Lecture 2: *Nuclear Phenomenology*

- Mass models & systematics
- Charge & matter distributions
- Moments & deformation
- Spin, parity, & isospin

Empirically-motivated & focused theory

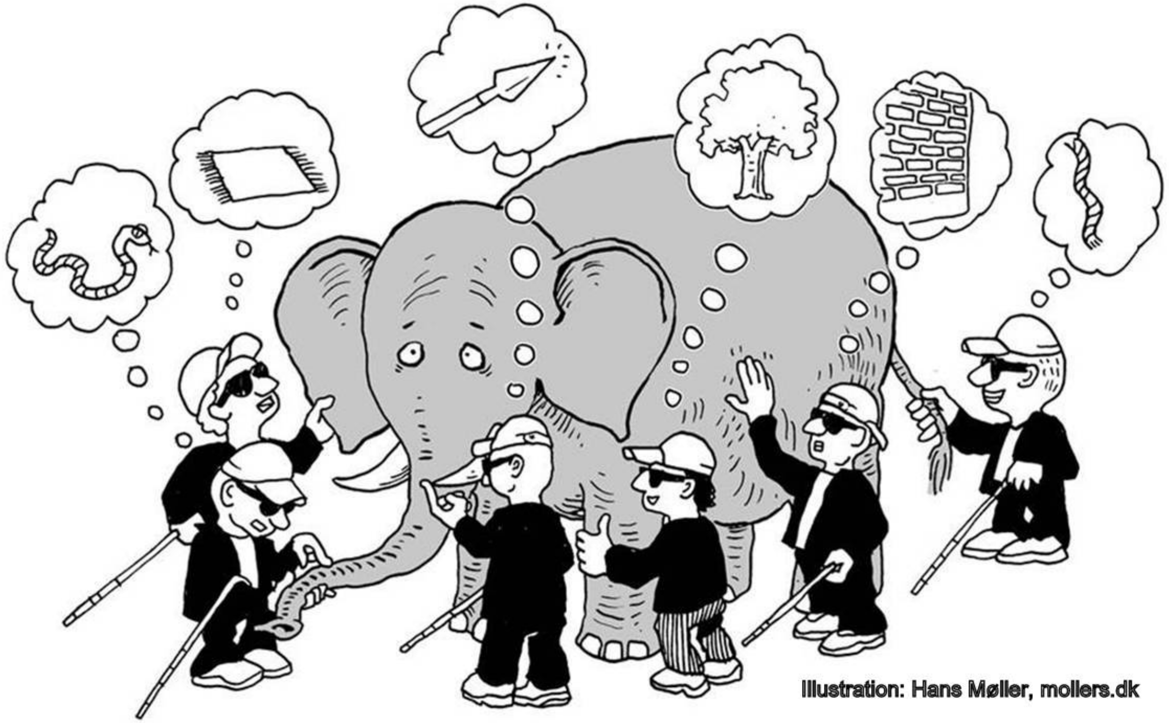
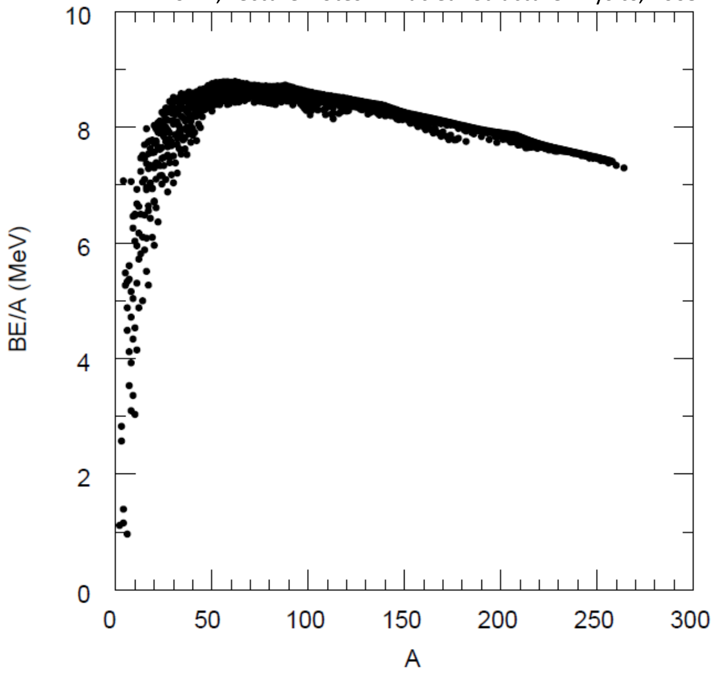


