

# Lecture 6: *Radioactive Decay*

- Decay Equations
- Multiple Species
- Equilibrium
- Natural Radioactivity & Dating
- Radiotracers



# Radioactive Decay: *Basic Concepts*

- It is often energetically favorable for nuclei to undergo transmutation, either converting a proton to a neutron (or vice versa), emitting some combination of nucleons, or splitting apart. This is radioactive decay.
- Nuclei are effectively insulated from the surrounding environment and so the rate at which radioactive decay proceeds is independent of the surrounding conditions (e.g. pressure, temperature)  
*[Though, see the fascinating exception of bound-state  $\beta$ -decay in extremely high-temperature environments (J. Bachall, Phys. Rev. 1961)]*
- In general, radioactive decay is an irreversible event  
*[Again, sort of an exception in some exotic astrophysical environments, namely, Urca cycling (Gamow & Schoenberg, Phys. Rev. 1941)]*
- Whether or not a particular nucleus undergoes decay depends on many stochastic processes, so the decay of several nucleons is described using statistical tools describing probability
- For large numbers of nuclei (pretty much every case:  $>10^{21}$  nuclei for a gram of almost anything), a statistical treatment is not only adequate, but also likely the only practicable way to describe decay kinetics

# Decay Equations for One Nuclear Species

- Since nuclear decay is (usually) independent of the surroundings, the rate at which a sample composed of a particular isotope will undergo decay will just be proportional to the number of that isotope in the sample
  - $(\text{Decay Rate of Sample}) \propto (\text{Decay Rate of Single Nucleus}) \times (\text{Number of Nuclei})$
- A single nucleus will be no more or less likely to decay at one time as opposed to another, so  $(\text{Decay Rate of Single Nucleus}) = \text{constant} \equiv \lambda$
- Therefore, the rate at which the sample decays,  
i.e. the rate at which the number of nuclei  $N$  decreases:  $-\frac{dN}{dt} = \lambda N$
- Keep in mind  $N$  is changing with time, since decay is proceeding:  $-\frac{dN}{dt} = \lambda N(t)$
- This is well and good, but we're interested in  $N(t)$ , which isn't obvious from the equation above

# Decay Equations for One Nuclear Species

- Solving for  $N(t)$  from the seemingly innocuous equation  $\frac{dN}{dt} = -\lambda N(t)$ , is actually pretty hard, unless one employs the Laplace transform, which turns our differential equation into an algebraic one
- For the LHS, we assume  $N(t)$  to be an exponential function (empirically a safe bet) and use the derivative property of Laplace transform [with complex variable  $s$ ]:
  - $\frac{dN}{dt} \rightarrow sN(s) - N(0)$
- For the RHS, simply swap-in  $s$  for time:  $-\lambda N(t) \rightarrow -\lambda N(s)$
- Therefore,  $sN(s) - N(0) = -\lambda N(s)$
- Which is re-written as:  $N(s) = \frac{N(0)}{(s+\lambda)}$
- Using one of several different methods (See E.g. [D.Pressyanov, Am.J.Phys. 2002](#), or a Math Methods book), the inverse Laplace transform can be employed, yielding the familiar relation:
  - $N(t) = N(0)e^{-\lambda t}$ , the number of nuclei existing at time  $t$
- Since the Activity (decays/second)  $A = \lambda N$ ,
  - $A(t) = A(0)e^{-\lambda t}$

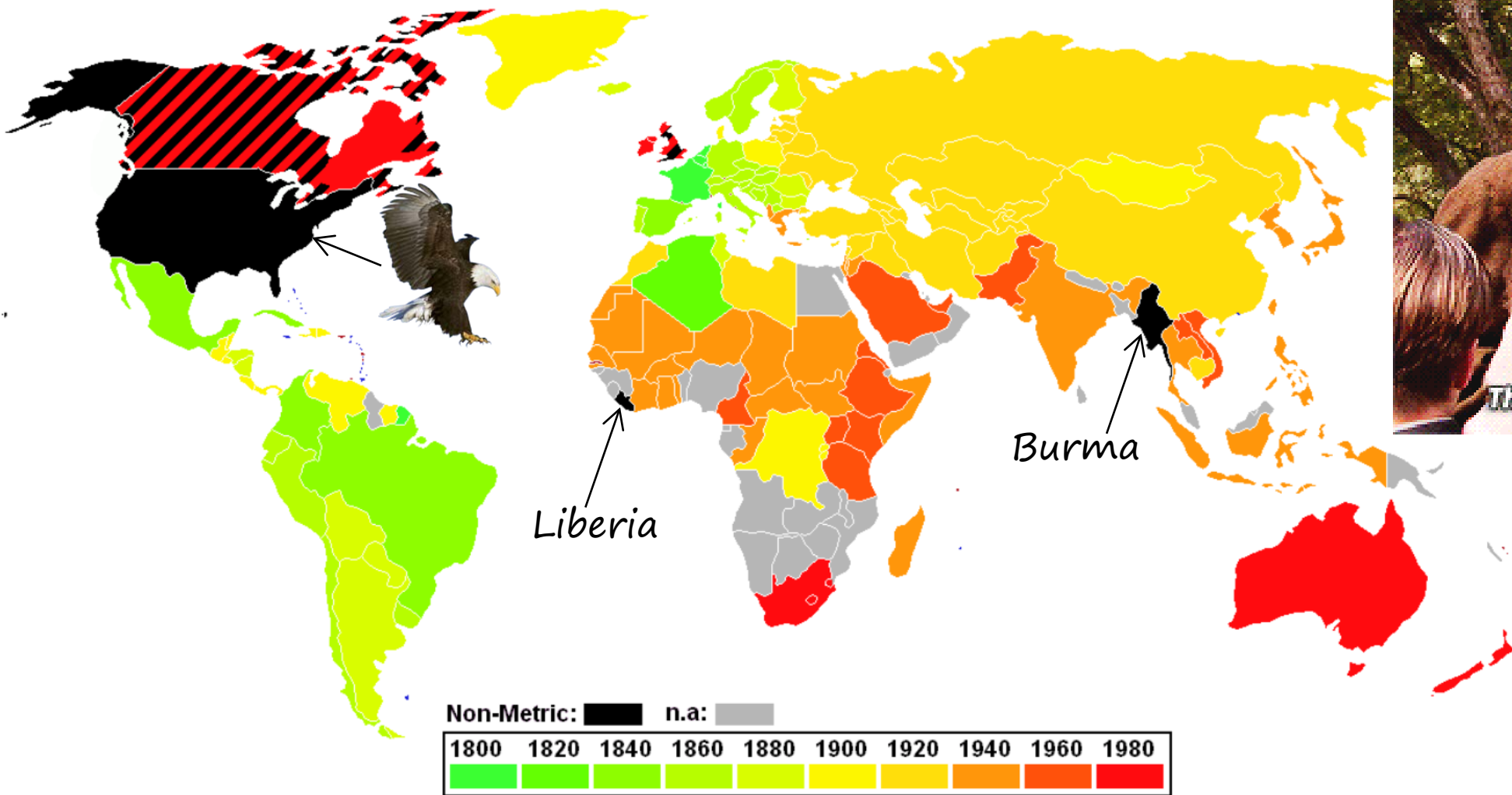
# Common Descriptors for Decay

- Aside from the decay constant  $\lambda$ , other more intuitive quantities are often used
- It is common to state the time at which half of the nuclei in a radioactive sample will have undergone decay, a.k.a. the half-life:  $t_{1/2}$ 
  - By definition:  $\frac{N(t_{1/2})}{N(0)} = \frac{1}{2}$  ... so,  $\frac{1}{2} = e^{-\lambda t_{1/2}}$
  - Re-write as  $2 = e^{\lambda t_{1/2}}$ , which makes it apparent that  $t_{1/2} = \frac{\ln(2)}{\lambda}$
- An alternative piece of trivia is the mean lifetime for the nuclei in the sample,  $\tau$ 
  - By definition:  $\tau = \frac{\int_0^\infty tN(t)dt}{\int_0^\infty N(t)dt} = \frac{\int_0^\infty tN(t)dt}{N(0)/\lambda} = \frac{N(0)/\lambda^2}{N(0)/\lambda} = \frac{1}{\lambda}$
  - Therefore  $t_{1/2} = \ln(2)\tau \approx 0.693\tau$
- We can re-state the lifetime in terms of the equivalent energy width using the Heisenberg uncertainty principle:
  - $\Delta E \cdot \Delta t \geq \hbar$
  - Taking the mean lifetime as the time uncertainty,  $\Delta E = \frac{\hbar}{\tau} \approx \frac{6.582 \times 10^{-22} \text{ MeV}}{\tau \text{ (in seconds)}}$

# Radioactive Decay Units

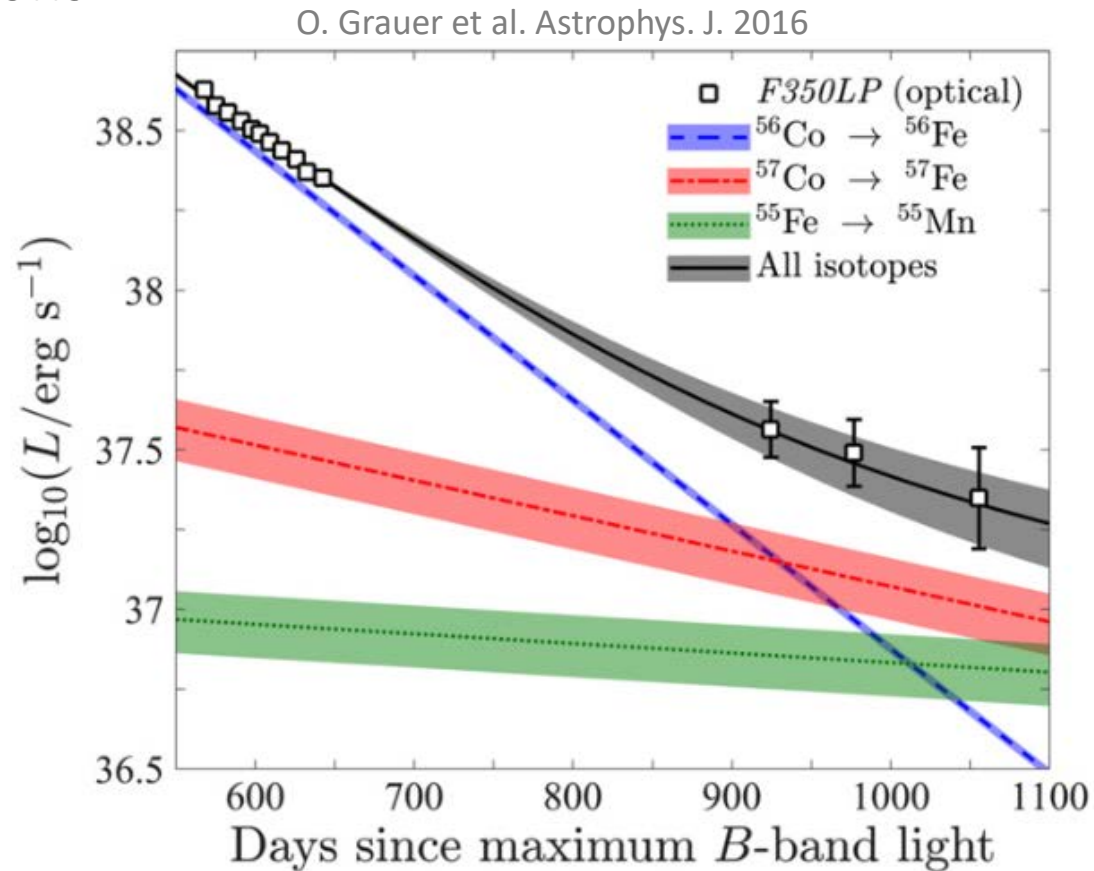
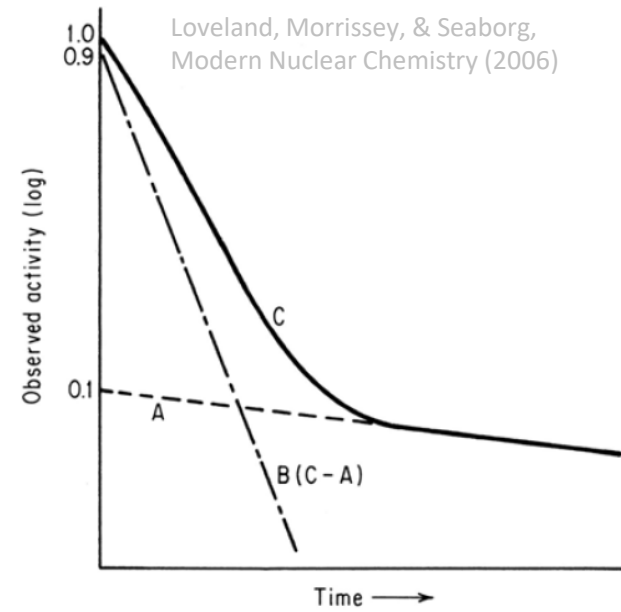
- For samples in the lab, we usually care about how many decay products a sample is emitting, i.e. the Activity
- A sensible unit is the Becquerel:  $1 \text{ Bq} \equiv 1 \text{ decay/second}$
- A historical unit, based on the decay rate from a gram of radium ([E. Rutherford, Nature \(1910\)](#)), is the Curie:  $1 \text{ Ci} \equiv 37 \text{ GBq}$
- For context:
  - Soil most places on earth has  $^{40}\text{K}$ ,  $^{238}\text{U}$ ,  $^{226}\text{Ra}$ , and  $^{232}\text{Th}$  for a total of a few-hundred Bq/kg ([IAEA Technical Report #1162](#))
  - The glassy residue from the July 1945 Trinity nuclear weapon test has an activity of  $\sim 100 \text{ Bq/g}$  (P.Parekh et al. J. Enviro. Rad. (2006))
  - A typical household smoke detector contains  $\sim 1 \text{ } \mu\text{Ci}$  (a.k.a.  $37 \text{ kBq}$ ) of  $^{241}\text{Am}$
- The activity per unit mass is called the “specific activity”

