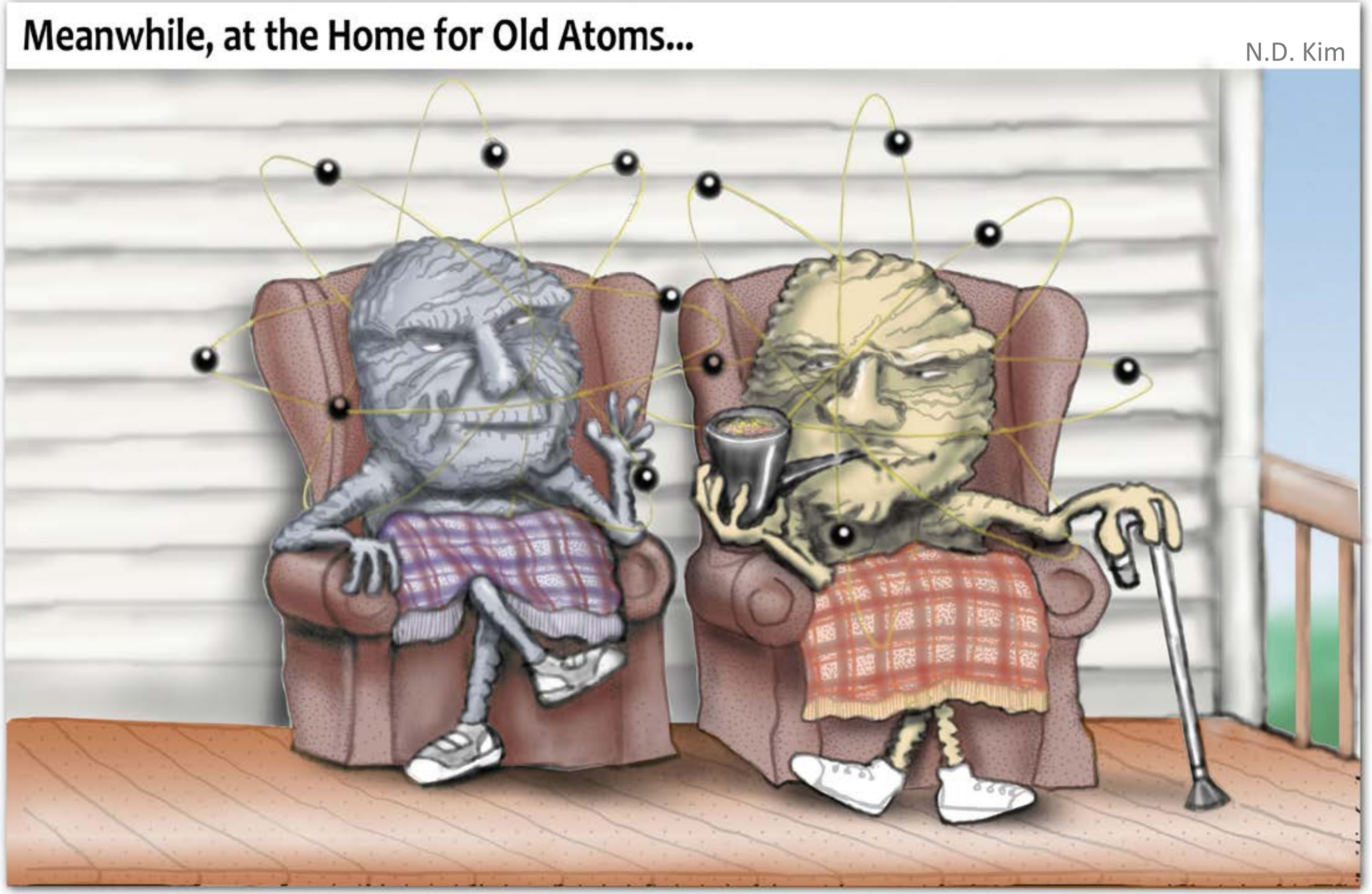
Lecture 7: *α Decay*

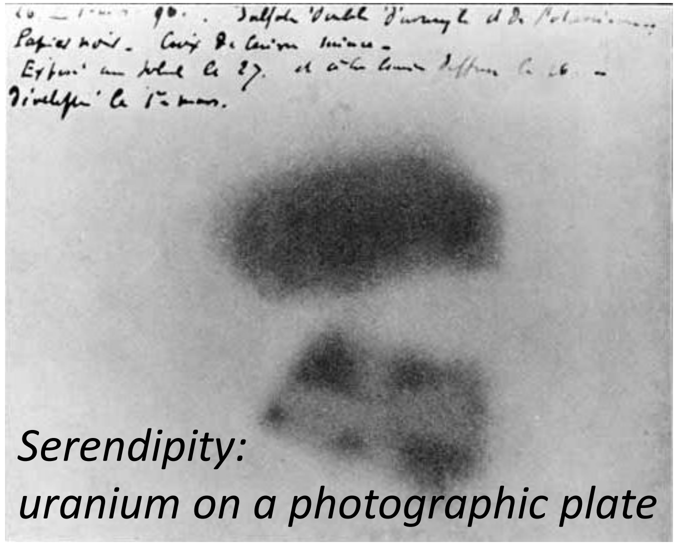
- Energetics
- Geiger-Nuttall
- Tunneling through a barrier
- Decay hindrance
- Why α emission?



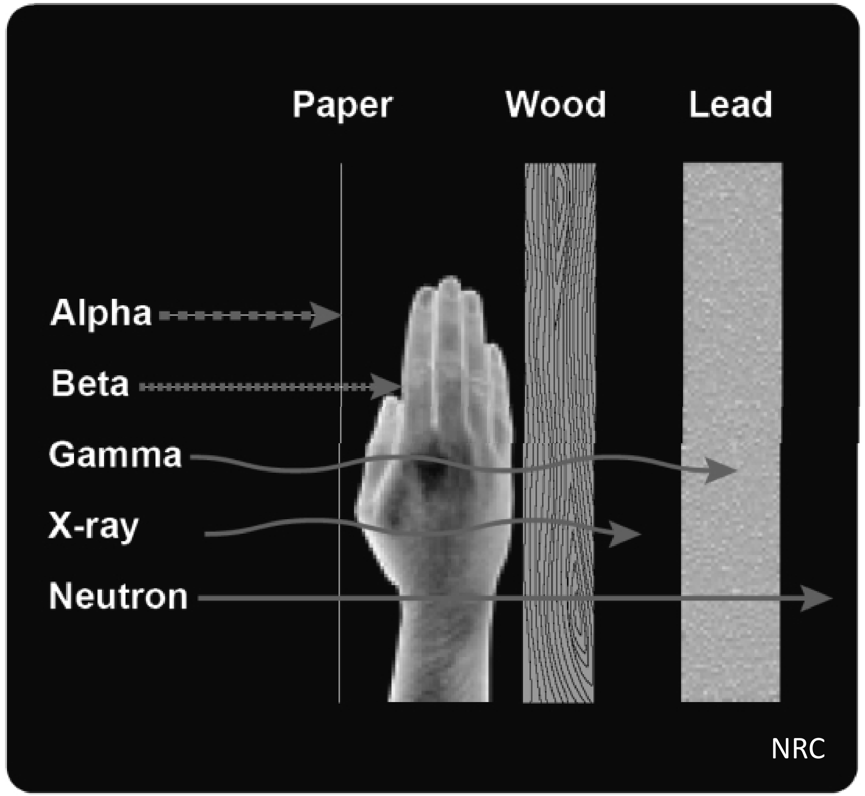
"When I was young I was an atom of uranium-238. I felt so alive, so dangerous! Then one day I accidentally ejected an alpha particle. Now look at me – a spent old atom of lead-206. All my life since then has been nothing but decay, decay, decay..."

The α-ray

- At the end of the 1800s, folks such as Becquerel, the Curies, and Rutherford got their hands on uranium and radium samples, they found emitted energetic particles
    - *As an aside, Becqurel was studying the phosphorescence of a uranium compound. On a cloudy day, he gave up and put his uranium and photographic plate in his desk drawer and went home. Later he found that with no external light source, the photographic plate had an image nonetheless.*
- In 1899 Rutherford was studying the penetrating power of radiation from uranium and he found some was stopped after a thin piece of material and some took much more material to do the stopping
    - Naturally, he ranked them: α, β
    - The next year, Villard found a more penetrating type: γ
- The “rays” were further differentiated by mass spectrometry and α’s were identified as helium nuclei a few years later
(though it took until 1914 to realize γ-rays were electromagnetic)



Serendipity:
uranium on a photographic plate

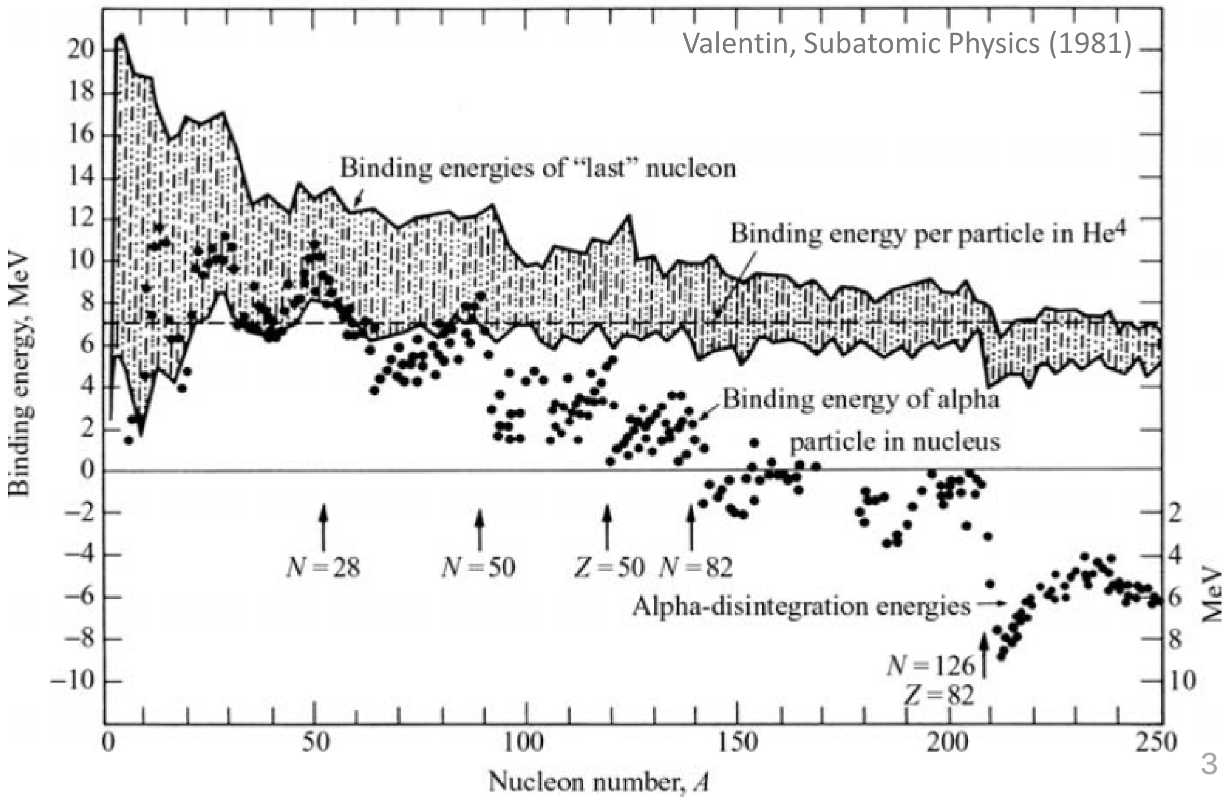


Energetics of α decay

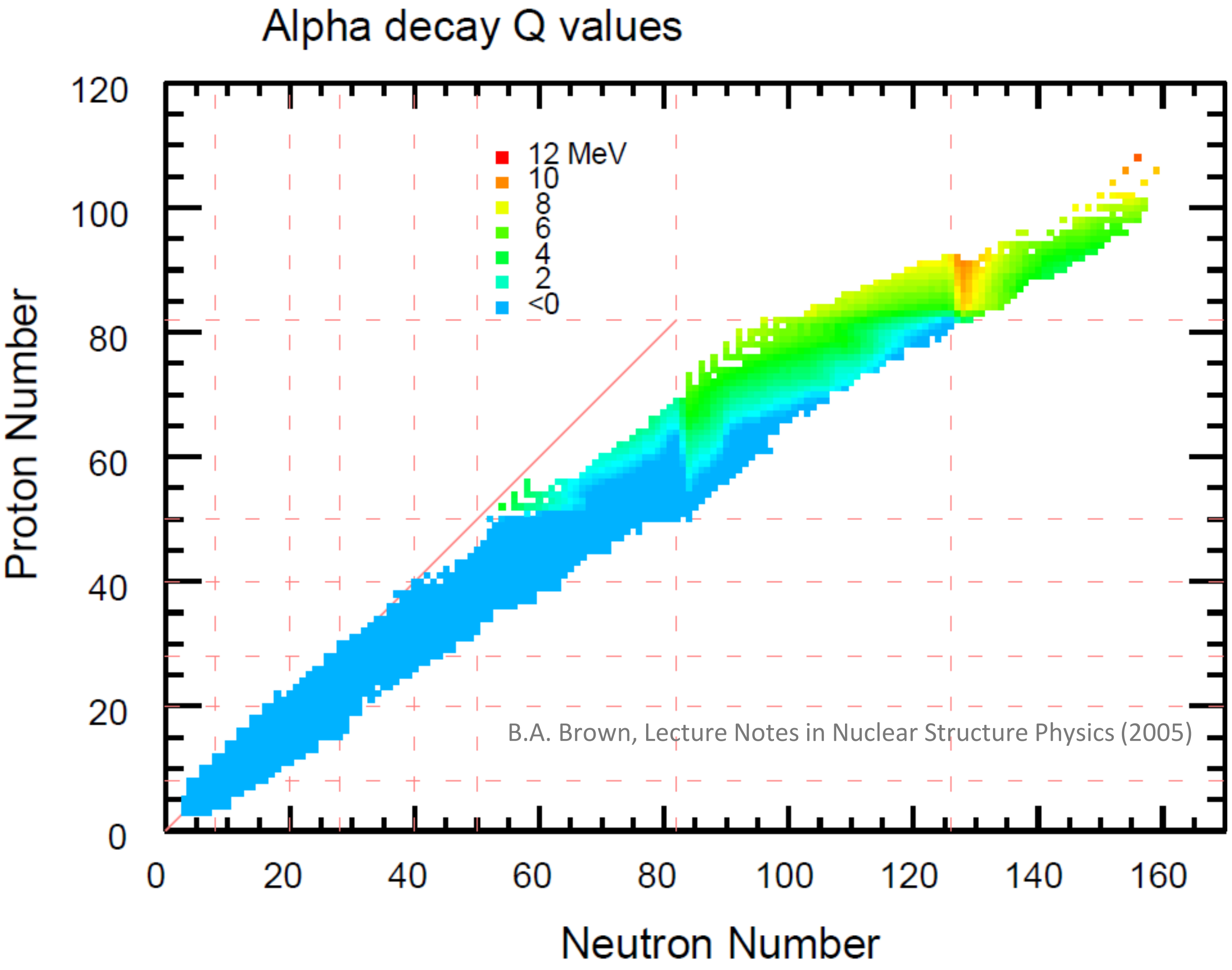
- Radioactive decay is a spontaneous process, caused by a system moving to a lower energy state
- As such, energy is released in the decay; i.e. it is exothermic
- The energy release is described by the Q-value: $Q = \sum_{reactants} ME(Z,A) - \sum_{products} ME(Z,A)$
- For α-decay: $Q_\alpha = ME_{parent} - ME_{daughter} - ME_{^4\mathrm{He}}$

