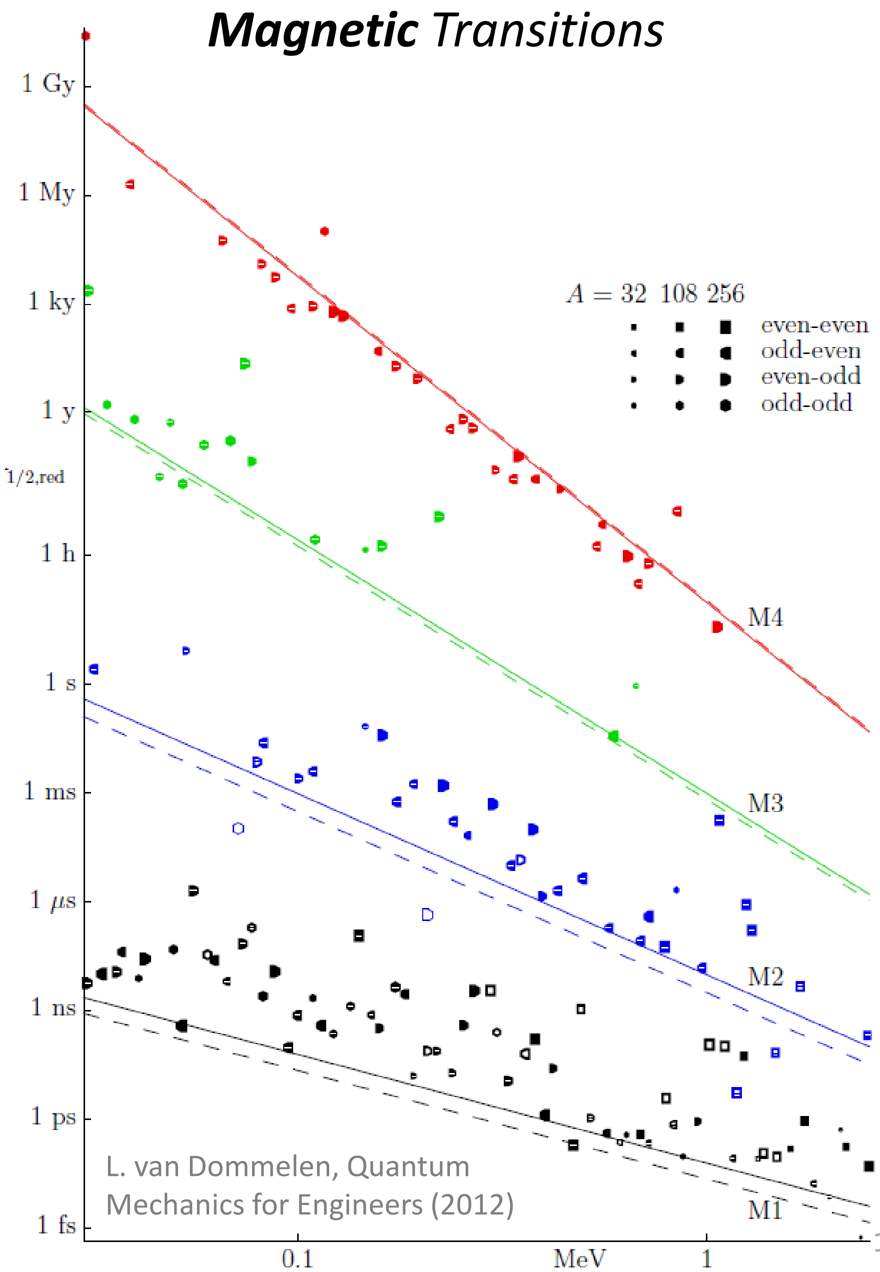
Note that E transitions of a given multipole and E_γ are ~100X faster than M transitions with the same E_γ, ℓ

Now we see how it is that low-energy high-spin states exist as isomeric states.

Weisskopf estimates for $t_{1/2}$ compared to data

- Single-particle estimates aren't too bad
- The general spread is explained away by the fact that transitions are often not going to be due to a single particle rearranging itself, but rather a collective effect
- $E2$ are thought to be faster because collective de-excitations *(rotation, vibration)* will favor $\Delta J = 2, \Delta\pi = no$
- However, other lower-order multipoles are thought to be suppressed because shape-changes will generally be complex and, in general, poorly described by smoothly varying Legendre polynomials (low l)