# Who uses “Curie”?



# Decay of Multiple Species in Parallel

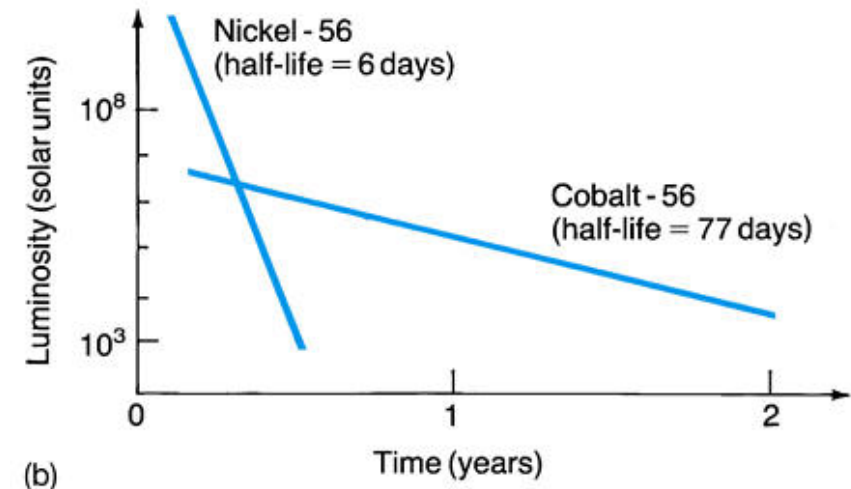
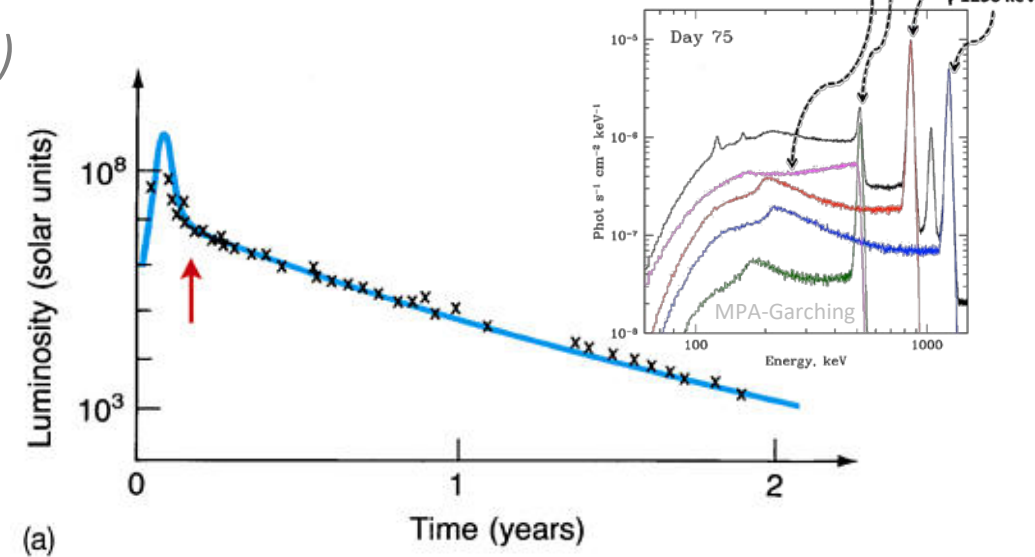
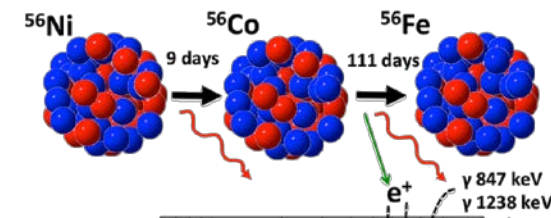
- If several nuclei are decaying at once in parallel (i.e. independently from each other), the species can be picked-out by fitting the resulting non-linear curve on a semi-log plot with multiple lines.
- This provides unique solutions for up to 3 parallel decays
- A neat example is the identification of  $^{57}\text{Co}$  decay following a Type Ia supernova





# Decay Sequences, a.k.a. Compound Decay

- For radioactive decays in series  
(e.g.  $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ , which powers Type Ia supernova light-curves)  
we have a set of coupled differential equations
- Say we want to describe how much of nucleus 2  
we have as a function of time for the decay sequence  
 $1 \rightarrow 2 \rightarrow \dots$
- Schematically,  
$$\frac{dN_2}{dt} = (2 \text{ Production Rate}) - (2 \text{ Destruction Rate})$$
- Therefore,  $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$
- Perhaps unsurprisingly  
(given our experience solving the 1 species case)  
solving this requires a bit of gymnastics



# “Daughter” Activity

- To solve  $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$  for  $N_2(t)$ , someone clever realized one way to do it is
  1. Rearrange the equation into:  $dN_2 + \lambda_2 N_2 dt = \lambda_1 N_1 dt$
  2. Substitute in the previous solution for  $N_1(t)$ :  $dN_2 + \lambda_2 N_2 dt = \lambda_1 N_1(0)e^{-\lambda_1 t} dt$
  3. Multiply both sides by  $e^{\lambda_2 t}$ :  $dN_2(e^{\lambda_2 t}) + \lambda_2 N_2(e^{\lambda_2 t})dt = \lambda_1 N_1(0)e^{-\lambda_1 t}(e^{\lambda_2 t})dt$
  4. And, harrumph harrumph, realize this is the same as:  $d(N_2 e^{\lambda_2 t}) = \lambda_1 N_1(0)e^{(\lambda_2 - \lambda_1)t} dt$
  5. Which can be integrated from time 0 to  $t$ :  $N_2 e^{\lambda_2 t} \Big|_0^t = \frac{\lambda_1 N_1(0)e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1} \Big|_0^t$
  6. Which is:  $N_2 e^{\lambda_2 t} - N_2(0) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)(e^{(\lambda_2 - \lambda_1)t} - 1)$
  7. Multiply both sides by  $e^{-\lambda_2 t}$ :  $N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_2(0)e^{-\lambda_2 t}$
- Therefore, the activity of the daughter nuclide in the decay sequence is:

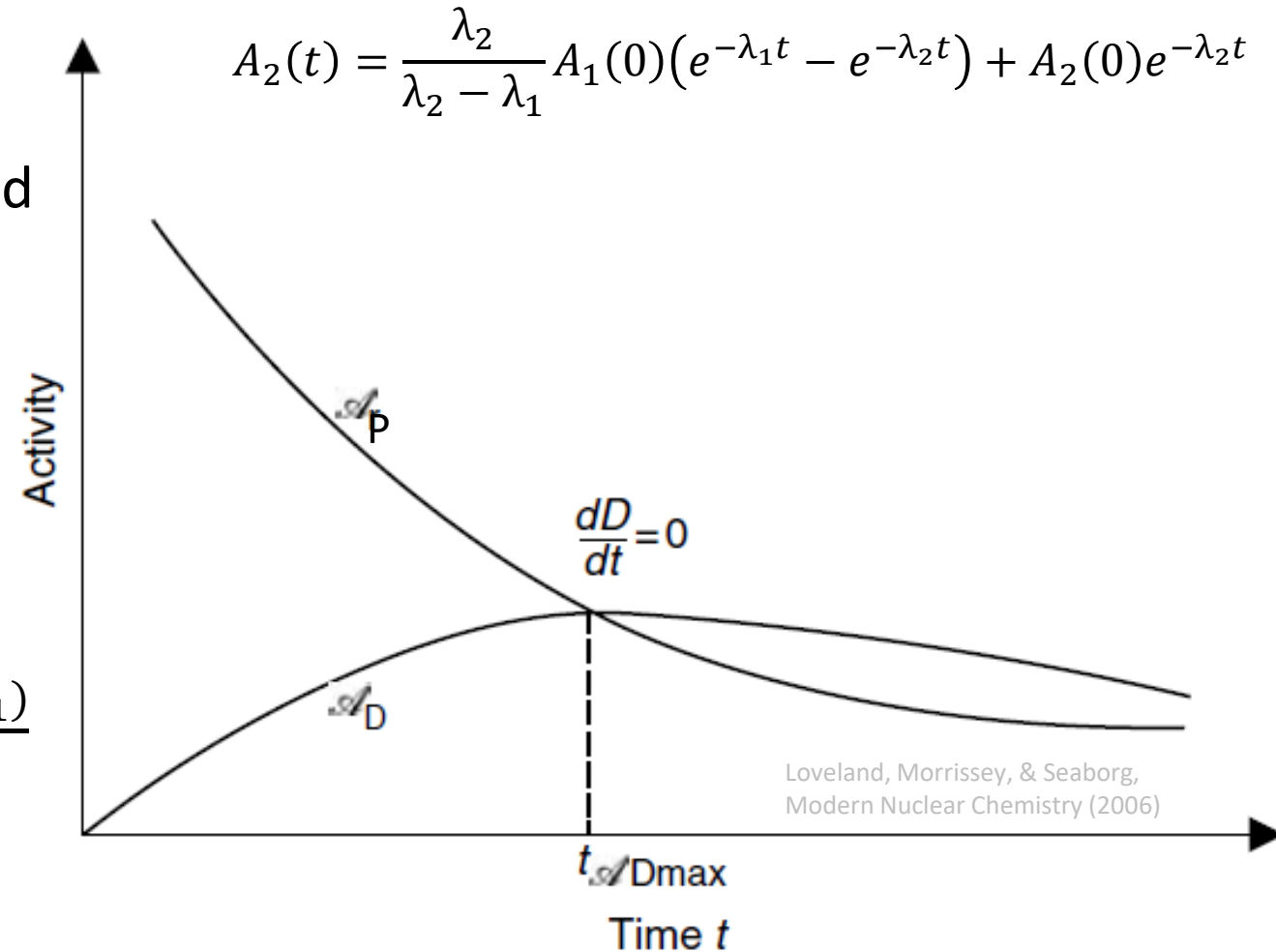
$$A_2(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_1(0)(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + \lambda_2 N_2(0)e^{-\lambda_2 t}$$

a.k.a. 
$$A_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(0)(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_2(0)e^{-\lambda_2 t}$$

