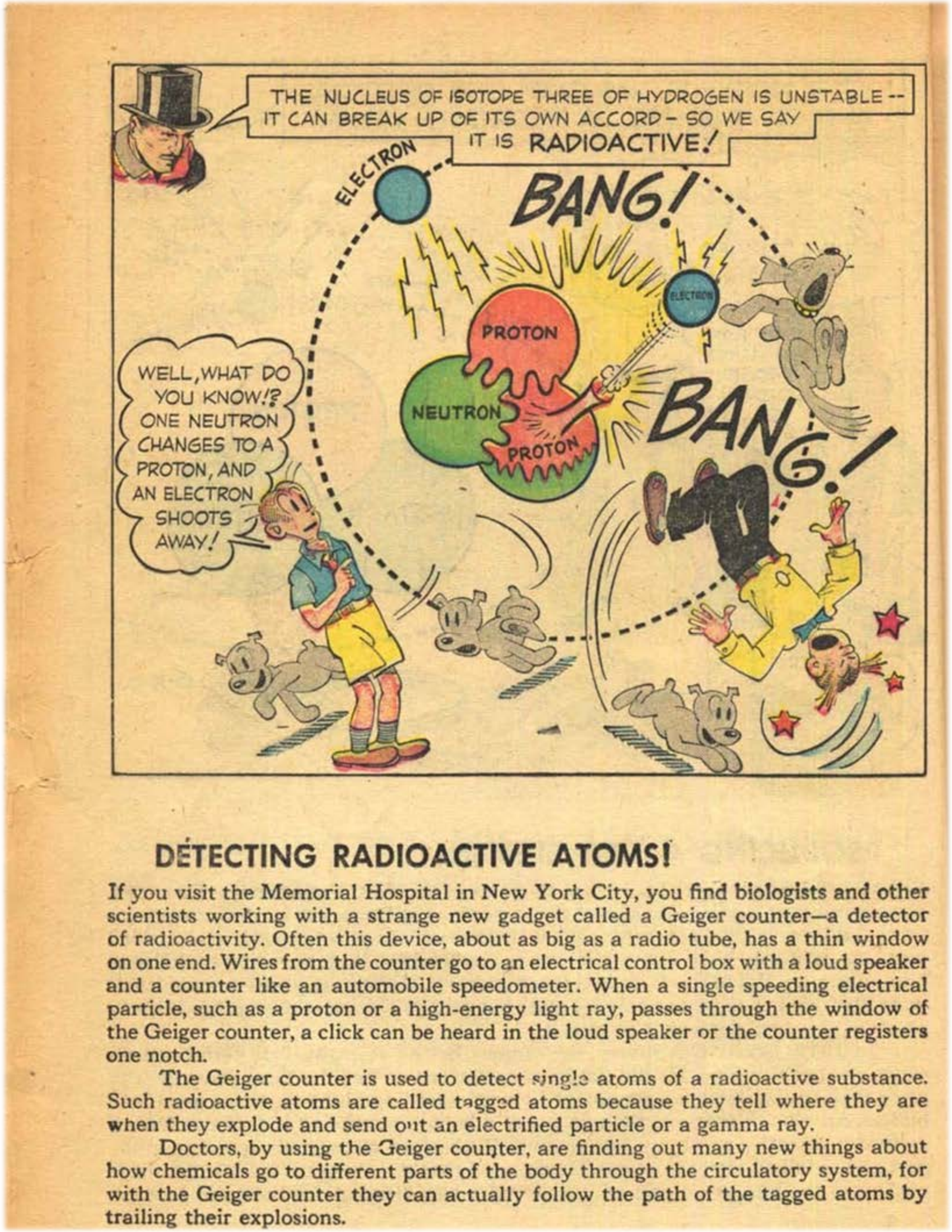
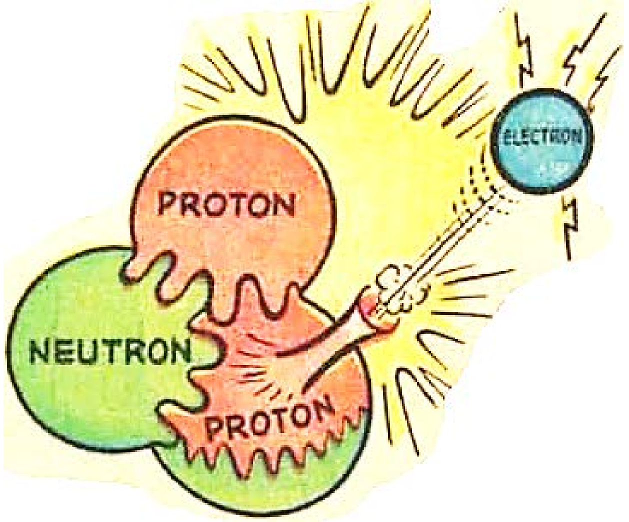
Lecture 8: β *Decay*

- Basic process & energetics
- Fermi theory
- *ft*-values
- Electron-capture
- Parity violation
- Special cases



What is β decay?



a) Nucleus perspective:

$$\beta^-:\ {}^A_ZX \rightarrow {}_{Z+1}^{\ \ A}X' + e^- + \bar{\nu}_e$$
$$\beta^+:\ {}^A_ZX \rightarrow {}_{Z-1}^{\ \ A}X' + e^+ + \nu_e$$
$$EC:\ e^- + {}^A_ZX \rightarrow {}_{Z-1}^{\ \ A}X' + \nu_e$$

e^- can be from atomic shells (terrestrial cases) or from surrounding electron gas (e.g. neutron star outer crust)

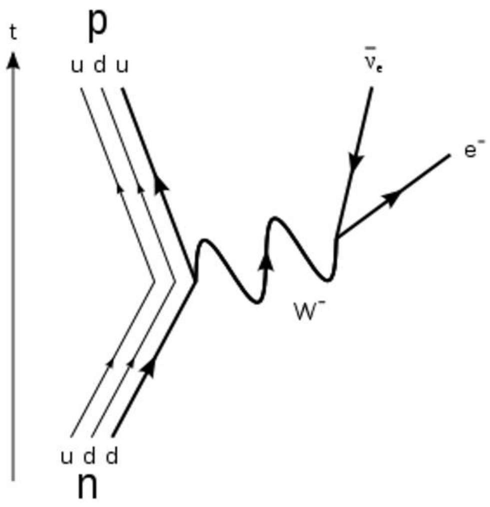
b) Nucleon perspective:

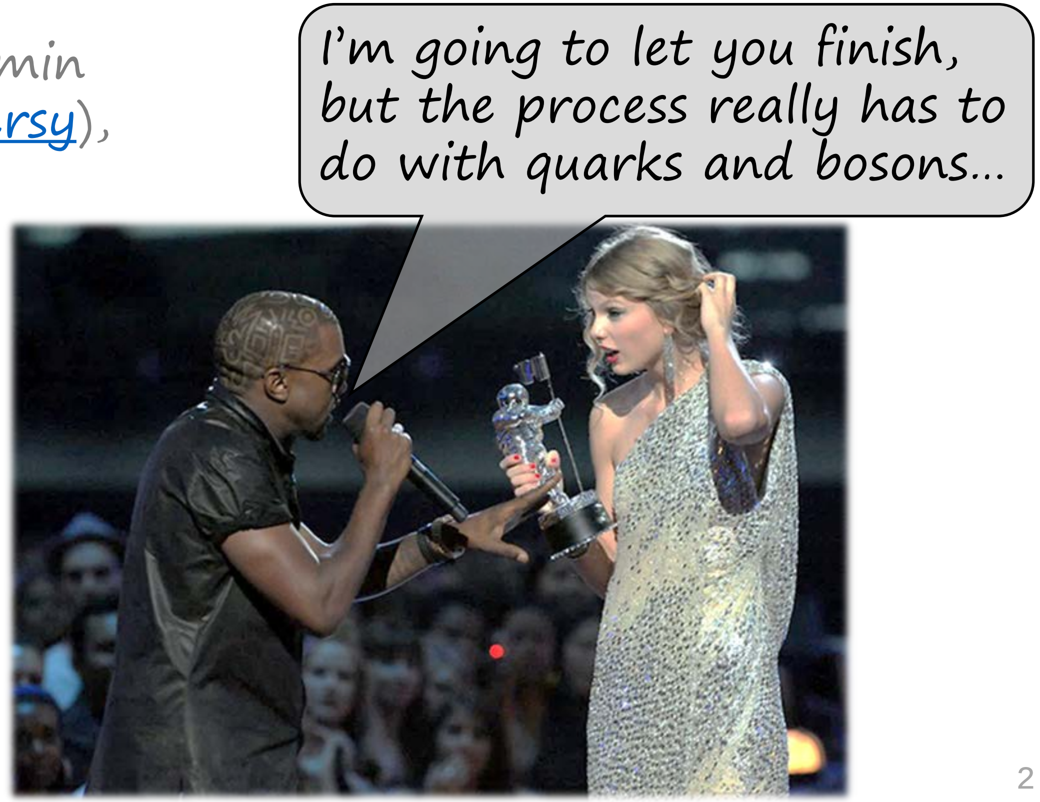
$$\beta^-: n \rightarrow p + e^- + \bar{\nu}_e$$
$$\beta^+: p \rightarrow n + e^+ + \nu_e$$
$$EC: e^- + p \rightarrow n + \nu_e$$

Free neutron decay takes ~15min (though there is some controversy), while $t_{1/2}(p)>10^{34}$yr.

c) Quark perspective:

$$\beta^-: d \rightarrow u + e^- + \bar{\nu}_e$$
$$\beta^+: u \rightarrow d + e^+ + \nu_e$$
$$EC: e^- + u \rightarrow d + \nu_e$$





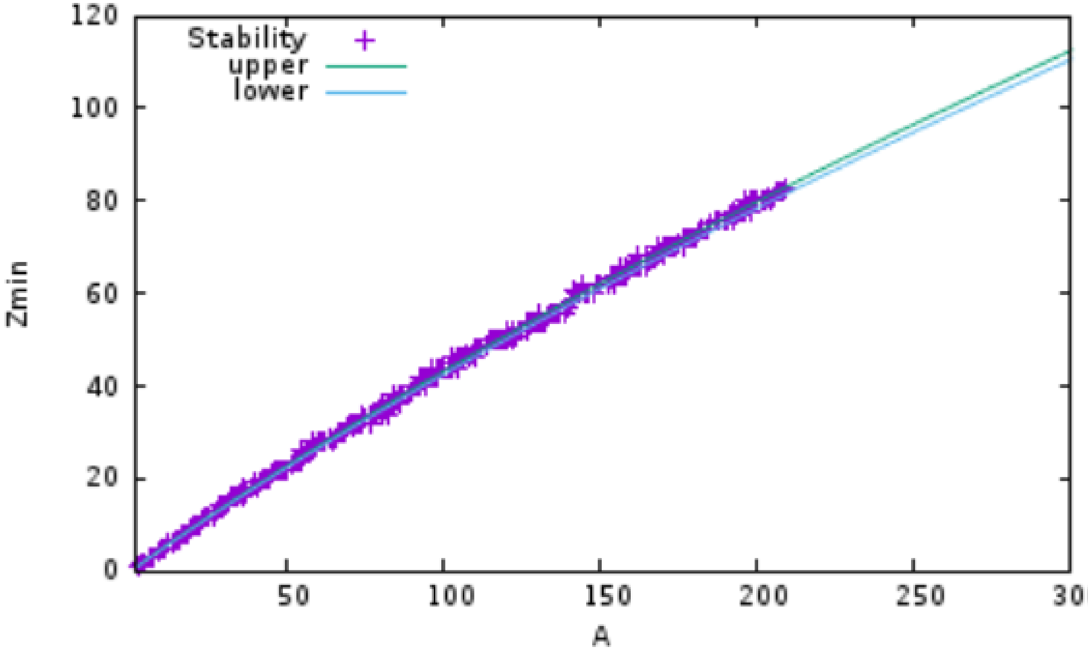
β decay energetics

- β-decay can proceed if energetics allow it ($Q_\beta > 0$):
  - $Q_{\beta^-} = ME(Z,A) - ME(Z+1,A)$
  - $Q_{\beta^+} = ME(Z,A) - ME(Z-1,A) - 2m_e$
  - $Q_{EC} = ME(Z,A) - ME(Z-1,A)$