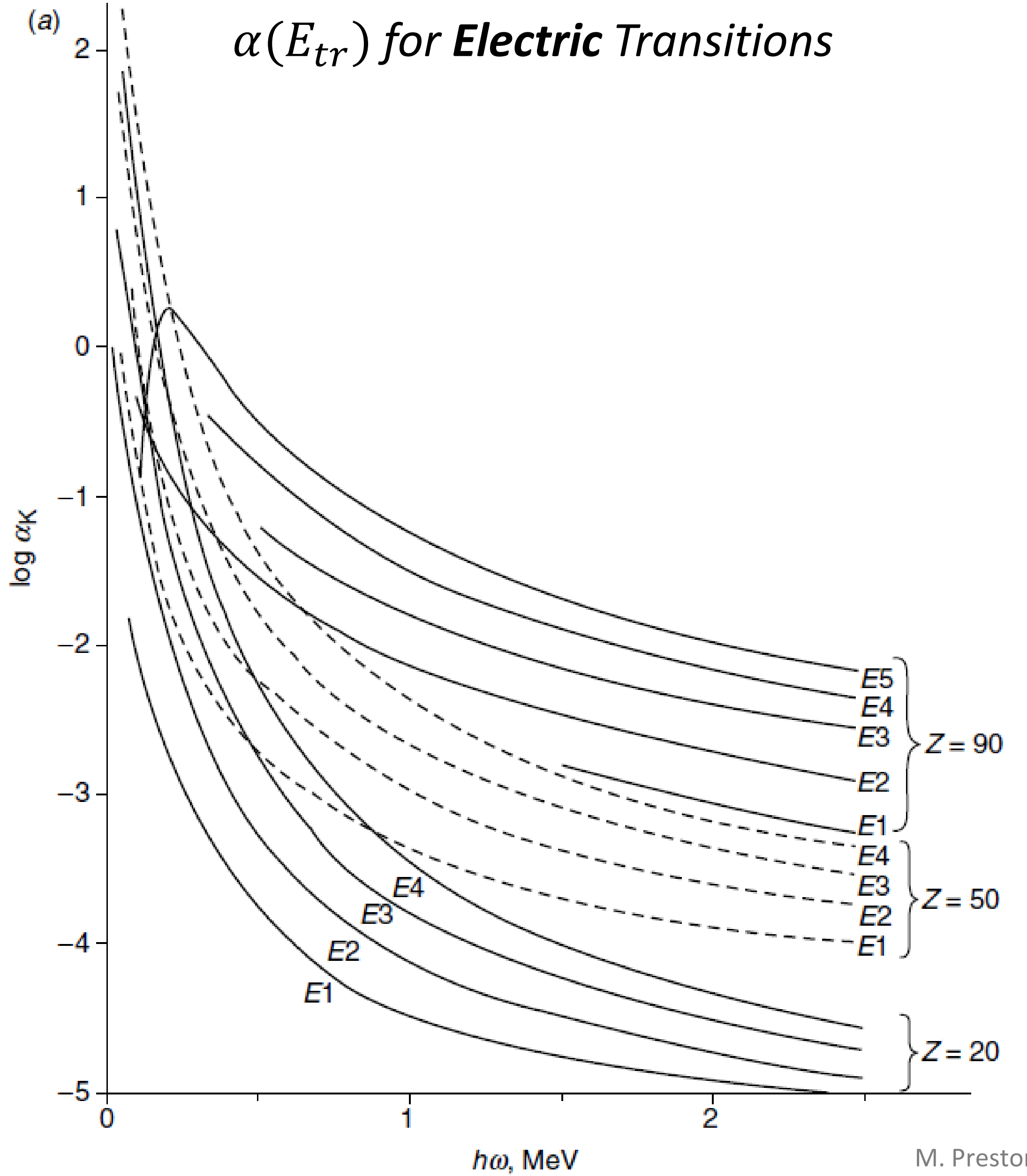


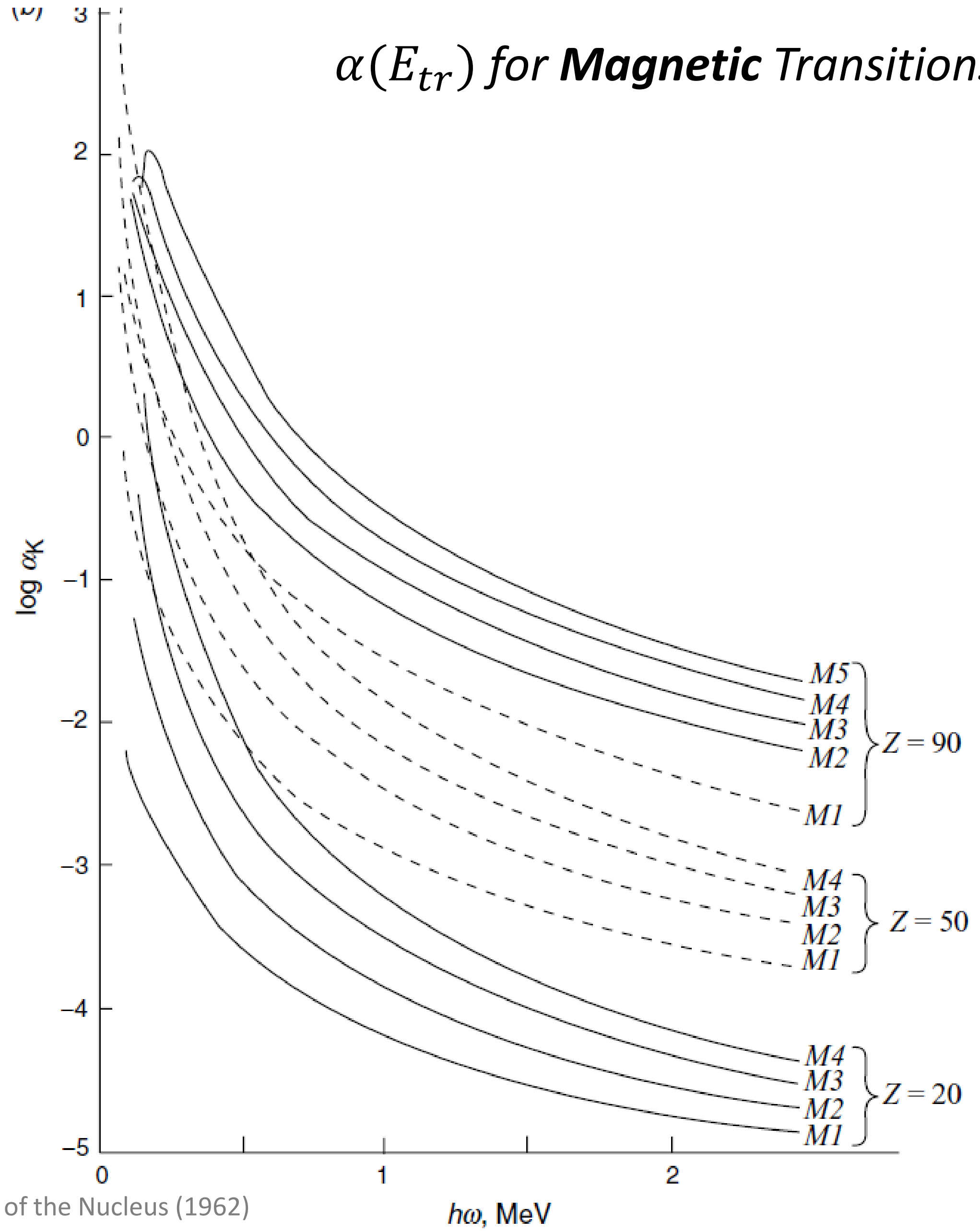
Internal conversion coefficient, α

- Don't forget about our old friend, internal conversion, which competes with γ decay
- Competition between the two is described by the internal conversion coefficient $\alpha = \frac{\lambda_{IC}}{\lambda_\gamma}$,
  so $\lambda = \lambda_{IC} + \lambda_\gamma = \lambda_\gamma(1+\alpha)$
- α depends on the density of electrons near the nucleus, and so some friendly atomic physicists have done the dirty work of calculating the following approximate formulas:
  - $\alpha(El) = \frac{Z^3}{n^3}\left(\frac{l}{l+1}\right)\alpha_{f.s.}^4\left(\frac{2m_ec^2}{Q}\right)^{l+5/2}$; $\alpha(Ml) = \frac{Z^3}{n^3}\alpha_{f.s.}^4\left(\frac{2m_ec^2}{Q}\right)^{l+3/2}$,
    where $\alpha_{f.s.} \approx \frac{1}{137}$, Q is the transition Q-value,
    and n is the principal quantum number of the orbital electron being ejected

  These rely on the Born approximation, so Z<<137 ought to apply