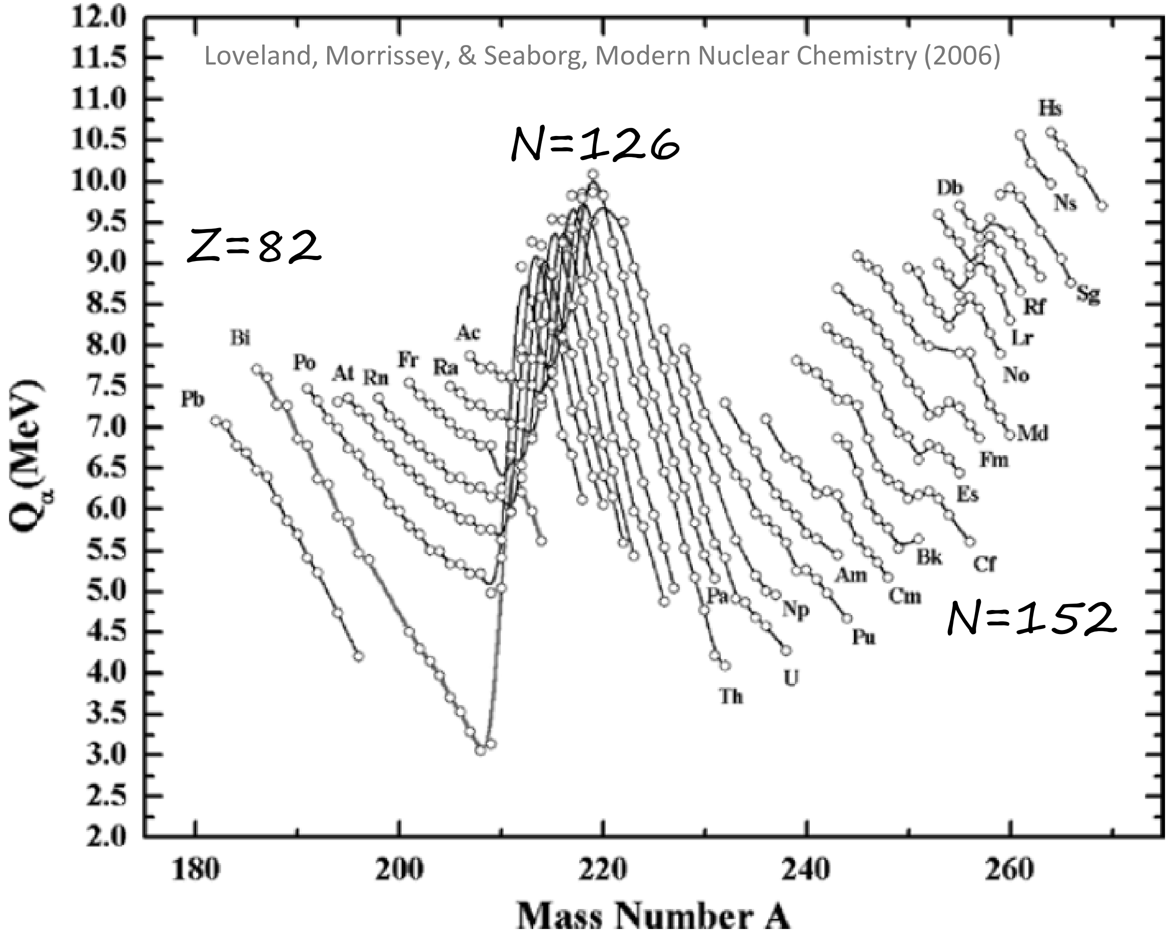
The change in atomic binding energy (helium leaves ionized and the daughter initially has 2 extra electrons) can be ignored

- Why would Q_α be positive?
  - In terms of the SEMF, losing the 2 protons lowers the Coulomb energy, doesn't impact asymmetry and pairing, and barely changes the surface and volume energies (per A)
  - While the last nucleon is tightly bound, so is a nucleon in a ^{4}He cluster rattling around in the nucleus.
  - The ^{4}He itself is not nearly as bound in the nucleus, making α-emission energetically favorable above A~150



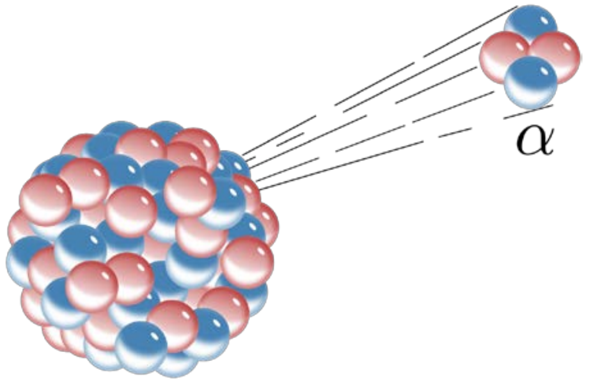
Q_α provides another signature for magic numbers





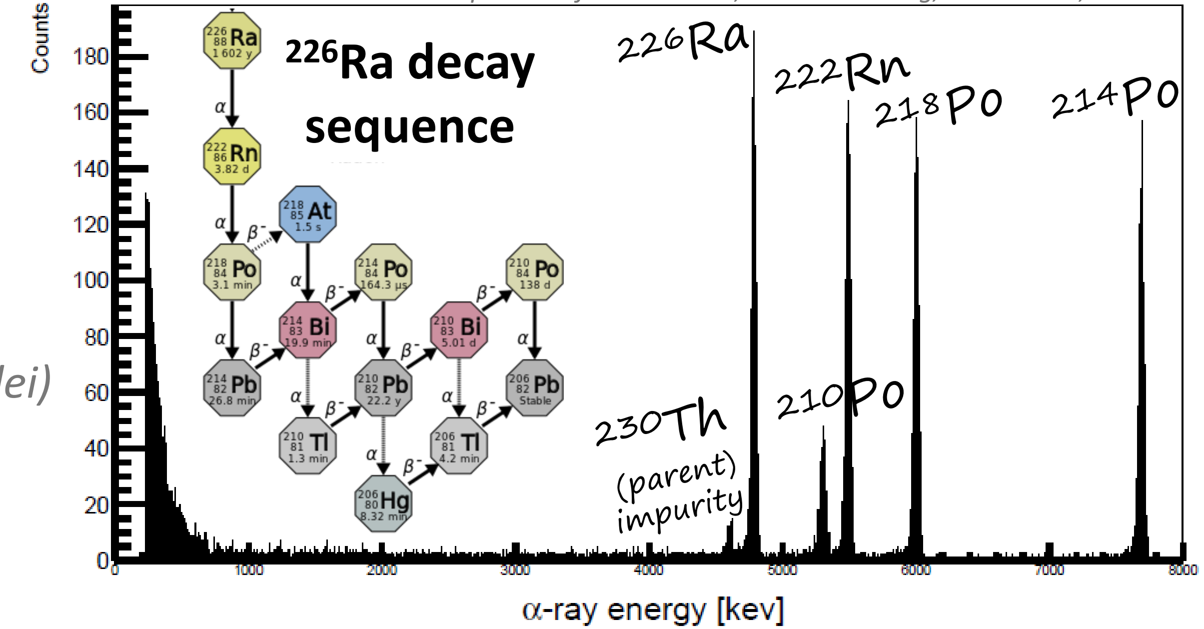
Wait a second, Zach! How is this different than measuring individual masses and taking mass differences, like the neutron separation energy, to find structure signatures?
We can get Q_α from the emitted α energy.

α energy from Q_α

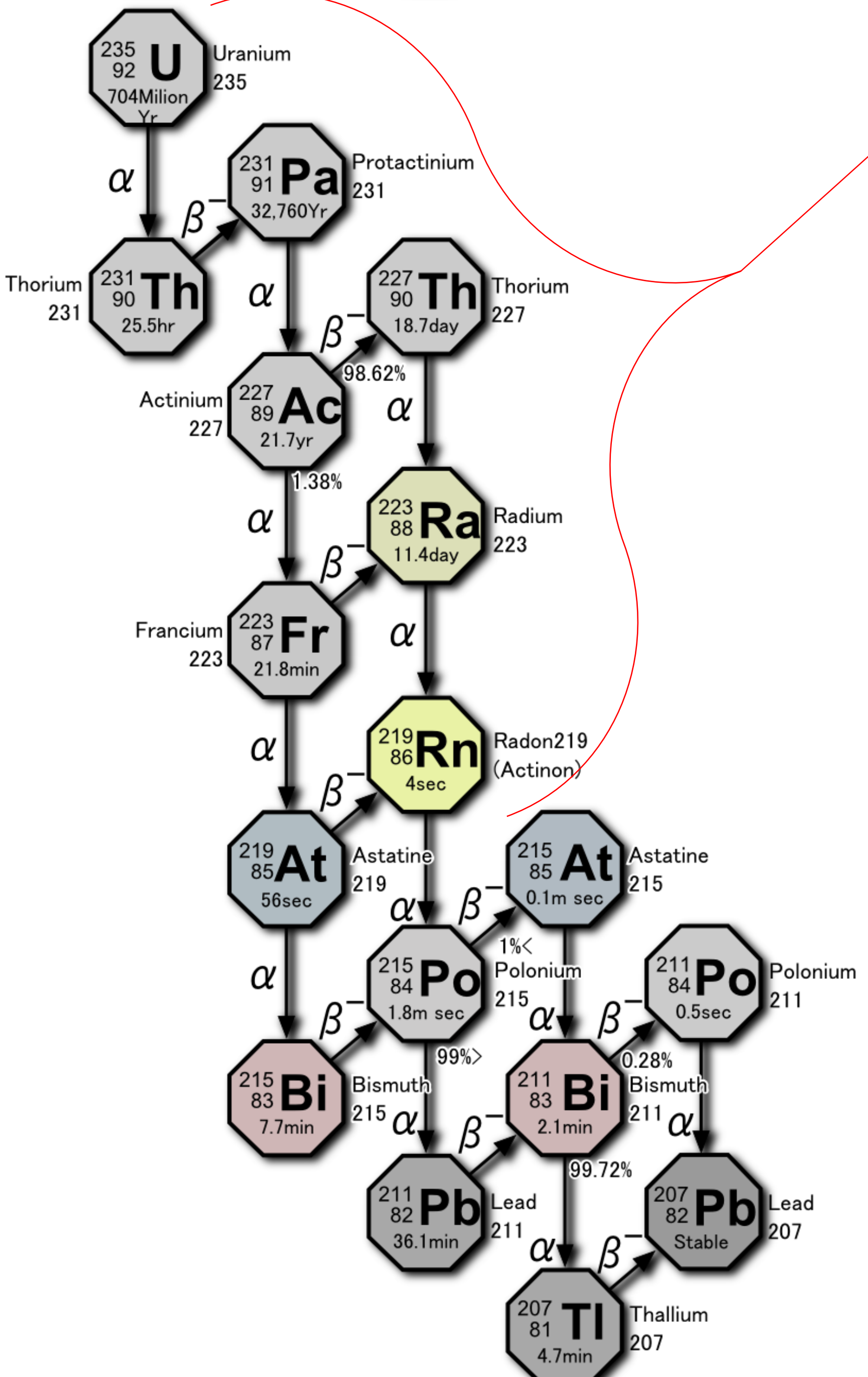


- When an α is emitted, it will share some energy with the heavy recoil, so KE_α isn't quite equal to Q_α
- We just need to employ conservation of momentum and energy
  - $\vec{p}_{parent} = \vec{p}_{daughter} + \vec{p}_\alpha$
  - Conveniently $p_{parent} = 0$, so the daughter and α will move in opposite directions and $p_{daughter} = -p_\alpha$
  - $\frac{p^2_{parent}}{2A_{parent}} + Q_\alpha = \frac{p^2_{daughter}}{2A_{daughter}} + \frac{p^2_\alpha}{2A_\alpha} = \frac{p^2_\alpha}{2A_{daughter}} + \frac{p^2_\alpha}{2A_\alpha} = \frac{A_\alpha}{A_{daughter}} KE_\alpha + KE_\alpha$
  - $Q_\alpha = \frac{A_\alpha + A_{daughter}}{A_{daughter}} KE_\alpha = \frac{A_{parent}}{A_{daughter}} KE_\alpha$
  - $KE_\alpha = \frac{A_{daughter}}{A_{parent}} Q_\alpha$
  - So it's a pretty small effect, but one you should include anyway
    (Not a small effect for β-delayed particle emission in lighter nuclei)
- Conveniently, α sources typically have several E_α from the decay chain, and so they provide several energy calibration points

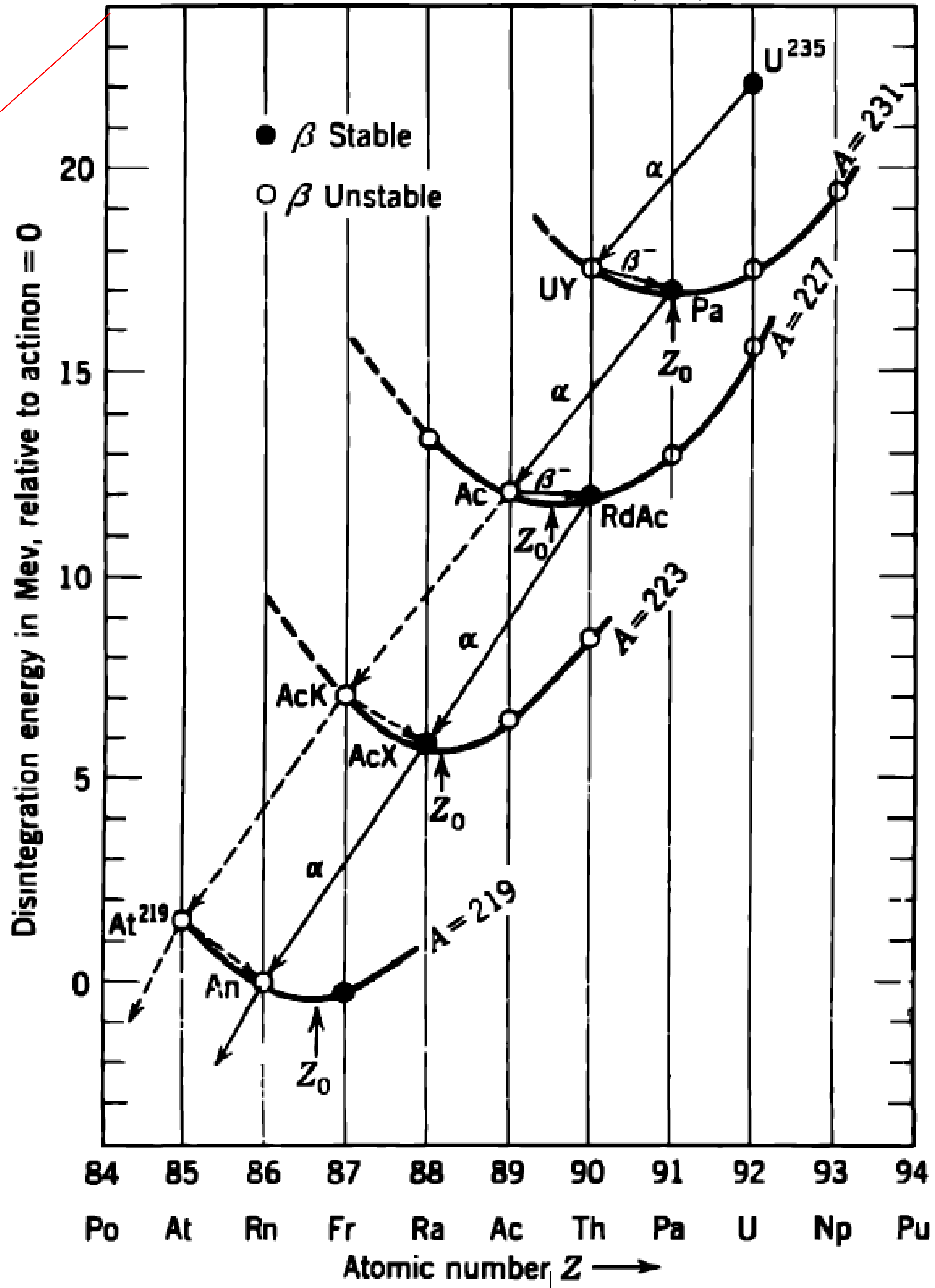
Spectrum from M.Mroz, K. Brandenburg, A. Mamum, & A. Pun