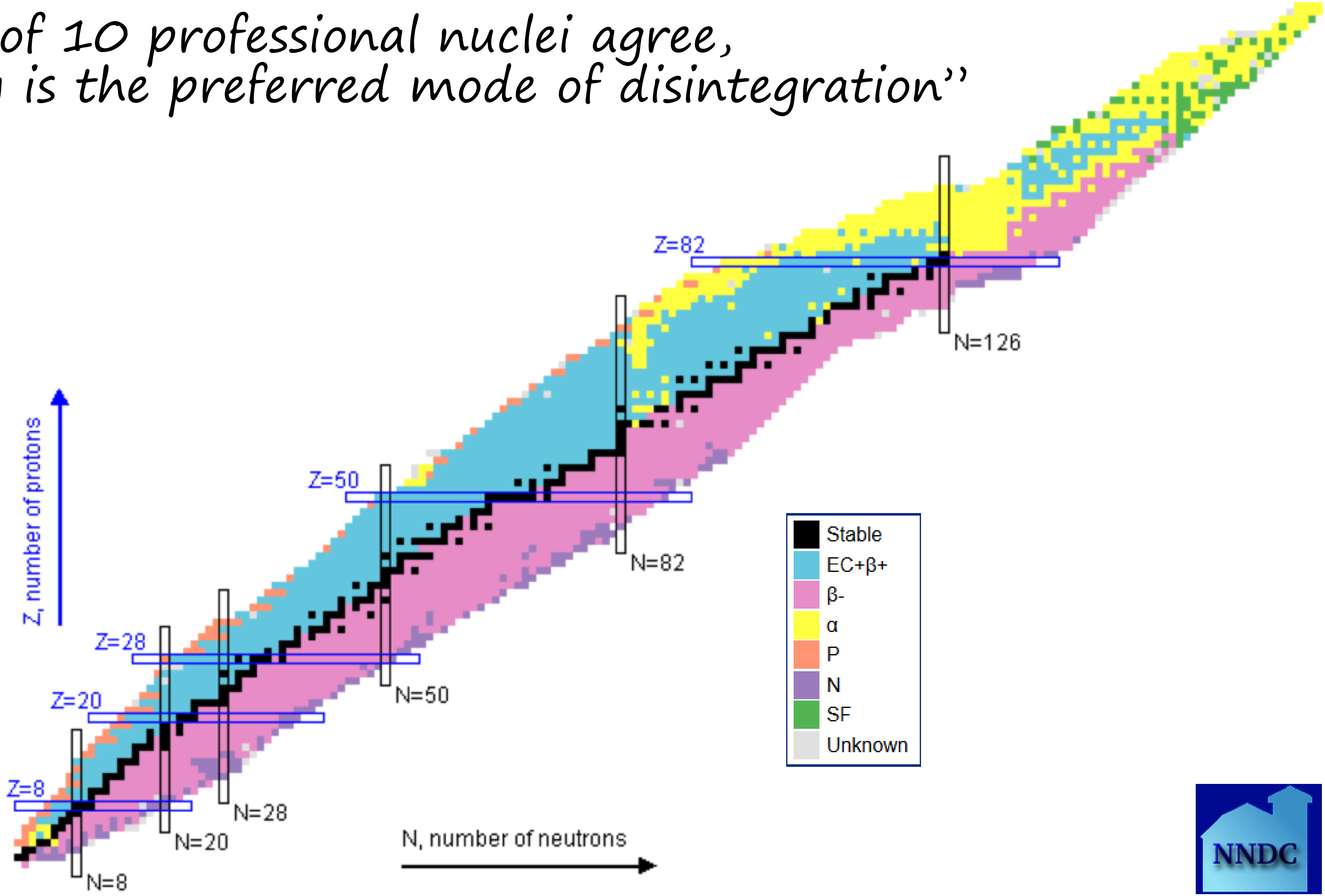
Atomic mass excesses have the electron masses included, so it's only for positron emission that we have to take into account that we will not be gaining an electron with the atom, but instead effectively losing one.

- For several cases near stability, more than one type is possible, e.g.

- An estimate for β instability for the nuclear landscape can be determined by finding the minimum Z for a given A using the semi-empirical mass formula *(See homework #3)*
  - Unsurprisingly, the agreement with the valley of β stability is excellent:

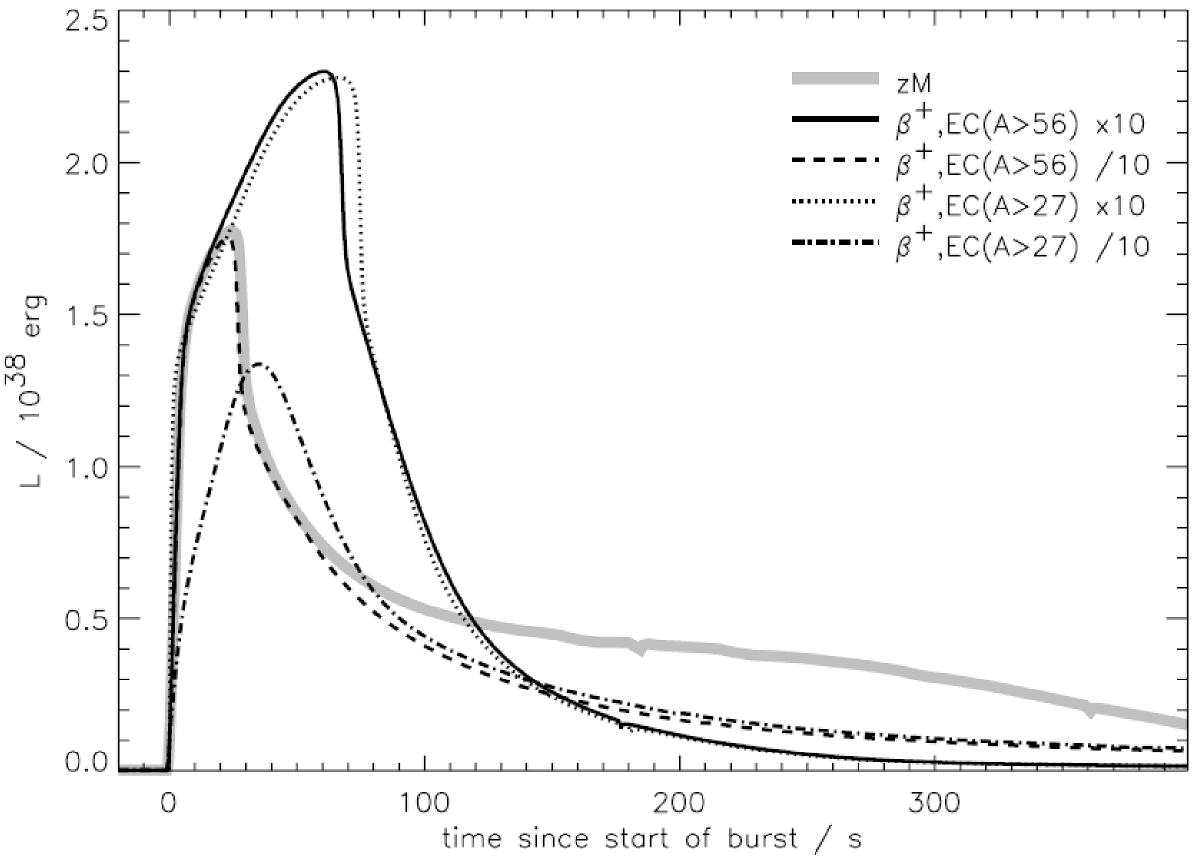


"9 out of 10 professional nuclei agree, β decay is the preferred mode of disintegration"



β decay in nuclear astrophysics (selected examples)

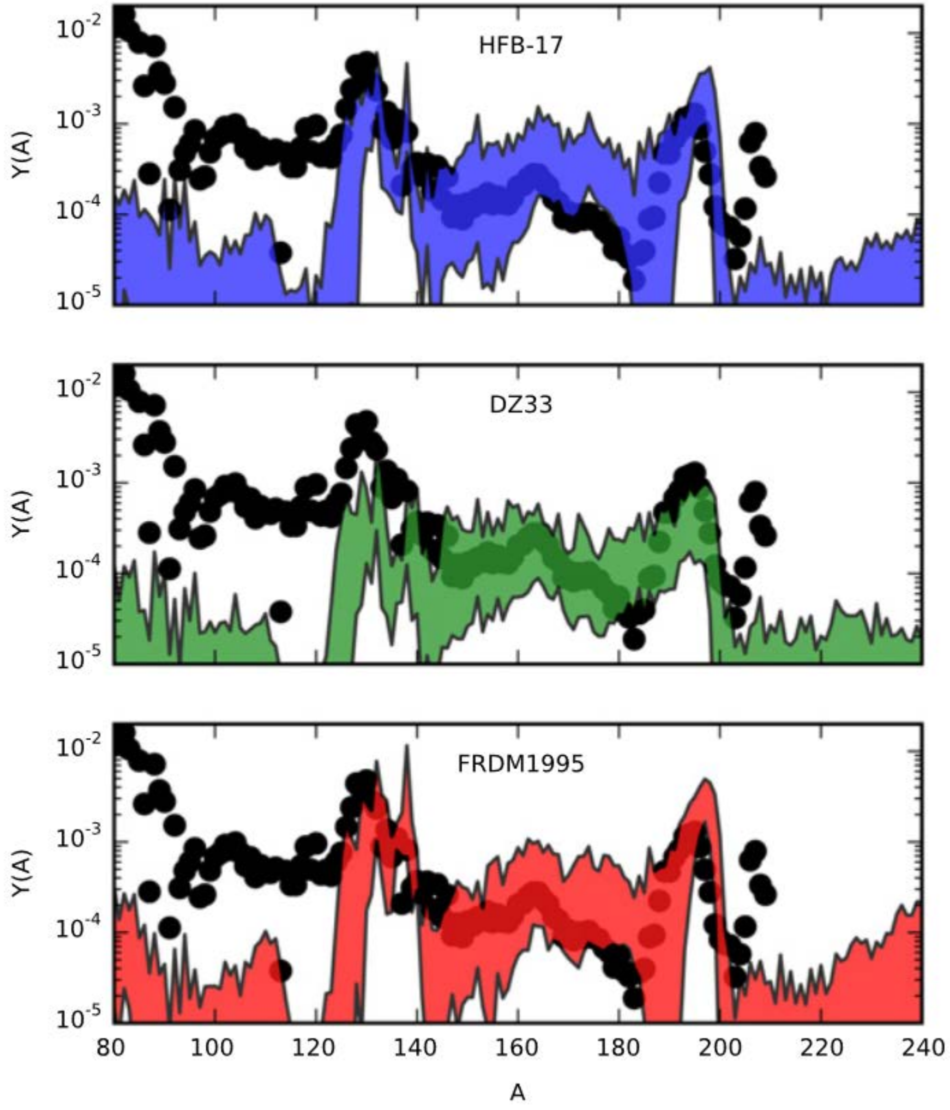
Type I X-ray bursts (Woosley et al. ApJS 2004)



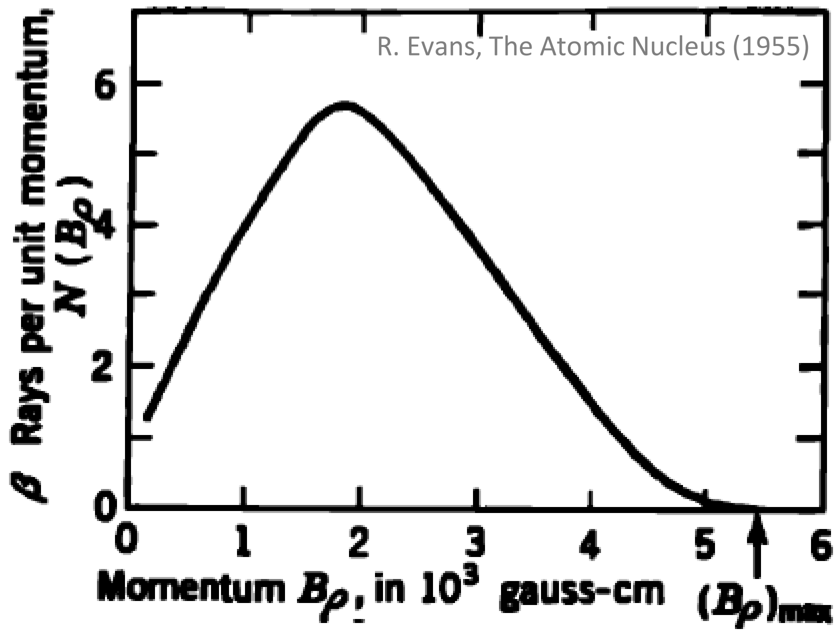
*all important half-lives for the rp-process have been determined experimentally to sufficient precision

r-process nucleosynthesis

(Mumpower et al. Prog. Nucl. Part. Phys (2016))



β decay spectrum, spin conservation, and the neutrino



- Early experiments investigating the "β ray" showed that it was not emitted with a singular energy, like the "α ray", but rather in a continuum of energies
    - Though the maximum energy is equal to the decay Q-value
- Furthermore, the reaction $n \rightarrow p + e^-$ doesn't conserve spin!
    - $J_n = J_p = J_e = \frac{1}{2}$ … so $0 \leq J_p + J_e \leq 1 \quad \neq \frac{1}{2}$
- To remedy this issue, Pauli proposed the involvement of a 3rd hypothetical particle, the neutrino ν *Like a proper old-timey physicist, he made this proposal not in a paper, but in [a letter](#) to physicist Lisa Meitner*
- Given the above considerations, it was postulated that ν is a spin-½ particle ("fermion") that it is massless* and electrically neutral
  (of course this isn't quite true, but true enough for our purposes)
- In one of his last works before switching to primarily performing experimental work, Fermi postulated (E. Fermi, Z. Phys. 1934) that nucleons could act as sources & sinks of electrons and neutrinos, in analogy to charged particles acting as sources and sinks of photons in quantum electrodynamics (the only successful theory of interactions between quantum particles at that point)