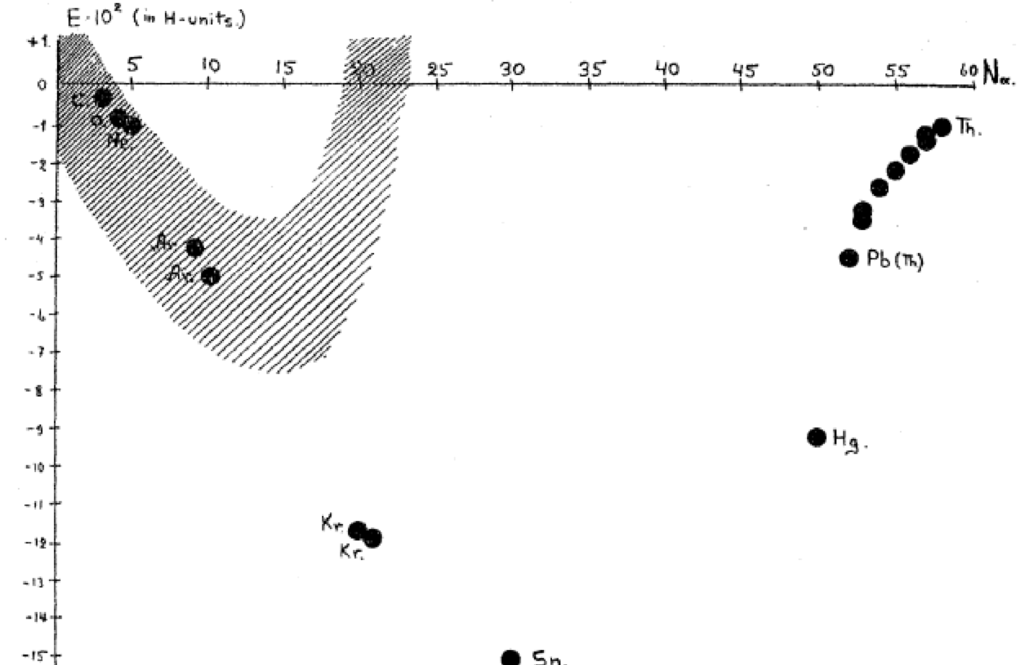
Illustration: Hans Møller, mollers.dk

How to explain the binding energy trend?

- Like, Gamow in 1929 & 1930 (Z. Phys. A), consider the nucleus as a group of nucleons (he assumed αs) in a close configuration
- Nucleons will bind together via some attraction force, like water molecules in a drop, but there will be some penalty for like-charges repelling each other.
- Comparing to data, do a pretty decent job, but clearly missing something(s)

B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.

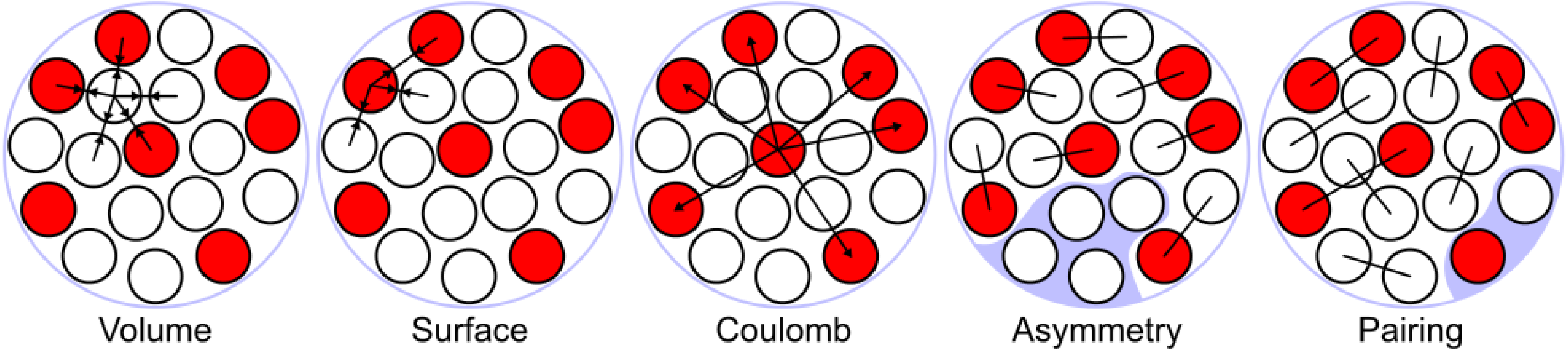




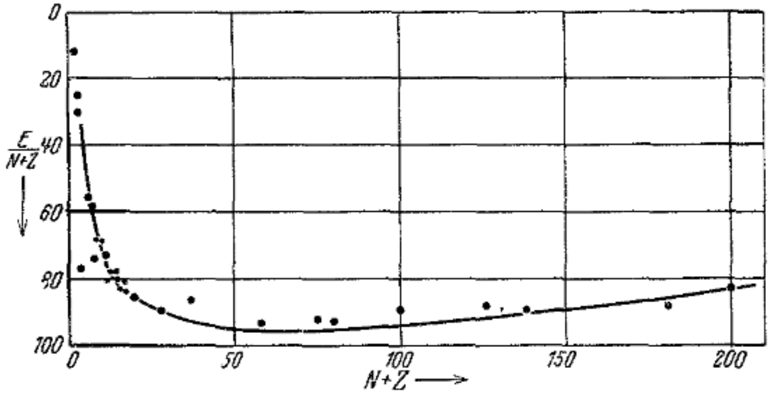
G. Gamow, Proc. Roy. Soc. A, 1930.