Aside: Decay sequences can be mapped by considering the mass excess of nuclides involved, E.g. ^{235}U



R. Evans, The Atomic Nucleus (1955)



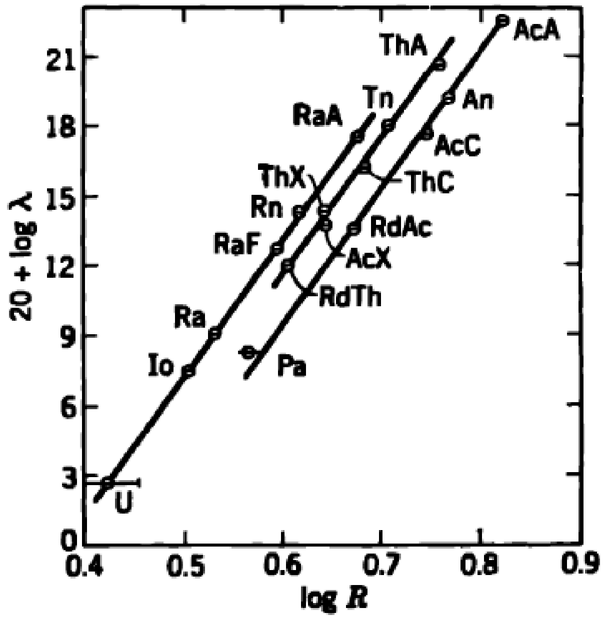
Geiger-Nuttall relation

- In an early effort to characterize α-decay, Geiger & Nuttall
  (H. Geiger & J.M. Nuttall, Philisoph. Mag. (1911, 1912))
  compared the range of α particles in a material vs $t_{½}$ of the α-source and found a linear relationship in log-log space
- In modern terms, using Q_α instead of range,
  we get the Geiger-Nuttall relation: $\log_{10}(t_{½}) = a + bZQ_\alpha^{-½}$
  (Range is the integral of stopping power ($\frac{dE}{dx} \equiv S(E) \propto \frac{1}{E}$):
  $\mathrm{R} = \int \frac{1}{S(E)} \propto E^2$)
- Obviously the α energy somehow impacts $t_{½}$

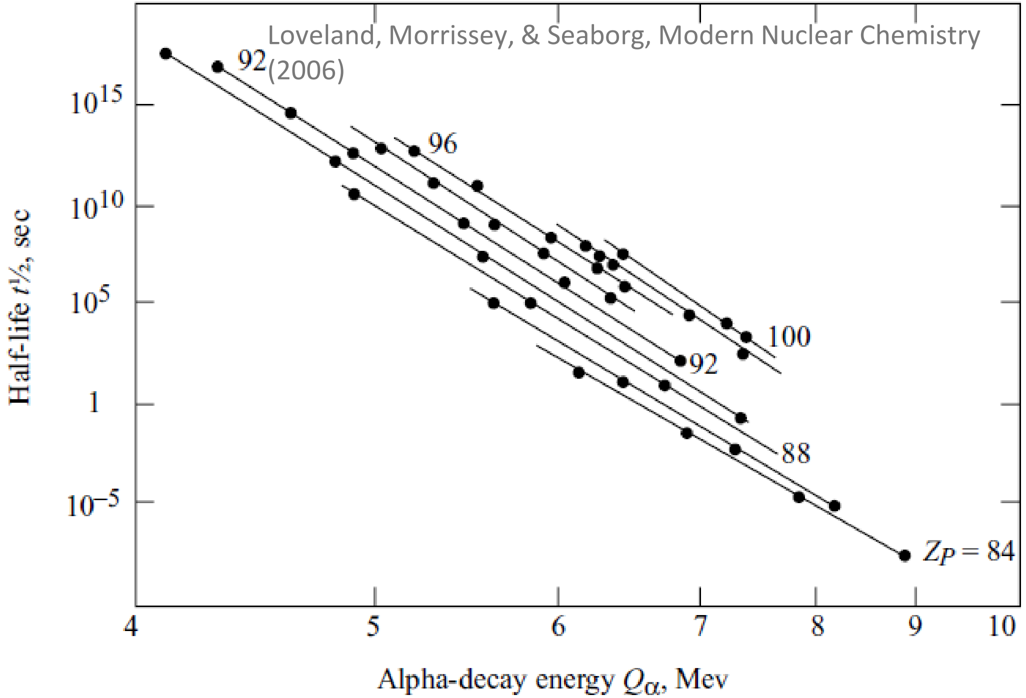
  …incredibly strongly
- For $\sim \times 2$ increase in Q_α,
  nearly 20 orders of magnitude decrease in $t_{½}$!!

What does this imply about useful α sources?
There's a relatively limited range of E_α available.
- *Large E_α sources aren't active for long enough,*
- *while low E_α sources require huge amounts to have an appreciable activity (A=λN).*

Geiger & Nuttall, Philisoph. Mag. (1912)



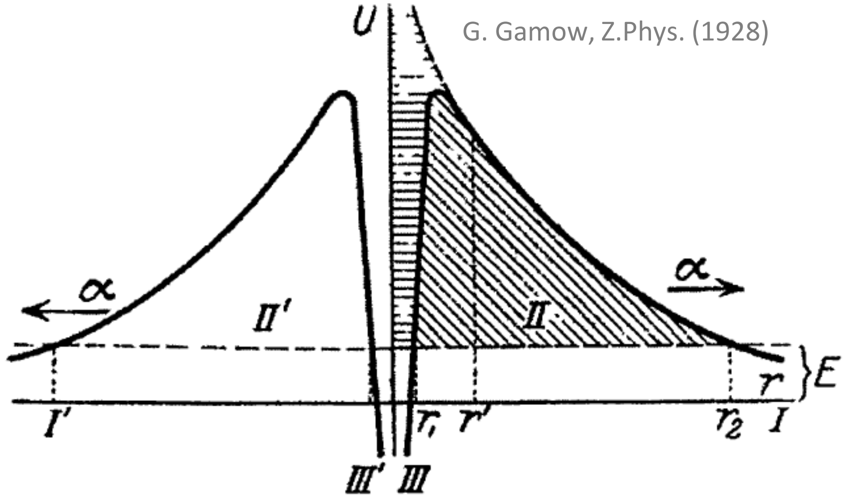
Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)



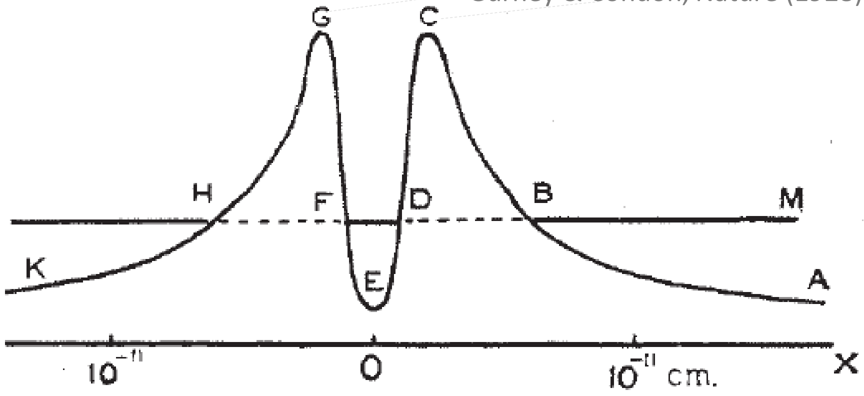
α decay from a quantum mechanical perspective

- Compared to the Coulomb barrier, E_α is pretty puny:
  - E.g. ^{226}Ra: $V_c = \frac{Z_\alpha Z_{Ra226}}{R}\frac{e^2}{\hbar c}\hbar c = \frac{2\cdot 88}{1.2\left(4^{1/3}+226^{1/3}\right)}\frac{197MeV fm}{137}$

    $\approx 27.5MeV$
  - Compare this to $E_\alpha \approx 5MeV$
- So, classically an α emission couldn't happen
- Gamow (and simultaneously Gurney & Condon) realized this problem could be neatly described by quantum mechanical tunneling
- The basic picture is that an α particle is rattling around in the nucleus, doing laps with a velocity $v_\alpha = \sqrt{2E_\alpha/m_\alpha}$
- Each time the α hits the Coulomb barrier, formed by the nuclear core [which is the daughter of the α decay], it has some probability of tunneling through
- Therefore, the challenge is to calculate this transmission probability



G. Gamow, Z.Phys. (1928)

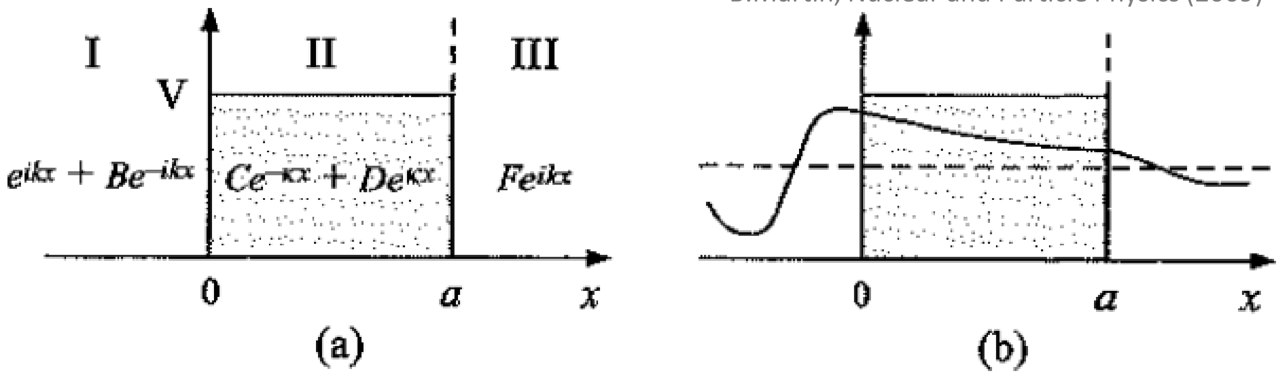


Gurney & Condon, Nature (1928)

Interestingly, Gamow figured this out on a summer research trip he took out of frustration with his thesis research. This is the same trip where he conceived the liquid drop model and the theory of nuclear fusion. (R. Stuewer, Plenary paper for 1997 Gamow Symposium)

Tunneling through a square 1D barrier

B.Martin, Nuclear and Particle Physics (2009)



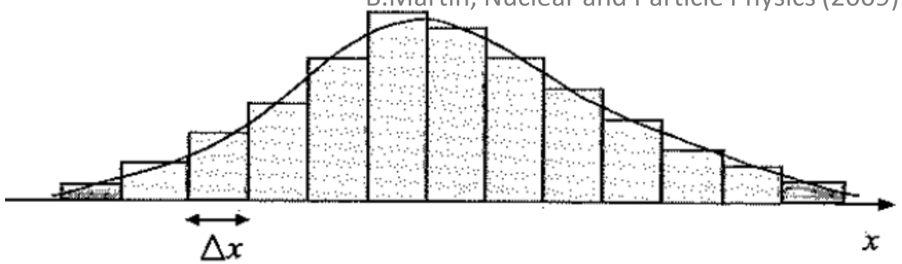
- Consider the simplest tunneling case, a plane wave penetrating a square barrier
- A particle with mass m and energy E hitting a barrier of with a and height $V > E$ can be described by three regions:
  - I: Incidence and reflection, II: decaying in barrier and decaying reflection, III: Outgoing
  - I: $\psi_I(x) = Ae^{ikx} + Be^{-ikx}$; II: $\psi_{II}(x) = Ce^{-\kappa x} + De^{\kappa x}$; III: $\psi_{III}(x) = Fe^{ikx}$
  - Where the wavenumber k is from $\hbar^2 k^2 = 2mE$ and the decay constant κ is from $\hbar^2 \kappa^2 = 2m(V - E)$
- The probability to make it through the barrier is the Transmission Coefficient $T \equiv |F/A|^2$
- Some time back in a quantum mechanics class, you found:

$$T = \left|\frac{2k\kappa e^{-ika}}{2k\kappa \cosh(\kappa a) - i(k^2 - \kappa^2)\sinh(\kappa a)}\right|^2$$

- Luckily, for large κa, $\sinh(\kappa a) \approx \cosh(\kappa a) \approx \frac{1}{2}e^{\kappa a}$ … so, $T \approx \left(\frac{4k\kappa}{k^2+\kappa^2}\right)^2 e^{-2\kappa a}$
- The exponent dominates, and so usually one writes $T \approx e^{-2\kappa a} \equiv e^{-2G}$

Tunneling through an arbitrary barrier

B.Martin, Nuclear and Particle Physics (2009)



- The result from the 1D barrier can be generalized by breaking an arbitrary barrier into a series of 1D barriers, which is a trick that goes by the name of the WKB approximation (Wentzel, Kramers, Brillouin)
- Replacing $2\kappa a$ with $2\sum\kappa(x)\Delta x$ and using an integral instead of a Riemann sum,

$$2\kappa a \rightarrow 2\int\sqrt{\left(\frac{2m}{\hbar^2}[V(x)-E]\right)}\,dx \;\;\ldots\;\; \text{generalizing to 3D: } 2G = \frac{2}{\hbar}\int\sqrt{(2m[V_c(r)-E])}\,dr$$

- Some details:
  - m actually needs to be a reduced mass, since the energies are in the center of mass system: $\mu = \frac{m_\alpha m_{daughter}}{m_\alpha + m_{daughter}}$
  - The limits for integration will be from the border of the potential well $R = r_0\left(A_\alpha^{1/3} + A_{daughter}^{1/3}\right)$ to the classical distance of closest approach b, where $E = Q_\alpha = \frac{Z_\alpha Z_{daughter}}{b}\frac{e^2}{\hbar c}\hbar c$ (i.e. $b = \frac{Z_\alpha Z_{daughter} e^2}{Q_\alpha}$)
  - $V_c(r) = \frac{Z_\alpha Z_{daughter}}{r}\frac{e^2}{\hbar c}\hbar c$

…though you could add a centrifugal barrier if Δℓ≠0 for the decay and the barrier shape could be from an optical potential determined by scattering

- So, $2G = \frac{2}{\hbar}\sqrt{2\mu Q_\alpha}\int_R^b\left(\frac{b}{r}\right)^{1/2}dr = 2\frac{e^2}{\hbar c}Z_\alpha Z_{daughter}\sqrt{\frac{2\mu c^2}{Q_\alpha}}\left[\cos^{-1}\left(\sqrt{\frac{R}{b}}\right) - \sqrt{\frac{R}{b}\left(1-\frac{R}{b}\right)}\right]$

Why show this gory detail? Now you can calculate the transmission coefficient for an arbitrary case.