- The atomic orbitals $K, L, M, N, O, \cdots$ correspond to $n = 1, 2, 3, 4, 5, \cdots$
- Clearly this process is favored for high-Z nuclei, …but also for $Q < 1.022MeV$ $l = 0$ transitions
- For $0^+ \rightarrow 0^+$ transitions, $\lambda_{E0} = 3.8 \cdot Z^3 A^{4/3} Q^{1/2}$, with Q in MeV and λ in s^{-1}

Internal conversion coefficient, α





M. Preston, Physics of the Nucleus (1962)

λ_{E0} predictions compared to data

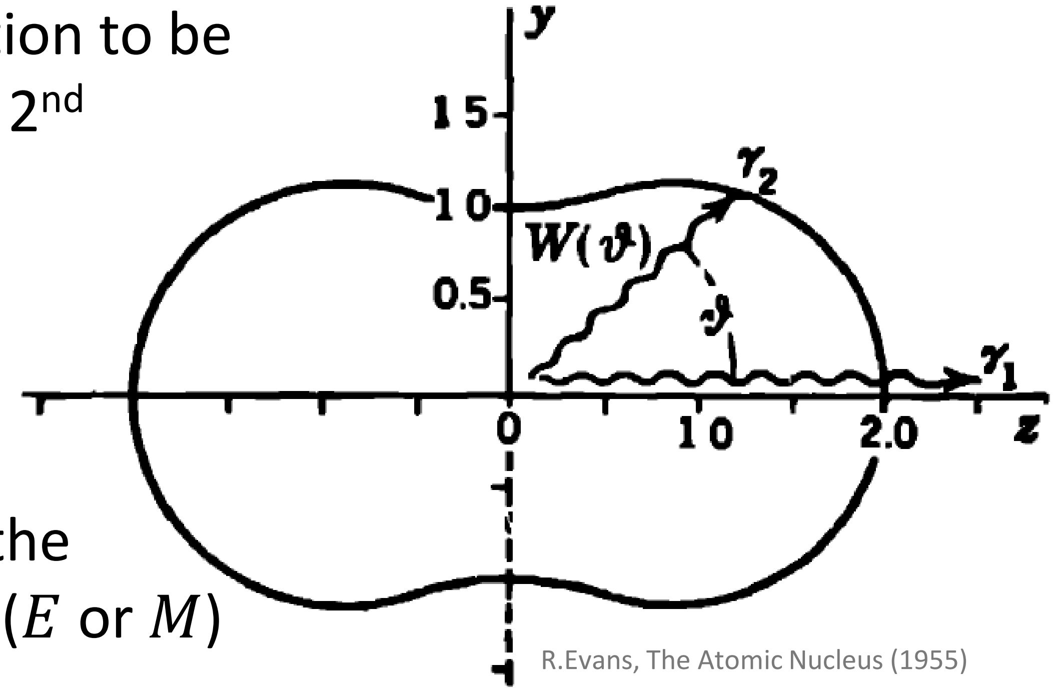
Nucleus	Q	$\tau_{1/2}$ μs	$\tau_{1/2,\mathrm{red}}$ μs	I.C. Theory μs
$^{184}_{80}$Hg	0.375	0.00062	0.80	0.72
$^{72}_{36}$Kr	0.671	0.0263	0.88	0.54
$^{72}_{32}$Ge	0.691	0.4442	10.	0.53
$^{98}_{42}$Mo	0.735	0.0218	1.8	0.51
$^{192}_{82}$Pb	0.769	0.00075	1.1	0.50
$^{98}_{40}$Zr	0.854	0.064	4.4	0.47
$^{194}_{82}$Pb	0.931	0.0011	1.6	0.45
$^{96}_{40}$Zr	1.582	0.0380	2.6	0.35
$^{90}_{40}$Zr	1.761	0.0613	3.8	0.33
$^{68}_{28}$Ni	1.770	0.2760	4.0	0.33
$^{40}_{20}$Ca	3.353	0.00216	0.0057	0.24
$^{32}_{16}$S	3.778	0.00254	0.0025	0.23
$^{16}_{8}$O	6.048	0.00007	0.000003	0.18