# “Daughter” Activity

- The total activity is some combination of the parent and daughter activity, where the exact time dependence will depend on the relationship between  $\lambda_1$  and  $\lambda_2$
- The relative magnitudes of  $\lambda_1$  and  $\lambda_2$  determine which type of “equilibrium” best describes the situation (covered in a moment)
- The time at which the daughter nucleus peaks in abundance (and activity) will be at  $\frac{dA_2(t)}{dt} = 0$ , which occurs at  $t_{max} = \frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$
- For the case of a stable daughter,  $\lambda_2 = 0$ , with zero initial abundance  

$$N_2(t) = N_1(0)(1 - e^{-\lambda_1 t})$$



# Decay “Equilibrium”

Consider 4 limiting cases:

a)  $\tau_{parent} < \tau_{daughter}$

- The daughter nucleus will buildup faster than it decays
- “No Equilibrium”

b)  $\tau_{parent} > \tau_{daughter}$

- Decay of the daughter is slightly faster than buildup
- Doesn’t get a name

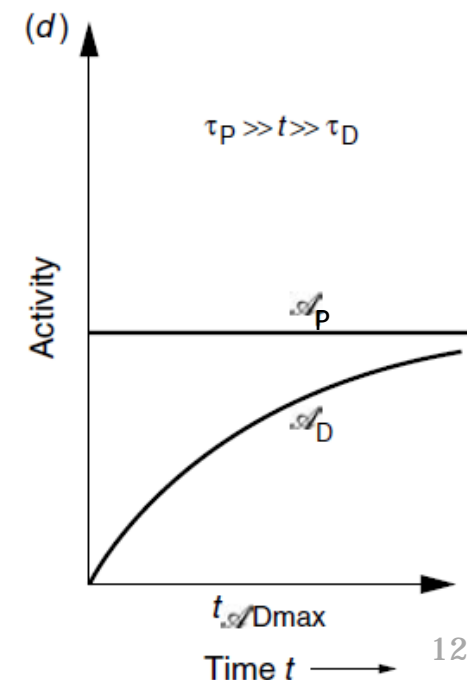
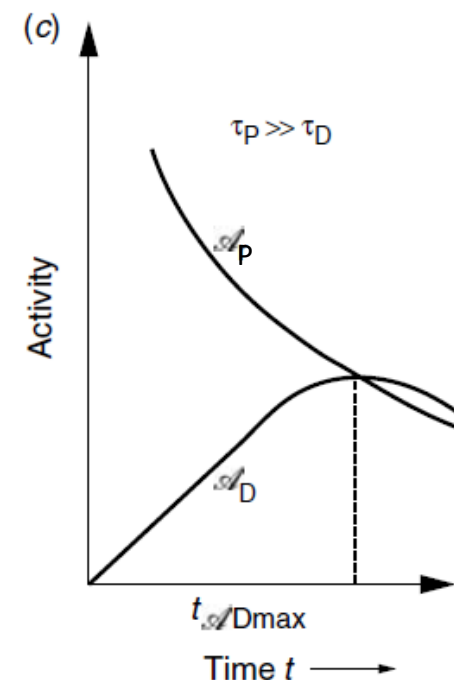
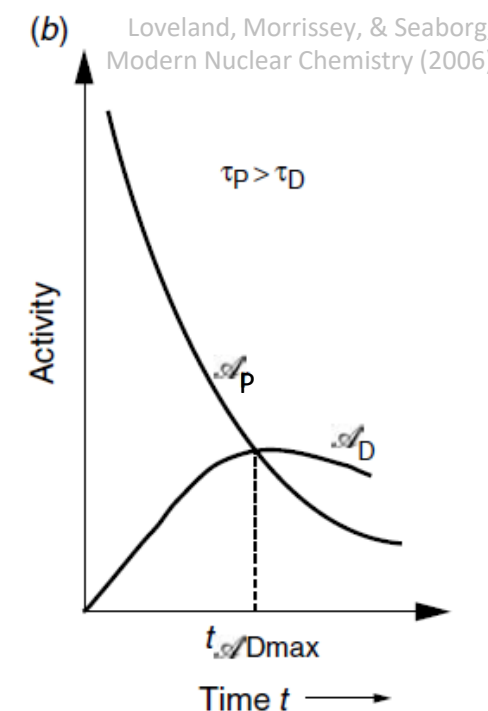
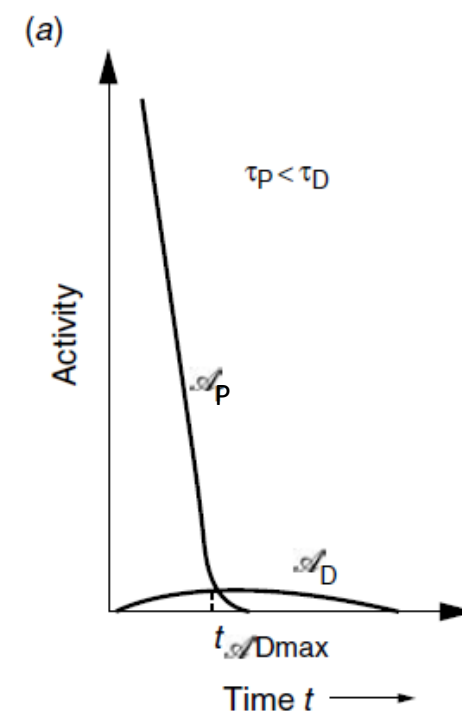
c)  $\tau_{parent} \gg \tau_{daughter}$

- The daughter decays almost immediately after being made
- “Transient Equilibrium”

d)  $\tau_{parent} \gg t \gg \tau_{daughter}$

(i.e. parent super-long-lived or infinite [beam])

- Parent is constantly replenished
- “Secular equilibrium”



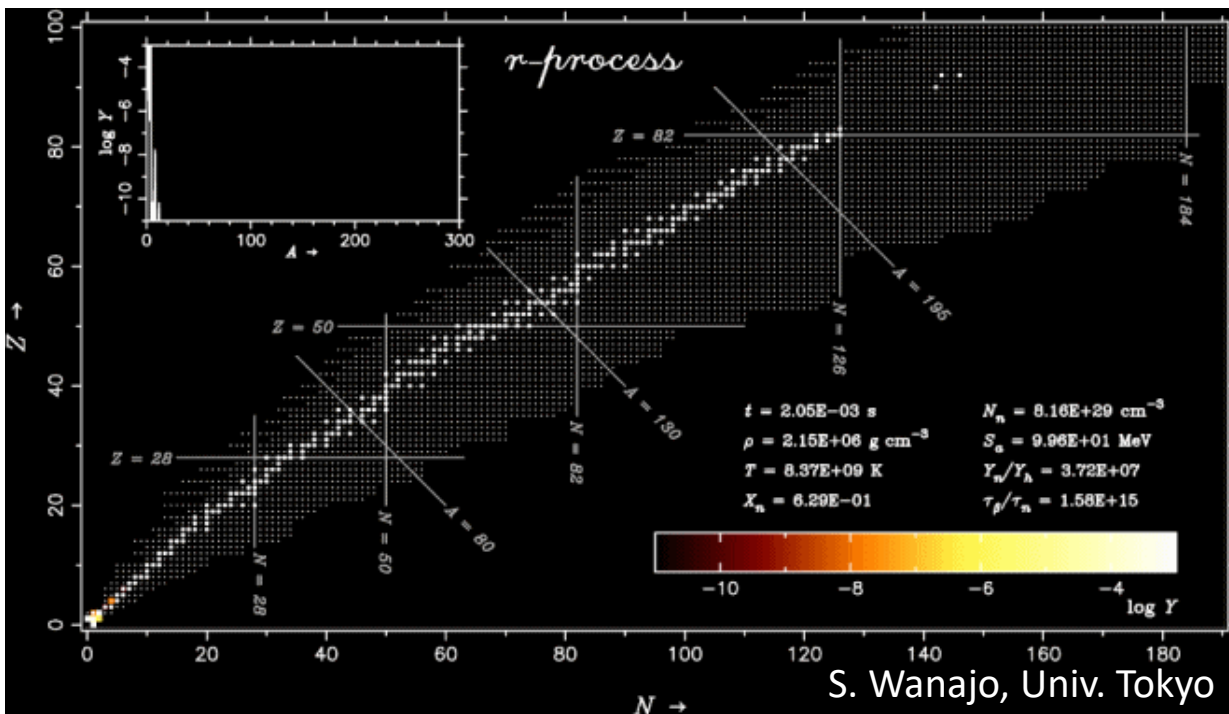
# No Equilibrium: $\tau_{parent} < \tau_{daughter}$

- This case will apply when exotic nuclides  $\beta$ -decay back to stability
- The decay can just be modeled as the daughter decay
- For example, this would be useful for modeling radioactivity due to core collapse supernova or neutron-star merger nucleosynthesis products