The Semi-Empirical Mass Formula (SEMF)

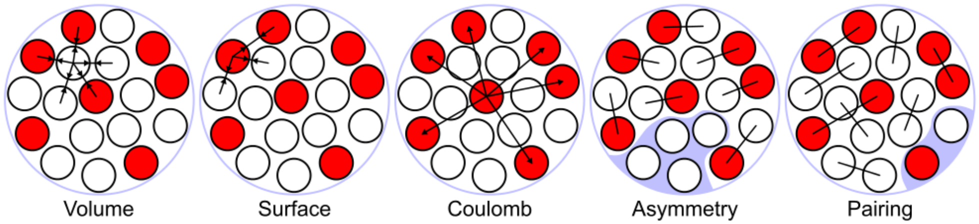
- More carefully consider all of the interactions going on in a nucleus:



- All nucleons are attracting each other via the strong force: generates some bulk binding energy
- However, nucleons near the surface don't have a neighbor: penalizes binding energy
- Protons are repel each other due to the Coulomb force: penalizes binding energy
- p-n attraction is stronger than p-p or n-n & p-n favored space-wise by Pauli exclusion: penalizes N-Z asymmetry
- Nucleons want a dance partner (make spin-0 pair): bonus for even-even, penalty for odd-odd, neutral for even-odd
- These considerations lead C. von Weizsäcker to develop the first usable theory for nuclear masses (Z.Phys. A 1935), now dubbed the semi-empirical mass formula, or SEMF if you're extra-cool



The Semi-Empirical Mass Formula



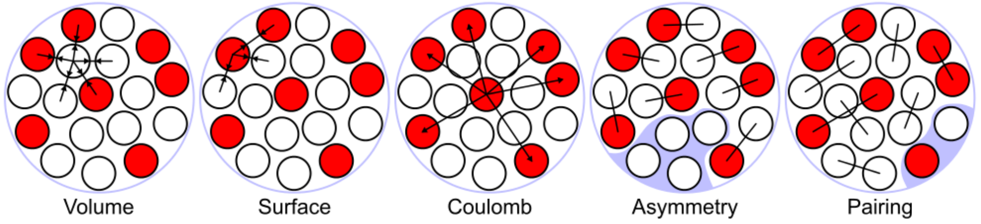
- BE(Z,A) = Volume - Surface - Coulomb - Asymmetry ± Pairing
- One mathematical parameterization* (*of many!*):

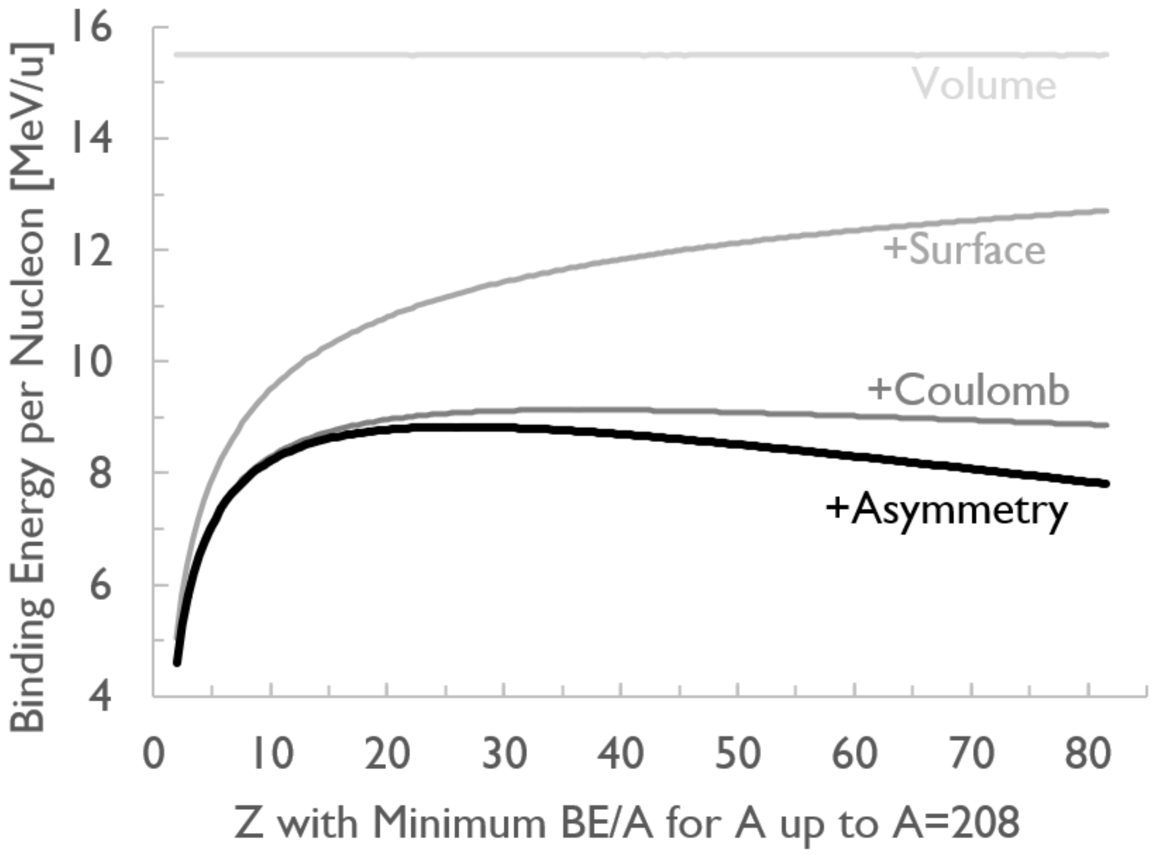
*from B. Martin, Nuclear and Particle Physics (2009)