For what it's worth, Nature rejected Fermi's paper for being "too remote from physical reality"

Fermi theory of β decay

- Fermi posited that small perturbative interaction precipitated the β decay
- The initial state is described by the wave function of the parent nucleus in state j, $\Psi_i = \psi_{p,j}$, whereas the final state is the product of the daughter nucleus in state k, $\psi_{d,k}$, the electron φ_e, and the neutrino* φ_ν, $\Psi_f = \psi_{d,k}\varphi_e\varphi_\nu$

**(of course it's an antineutrino, but that's cumbersome to say and it doesn't matter here)*

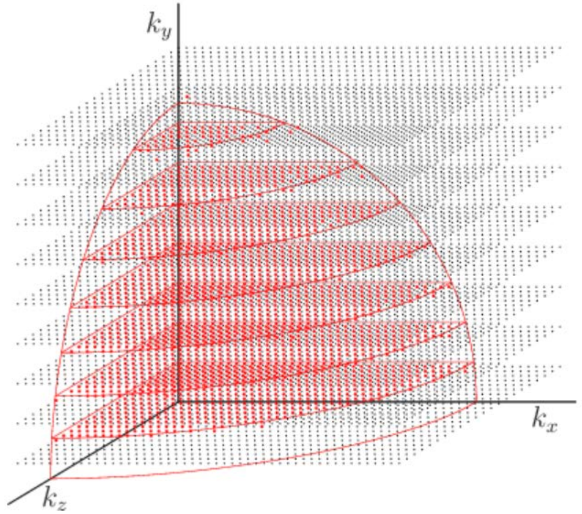
- The transition rate for such a case is derived by solving the Schrödinger equation to 1st order in time-dependent perturbation theory, where the result is known as the Second Golden Rule:
  (or often as Fermi's Golden Rule …though he didn't originally derive it)
  $\lambda = \frac{2\pi}{\hbar}\left|\langle\Psi_{final}|H'|\Psi_{initial}\rangle\right|^2 \rho(E) = \frac{2\pi}{\hbar}\left|\int \Psi^*_{final}H'\Psi_{initial}d\tau\right|^2 \rho(E_f)$
- The qualitative picture is that the transition rate is constant and depends only on two parts:
  - the density of final states to which the decay can proceed
    (i.e. # of ways e^- and ν can share the decay energy E_f)
  - the "matrix element" describing the interaction
    (i.e. description of initial state i to one of many possible final states f)
- $\rho(E_f)$ is a phase-space factor described by kinematics
- The matrix element describes the wave-function overlap between initial and final states, so a theoretical description requires calculating these wave functions

β decay phase space factor

- The decay phase-space $\rho(E_f) = \frac{dn}{dE}$ describes how many ways E_f can be split between e⁻ and ν
- The number final states available is:

$$dn = (\#\ electrons\ in\ volume\ V\ of\ momentum\ space\ with\ momentum\ between\ p_e\ and\ p_e + dp_e)$$
$$\times$$
$$(\#\ neutrinos\ in\ volume\ V\ of\ momentum\ space\ with\ momentum\ between\ p_\nu\ and\ p_\nu + dp_\nu)$$

- The number of available electron states between $p_e + dp_e$ is derived by solving for the combination of momentum components p_x, p_y, p_z that satisfy our momentum criterion:
(as we found for the nuclear level density when discussing the Fermi Gas Model)
  - $n(p_e)dp_e = \frac{V}{(2\pi\hbar)^3} 4\pi p_e^2 dp_e$
- Similarly, $n(p_\nu)dp_\nu = \frac{V}{(2\pi\hbar)^3} 4\pi p_\nu^2 dp_\nu$ …so $dn = \frac{V^2 16\pi^2}{(2\pi\hbar)^6} p_e^2 p_\nu^2 dp_e dp_\nu$
- For $m_\nu = 0$, $p_\nu = \frac{KE_\nu}{c} = \frac{Q - KE_e}{c}$ …so, for a fixed p_e, $dp_\nu = \frac{dQ}{c}$
- As such, $dn = \frac{V^2 16\pi^2}{(2\pi\hbar)^6 c^3} (Q - KE_e)^2 p_e^2 dp_e dQ$
- Meaning the change in the # of final states for a change in Q is: $\frac{dn}{dQ} = \frac{V^2 16\pi^2}{(2\pi\hbar)^6 c^3} (Q - KE_e)^2 p_e^2 dp_e$
- Which is the $\rho(E_f)$ we were after



L. van Dommelen, Quantum Mechanics for Engineers (2012)

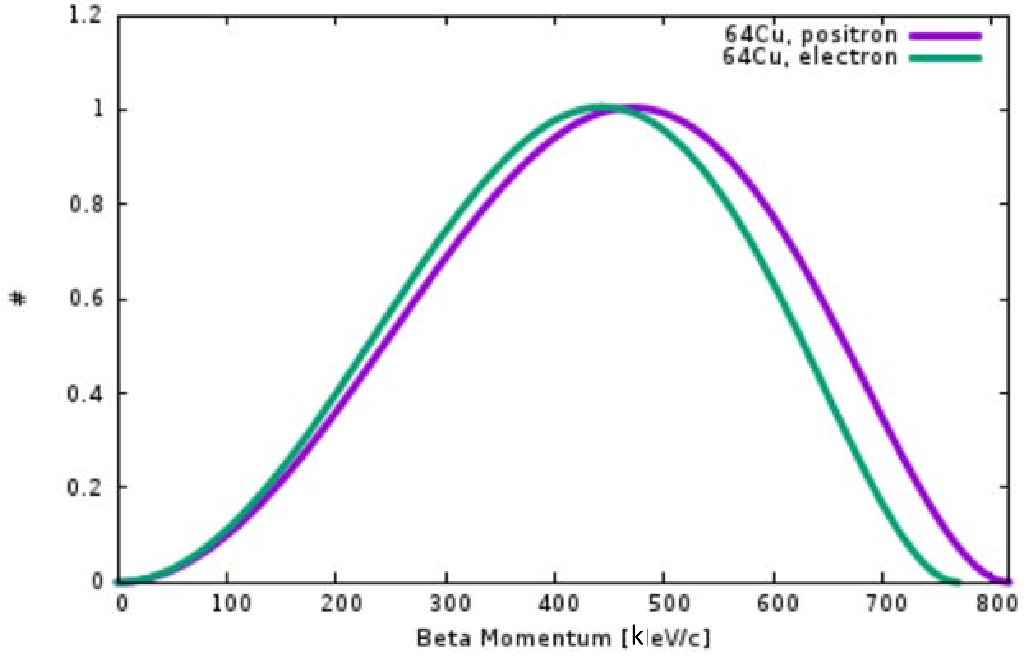
β decay phase space factor & the β energy spectrum

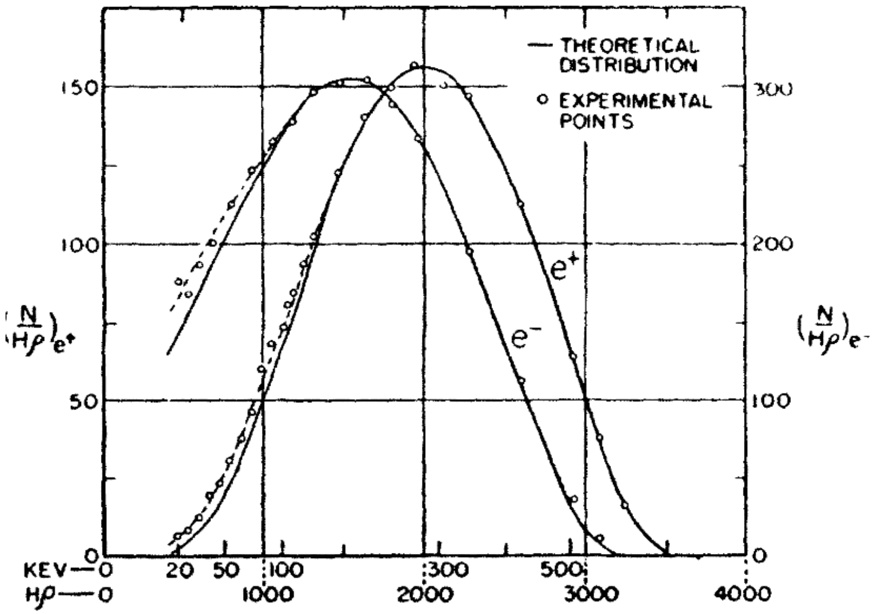
- Considering the β decay rate for an electron momentum within $p_e + dp_e$,

  $\lambda(p_e)dp_e = \frac{2\pi}{\hbar}\left|\langle\Psi_{final}|H'|\Psi_{initial}\rangle\right|^2\rho(E)$

- The matrix element is just some number, so the functional form is from $\rho(E)$

- Therefore, we expect $\lambda(p_e)dp_e \propto \left(Q - KE_p\right)^2 p_e^2 = (Q - KE_e)^2(2m_eKE_e) = \left(Q - \frac{p_e^2}{2m_e}\right)^2 p_e^2$





Wu & Albert, Phys. Rev (1949)

FIG. 2. Momentum spectra of Cu^{64} negatrons and positrons.

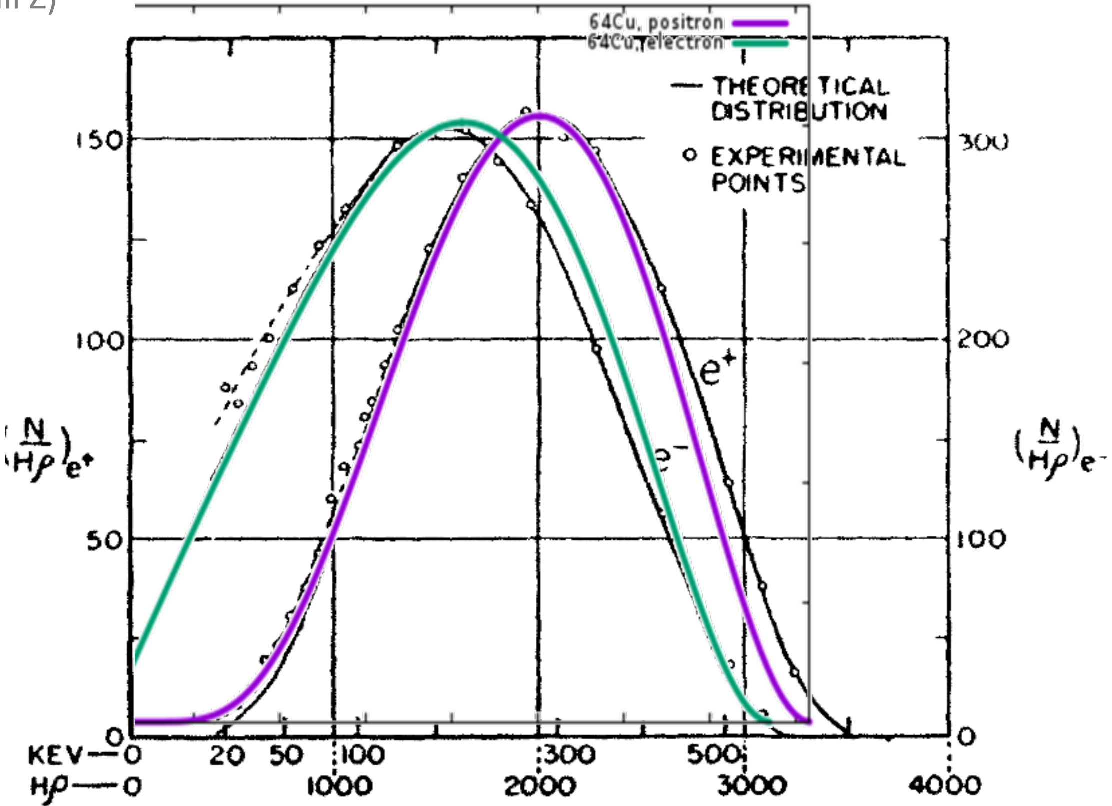
Not too bad, but what effect are we forgetting that will cause positrons and electrons to behave differently?

Coulomb repulsion!