For these decays e^+–e^- pair creation is a significant decay mode

L. van Dommelen, Quantum Mechanics for Engineers (2012)

γ angular correlations

- Radioactive decay, including γ decay, is isotropic when nuclei are oriented at random, which is generally the case in a laboratory setting
- Indeed, the subsequent decay following a preceding decay will also be isotropic
- However, the relative angle between the subsequent radiations will be correlated
- This is because the direction of the first radiation will be related to the angular momentum projection of the nuclear level that is populated prior to emission of the second radiation
- By arbitrarily choosing the direction of the first radiation to be $\theta_{CM} = 0$, the relative intensity to the intensity of the 2nd radiation at 90° $W(\theta) = \frac{Intensity\ at\ \theta}{Intensity\ at\ \theta_{cm}=90°}$, is described by a sum of several Legendre polynomials that I'm told results from some sophisticated and intimidating math
- It's important to note that W(θ) will only depend on the angular momentum of the radiation and not on type (E or M)



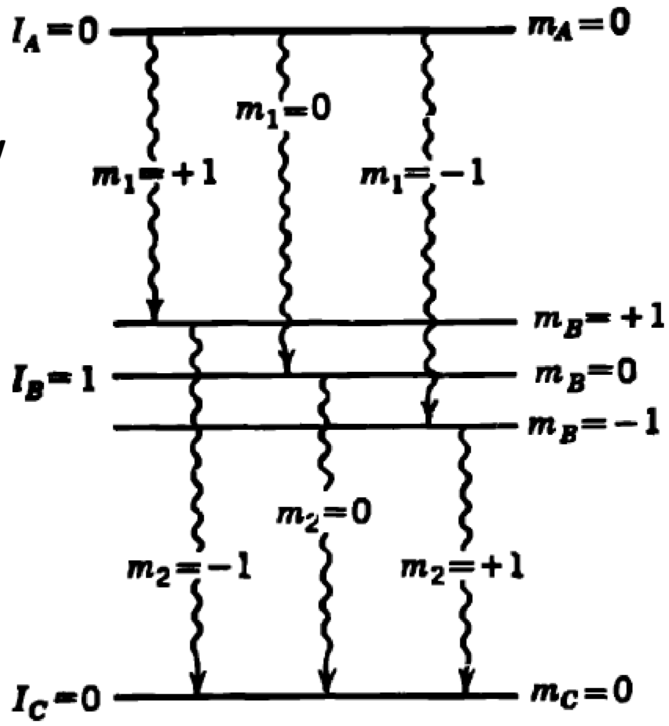
R.Evans, The Atomic Nucleus (1955)

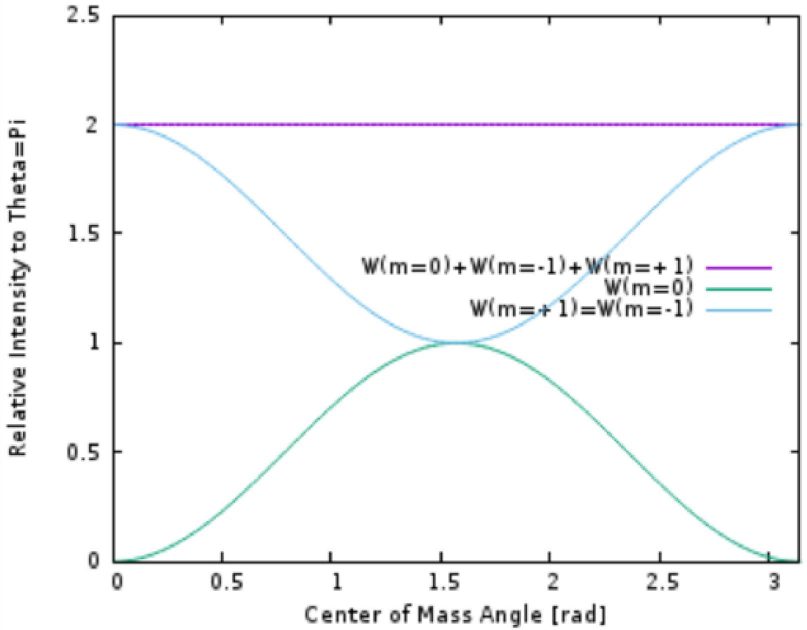
In all this and the following talk of W(θ), θ refers to the center of mass angle …however, as we showed earlier, the nuclear recoil is mostly negligible, so $\theta_{CM} \approx \theta_{lab}$.

γ angular correlations, dipole case

- Let's restrict ourselves to dipole-dipole ($l_{\gamma_1} = 1, l_{\gamma_2} = 1$) radiation for now
- The first decay will populate the $m = -1, 0, 1$ magnetic sub-states with equal probability
- The angular distribution for these three radiations are described by
  - $W_{l=1,\mathbf{m=0}}(\theta) \propto \sin^2(\theta)$
  - $W_{l=1,\mathbf{m=+1}}(\theta) = W_{l=1,\mathbf{m=-1}}(\theta) \propto\ (1 + \cos^2(\theta))$
- Their sum is an isotropic distribution
- However, for the second decay, we can gate on events where the 1st decay is located on our arbitrarily chosen $\theta = 0$ axis
- In that case, $W_{l=1,\mathbf{m=0}}(0) = 0$, so the second γ (which also has $l = 1$) will have an anisotropic distribution
- Thus, any γ-γ coincidence displaying such an angular correlation is indicative of dipole-dipole radiation
- This is mighty handy, since, if we know the ground-state J, (e.g. $J = 0$ for even-even nuclei) then we can build-up from there to get J for the preceding two levels