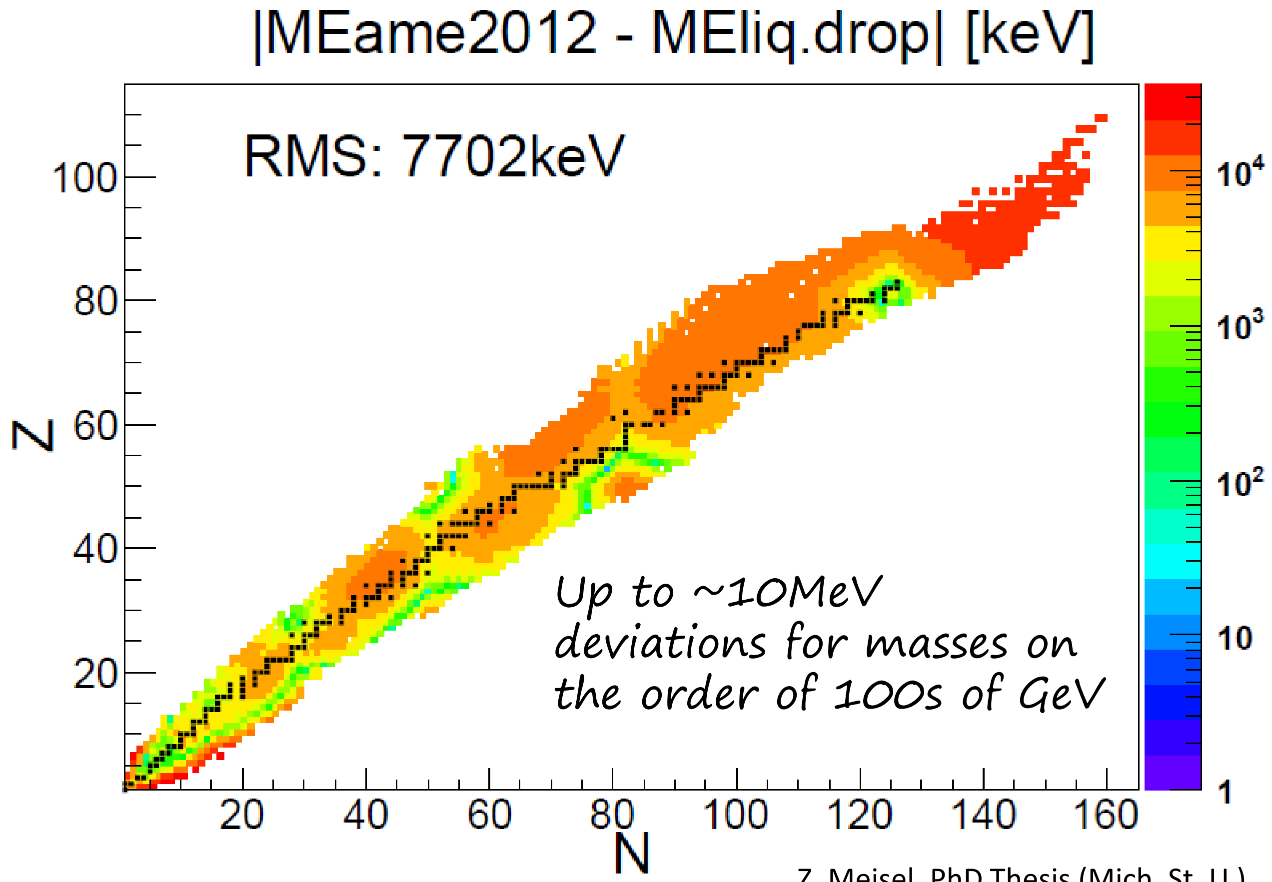
  - $BE(Z,A) = a_v f_v(A) - a_s f_s(A) - a_c f_c(Z,A) - a_a f_a(Z,A) + i a_p f_p(A)$
    - **Volume**: Nucleons cohesively bind, so: $\boldsymbol{f_v(A) = A}$
    - **Surface**: Since radius goes as $R \propto A^{1/3}$ and surface area goes as $SA \propto R^2$, $\boldsymbol{f_s(A) = A^{2/3}}$
    - **Coulomb**: Energy for a charged sphere goes as $\frac{q^2}{R}$ and $R \propto A^{1/3}$, so $\boldsymbol{f_c(Z,A) = \frac{Z(Z-1)}{A^{1/3}}}$
    - **Asymmetry**: Z=N favored (want Z=A/2) but lesser problem for large A, so $\boldsymbol{f_a(Z,A) = \frac{\left(Z-\frac{A}{2}\right)^2}{A}}$
    - **Pairing**: Favor spin-0 nucleon pairs & disfavor unpaired nucleons, empirically $\boldsymbol{f_p(A) = \left(\sqrt{A}\right)^{-1}}$
      - Even-Z, Even-N: $\boldsymbol{i = +1}$
      - Odd-Z, Odd-N: $\boldsymbol{i = -1}$
      - Even-Odd: $\boldsymbol{i = 0}$
  - a_i are fit to data

A mnemonic for remembering SEMF contributions is "VSCAP".