Coulomb distortion to the β spectrum

- Fermi realized that the protons in the daughter nucleus would repel e^+ and attract e^-, modifying the resultant spectrum by a factor $F\left(Z_{daughter}, p_e\right)$
  (in an unfortunate convention, this coulomb distortion factor is often called the Fermi function …or a better term is the Fermi screening factor)
- $F\left(Z_{daughter}, p_e\right)$ is pretty nasty to calculate, so numerical tables are generally used instead
  (See e.g. J.Reitz, Phys. Rev. (1949))
- A non-relativistic approximation that works for nuclides with $Z_{daughter} \ll \sqrt{2} \cdot 137$ is
  (See R. Evans, The Atomic Nucleus (1955) for the relativistic version that works well for all Z)

$F\left(Z_{daughter}, p_e\right) \approx \frac{2\pi y}{1-e^{-2\pi y}}$,
where $y \equiv \frac{\pm Z_{daughter}\alpha}{p_e c} E_e$, $\alpha \approx 1/137$,
$E_e = m_e c^2 + KE_e$,
and + is for β⁻ decay and − is for β⁺ decay



β decay matrix element

$\lambda = \frac{2\pi}{\hbar}\left|\int \boldsymbol{\Psi}^*_{final}\boldsymbol{H}'\boldsymbol{\Psi}_{initial}\boldsymbol{d\tau}\right|^{2}\rho(E_f)$

- Though the phase-space factor and Coulomb distortion give us the β spectrum, we still need to evaluate the initial/final wave function overlap to get the decay rate
- $\left|\int \Psi^*_{final}H'\Psi_{initial}d\tau\right|^2 = \left|\int \psi^*_{d,k}\varphi^*_e\varphi^*_\nu G_F\psi_{p,j}d\tau\right|^2$, where G_F is the constant describing the perturbation (the Fermi coupling constant $G_F \approx 8.9\times10^{-5}MeV\cdot fm^3$

  The dimensionless form of this coupling constant is $G_F*(M_Wc^2)^2/(\sqrt{2}*4\pi(\hbar c)^3) \equiv \alpha_W \sim 10^{-3}*\alpha$ …so it's "weak"

- The neutrino free-streams out, so it is treated as an outgoing plane wave $\varphi^*_\nu = \frac{1}{\sqrt{V}}e^{-i\vec{p}_\nu\cdot\frac{\vec{r}}{\hbar}}$
- The electron has more of an interaction with the nucleus, but the Coulomb part is taken care of by the Coulomb distortion from earlier, so it is also treated as a plane wave $\varphi^*_e = \frac{1}{\sqrt{V}}e^{-i\vec{p}_e\cdot\frac{\vec{r}}{\hbar}}$
- To first order in a Taylor expansion, $e^{-i(\vec{p}_\nu+\vec{p}_e)\cdot\frac{\vec{r}}{\hbar}} \approx 1 - -i(\vec{p}_\nu+\vec{p}_e)\cdot\frac{\vec{r}}{\hbar} + \cdots \approx 1$, (we'll consider higher orders later)

  so: $\left|\int \psi^*_{d,k}\varphi^*_e\varphi^*_\nu g\psi_{p,j}d\tau\right|^2 \approx \frac{G_F^2}{V^2}\left|\int \psi^*_{d,k}\psi_{p,j}d\tau\right|^2 \equiv \frac{G_F^2}{V^2}\left|M_{fi}\right|^2$

- $\left|M_{fi}\right|^2$ is the overlap between wave-function for state j of the parent nucleus and the wave-function for state k of the daughter nucleus and is known as the nuclear matrix element

β decay rate per electron momentum

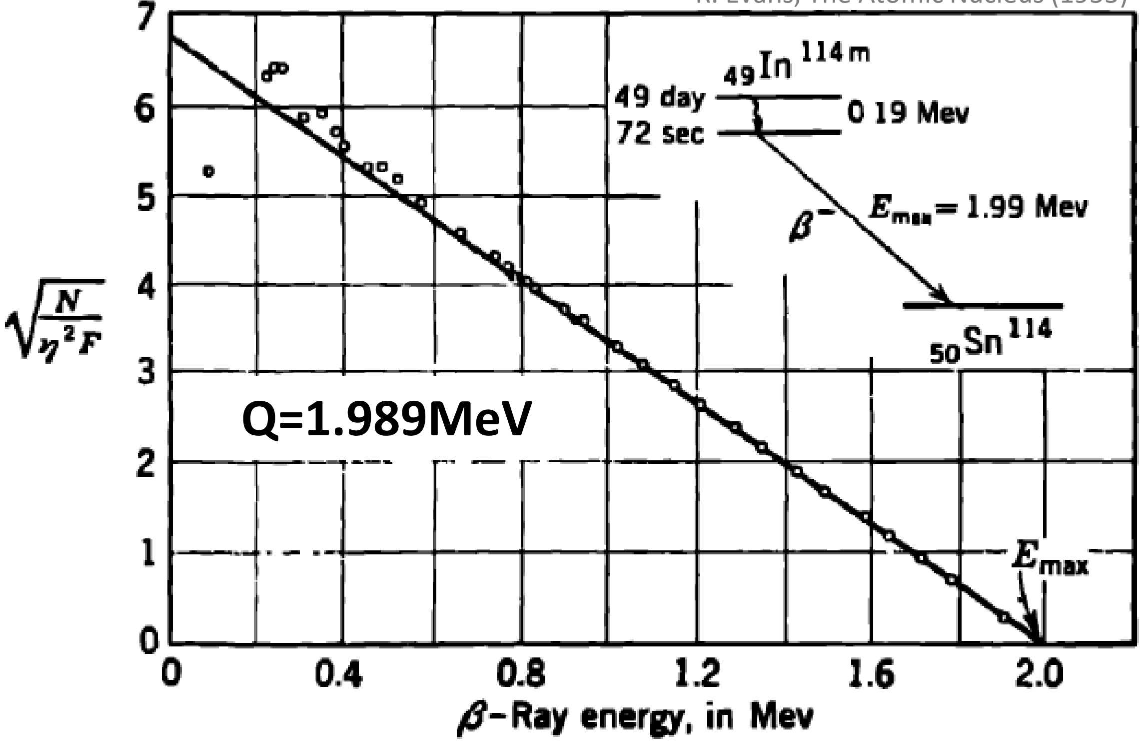
$$\lambda = \frac{2\pi}{\hbar}\left|\int \Psi^*_{final} H' \Psi_{initial} d\tau\right|^2 \rho(E_f)$$

- Combining all the pieces,
  $$\lambda(p_e)dp_e = \frac{2\pi}{\hbar}\frac{G_F^2}{V^2}\left|M_{fi}\right|^2 F(Z_d, p_e)\frac{V^2 16\pi^2}{(2\pi\hbar)^6 c^3}(Q - KE_e)^2 p_e^2 dp_e$$

You may have worried that things depended on the nuclear volume V, but now you can see it cancels

- Cancelling & consolidating: $\boldsymbol{\lambda(p_e)dp_e = \frac{G_F^2}{2\pi^3\hbar^7 c^3}\left|M_{fi}\right|^2 F(Z_d, p_e)(Q - KE_e)^2 p_e^2 dp_e}$
- This provides us with a neat way to ascertain the decay Q-value
- From the bold equation,
  we see $\sqrt{\frac{\lambda(p_e)}{p_e^2 F(Z_d, p_e)}} \propto \left|M_{fi}\right|^2 (Q - KE_e)^2$
- This yields a straight line which intersects the horizontal axis at $\boldsymbol{KE_e = Q}$
- This type of plot is known as a Kurie plot
- *Note that this simple form only applies to "allowed transitions"*
  …which we'll discuss more about in a moment

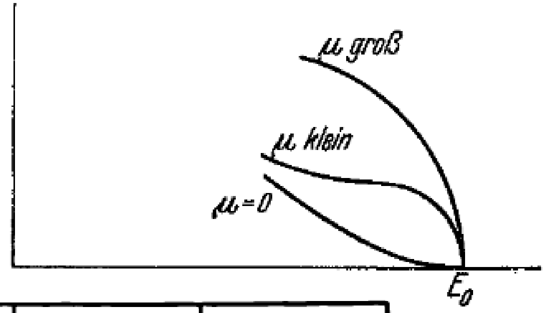
R. Evans, The Atomic Nucleus (1955)



Aside: ν mass from the Kurie plot

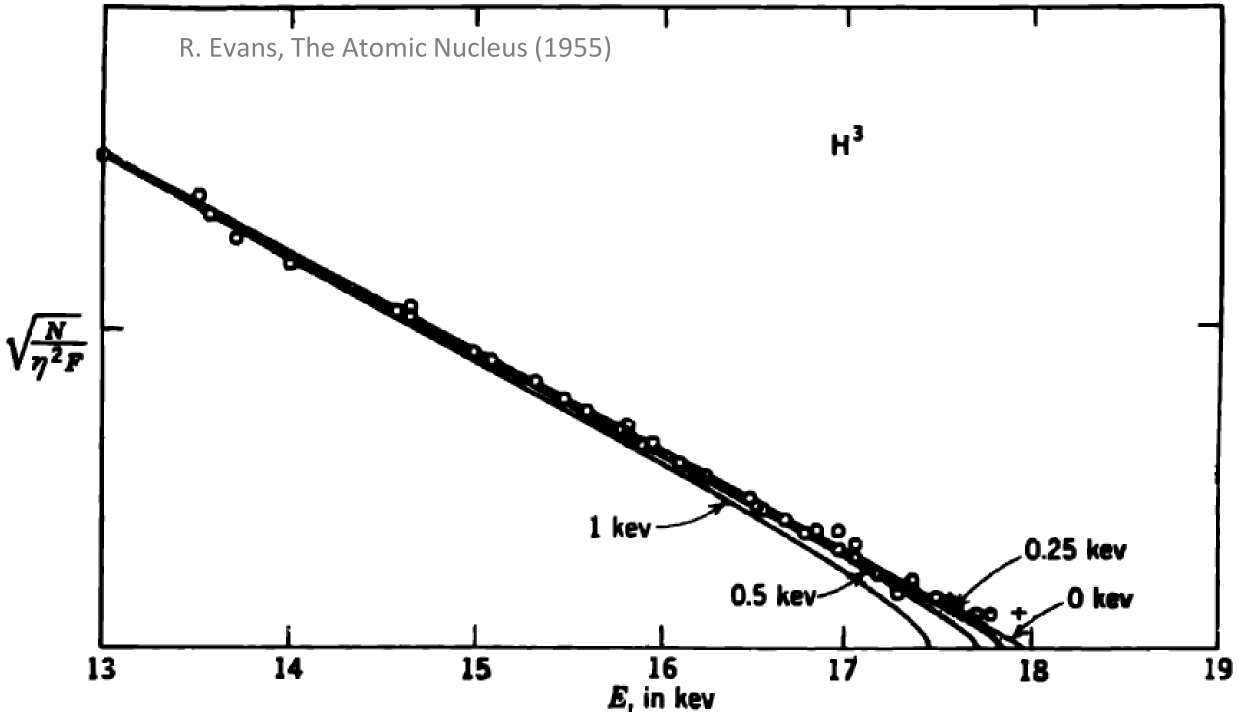
- It turns out that incorporating a non-zero neutrino mass into the phase-space factor calculation yields $\lambda(p_e)dp_e = \frac{G_F^2}{2\pi^3\hbar^7c^3}\left|M_{fi}\right|^2 F(Z_d, p_e)(Q - KE_e)^2 p_e^2 \sqrt{1 - \frac{m_\nu^2 c^4}{(Q-KE_e)^2}}\, dp_e$

E. Fermi, Z.Phys. (1934)



- Fermi realized one could possibly use this to determine m_ν
- The best shot of doing this would be for small Q, for which tritium ($Q \approx 18.6keV$) is the best case
- Limits from such measurements place $m_{\nu_e} < 2eV$ (Otten & Weinheimer, Rep.Prog.Phys (2009))

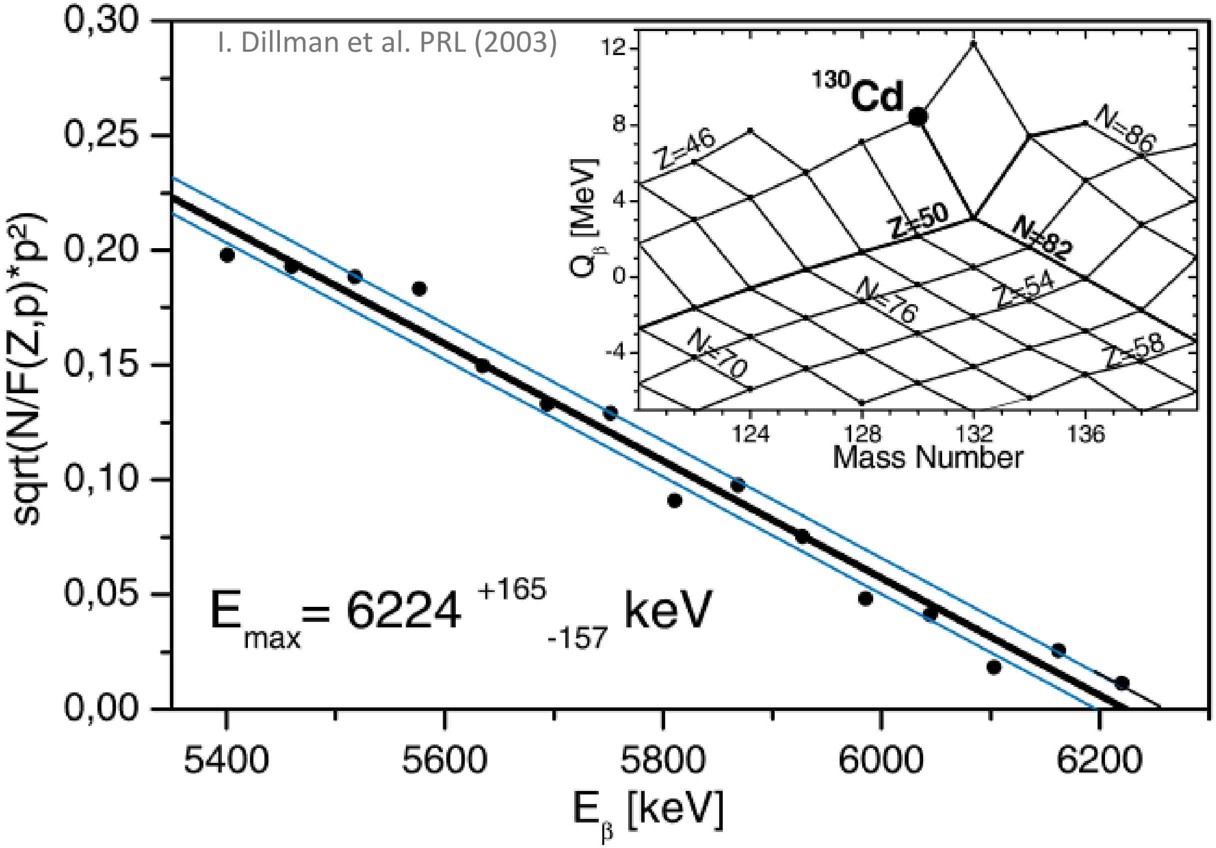
This is complementary to the more stringent limit placed by astronomical observations:
$\Sigma m_\nu < 0.3eV$
(A. Goobar et al. JCAP (2006))