Tunneling through a thick barrier

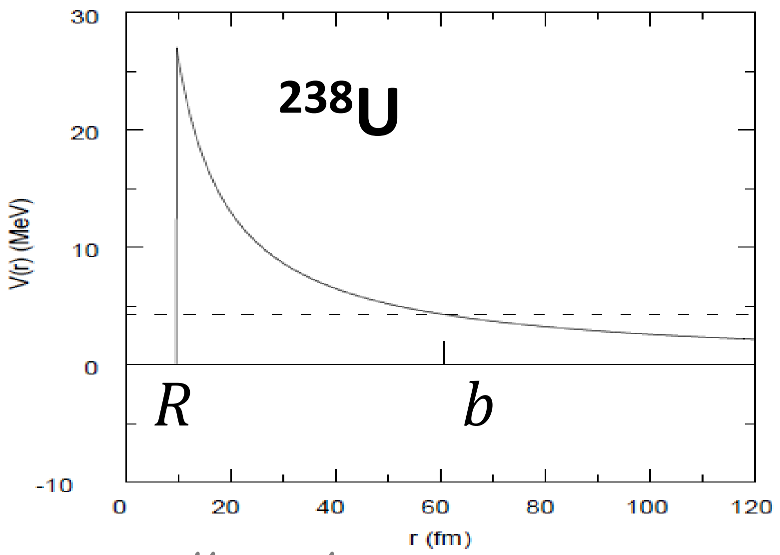
- $T = e^{-2G}$, where $2G = 2\frac{e^2}{\hbar c} Z_\alpha Z_{daughter} \sqrt{\frac{2\mu c^2}{Q_\alpha}} \left[cos^{-1}\left(\sqrt{\frac{R}{b}}\right) - \sqrt{\frac{R}{b}\left(1 - \frac{R}{b}\right)} \right]$ is pretty ugly
- Conveniently, for most cases $b \gg R$, so the stuff in [ ] $\rightarrow \frac{\pi}{2}\left(1 - \frac{4}{\pi}\sqrt{\frac{R}{b}}\right)$

  so the Gamow factor $2G \approx 2\pi \frac{e^2}{\hbar c} Z_\alpha Z_{daughter} \sqrt{\frac{\mu c^2}{2Q_\alpha}} \boxed{\left(1 - \frac{4}{\pi}\sqrt{\frac{r_0\left(A_\alpha^{1/3} + A_{daughter}^{1/3}\right)}{\frac{Z_\alpha Z_{daughter} e^2}{Q_\alpha}}}\right)}$

this correction looks small and is often neglected in text books, but a factor of ~0.5 in an exponent can be a big deal

- Since the decay half-life will be inversely proportional to the tunneling probability, $t_{½} \propto e^{2G} \propto e^{Z/\sqrt{Q_\alpha}}$
- You may notice this is what Geiger & Nuttall told us all along $\log_{10}(t_{½}) = a + bZQ_\alpha^{-½}$

B.A. Brown, Lecture Notes in Nuclear Structure Physics (2005)



Aside:
Gamow realized this formalism would work just as well for a charged-particle tunneling in (i.e. for nuclear fusion). For nuclear fusion, 2G is often written instead as 2πη, where η is the Sommerfeld parameter.
In nuclear astrophysics, T=P=exp(-2πη) is convolved with the Maxwell-Boltzmann distribution to get the "Gamow window", the energy-window of astrophysical relevance.

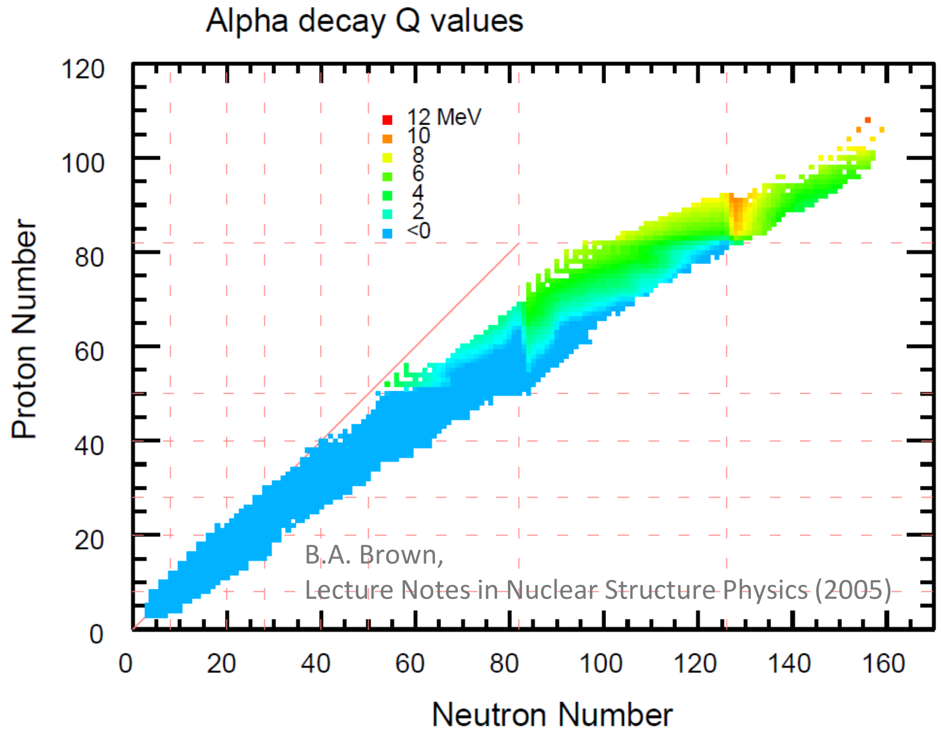
Qualitative implications

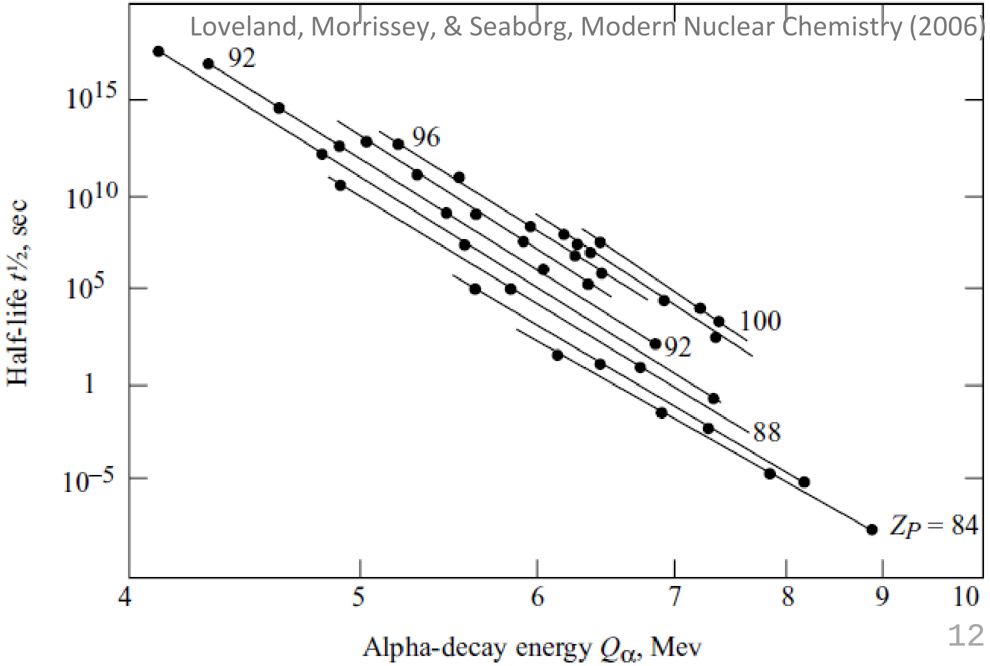
- Q_α generally increases with increasing Z because there is increased penalty in the liquid drop model binding for increased E_{Coul}
- Increasing Z means an increased Coulomb barrier height and therefore more of a barrier to tunnel through

  …for the same Q_α, increasing Z increases $t_{½}$
- Increasing Q_α means the α energy required to tunnel through the barrier is larger, therefore the α velocity is larger, therefore the α bombards the barrier more frequently

  …for the same Z, increasing Q_α decreases $t_{½}$

  i.e. decays to excited states should have longer half-lives, provided parity needn't be violated

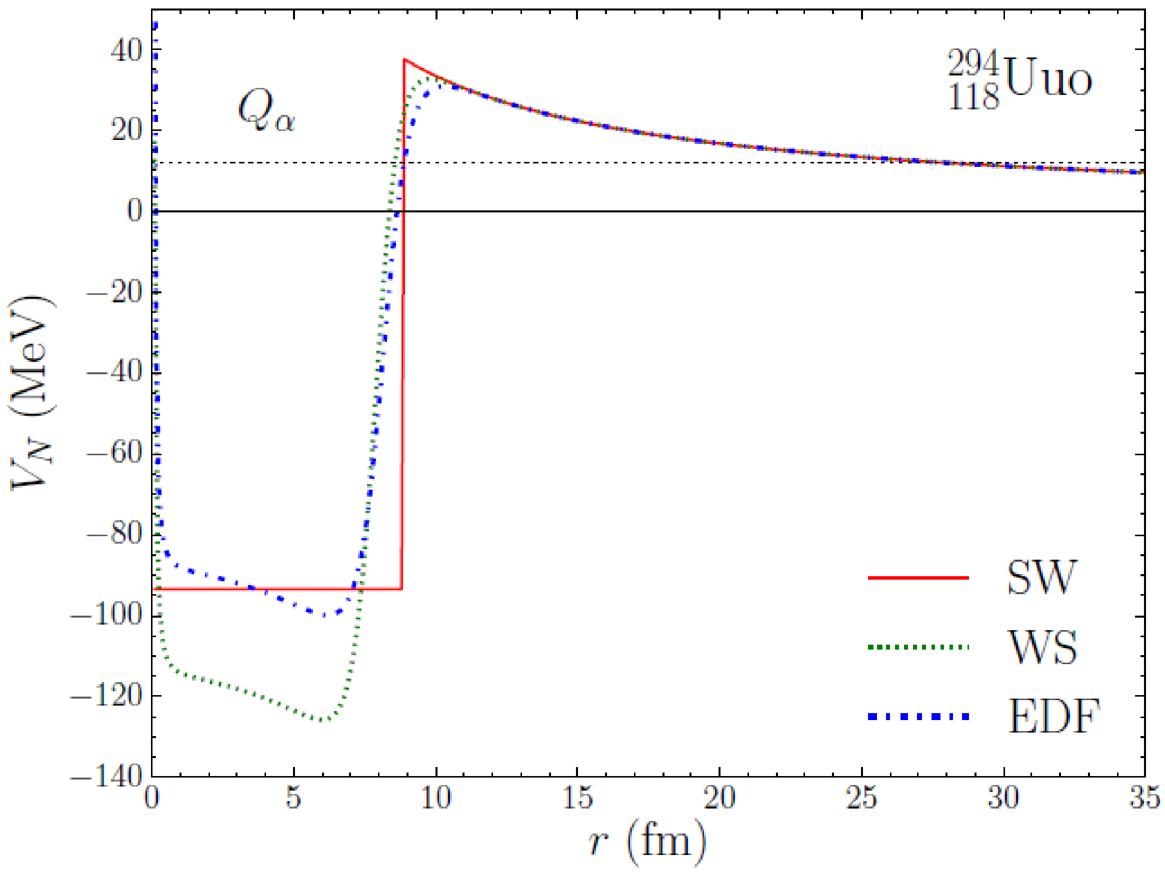




What about using a Woods-Saxon?

- Before we go too far, we may want to pause and check how big of an issue it is that we're using a square well potential + the Coulomb potential and not the Woods-Saxon we know and love
- It turns out, using the square well doesn't change the answer all that much

E. Shin et al. Phys. Rev. C (2016)

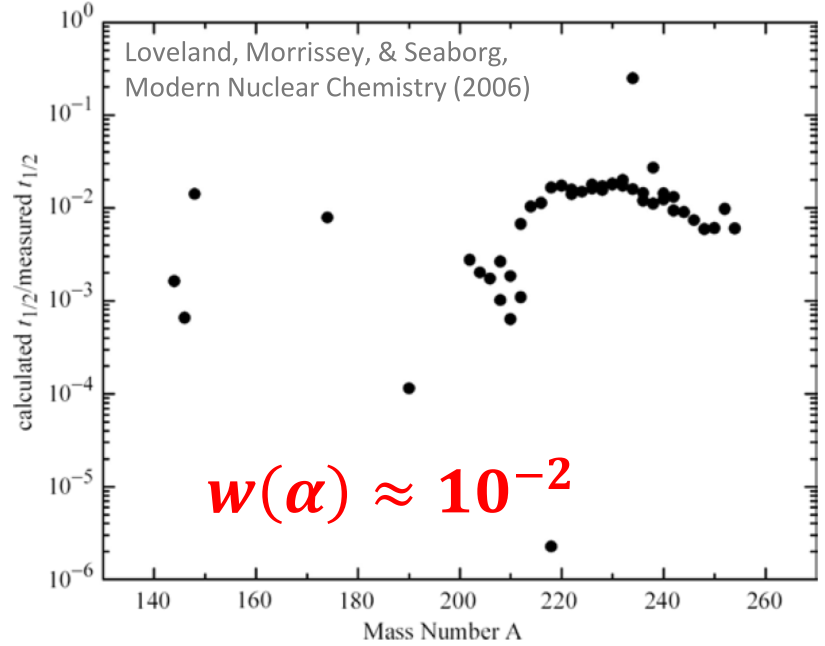


(Z, A)	$Q_\alpha^{\text{Expt.}}$ (MeV)	$T_{1/2}^{\text{Expt.}}$	$T_{1/2}^{\text{SW}}$	$T_{1/2}^{\text{WS}}$	$T_{1/2}^{\text{EDF}}$
(118, 294)	11.81 ± 0.06	$0.89^{+1.07}_{-0.31}$ ms	$1.48^{+0.52}_{-0.38}$ ms	$1.40^{+0.50}_{-0.37}$ ms	$0.40^{+0.15}_{-0.11}$ ms
(116, 293)	10.67 ± 0.06	53^{+62}_{-19} ms	161^{+68}_{-47} ms	243^{+104}_{-72} ms	52^{+23}_{-16} ms
(116, 292)	10.80 ± 0.07	18^{+16}_{-6} ms	78^{+38}_{-25} ms	80^{+40}_{-27} ms	25^{+13}_{-8} ms
(116, 291)	10.89 ± 0.07	$6.3^{+11.6}_{-2.5}$ ms	47^{+23}_{-15} ms	69^{+34}_{-22} ms	16^{+8}_{-5} ms
(116, 290)	11.00 ± 0.08	$7.1^{+3.2}_{-1.7}$ ms	$26.1^{+14.6}_{-9.3}$ ms	$26.2^{+15.0}_{-9.4}$ ms	$8.9^{+5.0}_{-3.3}$ ms
(115, 288)	10.61 ± 0.06	87^{+105}_{-30} ms	115^{+48}_{-34} ms	99^{+42}_{-29} ms	43^{+19}_{-13} ms
(115, 287)	10.74 ± 0.09	32^{+155}_{-14} ms	55^{+37}_{-22} ms	38^{+26}_{-15} ms	21^{+15}_{-8} ms
(114, 289)	9.96 ± 0.06	$2.7^{+1.4}_{-0.7}$ s	$2.7^{+1.3}_{-0.9}$ s	$4.3^{+2.1}_{-1.4}$ s	$1.1^{+0.5}_{-0.3}$ s
(114, 288)	10.09 ± 0.07	$0.8^{+0.32}_{-0.18}$ s	$1.2^{+0.67}_{-0.43}$ s	$1.32^{+0.74}_{-0.47}$ s	$0.48^{+0.27}_{-0.17}$ s
(114, 287)	10.16 ± 0.06	$0.48^{+0.16}_{-0.09}$ s	$0.79^{+0.36}_{-0.25}$ s	$1.21^{+0.56}_{-0.38}$ s	$0.32^{+0.15}_{-0.10}$ s
(114, 286)	10.33 ± 0.06	$0.13^{+0.04}_{-0.02}$ s	$0.29^{+0.13}_{-0.09}$ s	$0.30^{+0.13}_{-0.09}$ s	$0.12^{+0.05}_{-0.04}$ s
(113, 284)	10.15 ± 0.06	$0.48^{+0.58}_{-0.17}$ s	$0.40^{+0.18}_{-0.12}$ s	$0.61^{+0.28}_{-0.19}$ s	$0.28^{+0.13}_{-0.09}$ s
(113, 283)	10.26 ± 0.09	100^{+490}_{-45} ms	209^{+152}_{-87} ms	150^{+111}_{-63} ms	91^{+69}_{-39} ms
(113, 282)	10.83 ± 0.08	73^{+134}_{-29} ms	8^{+5}_{-3} ms	37^{+21}_{-13} ms	75^{+44}_{-28} ms
(112, 285)	9.29 ± 0.06	34^{+17}_{-9} s	49^{+26}_{-17} s	82^{+44}_{-28} s	23^{+12}_{-8} s
(112, 283)	9.67 ± 0.06	$3.8^{+1.2}_{-0.7}$ s	$3.8^{+1.9}_{-1.2}$ s	$6.1^{+3.0}_{-2.0}$ s	$1.8^{+0.9}_{-0.6}$ s
(111, 280)	9.87 ± 0.06	$3.6^{+4.3}_{-1.3}$ s	$0.49^{+0.22}_{-0.15}$ s	$2.6^{+1.2}_{-0.8}$ s	$6.0^{+2.9}_{-1.9}$ s
(111, 279)	10.52 ± 0.16	170^{+810}_{-80} ms	10^{+16}_{-6} ms	272^{+424}_{-163} ms	110^{+177}_{-67} ms
(111, 278)	10.89 ± 0.08	$4.2^{+7.5}_{-1.7}$ ms	$1.4^{+0.7}_{-0.5}$ ms	$6.3^{+3.5}_{-2.2}$ ms	$2.7^{+1.6}_{-1.0}$ ms
(110, 279)	9.84 ± 0.06	$0.20^{+0.05}_{-0.04}$ s	$0.27^{+0.12}_{-0.08}$ s	$0.42^{+0.19}_{-0.13}$ s	$0.13^{+0.06}_{-0.04}$ s
(109, 276)	9.85 ± 0.06	$0.72^{+0.97}_{-0.25}$ s	$0.12^{+0.05}_{-0.04}$ s	$0.63^{+0.29}_{-0.20}$ s	$0.29^{+0.14}_{-0.09}$ s
(109, 275)	10.48 ± 0.09	$9.7^{+46}_{-4.4}$ ms	$3.0^{+2.0}_{-1.2}$ ms	$12.3^{+8.3}_{-4.9}$ ms	$6.7^{+4.6}_{-2.7}$ ms
(109, 274)	9.95 ± 0.10	440^{+810}_{-170} ms	66^{+56}_{-30} ms	341^{+296}_{-157} ms	172^{+153}_{-80} ms
(108, 275)	9.44 ± 0.06	$0.19^{+0.22}_{-0.07}$ s	$0.71^{+0.35}_{-0.23}$ s	$1.15^{+0.56}_{-0.38}$ s	$0.39^{+0.20}_{-0.13}$ s
(107, 272)	9.15 ± 0.06	$9.8^{+11.7}_{-3.5}$ s	$2.3^{+1.2}_{-0.8}$ s	$13.3^{+6.9}_{-4.5}$ s	$7.0^{+3.7}_{-2.4}$ s
(107, 270)	9.11 ± 0.08	61^{+292}_{-28} s	$3.1^{+2.3}_{-1.3}$ s	124^{+94}_{-53} s	60^{+46}_{-26} s
(106, 271)	8.67 ± 0.08	$1.9^{+2.4}_{-0.6}$ min	$0.49^{+0.40}_{-0.22}$ min	$1.45^{+1.20}_{-0.65}$ min	$1.67^{+1.41}_{-0.76}$ min