The Semi-Empirical Mass Formula



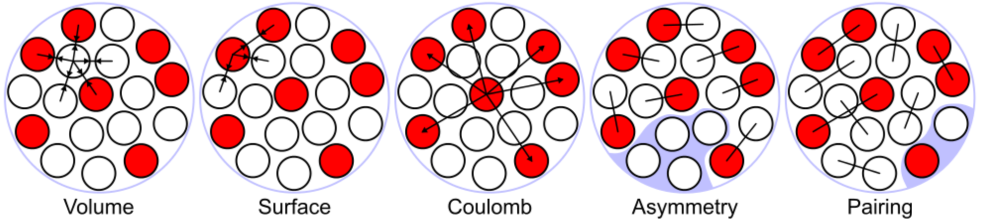




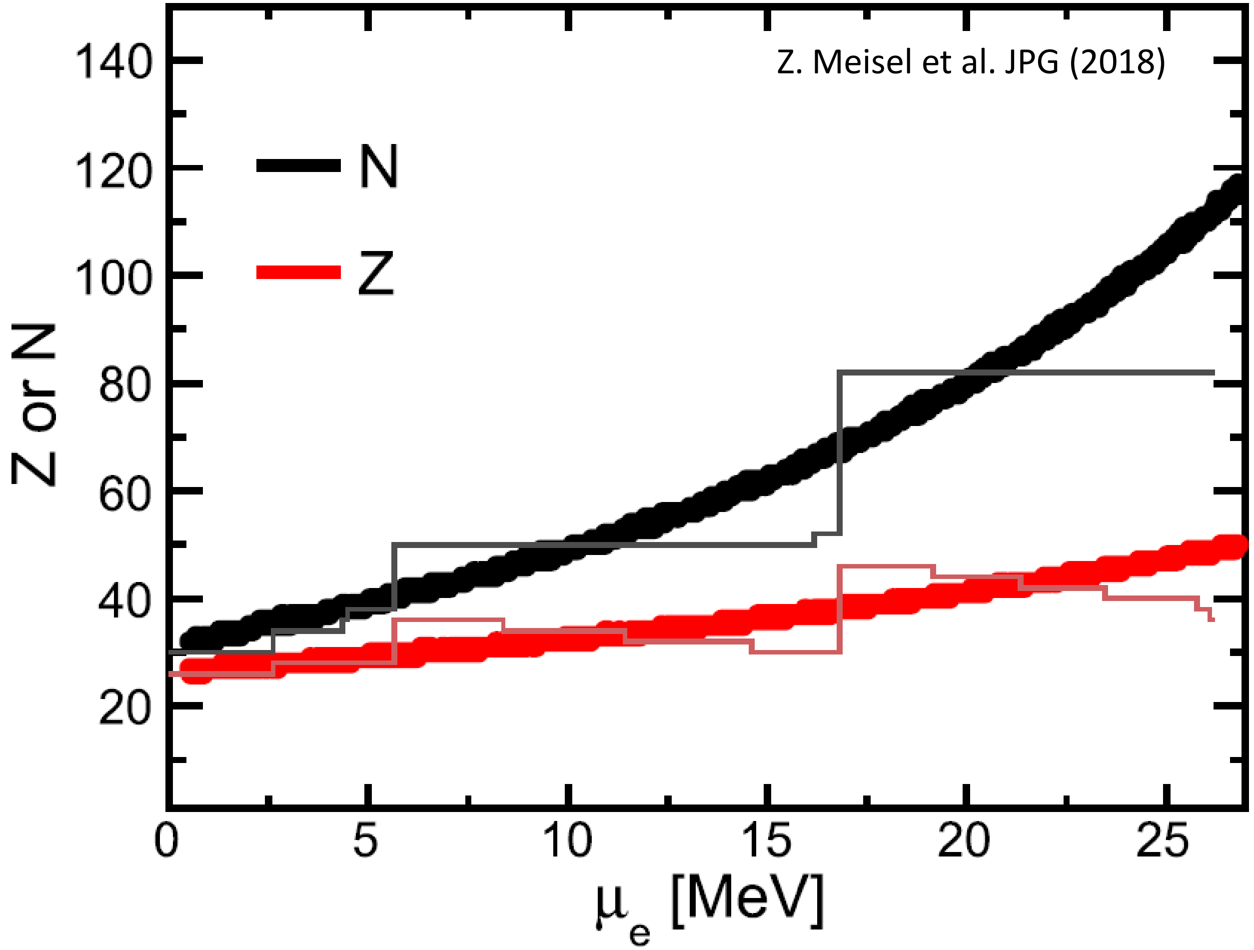
Z. Meisel, PhD Thesis (Mich. St. U.)