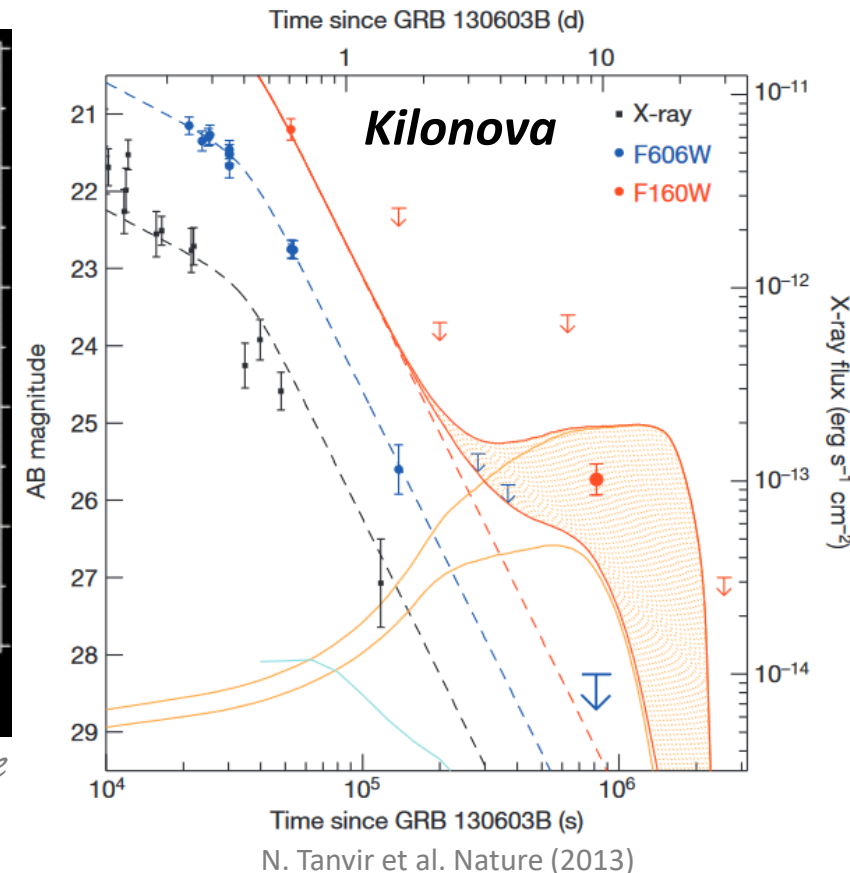
Activity

$\tau_p < \tau_D$

Time  $t \rightarrow$



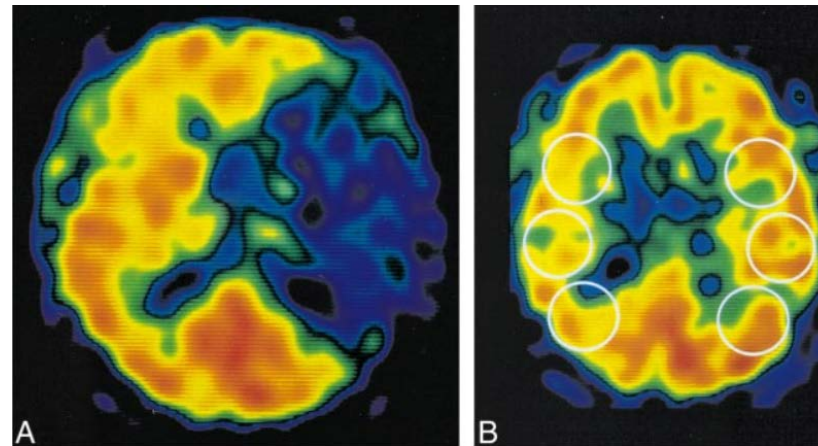
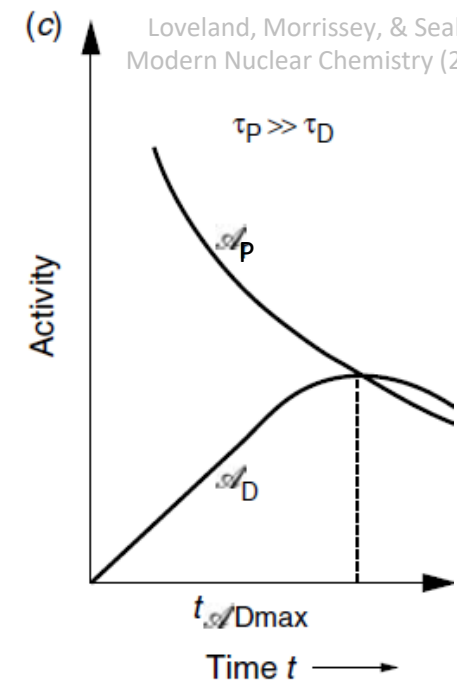
\*radioactivity isn't really consequential for the CCSN light-curve



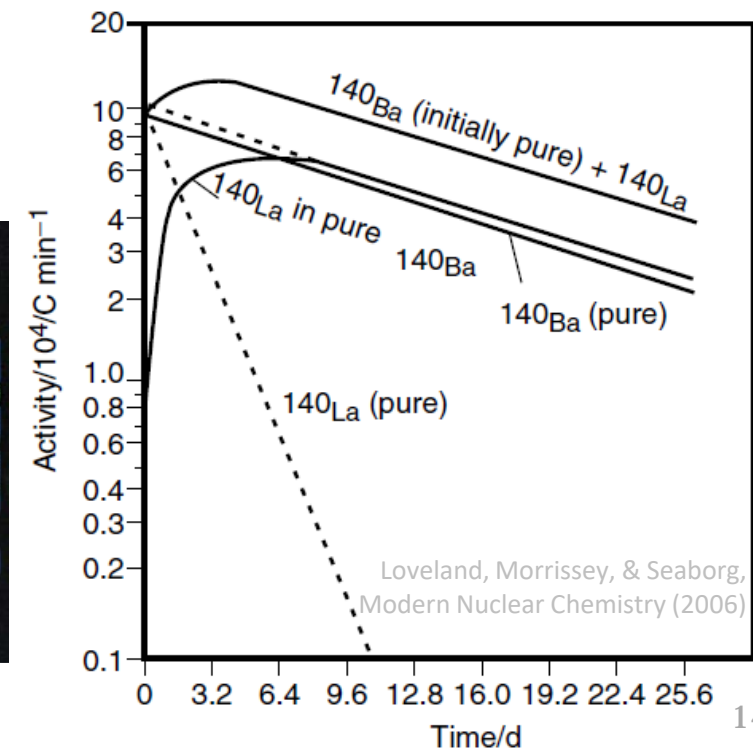


# Transient Equilibrium: $\tau_{parent} \gg \tau_{daughter}$

- When the parent is more than  $\sim 10\times$  longer-lived than the daughter nuclide, the daughter activity will effectively be that of the parent
- In this case,  $\lambda_2 > \lambda_1$ . So, at large  $t$ ,  $e^{-\lambda_2 t}$  will be much smaller than  $e^{-\lambda_1 t}$
- As such,  $A_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(0)(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_2(0)e^{-\lambda_2 t}$   
becomes:  $A_2(t) \approx \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(0)e^{-\lambda_1 t} = \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(t)$
- Therefore, in transient equilibrium at long times, the parent and daughter activity are related by a constant multiple,  $\frac{\lambda_2}{\lambda_2 - \lambda_1}$
- Example:  $^{99m}\text{Tc}$  generator
  - $^{99}\text{Mo}$  ( $t_{1/2} \approx 66\text{hrs}$ ) is produced off-site (from  $^{235}\text{U}$  or n-irradiation of  $^{98}\text{Mo}$ )
  - On-site chemistry techniques are used to extract  $^{99m}\text{Tc}$  ( $t_{1/2} \approx 6\text{hrs}$ ), which can then be used for SPECT



K. Ogasawara et al. American Journal of Neuroradiology (2001)

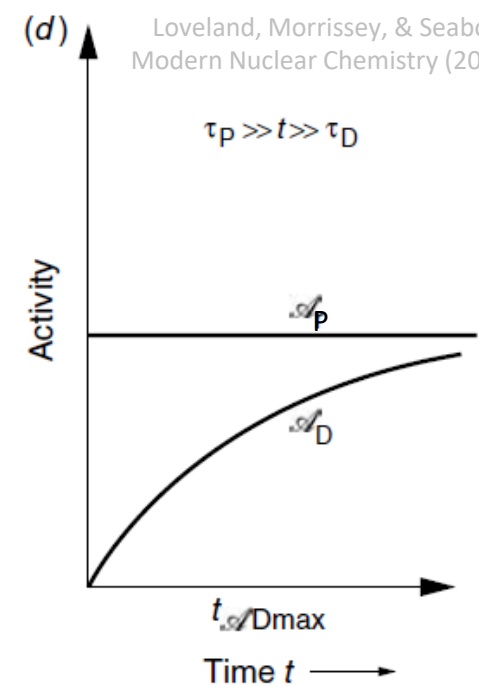
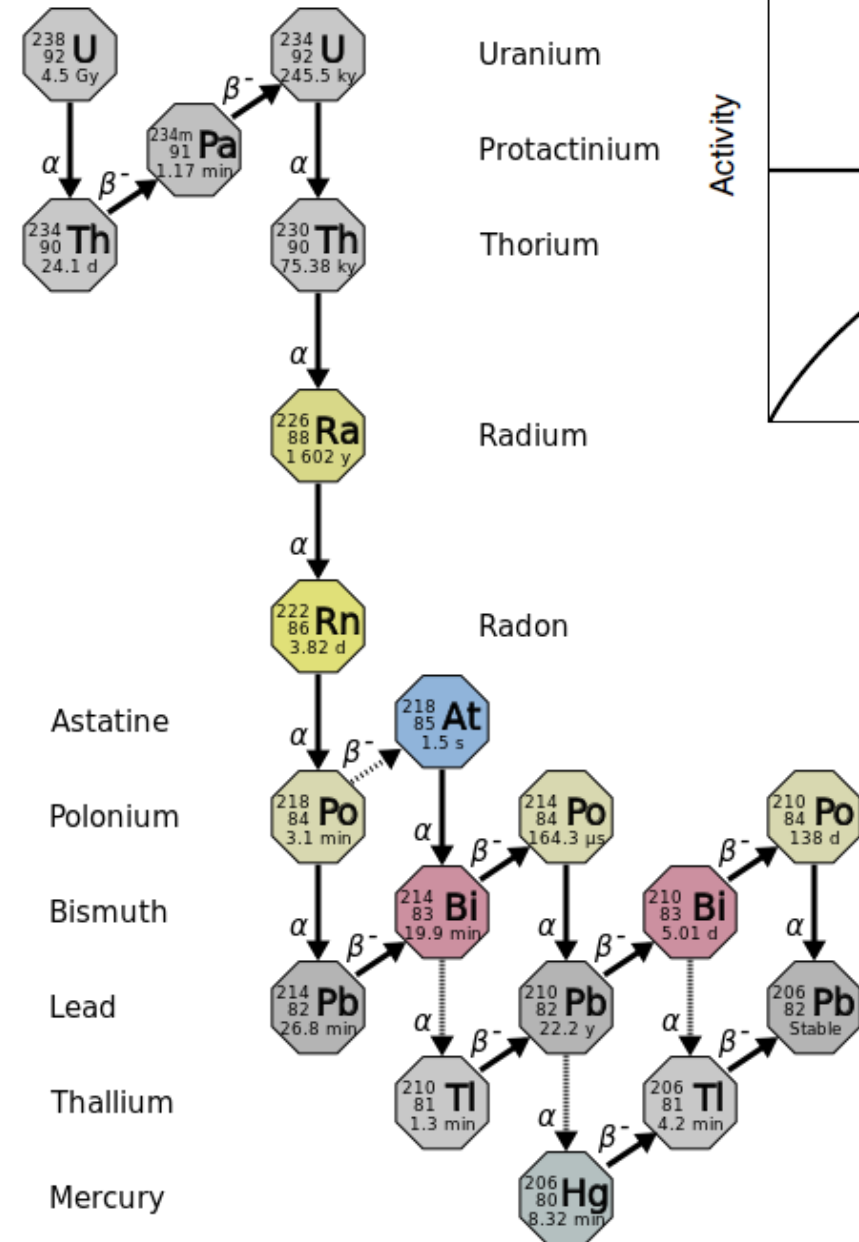