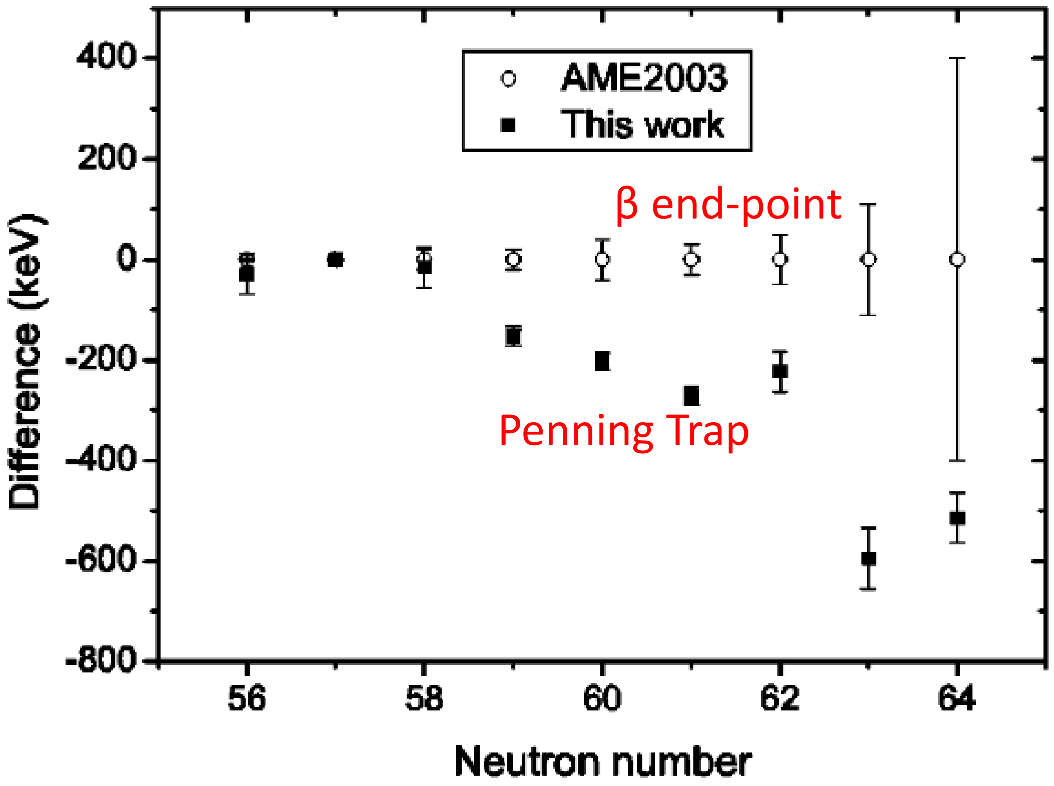
Aside: nuclear mass from the Kurie plot

- Of course the end-point of the Kurie plot can also be used to obtain nuclear masses, assuming the mass of the less-exotic nucleus in the decay is known
- This is known as the "β end-point method"

…which has fallen out of favor lately due to systematic discrepancies with higher-precision techniques



S. Rinta-Antila et al. PRC (2004)



Total β decay rate and the ft value

- All that's left is to integrate over the momentum distribution
  $\lambda = \frac{G_F^2}{2\pi^3\hbar^7 c^3}\left|M_{fi}\right|^2 \int_0^{p_{max}} F(Z_d, p_e)(Q - KE_e)^2 p_e^2 dp_e$, where $p_{max}c = \sqrt{Q^2 - m_e^2 c^4}$
- Unfortunately, life is hard and so is that integral
- The dimensionless Fermi integral is defined as:
  $f(Z_d, Q) \equiv \frac{1}{(m_e c)^3 (m_e c^2)^2}\int_0^{p_{max}} F(Z_d, p_e)(Q - KE_e)^2 p_e^2 dp_e$
  and numerical integration or tables of solutions are used to evaluate it

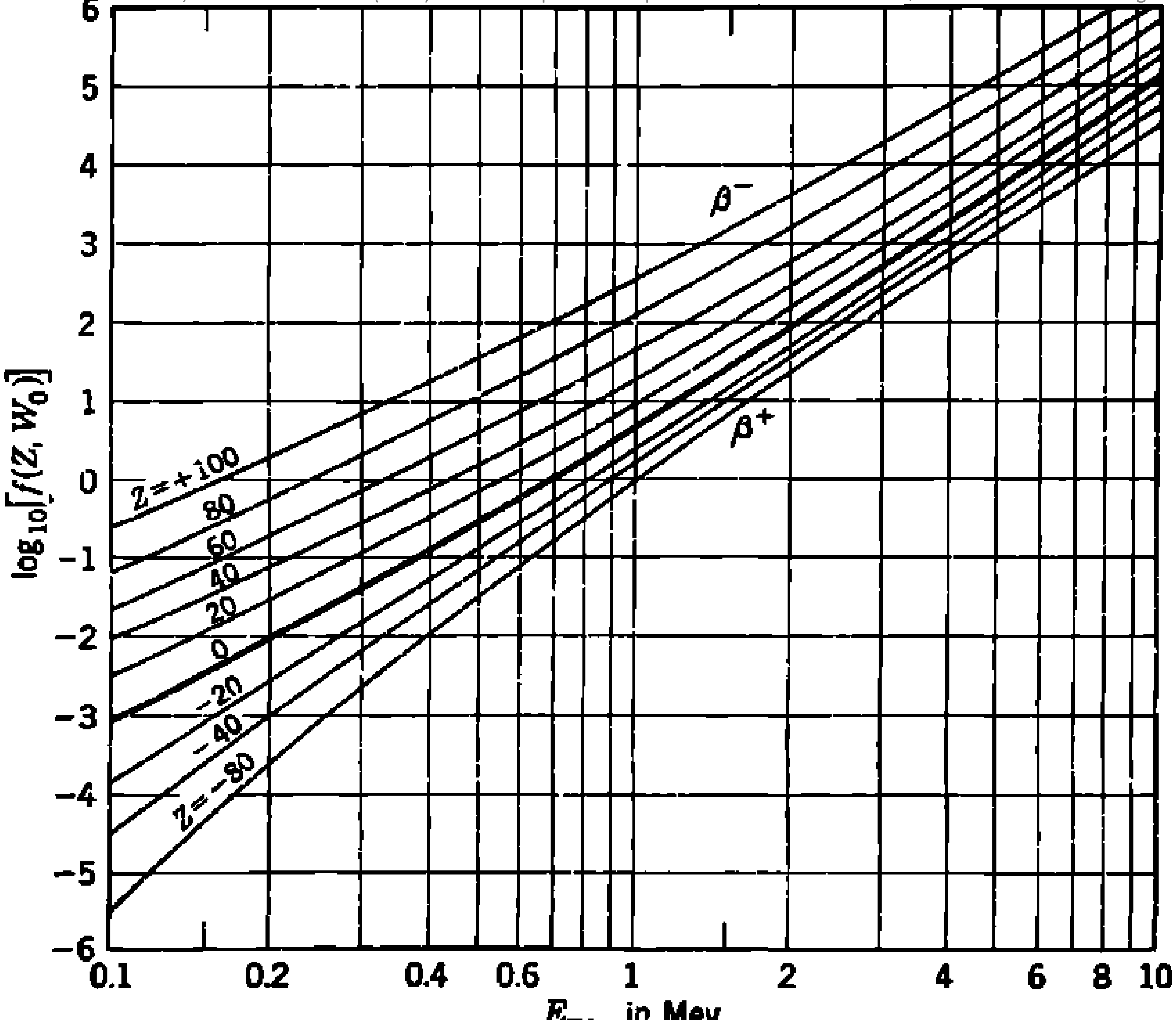
Evaluating the Fermi integral shows ***λ∝Q⁵****, which is known as "Sargent's Rule"*

- Our tidy expression for the total decay rate (a.k.a. the decay constant) is therefore,
  $\lambda = \frac{G_F^2 m_e^5 c^4}{2\pi^3\hbar^7}\left|M_{fi}\right|^2 f(Z_d, Q) = \frac{\ln(2)}{t_{1/2}}$
- Since Q and $t_{1/2}$ can be determined experimentally, the "comparative half-life" $f(Z_d, Q)t_{1/2}$ is used to determine the matrix element $ft = \frac{2\ln(2)\pi^3\hbar^7}{G_F^2 m_e^5 c^4}\frac{1}{\left|M_{fi}\right|^2}$
- Alternatively, a calculation of the wave-function overlap can be used to determine $t_{1/2}$ if experimental data or theoretical calculations are available for Q
- Because $t_{1/2}(\beta)$ for nuclei spans orders of magnitude for nuclei, $\log_{10}(ft)$ is often quoted

The Fermi integral

In case you find yourself needing to calculate the f(Zd,Q), use graph such as this one, which was determined numerically

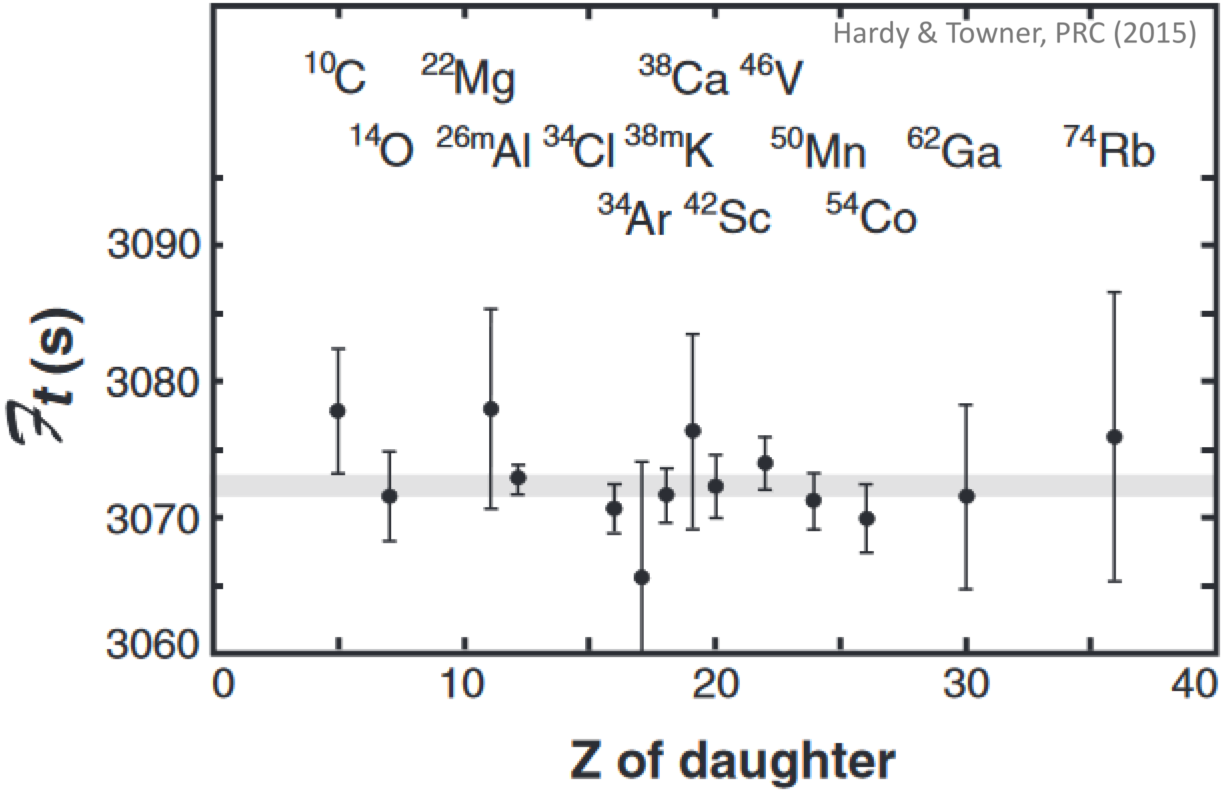
R. Evans, The Atomic Nucleus (1955) …a similar open-source plot is in L. van Dommelen's Quantum Mechanics for Engineers