- Gives a pretty remarkable reproduction of the data: ~1MeV deviations compared to ~8MeV/A

The Semi-Empirical Mass Formula



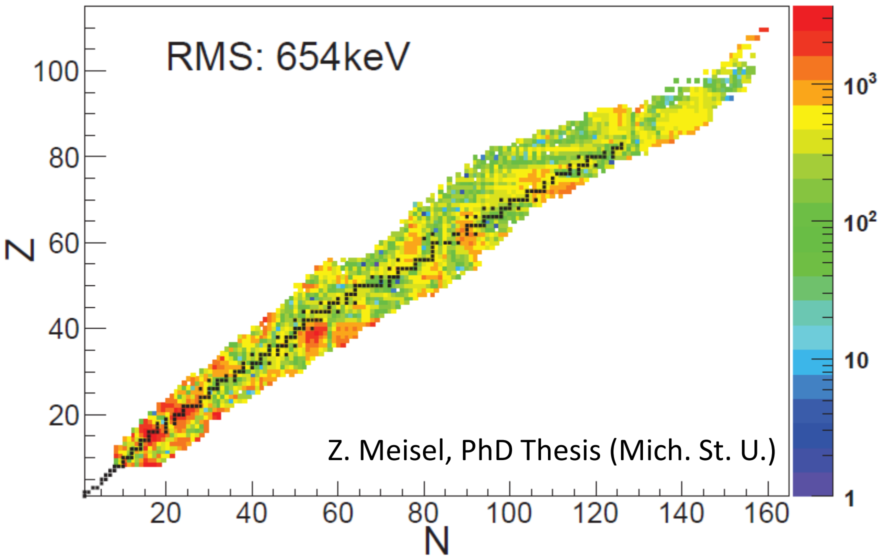
- Good enough for many modern applications, e.g. identifying the dominant effect setting the equilibrium composition of neutron star crusts



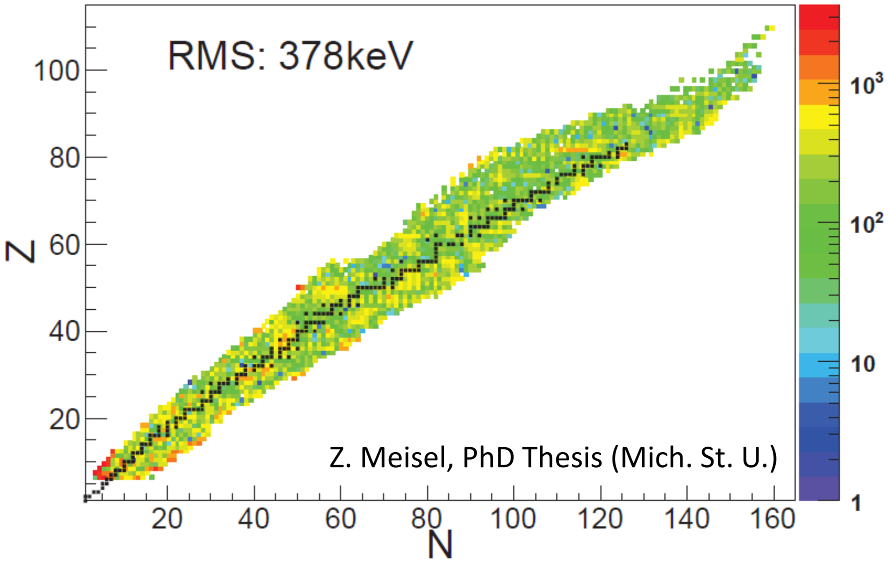
See also neutron star crustal heating (A. Steiner Phys. Rev. C 2012) and pristine crust composition (Roca-Maza & Piekarewicz PRC 2008)

Nuclear Mass Models: Common Recent-ish Global Models

FRDM: P.Möller et al., PRL (2012)



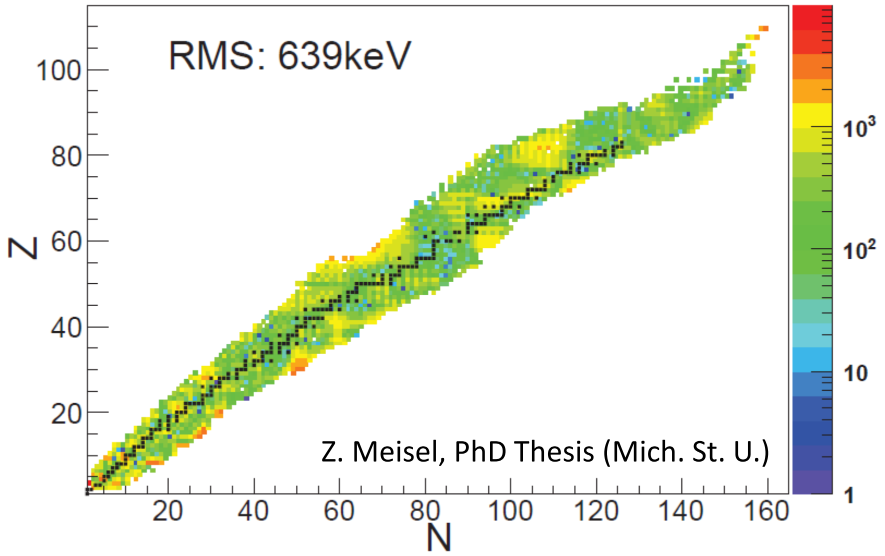
WS3: N.Wang & M.Liu, PRC R. (2011)