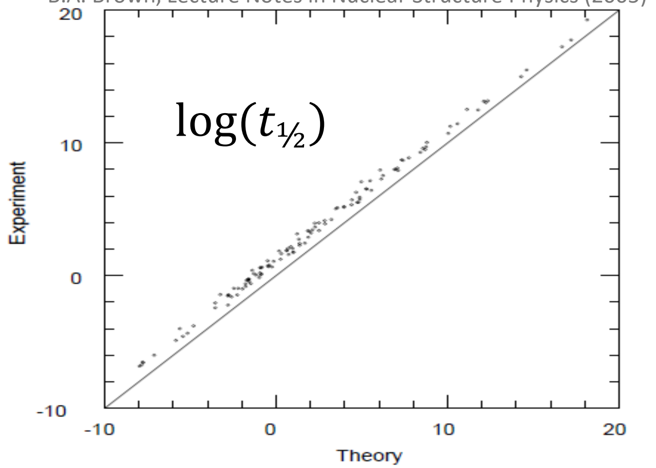
Estimate for α-decay λ

- The decay constant is a product of the frequency an α will bombard the barrier f, the tunneling probability T, and the probability of forming an α within the nucleus $w(\alpha)$
  - $\lambda = fTw(\alpha)$
- We discussed T already: $T = \exp\left(-2\pi\frac{e^2}{\hbar c}Z_\alpha Z_{daughter}\sqrt{\frac{\mu c^2}{2Q_\alpha}}\left(1-\frac{4}{\pi}\sqrt{\frac{r_0\left(A_\alpha^{1/3}+A_{daughter}^{1/3}\right)}{\frac{Z_\alpha Z_{daughter}e^2}{Q_\alpha}}}\right)\right)$
- f is just one over the time it takes for the α to travel across the nucleus $f = \frac{v_\alpha}{2R} = \sqrt{\frac{2(V_0+Q_\alpha)}{\mu}}\frac{1}{2r_0\left(A_\alpha^{1/3}+A_{daughter}^{1/3}\right)}$
- Setting $w(\alpha) = 1$ for the moment, consider ^{235}U
  - $V_0 \approx 30MeV$ (from optical model fits), $Q_\alpha \approx 4.68MeV$
  - $f = \frac{\sqrt{2(30MeV+4.68MeV)/(3.93amu*931.5MeV/c^2/amu)}}{2(9.3fm)} \approx 2.3\times10^{21}s^{-1}$
  - $T = \exp\left(-2\pi\frac{1}{137}(2)(90)\sqrt{\frac{(3.93)\left(931.5MeV/c^2\right)c^2}{2(4.68MeV)}}\left(1-\frac{4}{\pi}\sqrt{\frac{1.2fm\left(4^{\frac{1}{3}}+231^{\frac{1}{3}}\right)}{\frac{(2)(90)197MeVfm}{2(4.68MeV)137}}}\right)\right) \approx 1.08\times10^{-34}$
  - $\lambda \approx 2.4\times10^{-13}s^{-1}$ …i.e. $t_{½} \approx 9\times10^4 yr$

…actual: $7\times10^8 yr$



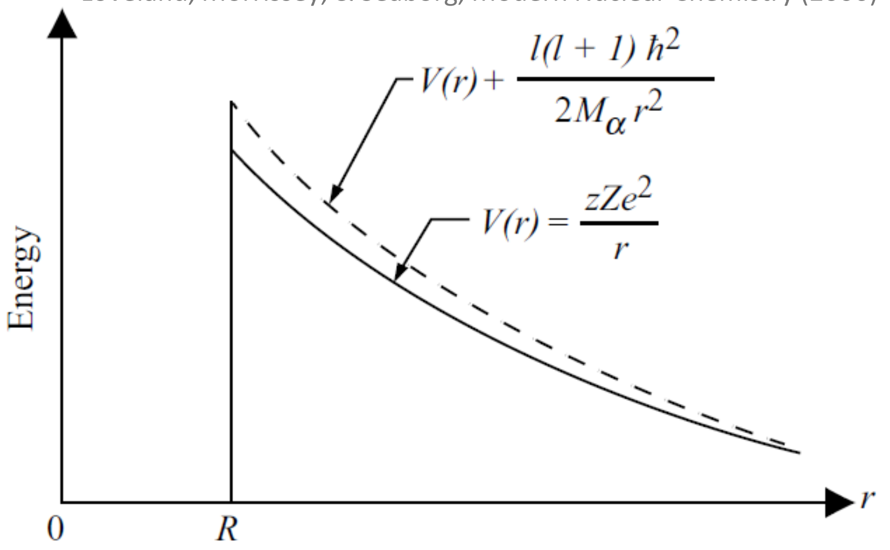
B.A. Brown, Lecture Notes in Nuclear Structure Physics (2005)



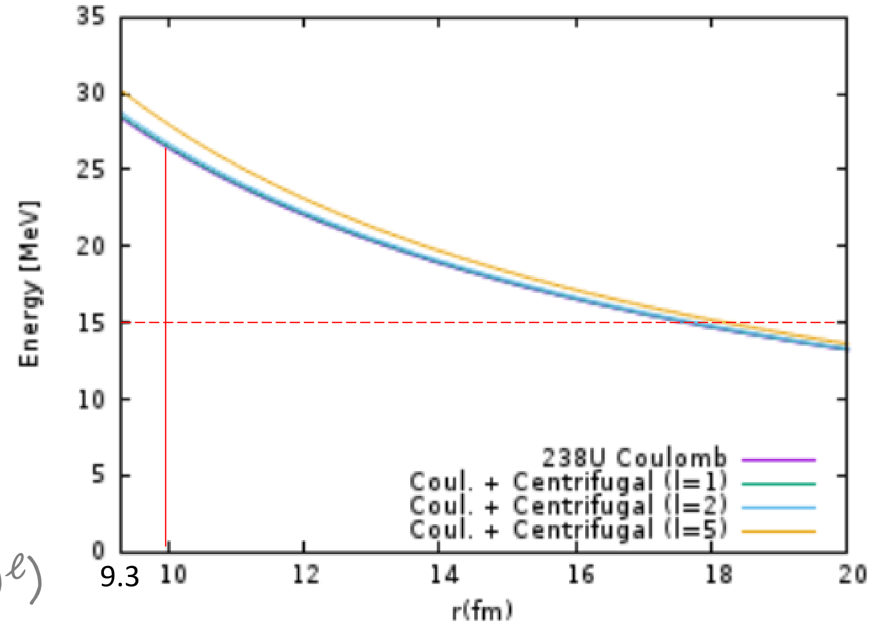
*results can be very sensitive to r_o

Including the centrifugal barrier

Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

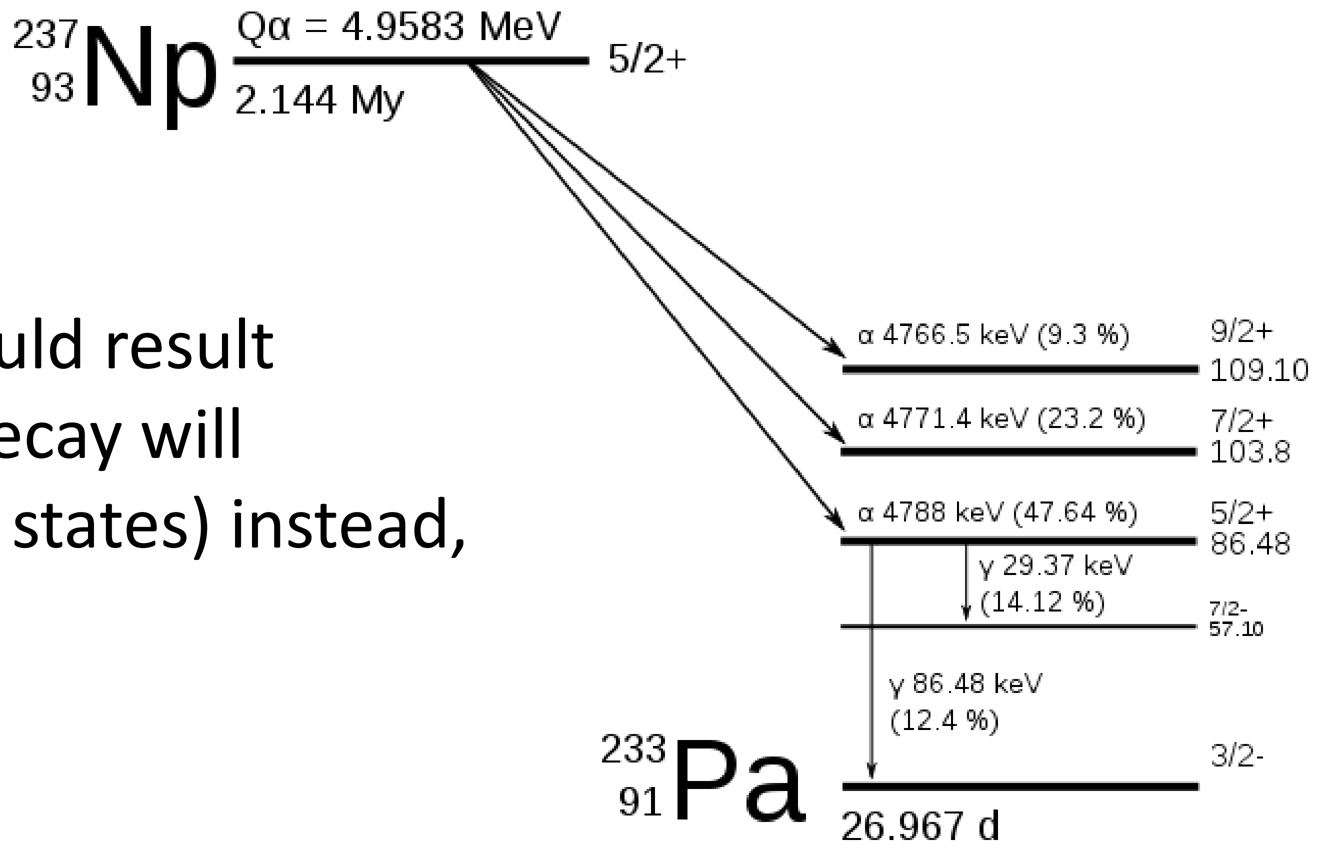


- α decay may involve transitioning from a nucleus with one J^π to another, meaning the α particle carries away angular momentum
- This means the α particle must tunnel through a centrifugal barrier: $V_l = \frac{l(l+1)\hbar^2}{2\mu R^2}$,
  where l is the angular momentum being carried away by the α
- This adds to the Coulomb barrier, creating a taller & thicker barrier for larger l
- However, practically speaking it is tiny
- It turns out (Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)), this correction is roughly:
  $\lambda_{l\neq 0} \approx \lambda_l e^{-\left(2.027(l(l+1))Z^{-1/2}A^{-1/6}\right)}$
- Compare this to the difference in barrier thickness corresponding to a fixed Q_α for a small change in R



However, parity conservation will affect the allowed $\Delta\ell$ ($\Delta\pi=(-1)^{\ell}$)