R.Evans, The Atomic Nucleus (1955)





γ angular correlations, general case

- Generally speaking, $W(\theta)$ for any γ-γ coincidence is defined by a sum of Legendre polynomials:
  - $W(\theta) = \sum_{i=0}^{i=l} a_{2i} P_{2i}(\cos\theta)$
  - i.e. $W(\theta) = 1 + a_2\cos^2(\theta) + a_4\cos^4(\theta) + \cdots a_{2l}\cos^{2l}(\theta)$, where the normalization is such that $W(90°) = 1$
- The coefficients a_i are fit to data and the results are checked against the expected results for particular combinations of J_i, J_i, J_f, l_1, l_2
- For common cases, pre-tabulated values are available to compare to

R.Evans, The Atomic Nucleus (1955)

γ-γ cascade $I_A(l_1)I_B(l_2)I_C$	$W(\vartheta)\,d\Omega = (1 + a_2\cos^2\vartheta + a_4\cos^4\vartheta)\,d\Omega$	
	a_2	a_4
0(1)1(1)0	1	0
1(1)1(1)0	$-\frac{1}{3}$	0
1(2)1(1)0	$-\frac{1}{3}$	0
2(1)1(1)0	$+\frac{1}{13}$	0
3(2)1(1)0	$-\frac{3}{29}$	0
0(2)2(2)0	−3	+4
1(1)2(2)0	$-\frac{1}{3}$	0
2(1)2(2)0	$+\frac{3}{7}$	0
2(2)2(2)0	$-\frac{15}{13}$	$+\frac{16}{13}$
3(1)2(2)0	$-\frac{3}{29}$	0
4(2)2(2)0	$+\frac{1}{8}$	$+\frac{1}{24}$

Arfken, Klema, & McGowan, Phys. Rev. Lett. (1952)

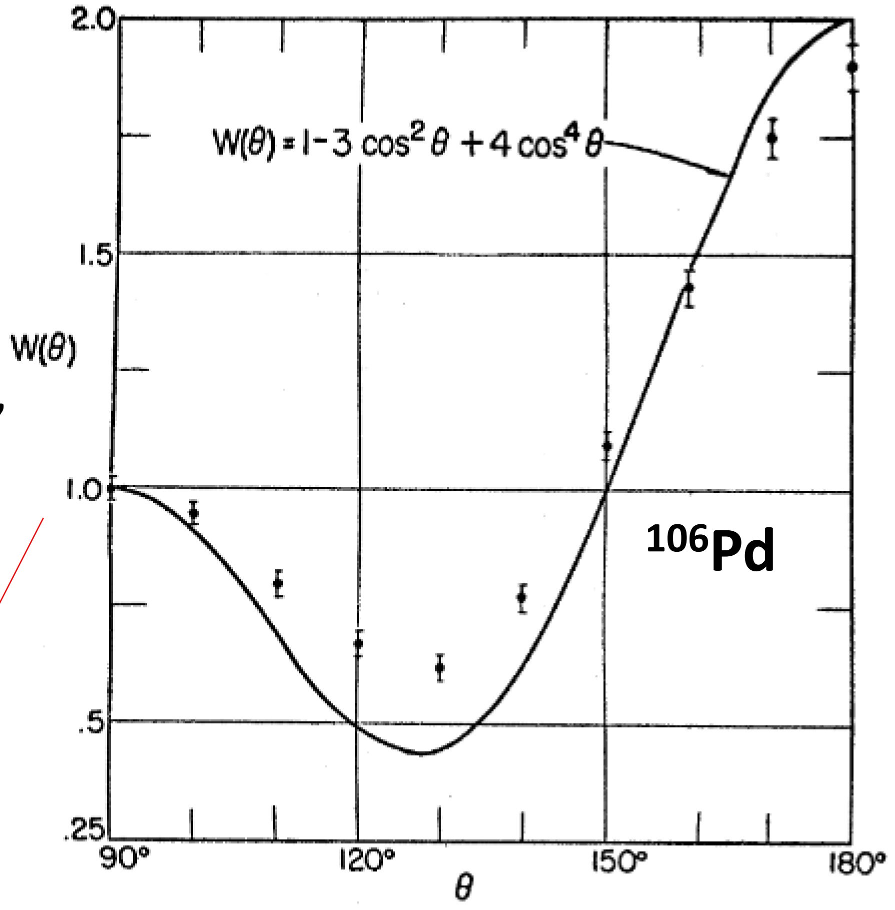
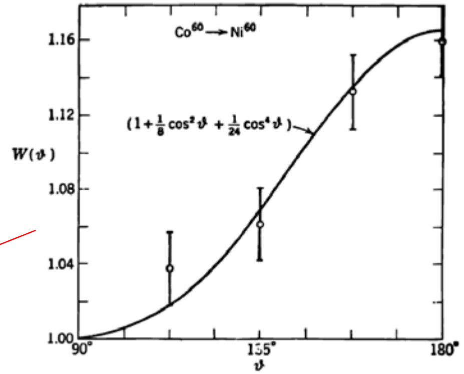
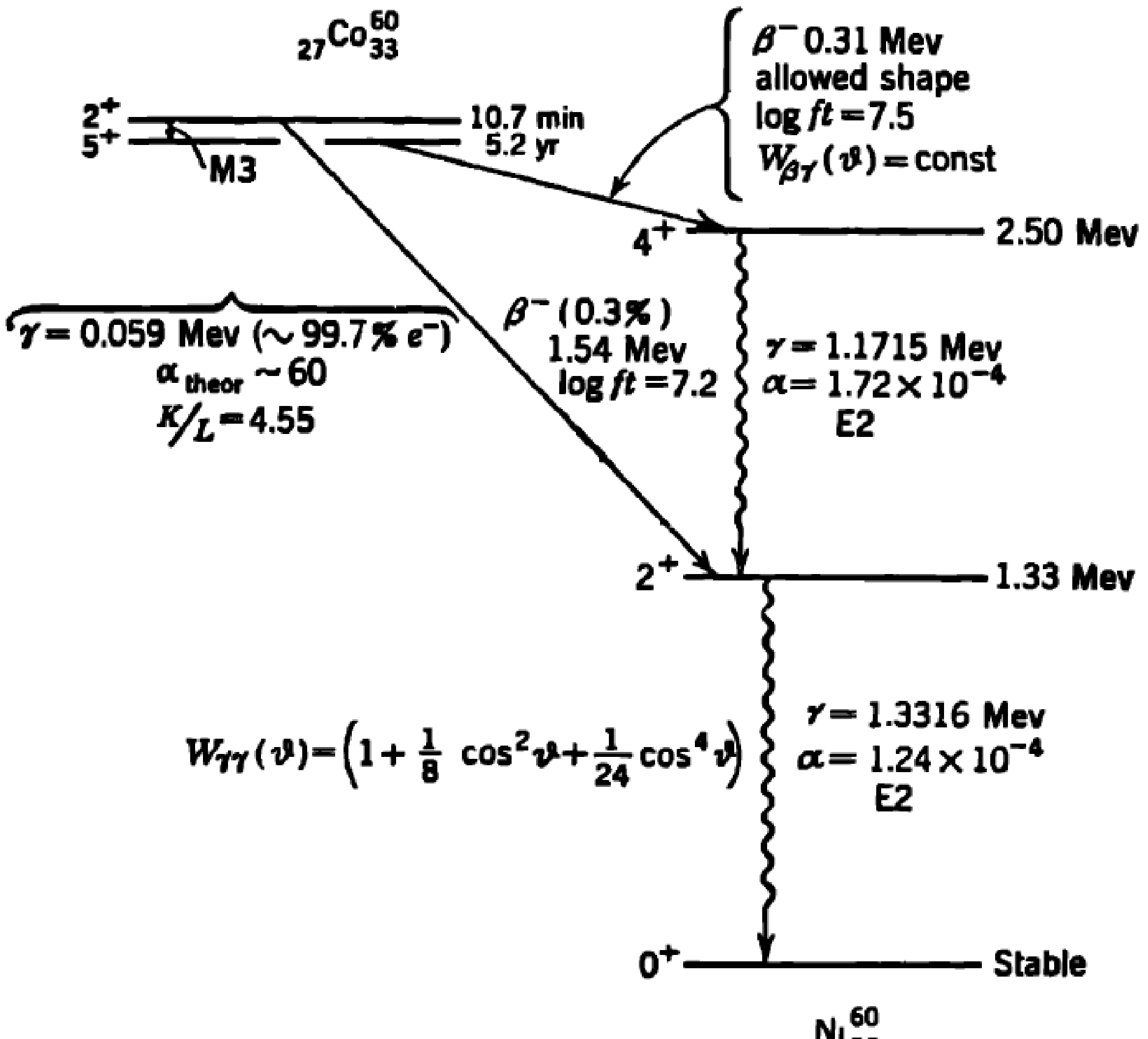