# Secular Equilibrium: $\tau_{parent} \gg t_{long} \gg \tau_{daughter}$

- Consider our previous relation

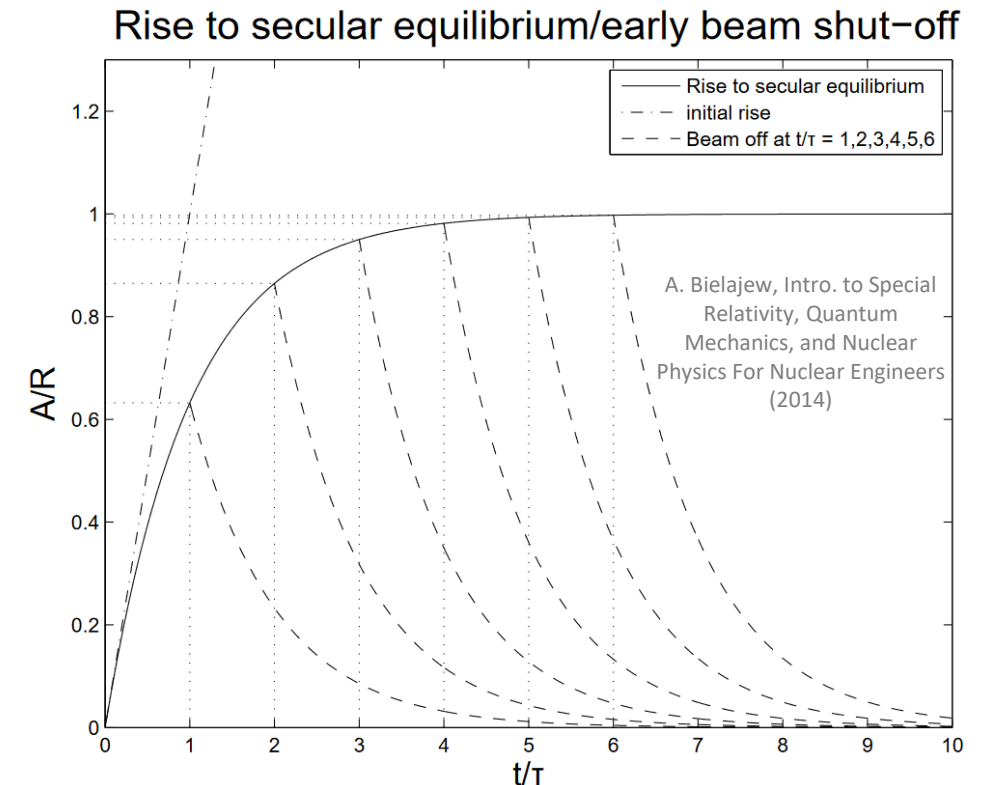
$$A_2(t) \approx \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(t) \text{ when } \lambda_2 \gg \lambda_1$$

- For very large half-life disparities,  $\lambda_2 \gg \lambda_1$ ,  $(\lambda_2 - \lambda_1) \approx \lambda_2$ , so  $A_2(t) \approx A_1(t)$
- This applies for the decay chain of some transuranic nuclides
- For example: Radon from Granite
  - Granite has relatively large concentrations of  $^{238}\text{U}$  ( $t_{1/2} \approx 4.5\text{Gyr}$ ), which wouldn't really be a problem, since it's trapped in the rock
  - However,  $^{238}\text{U}$  decay leads to  $^{222}\text{Rn}$  ( $t_{1/2} \approx 4\text{days}$ ), which is a gas that easily escapes the rock and gets the chance to  $\alpha$ -decay in your lungs
  - So, fancy countertops and state capitol buildings will be hazardous for perpetuity



# Secular Equilibrium & Radioactivity from Nuclear Reactions

- If a radioactive nucleus is made in a nuclear reaction (e.g. in a star, from cosmic rays, or using an accelerator), the constant replenishment of the daughter is similar to the case of a very long-lived parent
- In particular this is true if the reaction rate is much slower than the daughter decay  
...which is pretty much always the case
- Starting with  $N_2(0) = 0$  and recasting “decay” of  $1 \rightarrow 2$  as a reaction,  $R_{12} = \lambda_1 N_1(0) = A_1(0)$ ,  $A_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(0) (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_2(0) e^{-\lambda_2 t}$  becomes  $A_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} R_{12} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$
- Since  $\lambda_1 \approx 0$ ,  $A_2(t) \approx R_{12} (1 - e^{-\lambda_2 t})$ , which is known as the “activation equation”
- Linear growth of the daughter at short times, but have diminishing returns as time increases
- If you’re making a custom radioactive source on-site, this lets you know how long to bother performing the production reaction
- This relation allows nuclear reaction cross sections to be measured using the daughter decay, and it lets you know how long to perform the reaction





# Reaction Cross Section Measurements with Activation

- Assuming the decay of the daughter nucleus is understood, the reaction rate for the production reaction is extracted from  $R_{12} = A_2(t)/(1 - e^{-\lambda_2 t})$
- After performing the reaction  $1 \rightarrow 2$  for time  $t_{irr}$  and then measuring the activity a time  $t_{post-irr}$  after the irradiation has stopped:  $A_2(t_{post-irr})$ : 
$$R_{12} = \frac{A_2(t_{post-irr})e^{-\lambda_2 t_{post-irr}}}{(1 - e^{-\lambda_2 t_{irr}})}$$
- The rate of production  $R_{12}$  is related to the reaction probability, the “cross section”, by  
(Production Rate) = (Rate of Incident Nuclei) \* (Areal Number Density of Target Nuclei) \* (Cross Section)
  - $R_{12} = I_1 n_2 \sigma_{12}$
- So, once we’ve counted the number of projectiles, determined the areal target density, measured the activity and irradiation time, and taken into account detection efficiencies and time delays between activity measurements and the end of irradiation
  - $$\sigma_{12} = \frac{A_2(t_{post-irr})e^{-\lambda_2 t_{post-irr}}}{I_1 n_2 (1 - e^{-\lambda_2 t_{irr}})}$$

*e.g. If you activate for 66 hours and measure the activity 20 minutes after ceasing irradiation,  $t_{irr}=66\text{hr}$  and  $t_{post-irr}=20\text{min}$*

*One pit-fall is a lack of knowledge about the daughter decay due to unaccounted for decay branches. For a detailed discussion of other complications, see e.g. G.Kiss et al. Phys. Lett. B (2014)*

# Decay sequences with more than two species

- For a longer sequence of decays  $1 \rightarrow 2 \rightarrow 3 \rightarrow \dots \rightarrow n$ ,  $N_n(t)$  is determined using the Bateman equations  
(H.Bateman, Proc. Cambridge Philos. Soc. (1910)) ... or see [D.Pressyanov, Am.J.Phys. 2002](#) for a modern derivation)

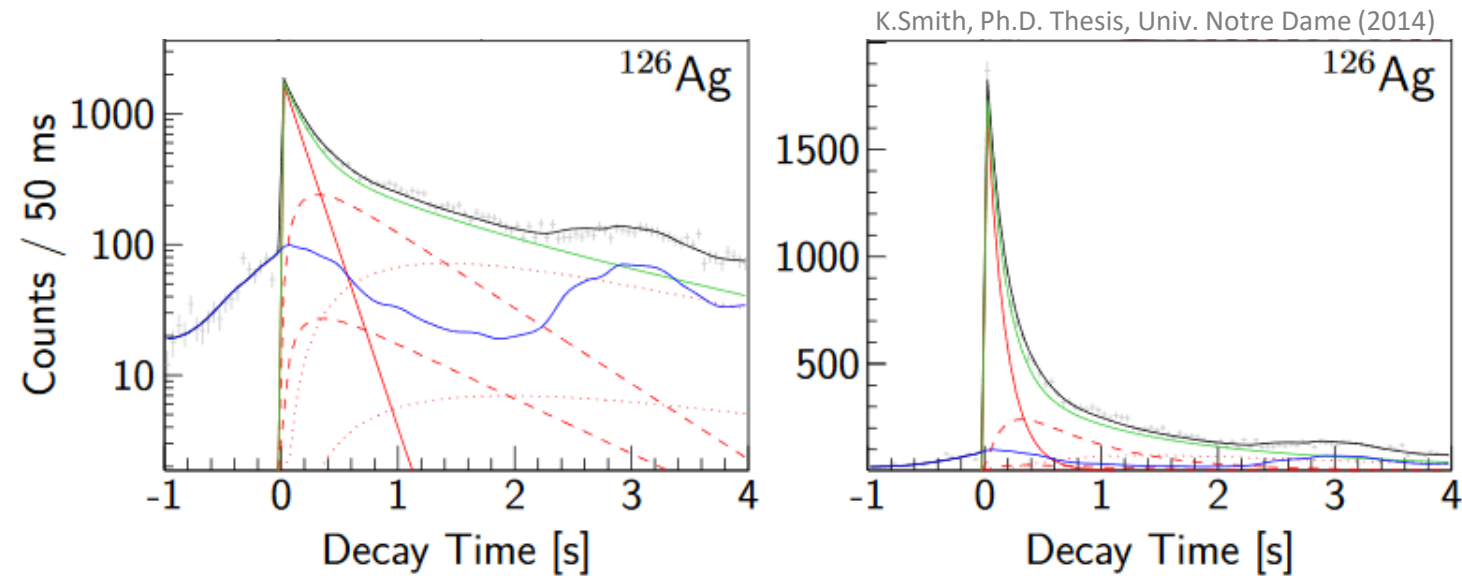
- For zero initial decay products, as for decays from a radioactive ion beam experiment,

$$N_n(t) = N_1(0)\lambda_{n-1}\lambda_{n-2}(\dots)\lambda_1 \sum_{i=1}^n \frac{e^{-\lambda_i t}}{\prod_{k=1(k \neq i)}^i (\lambda_k - \lambda_i)}$$

- A fit to experimental data can be used to determine several half-lives simultaneously for nuclides in a decay sequence

- Life is usually messier and nuclei can decay to different products, a.k.a “decay branches”

- For these cases, one can use “genealogically ordered exit-only decay chains” (Yuan & Kernan, J. Appl. Phys. 2007)



The diagram illustrates the decay chain of  $^{218}\text{Po}$ . The isotopes are represented by colored octagons with their atomic number (Z), mass number (A), and half-life (t<sub>1/2</sub>) listed inside. The decay paths are indicated by arrows with labels for the decay type (α or β<sup>-</sup>).

- $^{218}\text{Po}$  (Z=84, A=218, t<sub>1/2</sub>=3.1 min) decays via α to  $^{214}\text{Pb}$  (Z=82, A=214, t<sub>1/2</sub>=26.8 min) and via β<sup>-</sup> to  $^{218}\text{At}$  (Z=85, A=218, t<sub>1/2</sub>=1.5 s).
- $^{218}\text{At}$  decays via α to  $^{214}\text{Po}$  (Z=84, A=214, t<sub>1/2</sub>=164.3 μs).
- $^{214}\text{Po}$  decays via α to  $^{214}\text{Pb}$ .
- $^{214}\text{Pb}$  decays via β<sup>-</sup> to  $^{214}\text{Bi}$  (Z=83, A=214, t<sub>1/2</sub>=19.9 min) and via α to  $^{210}\text{Pb}$  (Z=82, A=210, t<sub>1/2</sub>=22.2 y).
- $^{214}\text{Bi}$  decays via β<sup>-</sup> to  $^{214}\text{Po}$  and via α to  $^{210}\text{Bi}$  (Z=83, A=210, t<sub>1/2</sub>=5.01 d).
- $^{210}\text{Bi}$  decays via β<sup>-</sup> to  $^{210}\text{Po}$  (Z=84, A=210, t<sub>1/2</sub>=138 d) and via α to  $^{206}\text{Tl}$  (Z=81, A=206, t<sub>1/2</sub>=4.2 min).
- $^{210}\text{Po}$  decays via α to  $^{206}\text{Pb}$  (Z=82, A=206, Stable).
- $^{210}\text{Pb}$  decays via β<sup>-</sup> to  $^{210}\text{Bi}$  and via α to  $^{206}\text{Tl}$ .
- $^{210}\text{Tl}$  decays via β<sup>-</sup> to  $^{206}\text{Pb}$  and via α to  $^{206}\text{Hg}$  (Z=80, A=206, t<sub>1/2</sub>=8.32 min).