Fundamental physics with super-allowed transitions

- For mirror nuclei (swapped N & Z), decays between states with identical J^{π} (always 0^+ to 0^+) the initial and final state wave functions are expected to be nearly identical, so $|M_{fi}|^2 = 1$
- Such transitions are called "super-allowed"
- E.g. $^{14}O(0^+) \rightarrow {}^{14}N(0^+) + e^- + \nu_e$, Q=2.831MeV
  - From the Fermi integral plot on the previous page, $f \approx 10^{2.5}$
  - So $t_{½} = \frac{2\ln(2)\pi^3\hbar^7}{G_F^2 m_e^5 c^4}\frac{1}{f} \approx \frac{2\ln(2)\pi^3(\hbar c)^7}{G_F^2(m_e c^2)^5 c}\frac{1}{f} \approx 20s$
  - The actual $t_{½} \approx 70s$ …*which isn't half bad for reading off of a steeply logarithmic plot!*
- It turns out that, given adequate corrections, super-allowed transitions all seem to obey $|M_{fi}|^2 = 1$, which is something known as the conserved vector current (CVC) hypothesis.
- Any deviation from this would imply physics beyond the standard model

several corrections are responsible for $f \rightarrow \mathcal{F}$



Gamow-Teller β decay transitions

$$\lambda = \frac{2\pi}{\hbar}\left|\int \Psi^*_{final} H' \Psi_{initial} d\tau\right|^2 \rho(E_f)$$

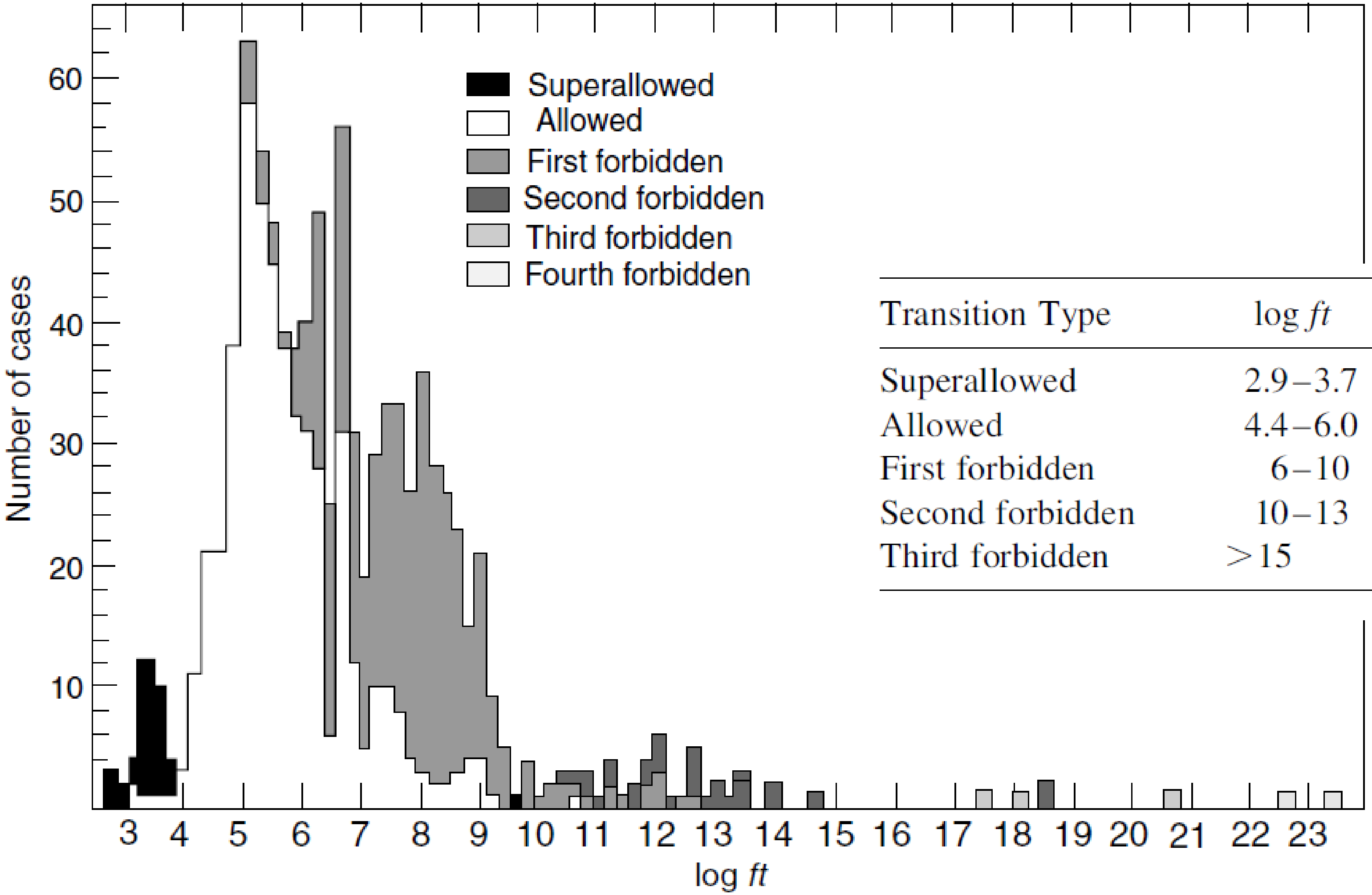
- Earlier, when we made the approximation for the electron and neutrino wave functions that we could take the first term of the Taylor expansion: $e^{-i(\vec{p}_\nu + \vec{p}_e)\cdot\frac{\vec{r}}{\hbar}} \approx 1$,
we consequently made the result that the β decay would result in no angular momentum transfer (because we ignored the terms with $\vec{p}\cdot\vec{r}$)
- And yet, some observed transitions with $\Delta J = 1$ (or with $\Delta J = 0$ from a non-0^+ state) yield lifetimes as if they're not reduced by the amount one would expect from the angular momentum transfer
- These cases, Gamow-Teller decays, happen because due to the coupling of spin between the electron and the neutrino, which can be anti-aligned ($S = 0$) or aligned ($S = 1$)
- For the anti-aligned case, zero angular momentum is carried away and parity is conserved …these are the Fermi decays we've covered thus far
- For the aligned case, 1 or 0 units of angular momentum can be carried away and parity is still conserved ….these are Gamow-Teller decays
- Note that GT decays cannot happen from 0^+ to 0^+,
since spin 0 (from the beta-decaying nucleus) and 1 (from $S = 1$) can only combine to be 1
- For GT decays, the wavefunction overlap won't be as good as for 0+ to 0+, so these decays are "allowed" but never "super allowed"

Decay selection rules and "forbidden" decays $\lambda = \frac{2\pi}{\hbar}\left|\int \Psi^*_{final} H' \Psi_{initial} d\tau\right|^2 \rho(E_f)$

- As was just alluded to, ignoring higher-order terms of the $e^{-i(\vec{p}_\nu+\vec{p}_e)\cdot\frac{\vec{r}}{\hbar}}$ Taylor expansion omits the possibility for angular momentum transfer
- If angular momentum transfer is to occur, higher-order terms need to be included and it will no longer be the case that $\left|M_{fi}\right|^2$ is independent of p_e
- In fact, for these cases $\Delta J > 1$ and/or $\Delta\pi = yes$, the leading-order overlap $\left|M_{fi}\right|^2 = 0$ and so a higher-order term will be necessary
- The order that's required will correspond to the angular momentum transfer of the decay ΔJ
- This combined with whether or not parity is changed is referred to as how "forbidden" a transition is…even though it's just a hindrance
    - $0^+ - 0^+ \rightarrow "super - allowed"$
    - $0^+ - 1^+ \; or \; \Delta J = 0 \; or \; 1 \; and \; \Delta\pi = no \rightarrow "allowed"$
    - $\Delta J = 0 \; or \; 1, \Delta\pi = yes \;\; \rightarrow "first \; forbidden"$
    - $\Delta J = 2, \Delta\pi = no \rightarrow "second \; forbidden"$
- For a given transition type, ft will typically be within an order of magnitude of some value

Empirical ft value distribution

W. Meyerhof, Elements of Nuclear Physics (1967)



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

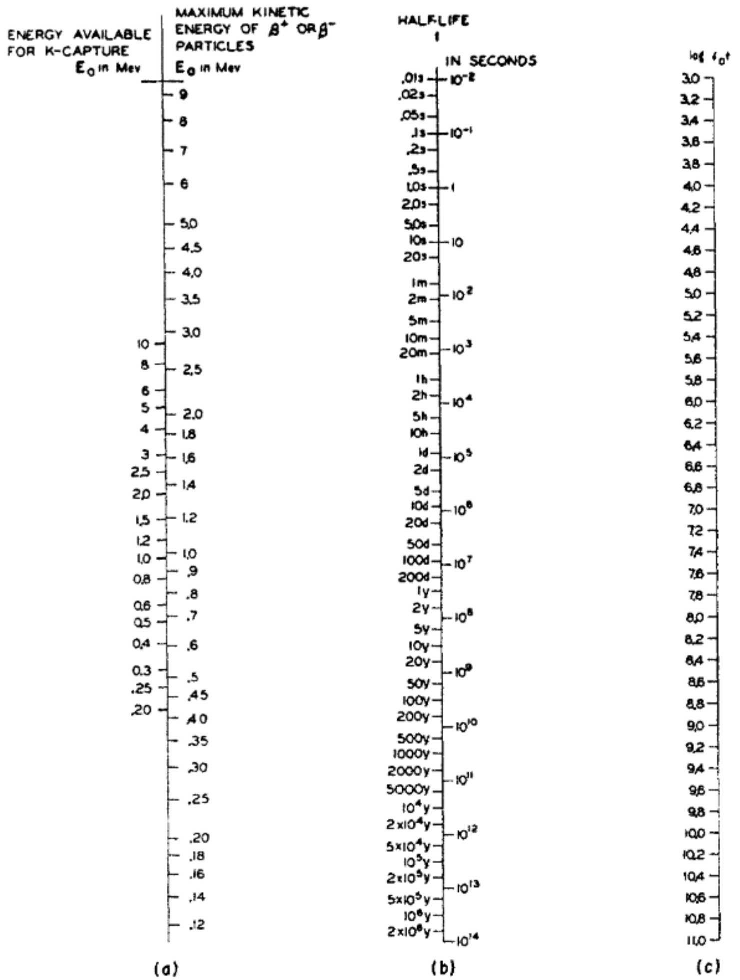
Transition Type	log ft	L_β	$\Delta\pi$	Fermi ΔI	Gamow–Teller ΔI
Superallowed	2.9–3.7	0	No	0	0
Allowed	4.4–6.0	0	No	0	0, 1
First forbidden	6–10	1	Yes	0, 1	0, 1, 2
Second forbidden	10–13	2	No	1, 2	1, 2, 3
Third forbidden	>15	3	Yes	2, 3	2, 3, 4

Empirical ranges for log(ft) given a particular $\Delta J, \Delta\pi$ are available in compilations, e.g. B. Singh et al. Nuclear Data Sheets (1998) .
For a given $\Delta J, \Delta\pi$ plausible values of log(ft) often range within ±1.5 of the median, i.e. this gets you a ballpark value.