FIG. 1. Angular correlation of the 513- and 624-kev gamma-ray cascade in Pd106 (not corrected for finite solid angles).



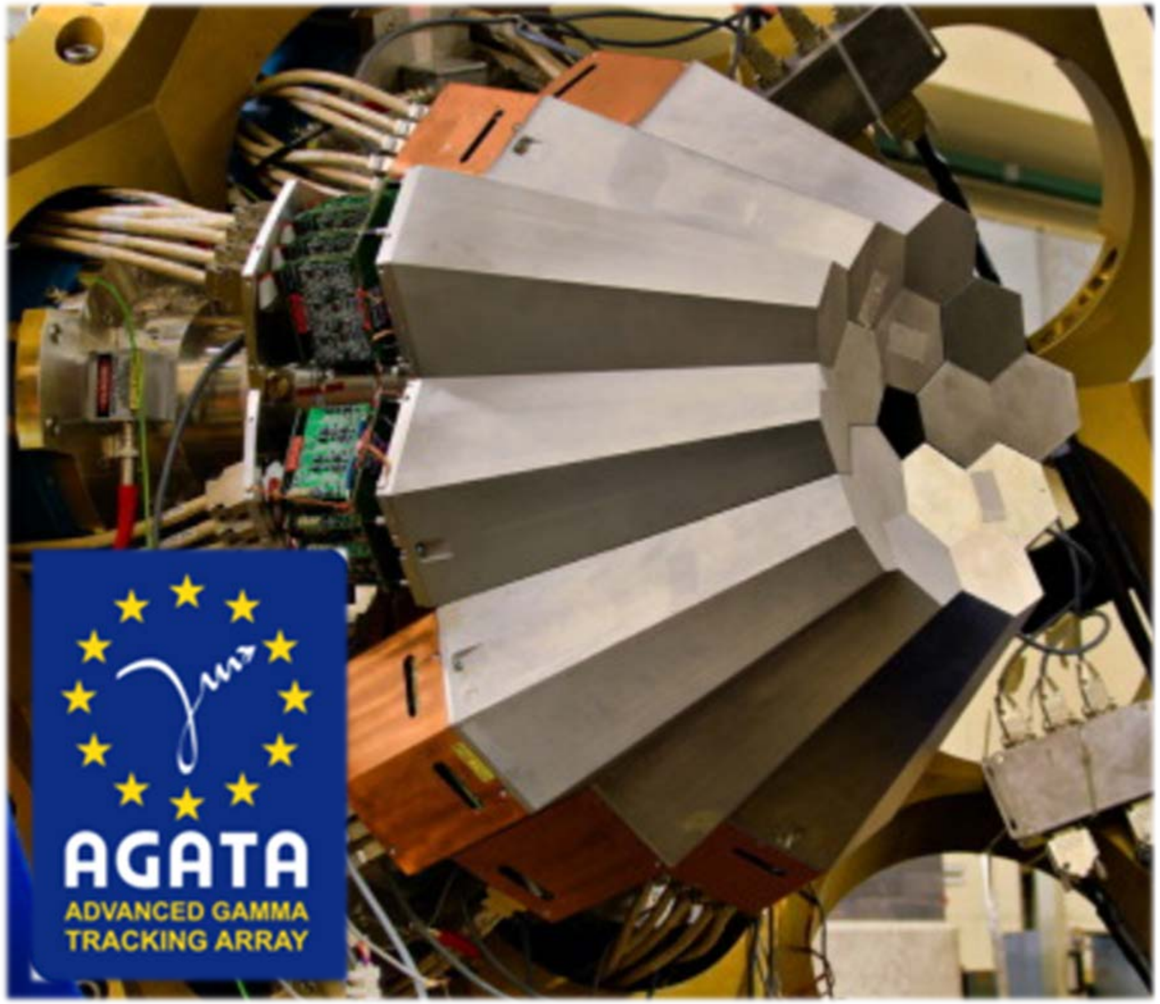
Spectroscopy with γ decay

- Particularly when combined with ft from β-decay, λ_γ and/or $W(\theta)$ provide powerful tools to determine or at least constrain J^π for nuclei



R.Evans, The Atomic Nucleus (1955)

Spectroscopy with γ decay: Modern Tools



γ strength function, γSF

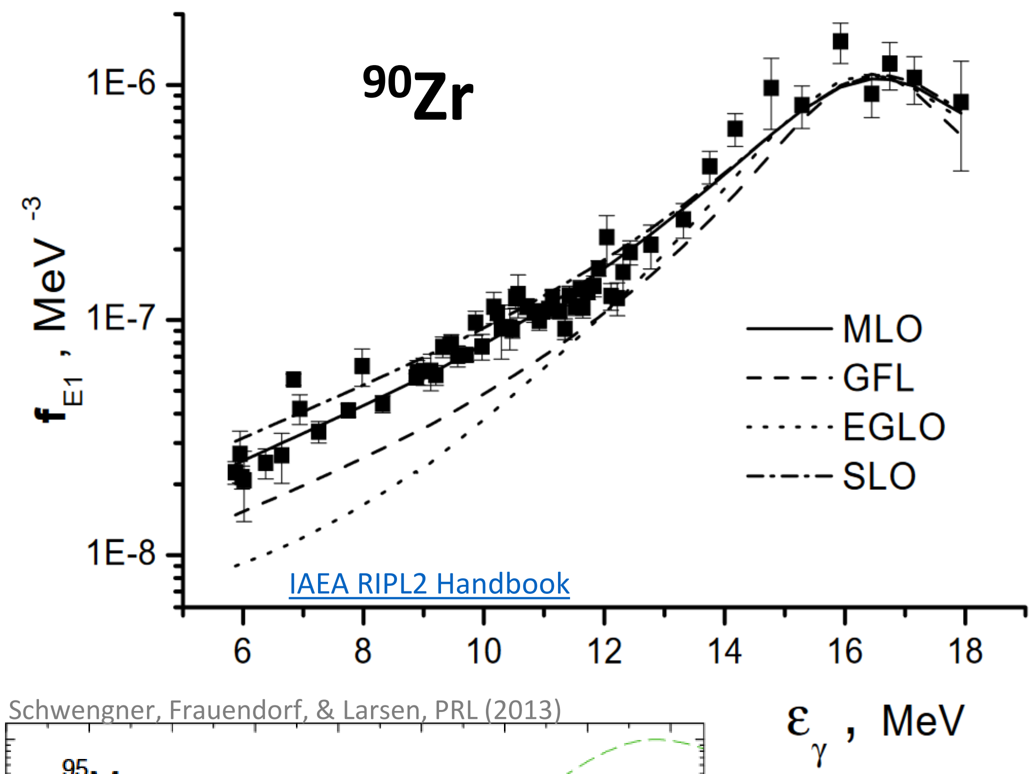
- Recall from a few lectures ago, back from when we were young and care free, that it's often in our best interest to consider the statistical properties of the nucleus
- For instance, a decay or capture reaction might populate a high excitation energy of a nucleus, up where there are a large number of nuclear levels
- The relevant question is then, *what is the probability of emitting a γ ray for a particular energy?*
- This is encapsulated in the Transmission Coefficient, in analogy to the transmission coefficient we saw for nucleons (α particles, specifically) earlier, which here is related to λ_γ
- The transmission coefficient for emitting a γ of energy E_γ and multipolarity Xl is:

$$T_{Xl}\left(E_\gamma\right) = 2\pi E_\gamma^{2l+1} f_{Xl}(E_\gamma)$$

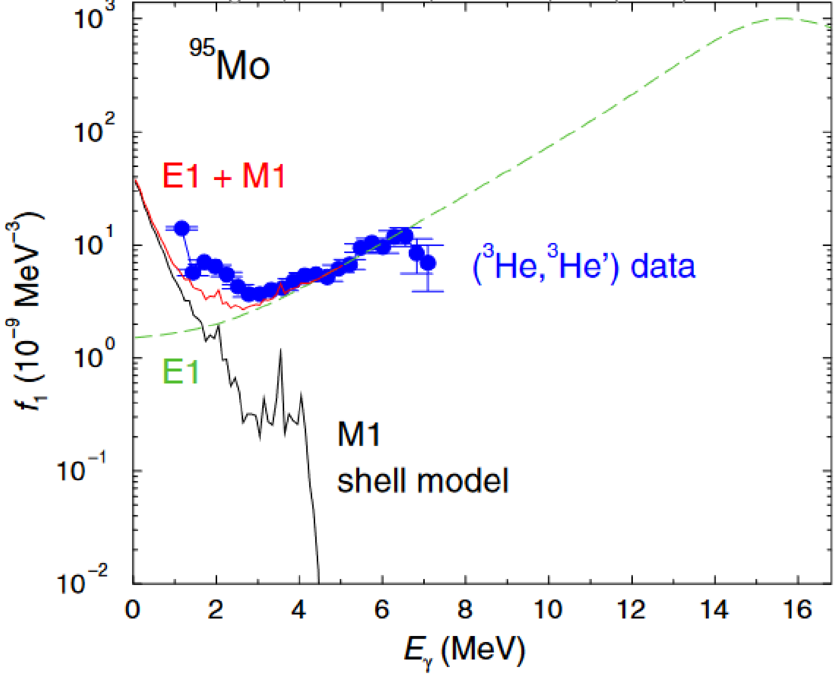
- Here you'll recognize the factor E_γ^{2l+1}, which is a scaling for the decay rate common to all λ_{Xl}
- $f_{Xl}(E_\gamma)$ is the "gamma strength function", γSF, which captures the E_γ probability distribution, as well as all the realistic features that make λ_{Xl} deviate from the single-particle model
- That $f_{Xl}(E_\gamma)$ only depends on excitation energy is known as the "Brink-Axel hypothesis"

γ strength function, γSF

- What functional form should we use for $f_{Xl}(E_\gamma)$?
- Just like our α transmission coefficients from earlier, T_γ describes the process of the γ going out of *or into* the nucleus
- So, we can make a guess at T_γ using photoabsorption cross section data, from which we're guided towards a Lorentzian shape
- Recent experimental and theoretical results show that M1 strength function has a special enhancement, called the "up-bend", at low E_γ



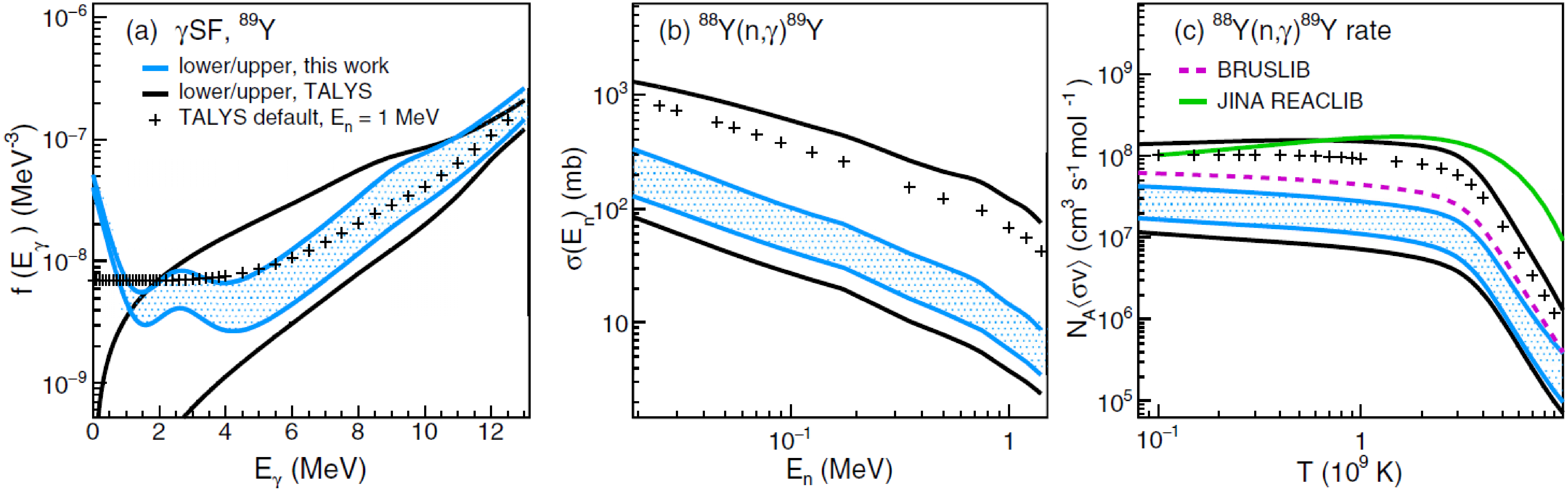
Schwengner, Frauendorf, & Larsen, PRL (2013)



γSF impact (selected examples)

(γ,α), (γ,p), (γ,n) for the p-process, (p,γ) for the rp-process, (n,γ) for the r- and s- processes

A.C. Larsen et al. Phys. Rev. C (2016)



Further Reading

- Chapter 9: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapter 7: Nuclear & Particle Physics (B.R. Martin)
- Chapter 14, Section : Quantum Mechanics for Engineers (L. van Dommelen)
- Chapter 4: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapter 6, Section 4: The Atomic Nucleus (R. Evans)
- Chapter 7: IAEA RIPL2 Handbook