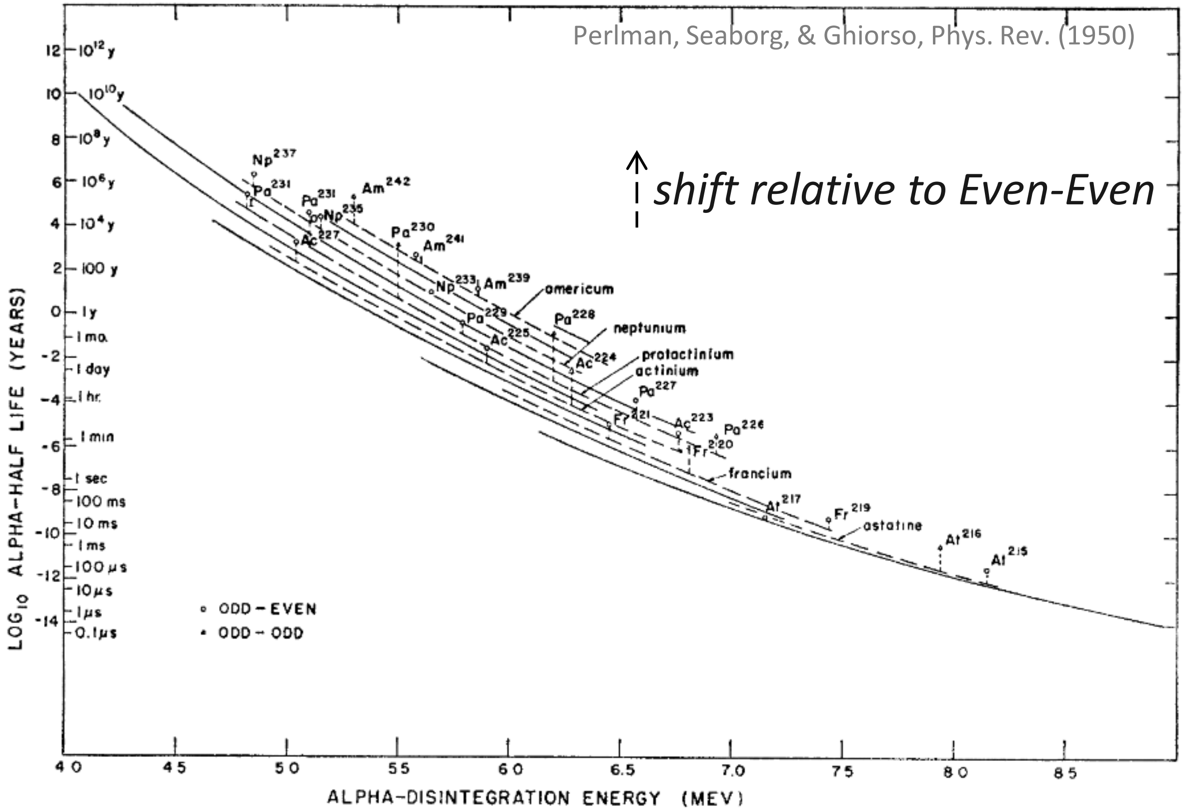
Impact of parity change on α decay

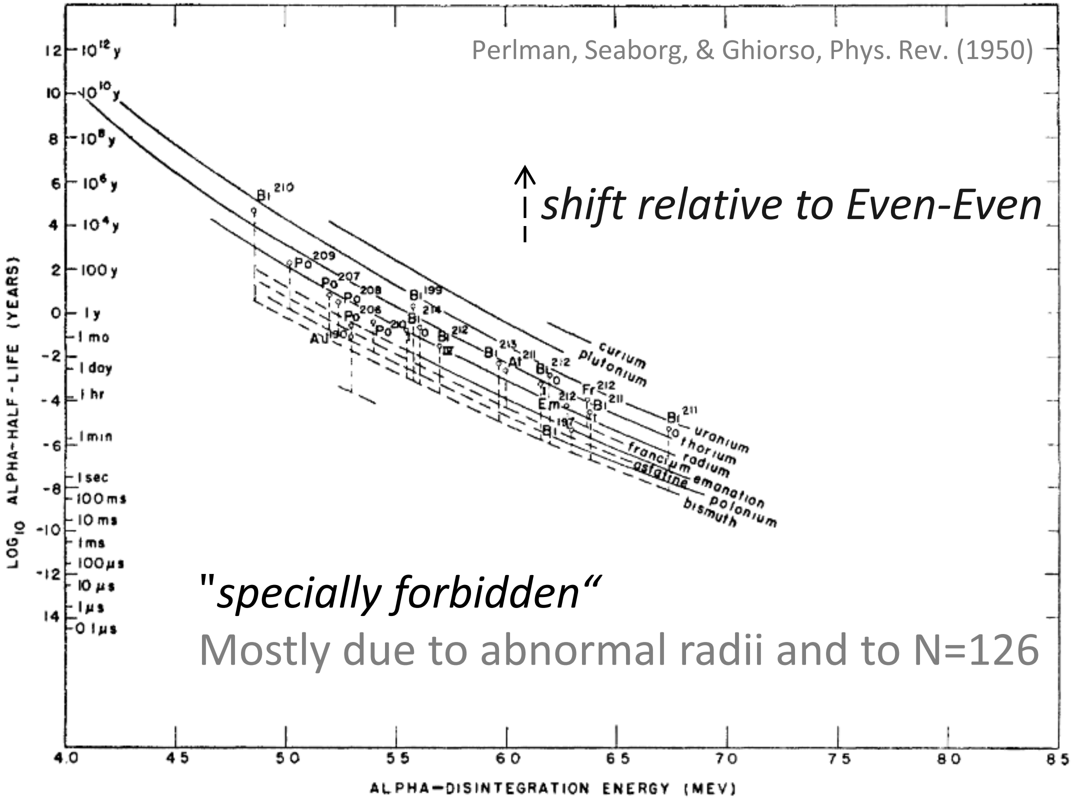


If an α decay to the ground-state would result in a parity change, often times the decay will proceed through an excited state (or states) instead, even though Q_α is lower

Hindrance factors

- To now, what we've done is valid for even-Z even-N nuclei
- For odd nuclei, the odd nucleon messes up α pre-formation, hindering the α decay by a factor of anywhere from a few to >1000, depending on the conditions
- E.g. If the odd nucleon of the parent & daughter is in the same orbit, λ is reduced by $\sim \times 4$
- If the parity must change, λ is reduced by $\sim \times 100$ ..if spin & parity change, $\sim \times > 1000$

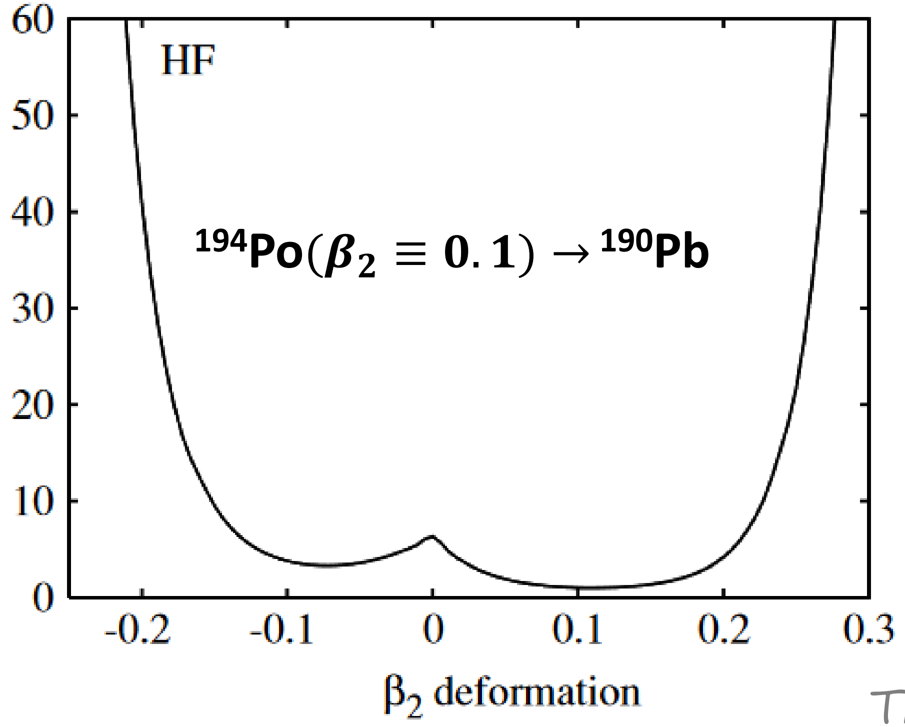




Hindrance factor from deformation

- Aha! You've forgotten life is a lie and nothing matters!
  Even-even nuclei can have hindrance factors too!
- The overlap in the wave functions for before and after the α-decay needs to be appreciable
- Since the wave function describes the probability for nucleons to be in a given location, there is obviously not going to be much overlap if the decay is from a non-deformed parent to a highly deformed daughter

D. Karlgren et al. Phys. Rev. C (2006)



Mother nucleus	$HF_{The.}$	$HF_{Exp.}$
^{198}Po	3.8	3.2(5)[a]
^{196}Po	3.0	2.6(2)[a]
^{194}Po	3.2	1.2(2)[a]
^{188}Po	1.3E-5	0.08(3)[a]
^{202}Rn	7.2	19(6)[b]
^{200}Rn	13	85(7)[c]

This can be turned on its head and one can infer the deformation of a nucleus based on the measured hindrance factor.

Improved empirical relations

- Now that you have an appreciation for how difficult it is to predict accurate λ_α, you can see the appeal of improved empirical relationships
- Fits exist using a modern form of the Geiger-Nuttall equation
- For example, for a fit to Q_α calculated with the 2003 Atomic Mass Evaluation, one finds:

$\Delta l = 0$	Z	N	$\Delta l \neq 0$
$\log_{10}[T] = -25.752 - 1.15055A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5913Z}{\sqrt{Q}}$	*Even*	*Even*	
$\log_{10}[T] = -34.156 - 0.87487A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6923Z}{\sqrt{Q}}$	*Even*	*Odd*	$\log_{10}[T] = -27.750 - 1.1138A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6378Z}{\sqrt{Q}} + \frac{1.7383 \times 10^{-6} ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.002457A[1-(-1)^l]$
$\log_{10}[T] = -32.623 - 1.0465A^{\frac{1}{6}}\sqrt{Z} + \frac{1.7495Z}{\sqrt{Q}}$	*Odd*	*Even*	$\log_{10}[T] = -27.915 - 1.1292A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6531Z}{\sqrt{Q}} + \frac{8.9785 \times 10^{-7} ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.002513A[1-(-1)^l]$
$\log_{10}[T] = -31.186 - 0.98047A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6744Z}{\sqrt{Q}}$	*Odd*	*Odd*	$\log_{10}[T] = -26.448 - 1.1023A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5967Z}{\sqrt{Q}} + \frac{1.6961 \times 10^{-6} ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.00101A[1-(-1)^l]$

G. Royer, Nuc. Phys. A. (2010)

Improved empirical relations

- Now that you have an appreciation for how difficult it is to predict accurate λ_α, you can see the appeal of improved empirical relationships
- Several other Geiger-Nuttall-esque parameterizations exist, E.g.
  - Denisov & Khudenko, Atom. Dat. Nuc. Dat. Tab. (2009)

$$\log_{10}[T_{1/2}(s)] = -a - \frac{bA^{1/6}\sqrt{Z}}{\mu} + \frac{cZ}{\sqrt{Q_\alpha}} + \frac{d\sqrt{\ell(\ell+1)}}{QA^{-1/6}} - e\left((-1)^{\ell} - 1\right)$$

gs-gs, all cases

$$\mu = \left(A/(A-4)\right)^{1/6}$$

clearly inspired by higher-order corrections to the tunneling probability we saw earlier

  - Hatsukawa, Nakahara, & Hoffman, Phys. Rev. C (1990)

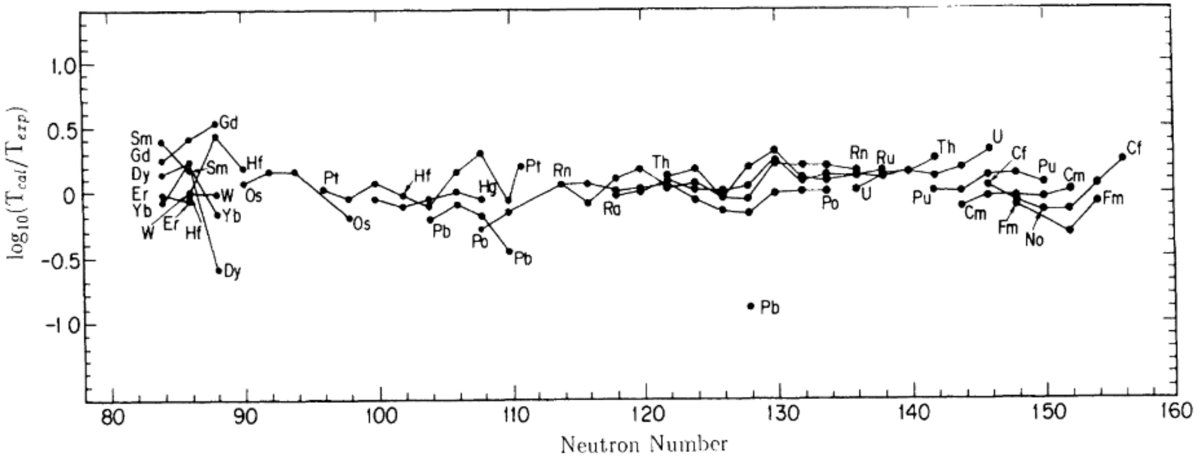
$$\log_{10}(t_{1/2}) = A(Z) \times \left[\frac{A_d}{A_p Q_\alpha}\right]^{1/2} \times \left[\arccos\sqrt{X} - \sqrt{X(1-X)}\right] - 20.446 + C(Z,N)$$

$$C(Z,N) = 0$$
for ordinary regions outside closed shells

$$C(Z,N) = [1.94 = -0.020(82-Z) - 0.070(126-N)]$$
for $78 \le Z \le 82,\ 100 \le N \le 126$

$$C(Z,N) = [1.42 - 0.105(Z-82) - 0.067(126-N)]$$
for $82 \le Z \le 90,\ 110 \le N \le 126.$

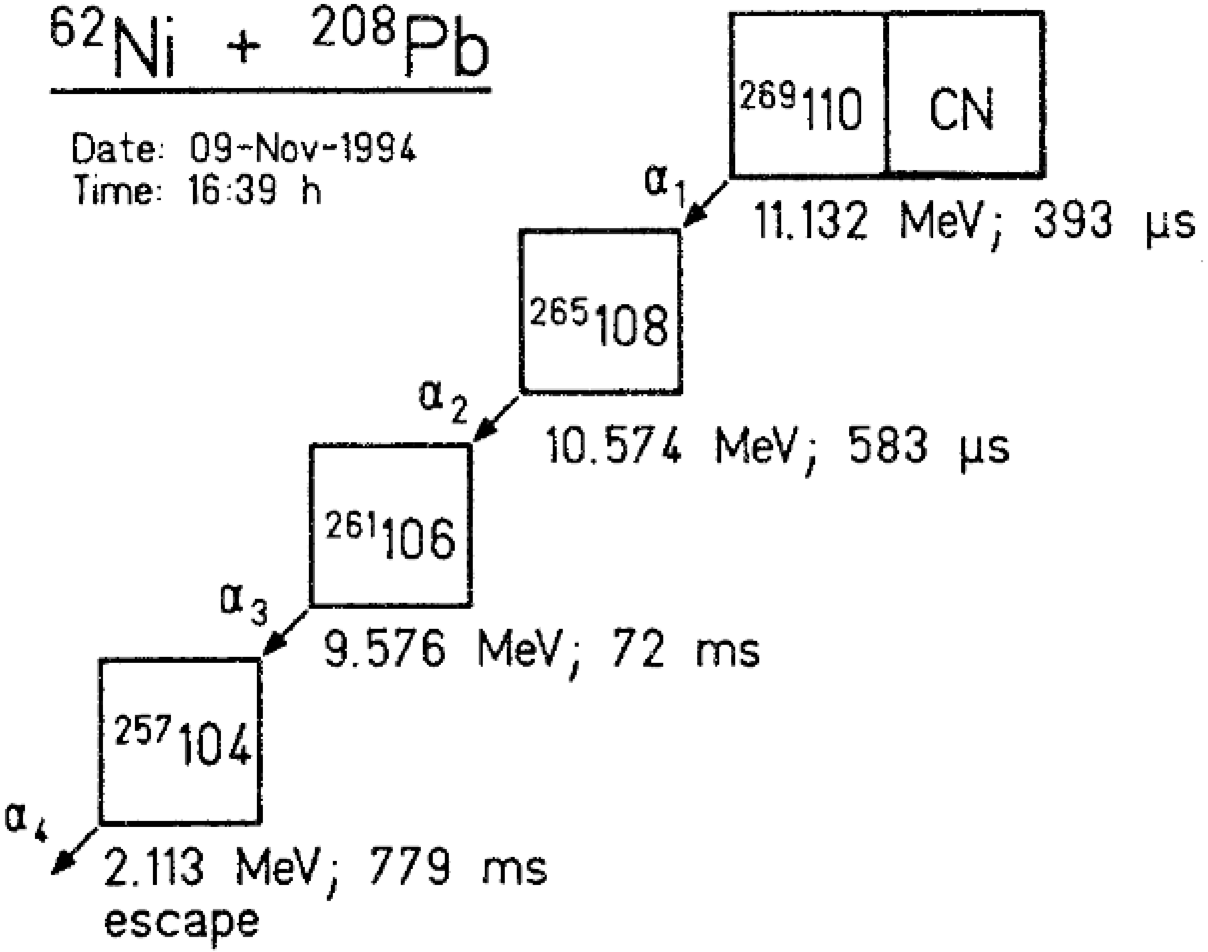
$$X = 1.2249(A_d^{1/3} + 4^{1/3}) \times \frac{Q_\alpha}{2Z_d e^2}$$



Why does the $t_{½}$-Q_α relationship matter? Superheavies

- The discovery of new elements and heavy isotopes typically relies on detecting several sequential (or coincidental) α decays
- Theoretical predictions allow one to know what energies and time windows to look for
- Measured Q_α and $t_{½}$ enable properties of the newly discovered elements to be inferred by looking at the departure from the even-even relationship