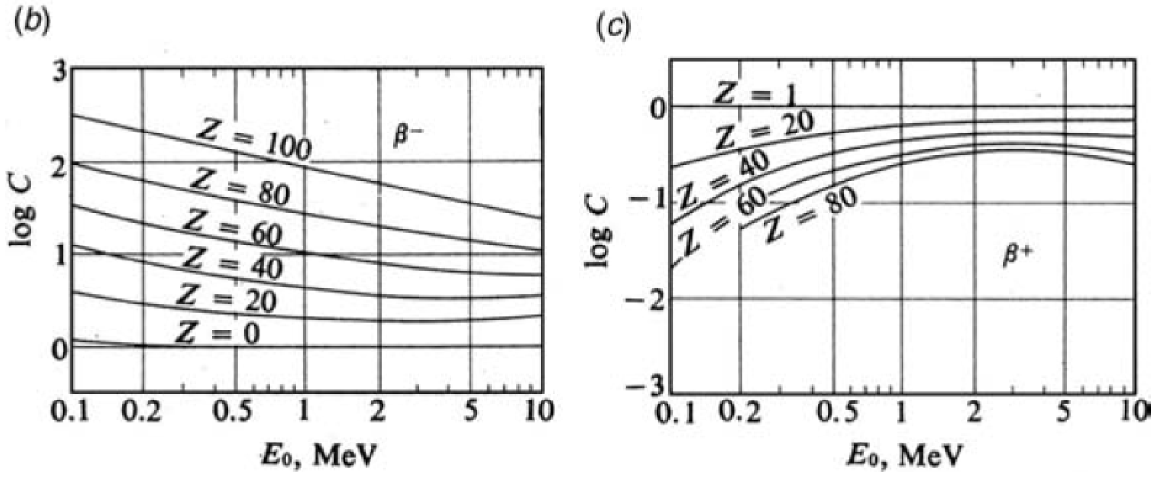
Tabular method to find ft value

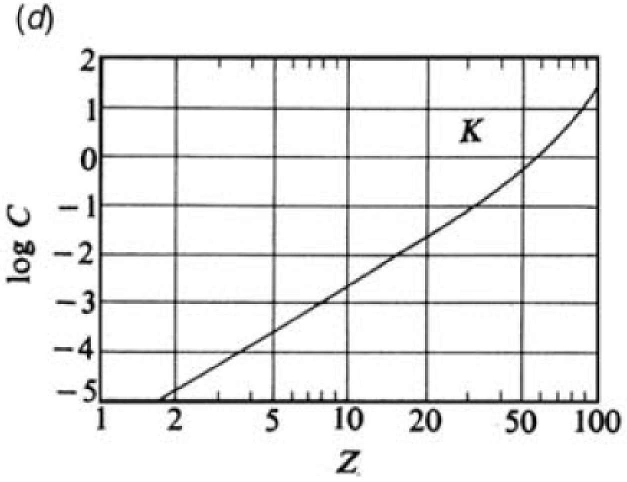
- Supposing you know the (state-to-state) Q-value and half-life, ft can be evaluated using a graphical/tabular method that summarizes the result of a large number of analytic calculations (S.Moszkowski, Phys.Rev. (1951))
- $\log(ft) = \log(f_0 t) + \log(C) + \Delta\log(ft)$, where $\log(f_0 t)$ is from (1), $\log(C)$ from (2), and $\Delta\log(ft)$ from (3)

(1)



(2)



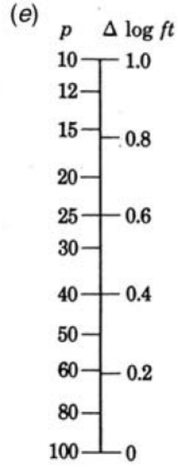


III. RANGE OF USEFULNESS OF THE FIGURES

The figures can be used for $\beta^{\pm}$ maximum energy of 120 kev to 9 Mev, and give results accurate to within 0.1 of the true value of log(ft). For K-captures, results to within 0.2 are obtained for energies of 200 kev to 10 Mev, with certain qualifications which are due to the approximations for f_0 made in Sec. II (a). Let $E_0'(Z)$ be the value of E_0 below which log(ft) for K-capture, as obtained from the figures, deviates by more than 0.2 from the correct value. Table I shows E_0' for various values of Z.

The branching correction obtainable from Fig. 3 is not restricted to values of $p > 10$, since if p is replaced by $10^{-n}p$, $\Delta\log(ft)$ is replaced by $n + \Delta\log(ft)$.

(3)

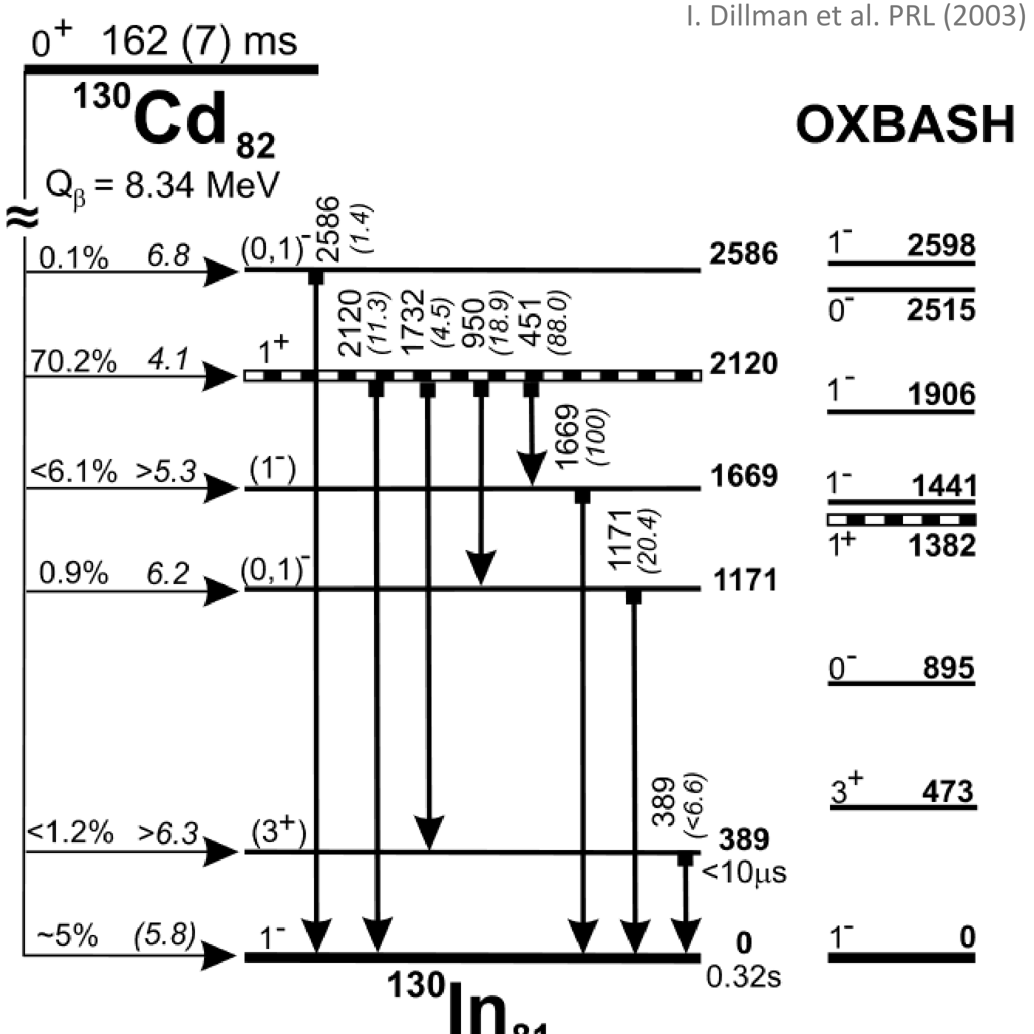


p=branching %

Using ft for spectroscopy

- Since we established ft is linked to a particular $\Delta J, \Delta\pi$, a measurement of ft can constrain J^π of an unknown state if the β decay proceeds from a state with known J^π
- Nuclear masses combined with γ ray energy measurements provide the transition Q-value, i.e. f
- The % of the time the decay proceeds through a given state (the branching ratio) gives the partial half-life for that decay ($t_{½}' = \frac{t_{½}}{Branching}$), i.e. t
- Given knowledge of the parent J^π (always 0+ for even-even nuclei), one can infer J^π for the daughter state

I. Dillman et al. PRL (2003)



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

Transition Type	log ft	L_β	Δπ	Fermi ΔI	Gamow–Teller ΔI
Superallowed	2.9–3.7	0	No	0	0
Allowed	4.4–6.0	0	No	0	0, 1
First forbidden	6–10	1	Yes	0, 1	0, 1, 2
Second forbidden	10–13	2	No	1, 2	1, 2, 3
Third forbidden	>15	3	Yes	2, 3	2, 3, 4

λ for Electron capture

$\lambda = \frac{2\pi}{\hbar}\left|\int \Psi^*_{final} H' \Psi_{initial} d\tau\right|^2 \rho(E_f)$

- Rather than a nucleon undergoing transmutation by its lonesome, instead e⁻-capture can occur
- This is either due to a capture of a low-lying (usually the "K-shell") electron or due to the electron Fermi energy in an electron-degenerate environment being high enough to overcome the electron-capture Q-value
- The decay constant for electron-capture decay is a bit different than for β decay, because the final state only consists of a nucleon and a neutrino … i.e. $KE_e = 0$ and $\Psi_f = \psi_{d,k}\varphi_\nu$
- The decay constant is then: $\lambda = \frac{G_F^2}{2\pi^3\hbar^3 c^3}\left|M_{fi}\right|^2 T_\nu^2 |\varphi_K(0)|^2$,
  where $\varphi_K(0)$ is the wave-function for the inner-most atomic electron (the one in the "K-shell")
- You may recall from your Quantum class, $\varphi_K(0) = \frac{1}{\sqrt{\pi}}\left(\frac{Z m_e e^2}{4\pi\varepsilon_0\hbar^2}\right)^{3/2}$
- As such, the ratio of electron-capture to β⁺ decay for a nucleus goes as $\frac{\lambda_K}{\lambda_{\beta+}} \propto Z^3$

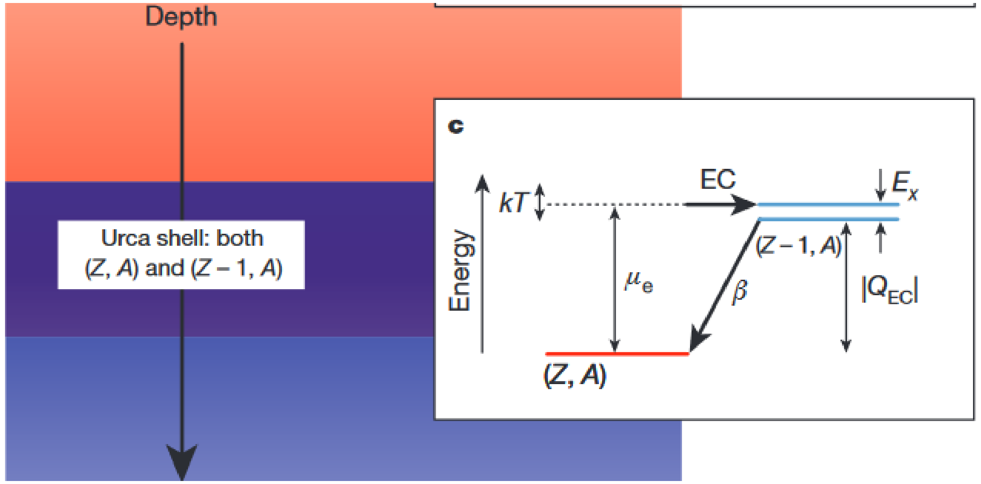
  (of course, $Q_\beta > 2m_e$ is a requirement for β⁺ decay to be possible in the first place)

Since EC decay only emits a neutrino, which will be almost impossible for us to detect, how do you figure EC decay is usually detected?

X-ray and Auger electron emission due to atomic electrons filling the vacated orbital

Urca cooling: EC-β^- cycling

H.Schatz et al Nature (2014)

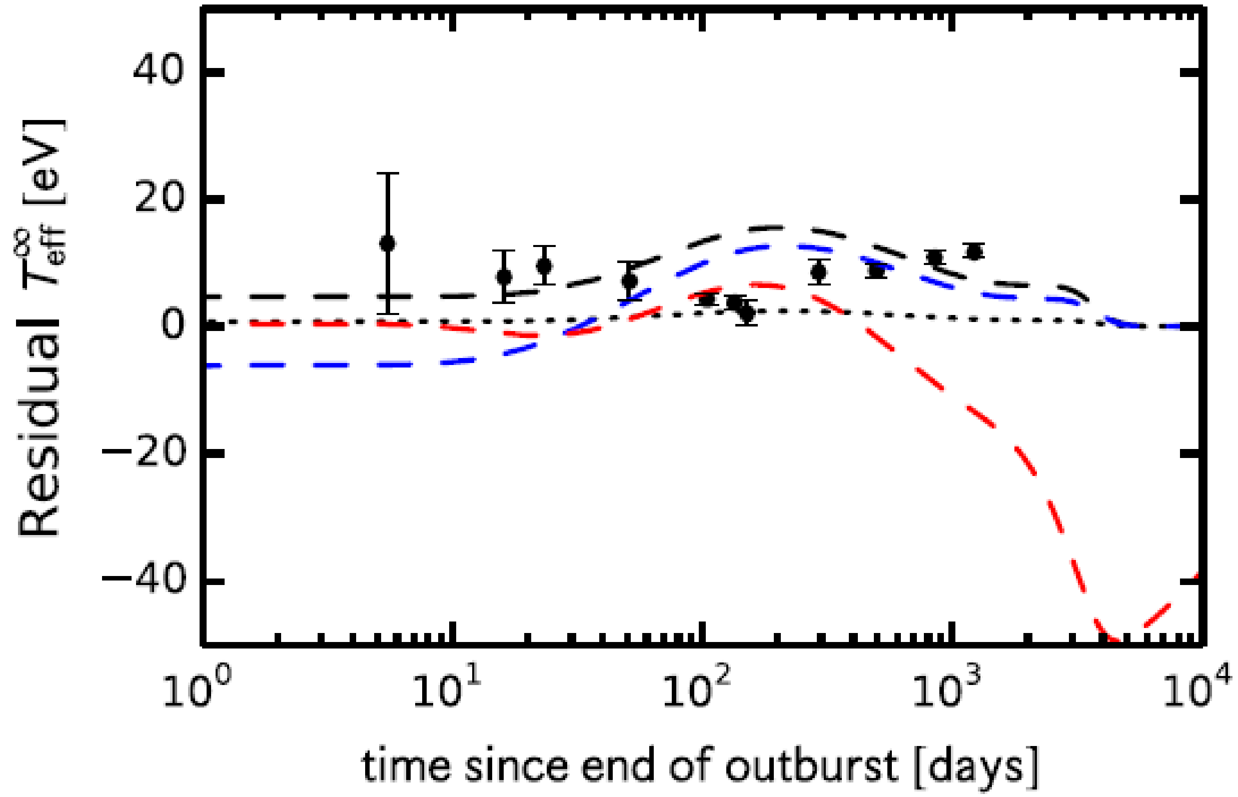


- In extremely dense environments, electrons are degenerate, meaning that electrons are available with an energy equal to the electron fermi energy $E_{F,e}$
- When $E_{F,e} \approx Q_{EC}$, electron capture will proceed
- However, when $\left|E_{F,e} - Q_{EC}\right| \lesssim k_B T$, there is some phase-space open near the Fermi surface for β^- decay to occur *[note that normally an electron would be forbidden from re-entering the environment due to Pauli exclusion]*
- In these sweet spots, EC-β^- cycling occurs, releasing two neutrinos with each cycle
- The neutrinos carry away energy with them and hence the phenomenon is known as Urca cooling
- $L_\nu \propto \frac{Q^5 T^5}{ft\prime}$, where $ft' = \frac{ft_{EC}+ft_\beta}{2}$ *B. Paxton et al. ApJS 2016*
  [The ft-values are different because of the different spin-degeneracy of the parent state. This is usually negligible because ft is often uncertain by orders of magnitude.]