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# Environmental Radioactivity

- Radioactivity is everywhere, coming from various sources:

- **Primordial:** Nuclei remaining since their formation

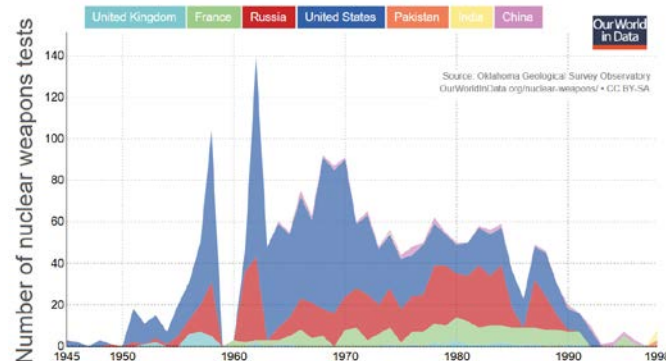
- Ex. 1: U, Th, and K that provide most of the Earth's internal heat
- Ex. 2: Deep-sea  $^{244}\text{Pu}$  from supernovae [A.Wallner et al. Nature Comm. (2015)]

- **Cosmogenic:** Nuclei produced from cosmic ray interactions

- Energetic particles from events out in space interact with nuclei in the atmosphere to make radioactive products
- Important examples include  $^3\text{H}$  ( $t_{1/2} \sim 12\text{yr}$ ),  $^7\text{Be}$  ( $t_{1/2} \sim 53\text{d}$ ), and  $^{14}\text{C}$  ( $t_{1/2} \sim 5700\text{yr}$ )

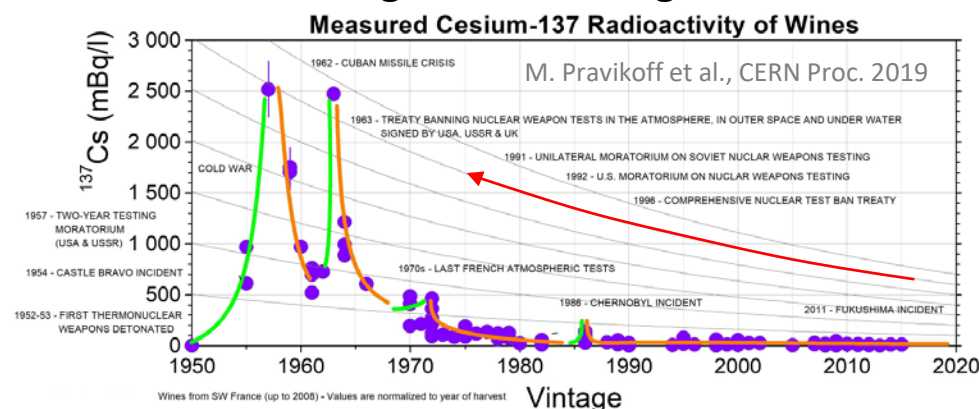
- **Anthropogenic:** Nuclei produced by human activities

- Mostly from nuclear reactors and weapons tests

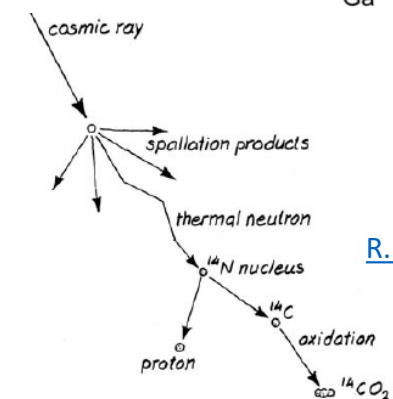
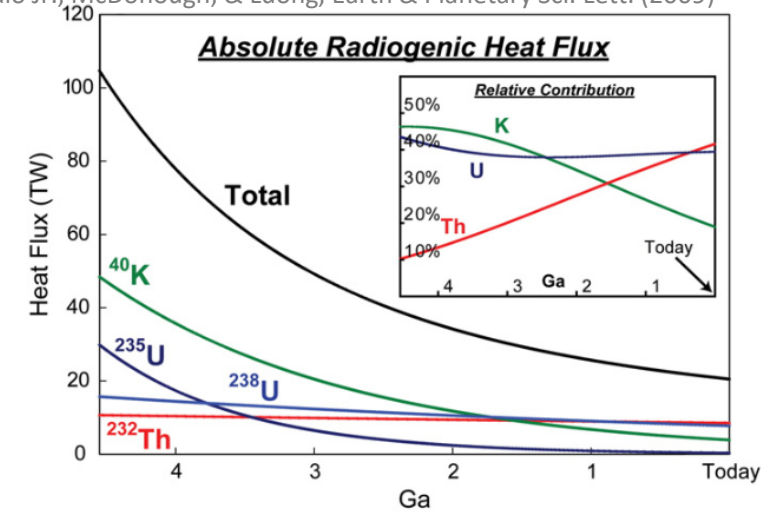


Neat time-lapse of weapons tests  
by location: <https://youtu.be/LLCF7vPanrY>

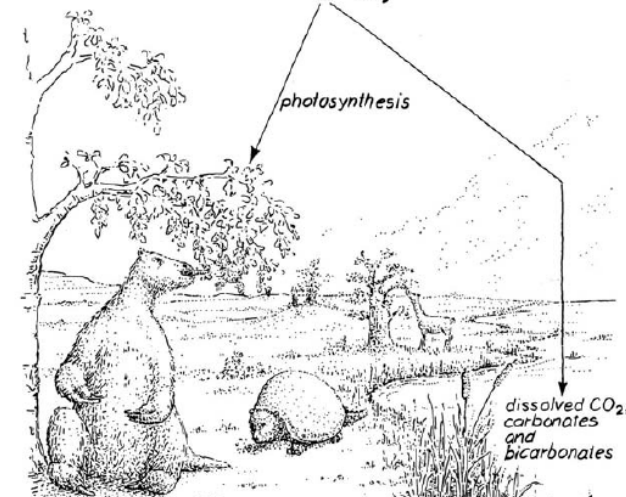
- ...can be taken advantage of for dating:



Arevalo Jr., McDonough, & Luong, Earth & Planetary Sci. Lett. (2009)