S.Hofmann et al. Z.Phys.A 1995

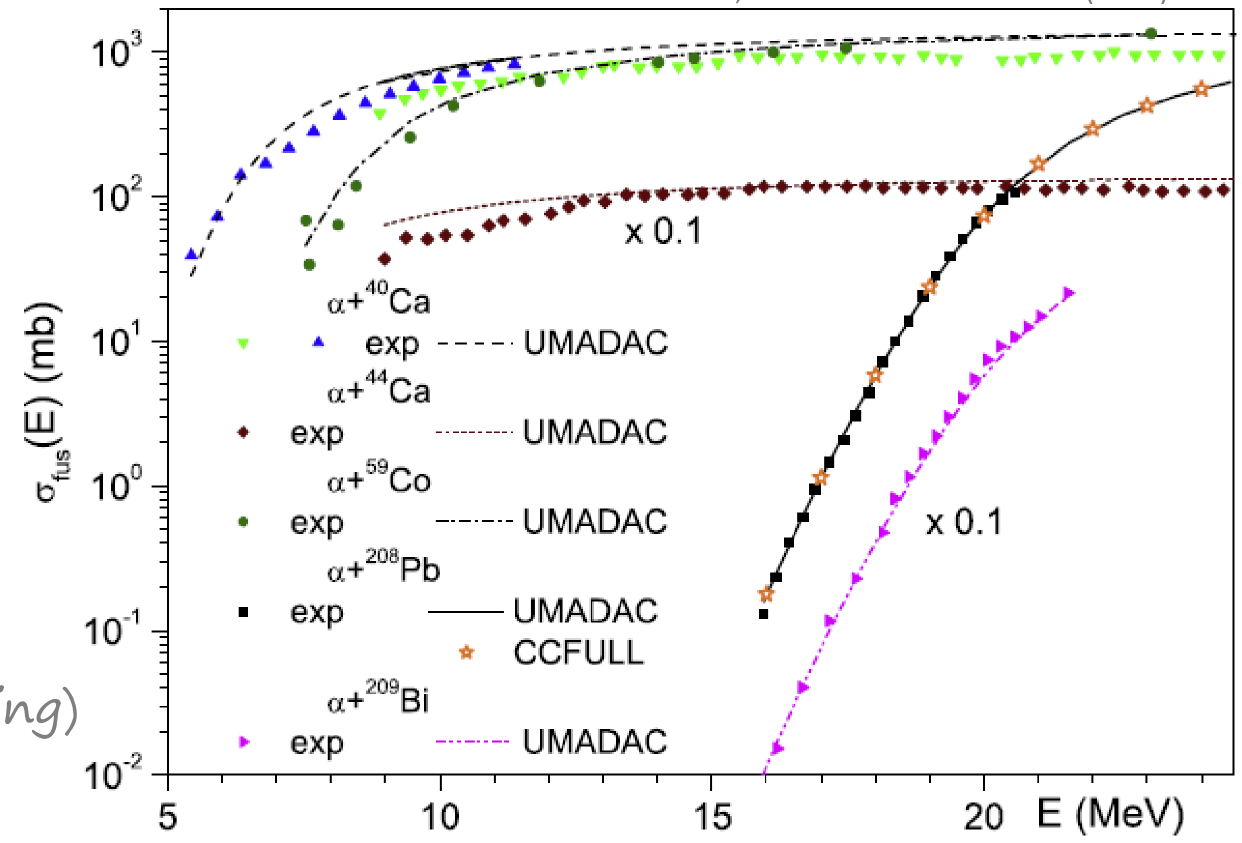


"Superheavy" nuclei are those that exist only due to the presence of shell structure, namely proximity to the Z=114 shell

Why does the $t_{½}$-Q_α relationship matter? α capture

- When the theory of α decay was first postulated, Gamow realized that tunneling in through the barrier should be no different than tunneling out
- i.e. α capture should be described by the same model
- α decay measurements can be used to infer information about the potential describing the interaction between the α and the nucleus (the "α optical potential")
- Cross section predictions using α optical potentials inferred from α decay measurements do a pretty decent job

Denisov & Khudenko, Atom. Dat. Nuc. Dat. Tab. (2009)



Why would this help?
- *Provide one more piece of data (e.g. with α scattering)*
- *Direct measurements might not be possible (short-lived nuclides)*

…why would we care about $\sigma_{\alpha\text{-capture}}$ for short-lived nuclides?
transmutation of material within reactors

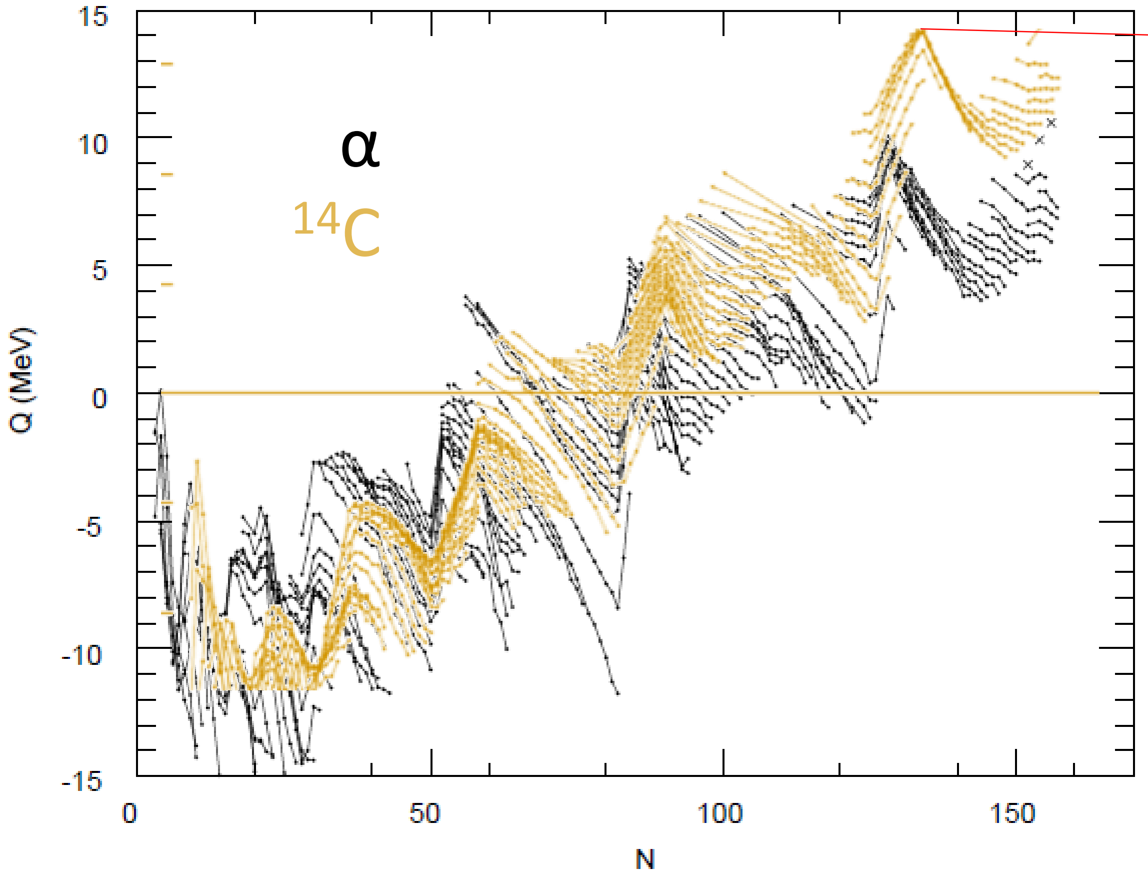
Why is it α particles that are being emitted?

- So far we've been smugly pleased with ourselves about our ability to describe α decay …but why α decay? Why not proton decay, or ^{3}He decay, or ^{12}C decay?
- The short answer is Q-values, Coulomb barriers, and clustering probabilities
  - *Q-value*: The cluster decay must be energetically favorable
  - *Coulomb barrier*: Higher-Z particles will have a larger barrier to tunnel through
  - *Clustering probability:* It's less likely for more nucleons to congregate within a nucleus

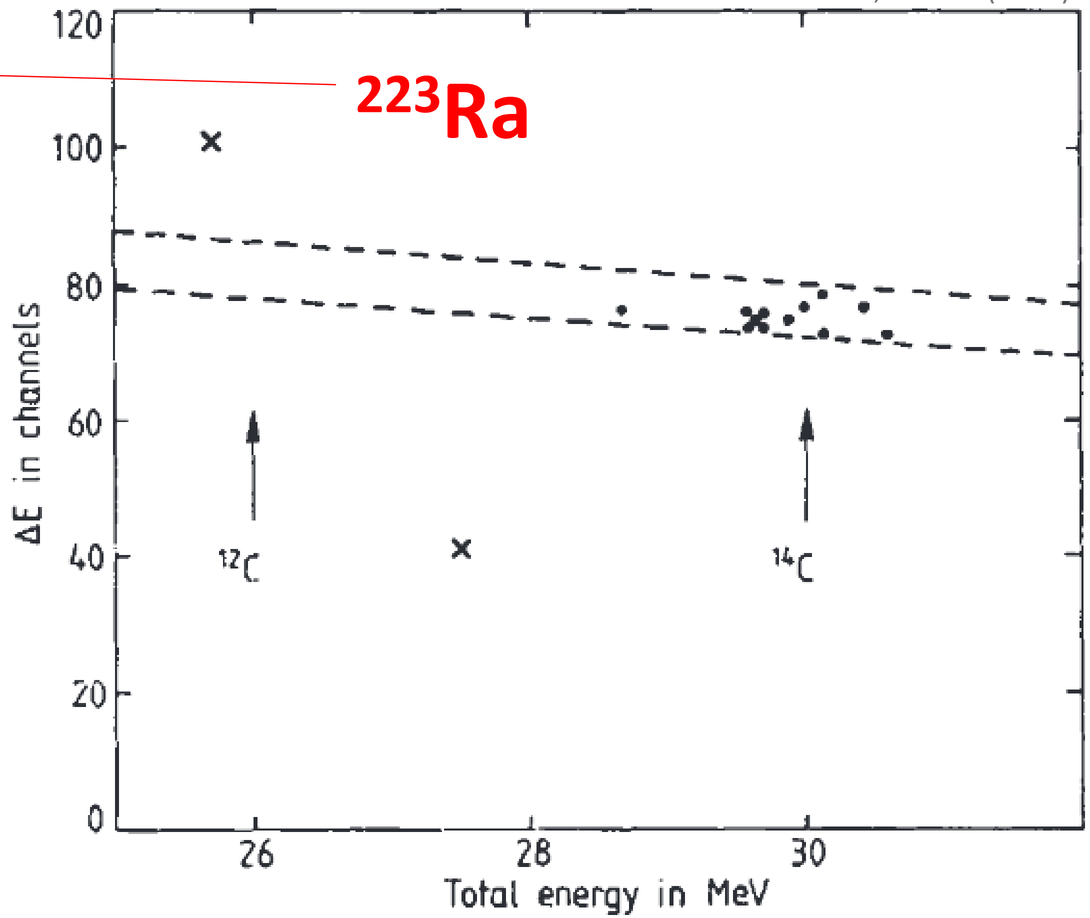
Why is it α particles that are being emitted? Q-value

- Decay is a spontaneous process that only occurs because there's a lower energy state that is available; i.e. a positive Q-value is required for a decay to occur
- The more positive the better (since this means more energy to tunnel)

B.A. Brown, Lecture Notes in Nuclear Structure Physics (2005)

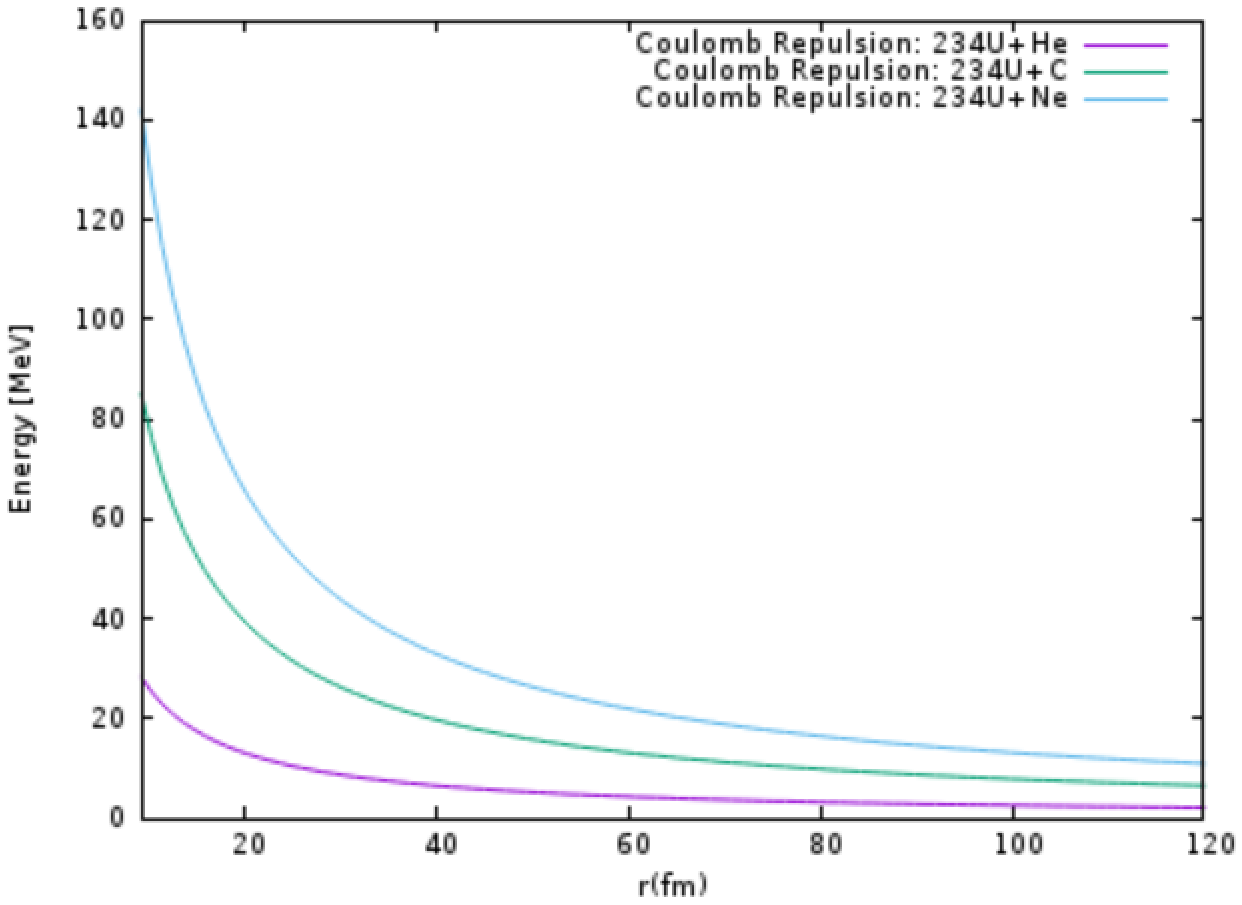


Rose & Jones, Nature (1984)