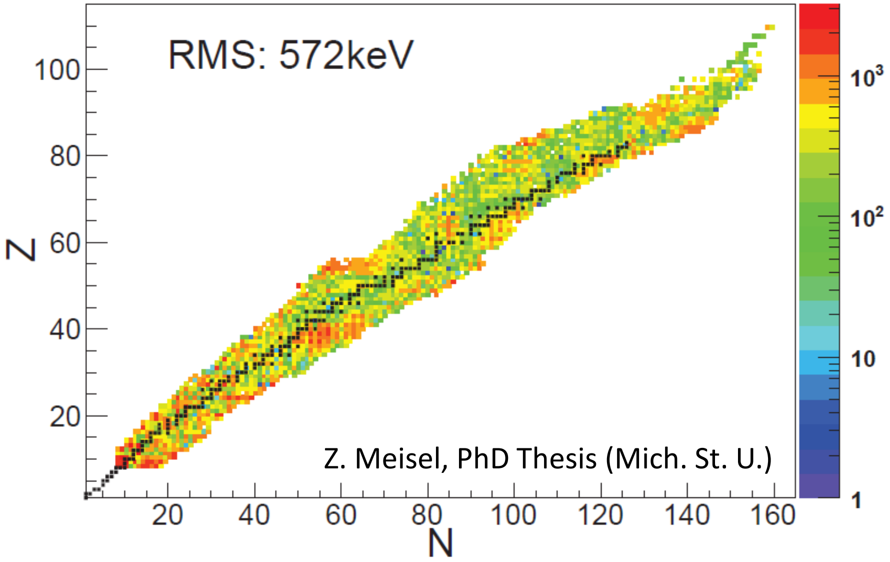
Others:

- Energy density functionals (EDF), e.g. M.Kortelainen et al. PRC (2012) (see the FRIB Mass Explorer)
- KTUY from Koura, Tachibana, Uno, Yamada, Prog. Theor. Phys. (2005)
- Local, algebraic relations:
  - Garvey-Kelson
  - Isobaric Mass Multiplet Equation
- Smooth mass-surface extrapolations from the AME
- many more …

Duflo-Zuker: J.Duflo & A.Zuker, PRC R. (1995)



HFB: S.Goriely, N.Chamel, J.Pearson PRC (2010)



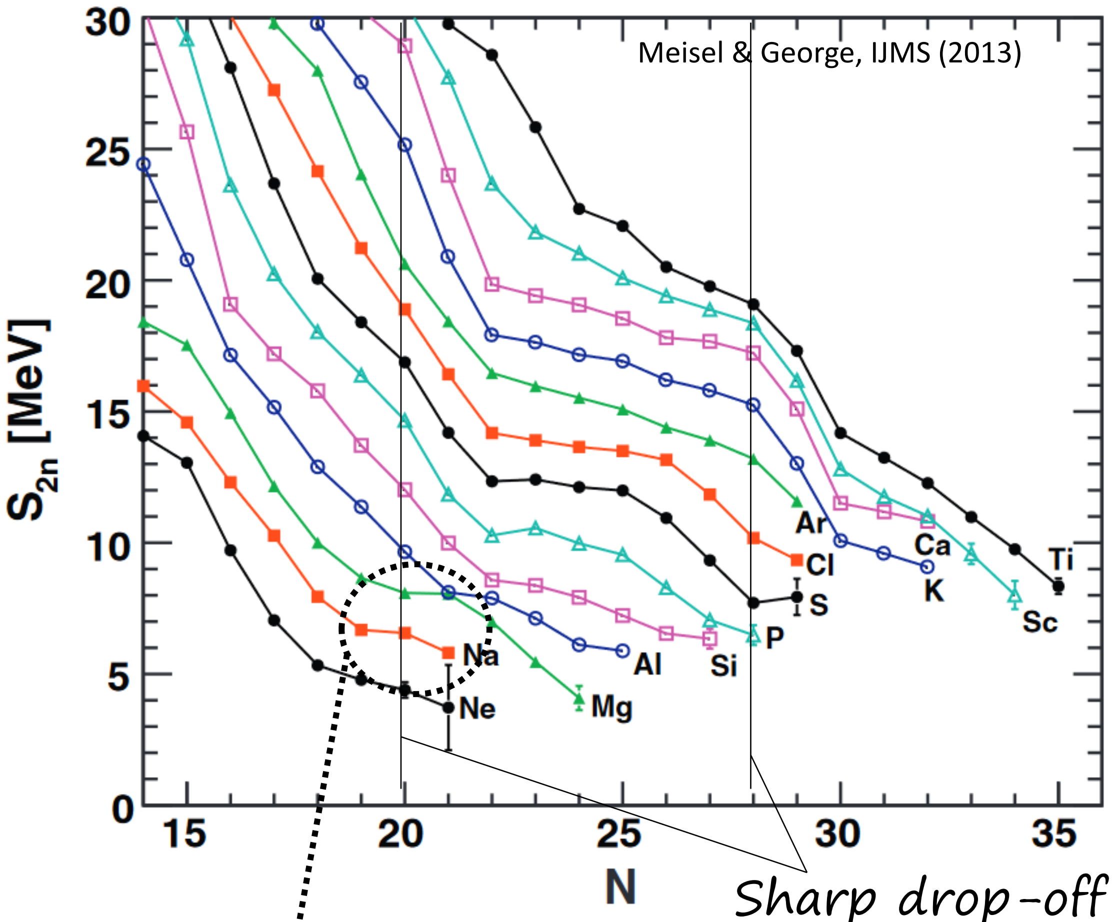
The main point is that even the best theoretical descriptions struggle to do better than the ~100keV level. Structure & astrophysics often need keV-level precision.