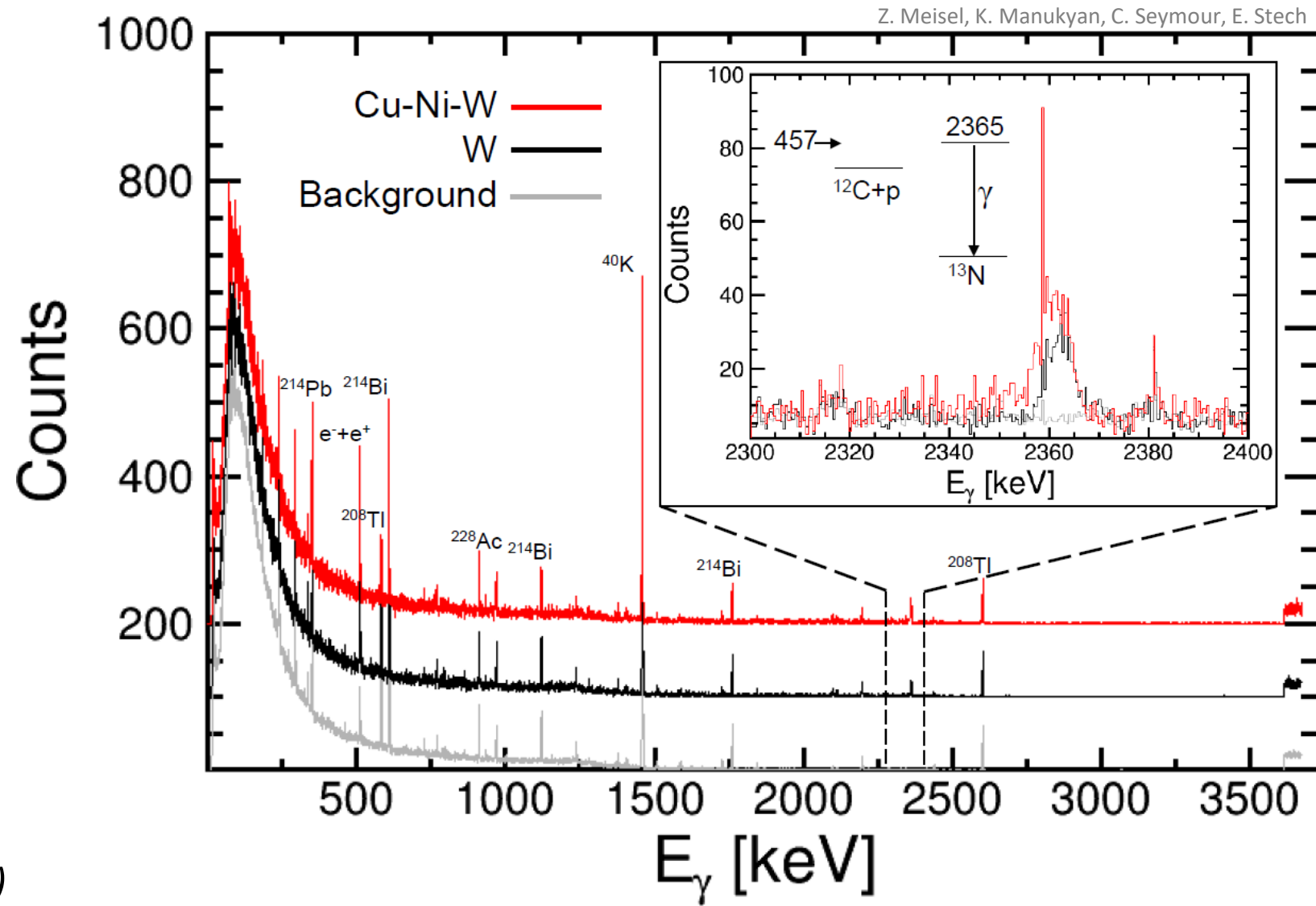


R. Taylor, Am. Sci. (2000)



# $\gamma$ -ray Background

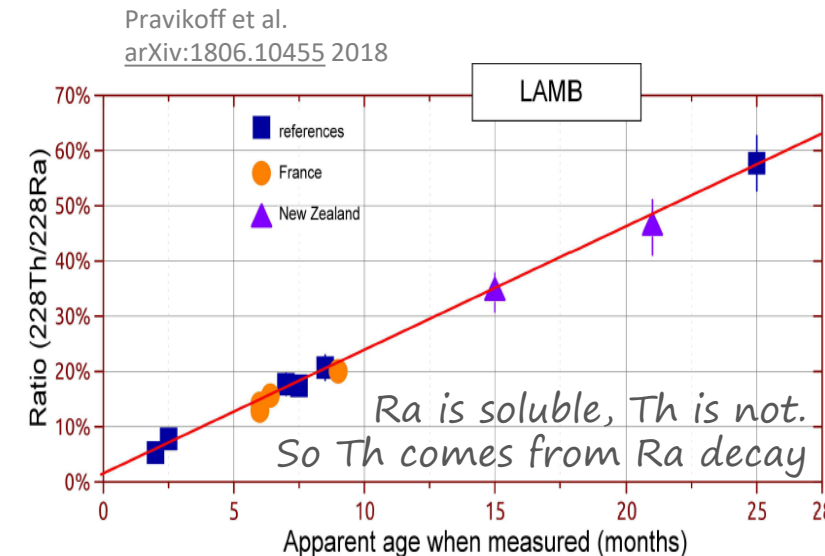
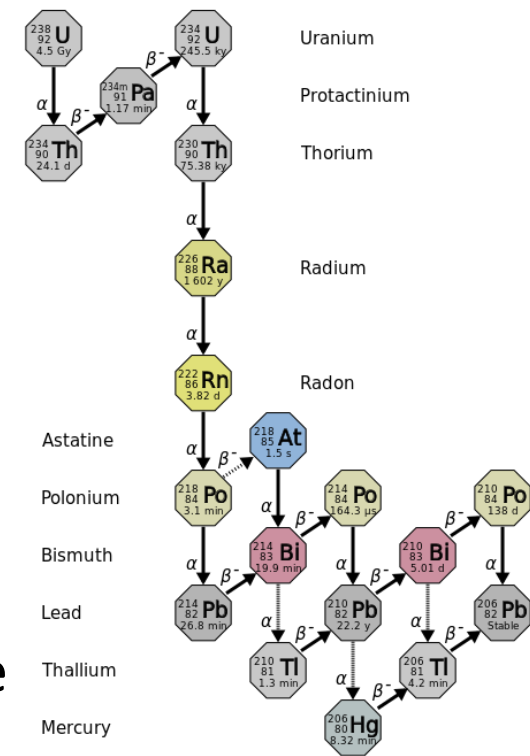
- Radiation from environmental radioactivity is readily seen in high-resolution  $\gamma$ -spectra
- Long-lived radioactivity, such as  $^{40}\text{K}$  can be seen, as can decay products from actinide decay series
  - E.g.  $^{208}\text{Tl}$  &  $^{228}\text{Ac}$  from  $^{232}\text{Th}$
  - E.g.  $^{214}\text{Pb}$  &  $^{214}\text{Bi}$  from  $^{238}\text{U}$
- Practically, the local background must be measured  
(which has the benefit of providing high-precision energy calibration points)
- Reference catalogs, such as [Appendix D of Gordon Gilmore's Practical Gamma Ray Spectrometry](#), are essential for background peak identification



# Radionuclide Dating

- If an object contains or contained a radioactive nucleus, we can use this to tell how old it is
  - From the equation for decay of a single species  $N(t) = N(0)e^{-\lambda t}$ , we can solve for the time  $t = \frac{\ln(N(t)/N(0))}{-\lambda}$
  - However, while determining  $N(t)$  just requires counting ( $A(t) = \lambda N(t)$ ), often we have no way to know  $N(0)$
  - The solution is to keep in mind that for the decay sequence, e.g.  $1 \rightarrow 2$ , the total number of nuclei in the decay sequence is constant:  

$$N_1(t) + N_2(t) = N_{tot} = \text{constant}$$
  - As such, assuming  $N_2(0) = 0$ , then  $t = \frac{1}{\lambda} \ln \left( 1 + \frac{N_2(t)}{N_1(t)} \right)$ , so we only need the ratio of daughter to parent nuclides in the sample
  - The “parent” nuclide need not be the direct decay parent of the “daughter”; it just has to be earlier in the decay sequence
    - e.g.  $^{206}\text{Pb}/^{238}\text{U}$  works well for old rocks
    - $^{228}\text{Ra} \rightarrow ^{228}\text{Ac} \rightarrow ^{228}\text{Th}$  can be used for meat
- 
- Pravikoff et al  
arXiv:1806.10



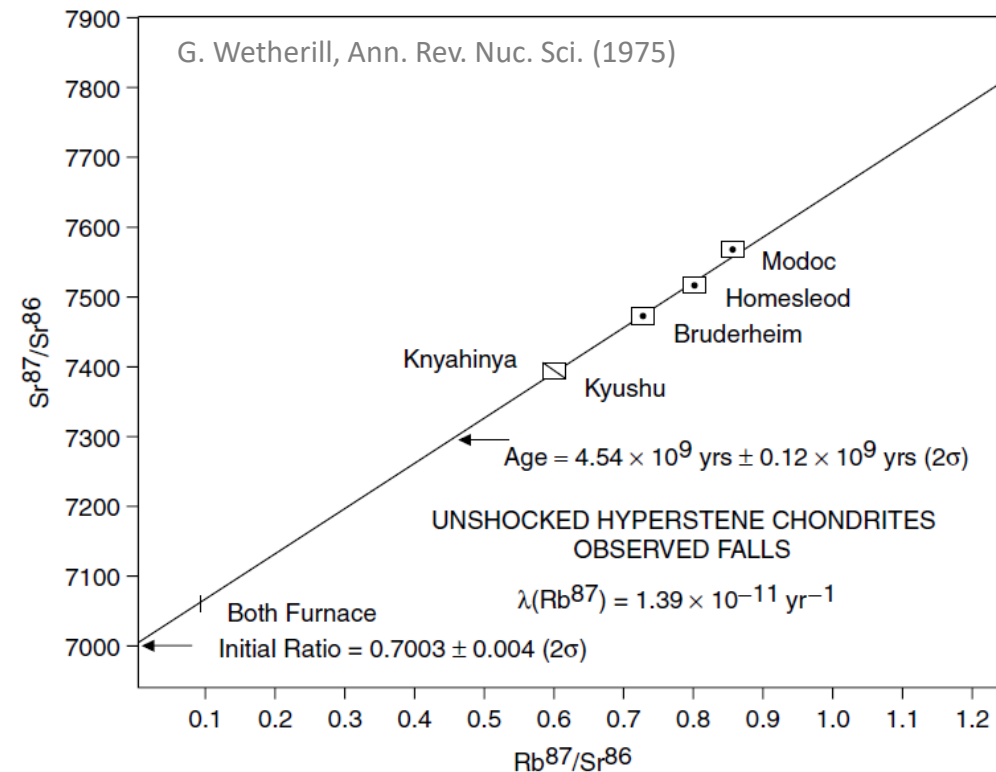


# Geochronometers

- Since the assumption  $N_2(0) = 0$  often has to be relaxed, another trick has to be applied
- If a stable isotope of the daughter element originally existed in the sample and isn't formed by the decay of something long-lived, then we at least know the ratio between the total number of nuclei in the decay chain to the number of nuclei of that stable isotope remained constant:

$$\frac{N_1(t) + N_2(t)}{N_S} = \frac{N_1(0) + N_2(0)}{N_S}$$

- Since  $N_1(0) = N_1(t)e^{\lambda_1 t}$ ,  $\frac{N_2(t)}{N_S} = \frac{N_2(0) + N_1(t)(1 + e^{\lambda_1 t})}{N_S}$
- Plotting  $\frac{N_2(t)}{N_S}$  vs  $\frac{N_1(t)}{N_S}$  for a sequence of measurements creates a straight line which  $N_2(0)$  and  $t$  can easily be determined from
- Common isotopic ratios used for dating earth rocks and meteorites include  $^{40}\text{Ar}/^{40}\text{K}$  ( $^{36}\text{Ar}$ ),  $^{207}\text{Pb}/^{235}\text{U}$  and  $^{206}\text{Pb}/^{238}\text{U}$  ( $^{204}\text{Pb}$ ),  $^{87}\text{Sr}/^{87}\text{Rb}$  ( $^{86}\text{Sr}$ )
- Modeling long-term chemistry of the elements within the sample is one challenge of this method

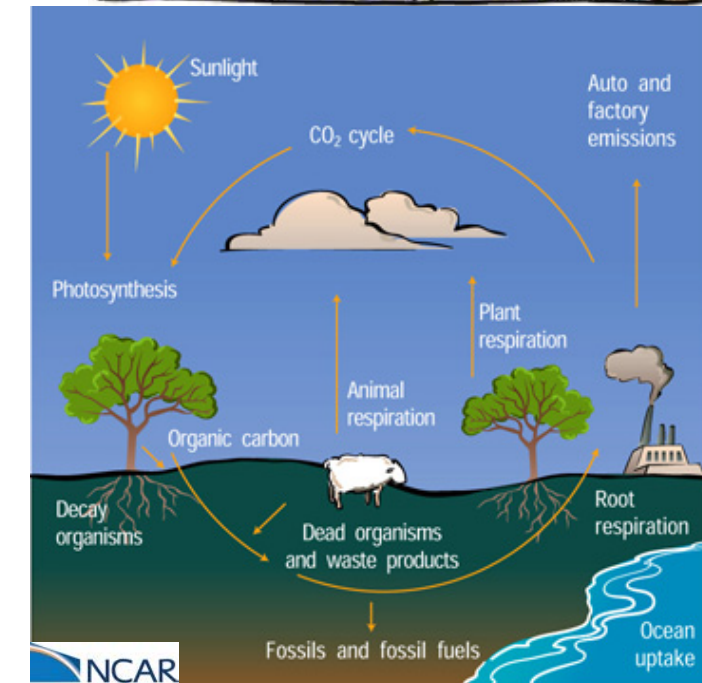
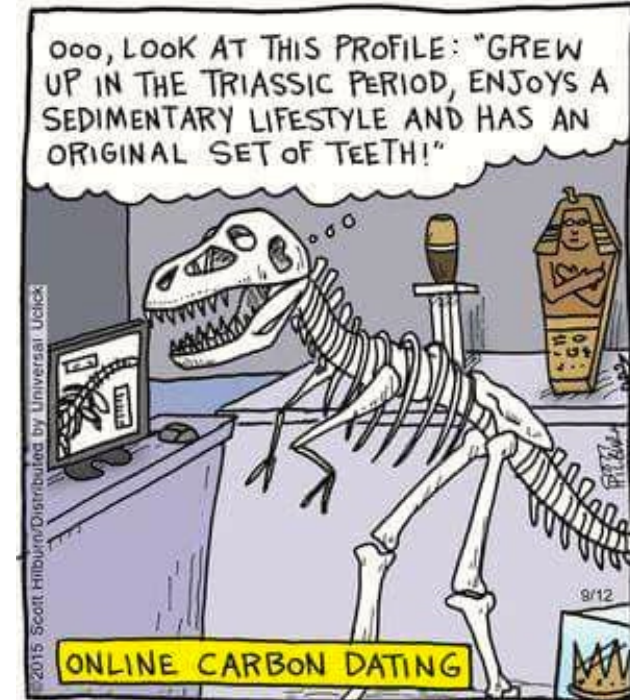


# Carbon Dating

- The most famous radionuclide dating method employs cosmogenic  $^{14}\text{C}$
- $^{14}\text{C}$  is constantly being created in the upper atmosphere by  $^{14}\text{N}(\text{n},\text{p})$ , induced by cosmic rays, and so it is in secular equilibrium
- Living organisms exchange carbon with the environment. Dead organisms do not.
- Thus, the  $^{14}\text{C}$  content decays away starting when the organism dies
- The  $^{14}\text{C}/^{12}\text{C}$  ratio is commonly used to date organic objects
- Activity measurements are good to  $\sim 10t_{1/2}(^{14}\text{C}) \approx 57\text{kyr}$
- Beyond this, accelerator mass spectrometry can be used to directly count carbon atoms
- Weapons tests complicated this a bit, since they produced  $^{14}\text{C}$ , but the amount created is relatively well known. Also, modulations in the cosmic ray flux must be accounted for.