Why is it α particles that are being emitted? Coulomb barrier

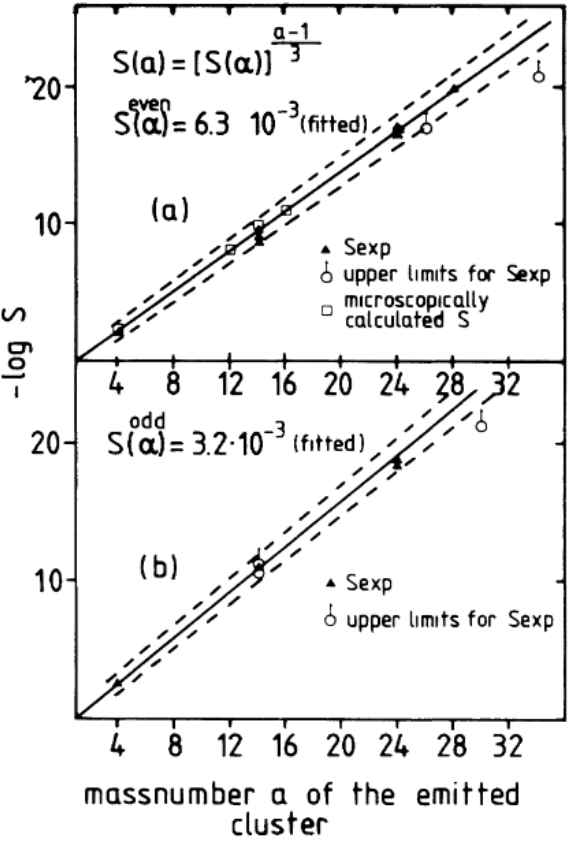
- The Coulomb barrier height scales with the charge of the particle being emitted
- It takes a much larger Q-value to make larger Z decay have any chance at tunneling through



Why is it α particles that are being emitted? Clustering probability

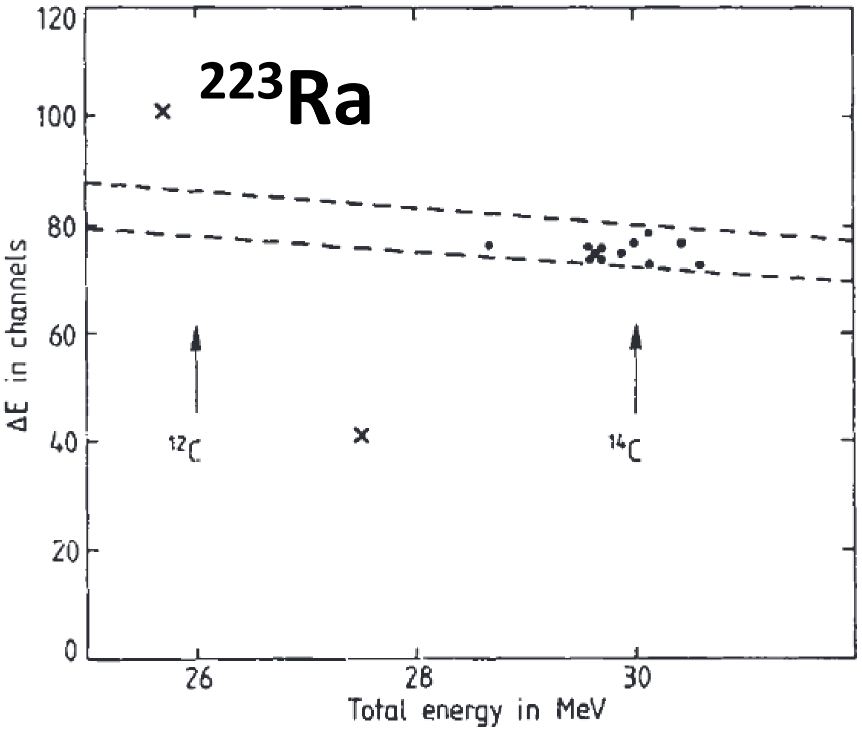
- The likelihood of forming a cluster of nucleons within a nucleus is the preformation factor
- Fancy calculations (which agree with some measurements) show that the cluster preformation probability relative to clustering for an α (for $A_c<28$) goes as (Blendowske & Walliser, Phys. Rev. Lett. (1988)):
  - $w(A_c) = w(\alpha)^{(A_{cluster}-1)/3}$, where $w_{even}(\alpha) = 6.3 \times 10^{-3}$ and $w_{odd}(\alpha) = 3.2 \times 10^{-3}$

Blendowske & Walliser, Phys. Rev. Lett. (1988):



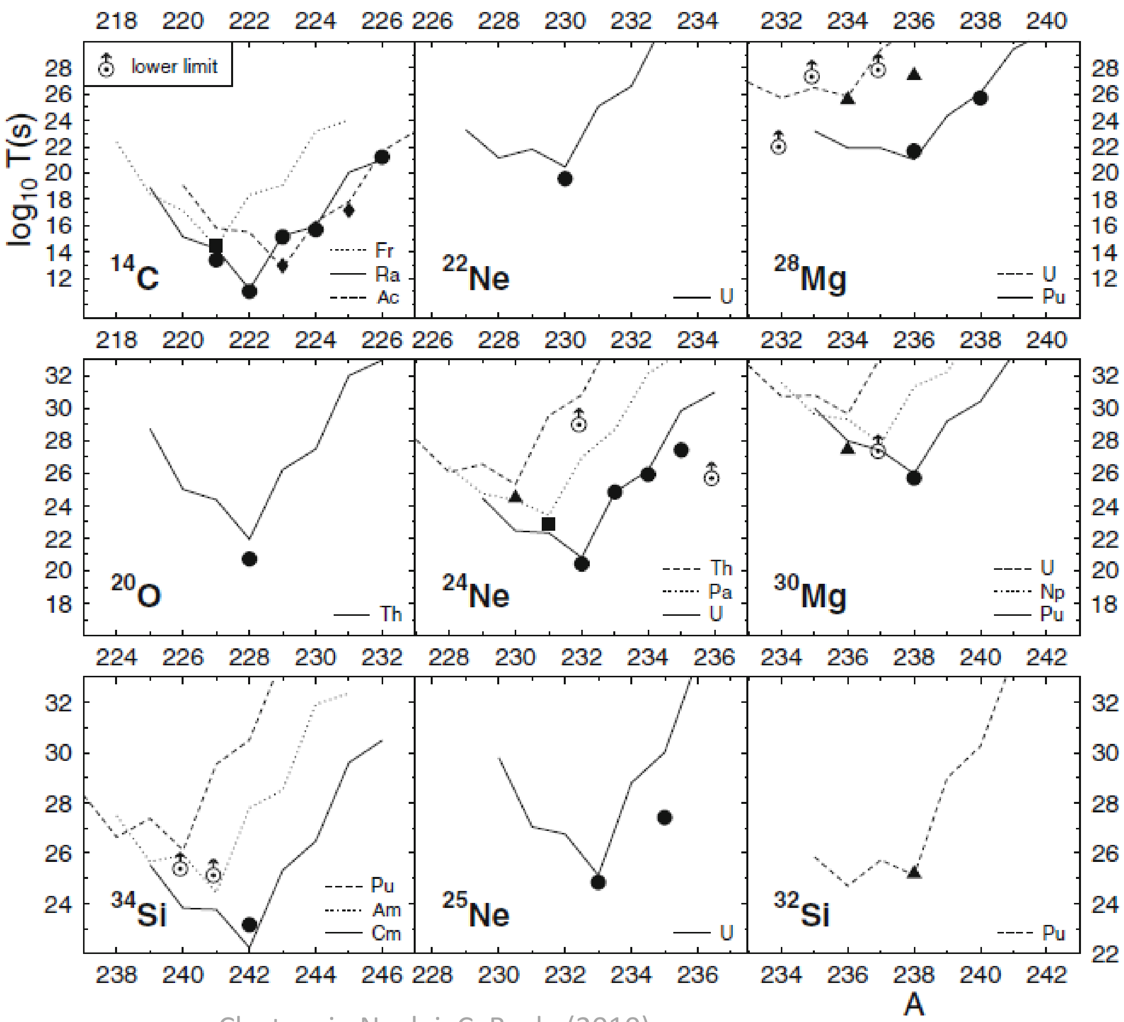
…though neutron-richness matters, since heavy nuclides will favor more neutron-rich decay products, e.g. the lack of ^{12}C emission from ^{223}Ra

Rose & Jones, Nature (1984)



…that said, pretty exotic cluster emission can happen

- Note that some pretty exotic cluster emission can happen, but it's usually a tiny decay branch
- However, some superheavies are predicted to favor cluster emission

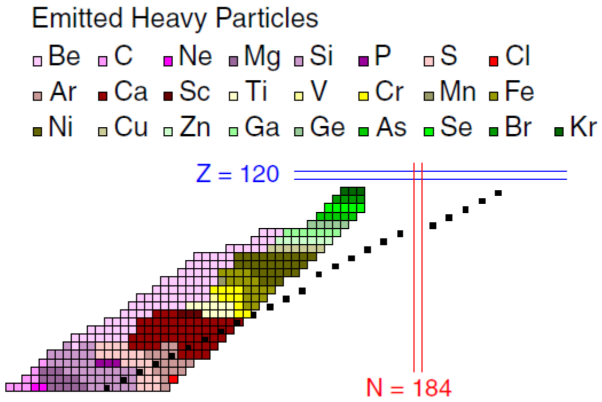


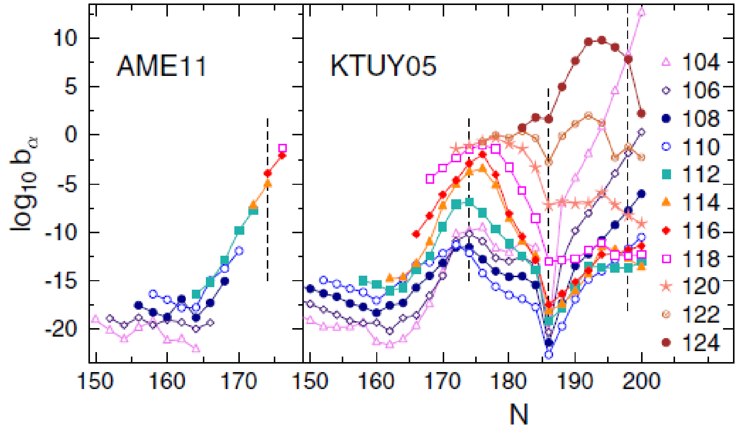
Clusters in Nuclei, C. Beck, (2010)

Clusters in Nuclei, C. Beck, (2010)

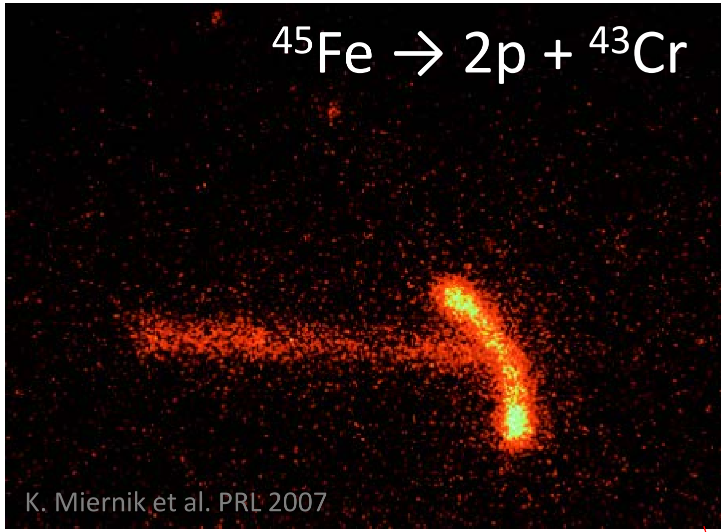
Decay	$Q_{Expt.}$	R_a	Expt.	$log_{10}(t_{½}(s))$ α decay
$^{231}Pa \rightarrow {}^{23}F + {}^{208}Pb$	51.84	R_t + 0.25	26.02	12.0
$^{231}Pa \rightarrow {}^{24}Ne + {}^{207}Tl$	60.41	R_t + 0.25	23.23	
$^{230}U \rightarrow {}^{22}Ne + {}^{208}Pb$	61.39	R_t	19.57	6.3
$^{230}U \rightarrow {}^{24}Ne + {}^{206}Pb$	61.35	R_t	>18.2	

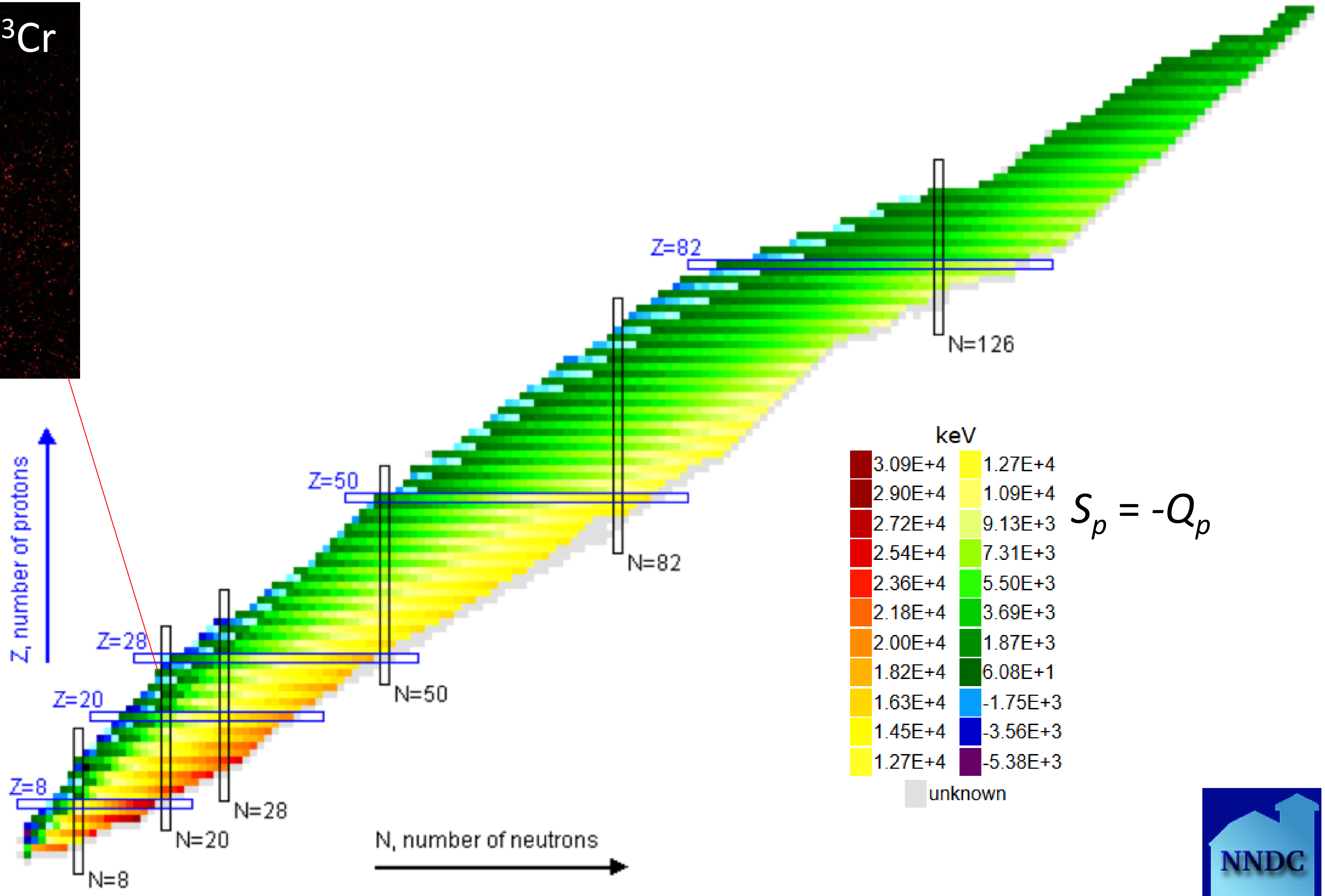
Poenaru, Gherghescu, & Greiner, Phys. Rev. Lett. (2011)





…and proton emission is a thing for very proton-rich nuclei





$S_p = -Q_p$

Further Reading

- Chapters 7: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapter 7: Nuclear & Particle Physics (B.R. Martin)
- Chapter 14, Section 11: Quantum Mechanics for Engineers (L. van Dommelen)
- Chapter 14: Introduction to Special Relativity, Quantum Mechanics, and Nuclear Physics for Nuclear Engineers (A. Bielajew)
- Chapter 4: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapter 16: The Atomic Nucleus (R. Evans)
- Clusters in Nuclei, C. Beck