Urca cooling in astrophysics (selected examples)

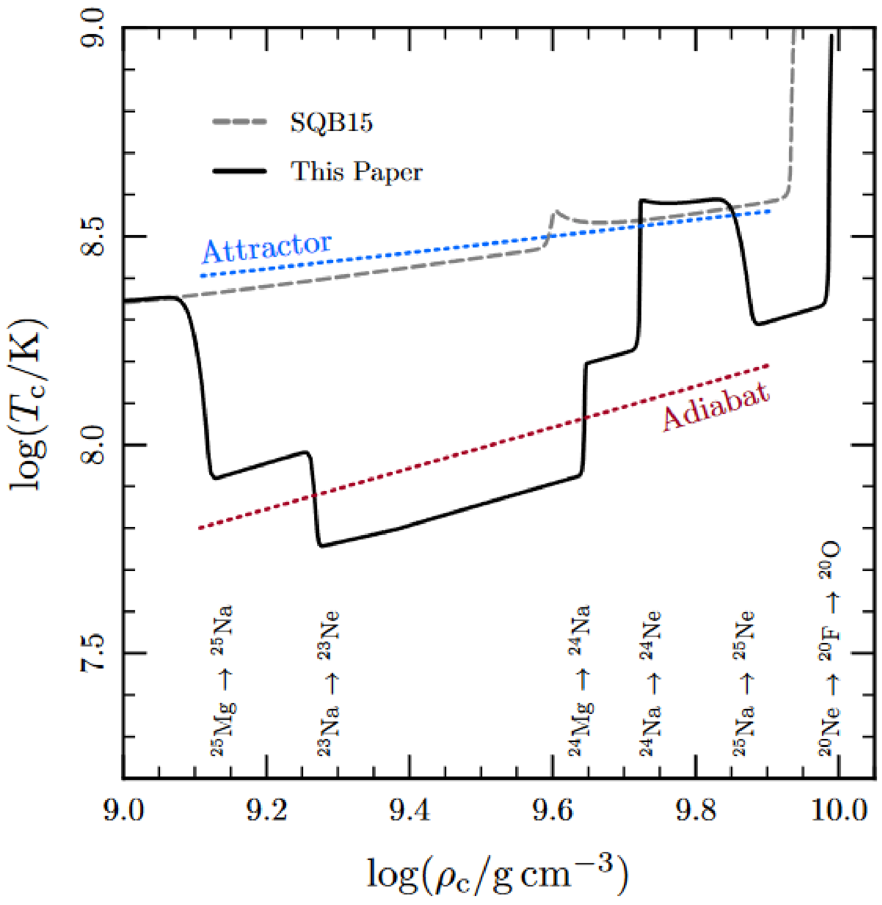
Light curves of neutron star cooling following accretion turn-off

(Meisel & Deibel, Astrophys. J. (2017))



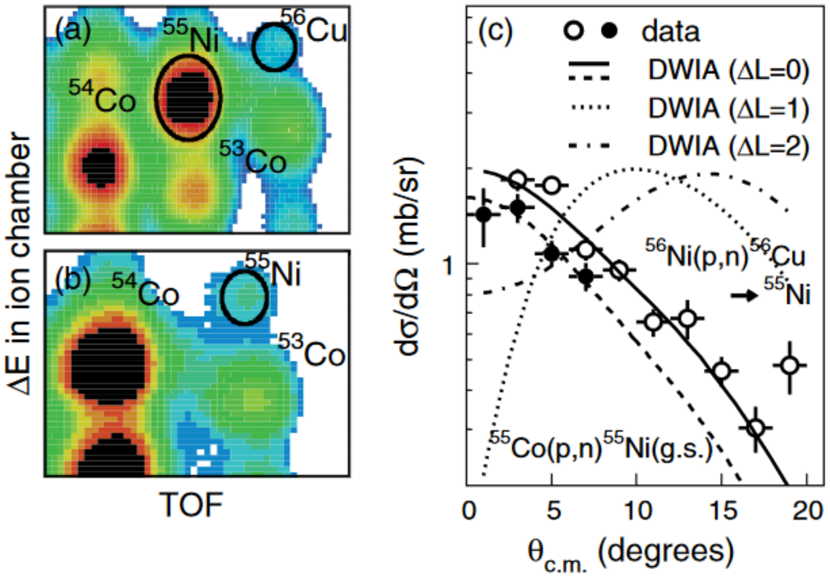
Core temperature evolution for accreting ONe WDs

(Schwab, Bildsten, & Quataert MNRAS (2017))

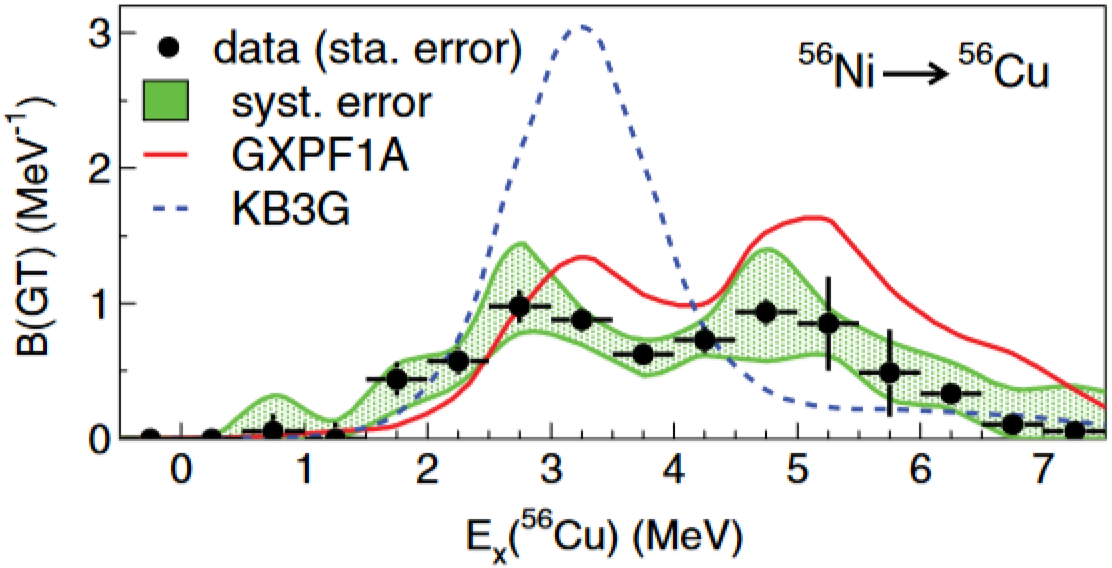


Transition Strengths: B(F) and B(GT)

- Theoretical calculations of weak transition rates characterize such rates with transition strengths, where B(F) and B(GT) are the Fermi and Gamow-Teller transition strengths, respectively
- Each is described by he modulus-square of the expectation value of the relevant transition operator
  [See e.g. : Lecture Notes in Nuclear Structure Physics (B.A. Brown)]
- Their sum is inversely proportional to the comparative half-life: $ft \propto \frac{1}{B(F)+B(GT)}$
- B(GT) is also handy because it can be deduced from charge-exchange measurements:

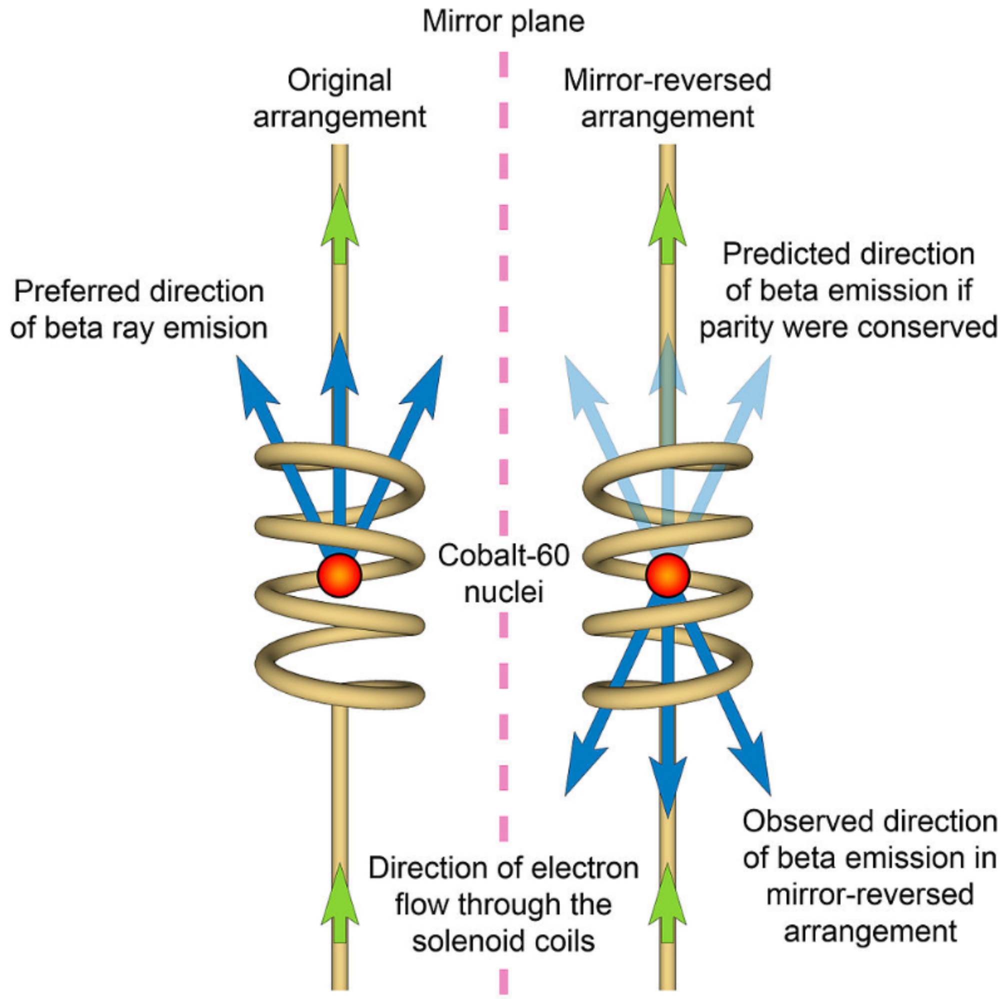


M. Sasano et al. PRL (2011)



Parity Non-Conservation

- For α-decay, parity was an important consideration for selection rules
- Transitions were only possible for which the parity change was $\Delta\pi = (-1)^l$
- For weak transitions, this is not the case
- It was demonstrated by observing the ^{60}Co β-decay angular distribution for ^{60}Co with its spin aligned along and against a magnetic field
- If parity were conserved, reflecting the spatial coordinates (by preparing ^{60}Co as spin down instead of spin up) shouldn't change the β angular distribution…but it did
- This showed weak transitions don't conserve parity



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Further Reading

- Chapter 8: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapter 7: Nuclear & Particle Physics (B.R. Martin)
- Chapter 14, Section 19: Quantum Mechanics for Engineers (L. van Dommelen)
- Chapter 15: Introduction to Special Relativity, Quantum Mechanics, and Nuclear Physics for Nuclear Engineers (A. Bielajew)
- Chapter 4: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapter 17: The Atomic Nucleus (R. Evans)
- Chapter 16: Elementary Nuclear Theory (H. Bethe)