*How do we date historical documents and paintings?  
We actually date when the plant used for the paper/canvas was killed.*

*For example, this is how we know the Shroud of Turin is only from  $\sim 1300\text{AD}$  (P.E.Damon et al. Nature 1989)*



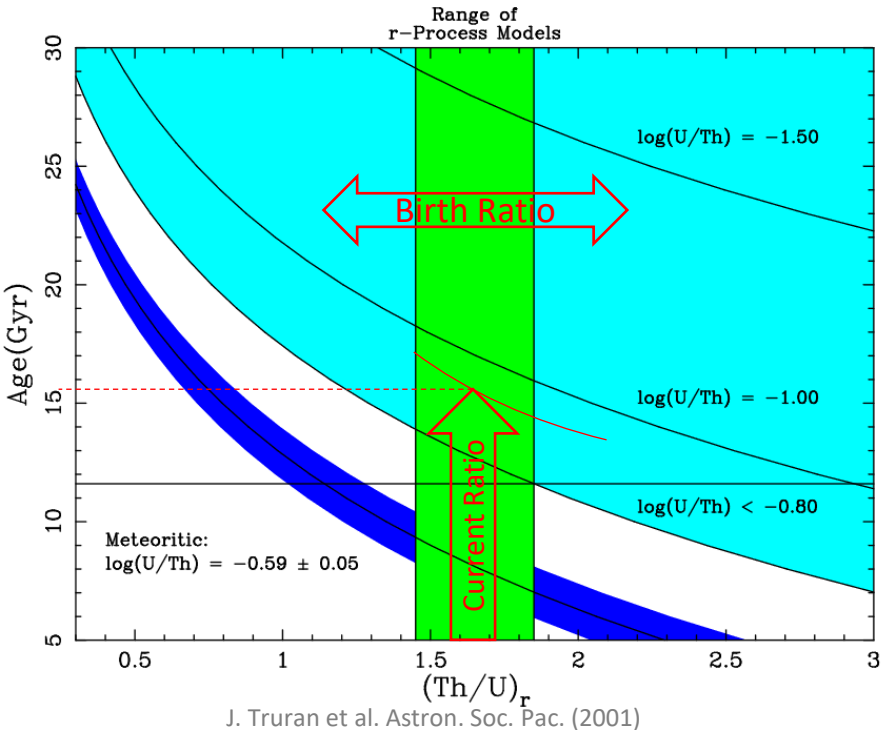
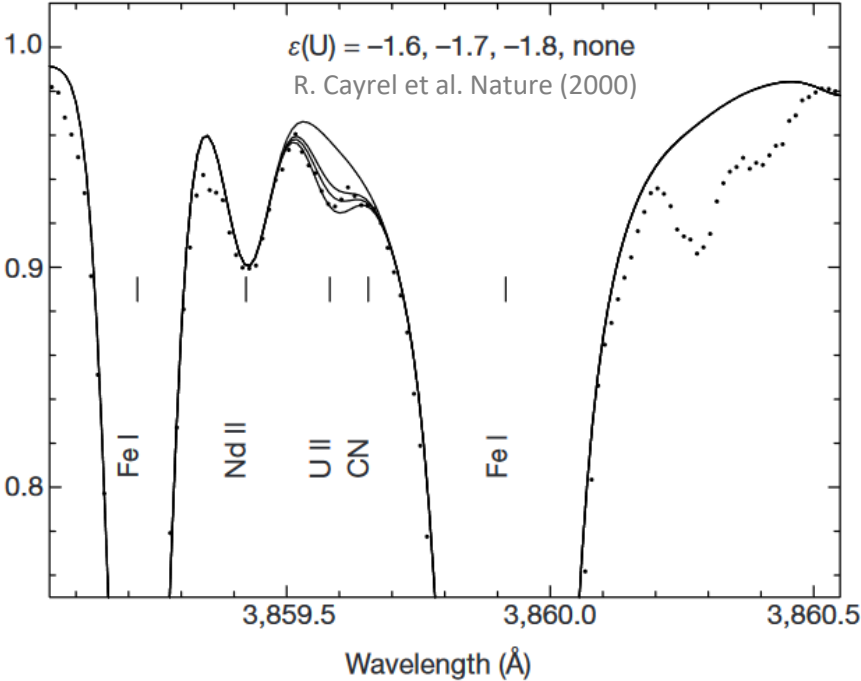


# Cosmochronometers

- Several long-lived nuclides have half-lives on the order of billions of years or more, i.e. cosmic timescales
- As such, abundance ratios can be used to date stars and therefore place limits on the age of the universe
- Stellar abundances are obtained by measuring absorption lines in stellar spectra
- Astrophysics model calculations are used to determine the original abundance ratios and the age of the star is obtained by determining how long it would take to result in the current abundance ratios
- For example, for a metal poor star with only r-process abundances, Os and/or Ir can be used as the stable nucleus that  $^{232}\text{Th}$  and  $^{238}\text{U}$  are referenced to:

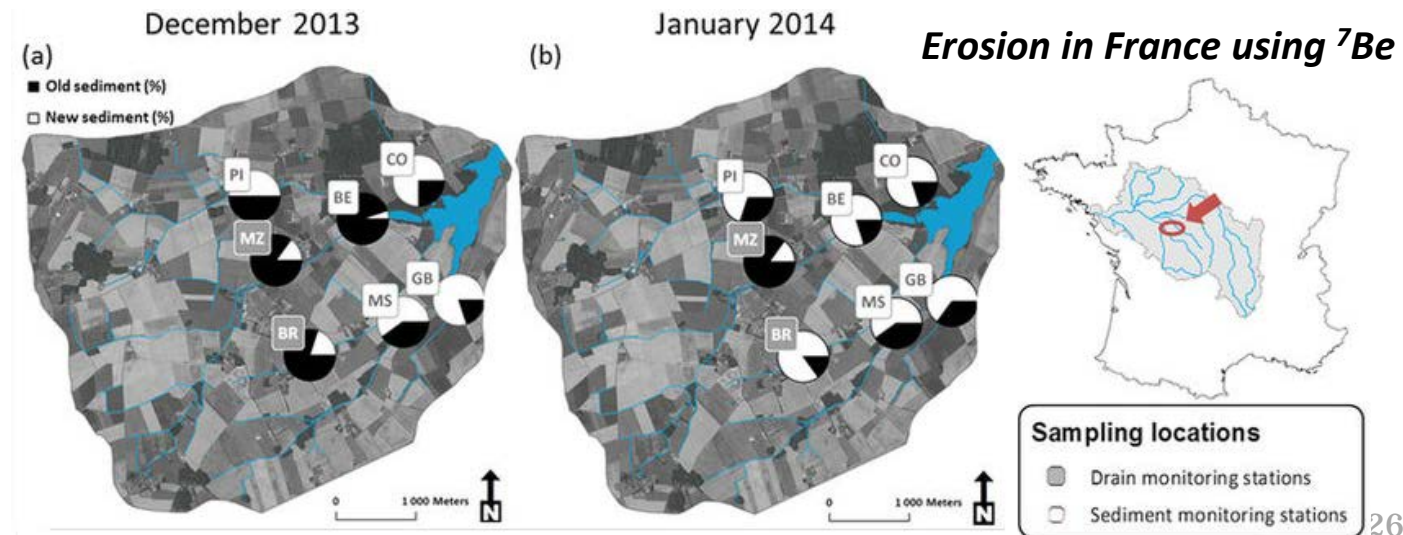
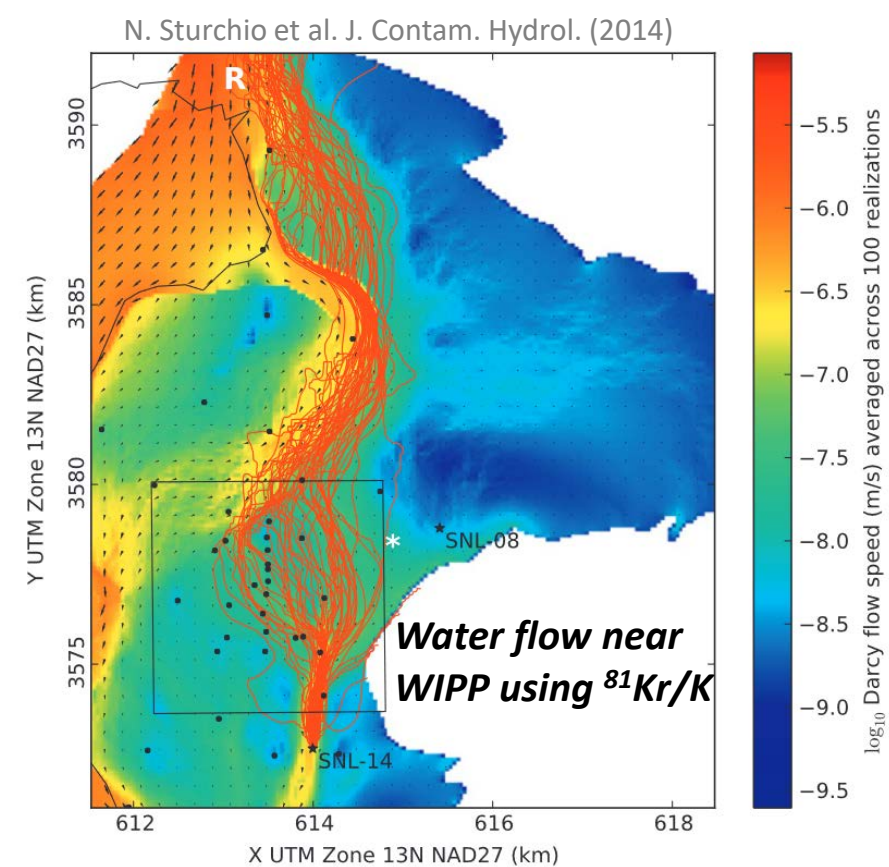
**Table 1 Ages derived for CS31082-001 as a function of production ratios**

Element pair	log (production ratio)	Ref.	log (observed ratio)	Derived age (Gyr)
U/Th	-0.255	4	$-0.74 \pm 0.15$	$10.6 \pm 3.3$
U/Th	-0.10	17	$-0.74 \pm 0.15$	$14.0 \pm 3.3$
U/Os	-1.27	4	$-2.19 \pm 0.18$	$13.6 \pm 2.7$
U/Ir	-1.30	4	$-2.10 \pm 0.17$	$11.8 \pm 2.5$



# Radiotracers

- Radioactive nuclei have the same chemical behavior as stable isotopes of that element, but are far easier to count
- Intentional or natural introduction of radioactive nuclei into environments can be used to track the location and/or movement of that particular element or the molecule the element is bound to
- A large range of applications are common, where the isotope employed depends on a range of factors, including availability, chemistry, half-life, and decay radiation
- Examples include:
  - $^3\text{H}$  for tracing young ground water flow
  - $^{18}\text{F}$  for PET scans
  - $^{24}\text{Na}$  for finding oil pipeline leaks
  - $^{32}\text{P}$  for DNA sequencing
  - $^{65}\text{Zn}$  for tracking heavy metals
  - $^{81}\text{Kr}$  for tracing old ground water flow
  - $^{99\text{m}}\text{Tc}$  for SPECT imaging
  - $^{192}\text{Ir}$  for checking welds



# Further Reading

- Chapters 3 & 4: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapter 2: Nuclear & Particle Physics (B.R. Martin)
- Chapter 14, Section 6: [Quantum Mechanics for Engineers \(L. van Dommelen\)](#)
- Chapter 5: Mathematics for Physicists (S. Lea)
- Chapter 13: [Introduction to Special Relativity, Quantum Mechanics, and Nuclear Physics for Nuclear Engineers \(A. Bielajew\)](#)