Nuclear Mass Differences

- The energy released in a nuclear reaction is the “Q-value”
  - $Q = \sum_{reactants} ME(Z, A) - \sum_{products} ME(Z, A)$,
  - For example, $Q_{^{68}Se(p,\gamma)^{69}Br} = ME\left({}^{68}Se\right) + ME(p) - ME\left({}^{69}Br\right)$
  - $= (-54.189 MeV) + (7.288 MeV) - (-46.260 MeV)$
  - $= -0.641 MeV$
- Considering the case above, we calculated the energy released by adding one proton to ^{68}Se, which corresponds to the energy it takes to remove one proton from ^{69}Br, a.k.a. the “proton separation energy”, S_p
- Similarly, can calculate the energy to remove 1-neutron S_n, two-protons S_{2p}, or two-neutrons S_{2n}
  - $S_p(Z, N) = ME(Z-1, N) + ME(p) - ME(Z, N)$
  - $S_n(Z, N) = ME(Z, N-1) + ME(n) - ME(Z, N)$
  - $S_{2p}(Z, N) = ME(Z-2, N) + 2 * ME(p) - ME(Z, N)$
  - $S_{2n}(Z, N) = ME(Z, N-2) + 2 * ME(n) - ME(Z, N)$

Nuclear Masses & Nuclear Structure

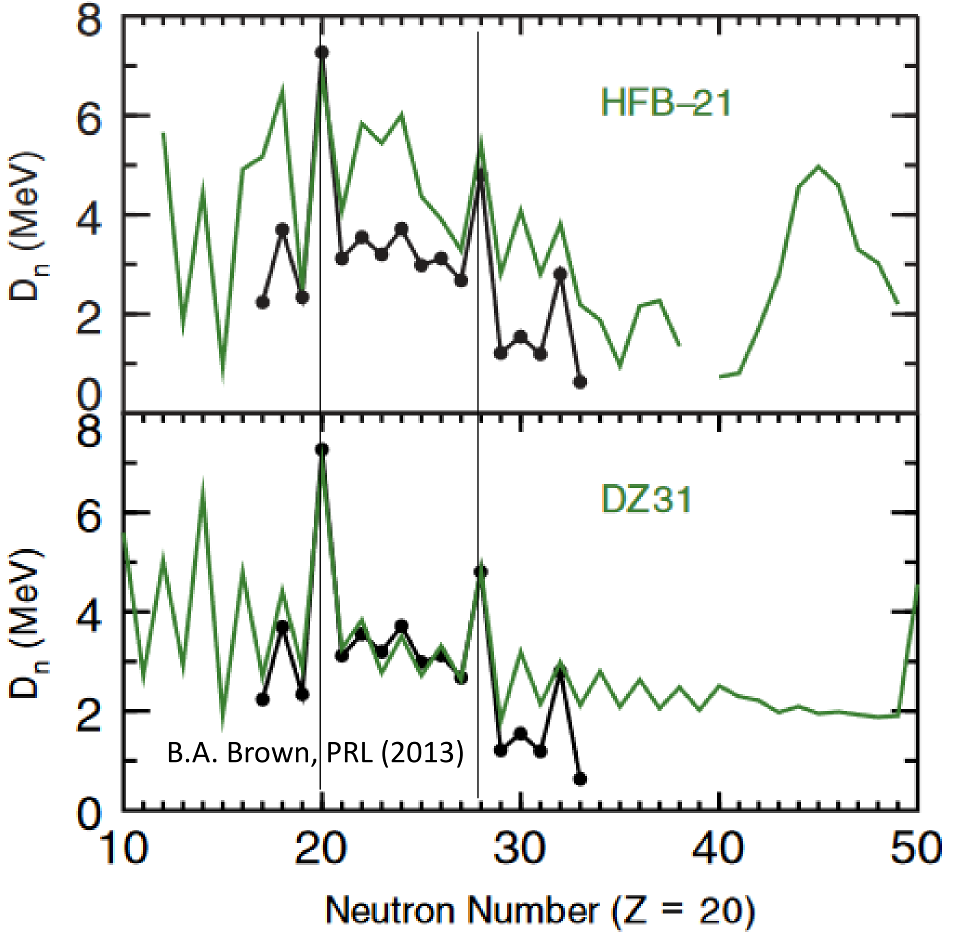
Information is encoded in separation energies
(& separation energy differences)



Sharp drop-off after magic numbers

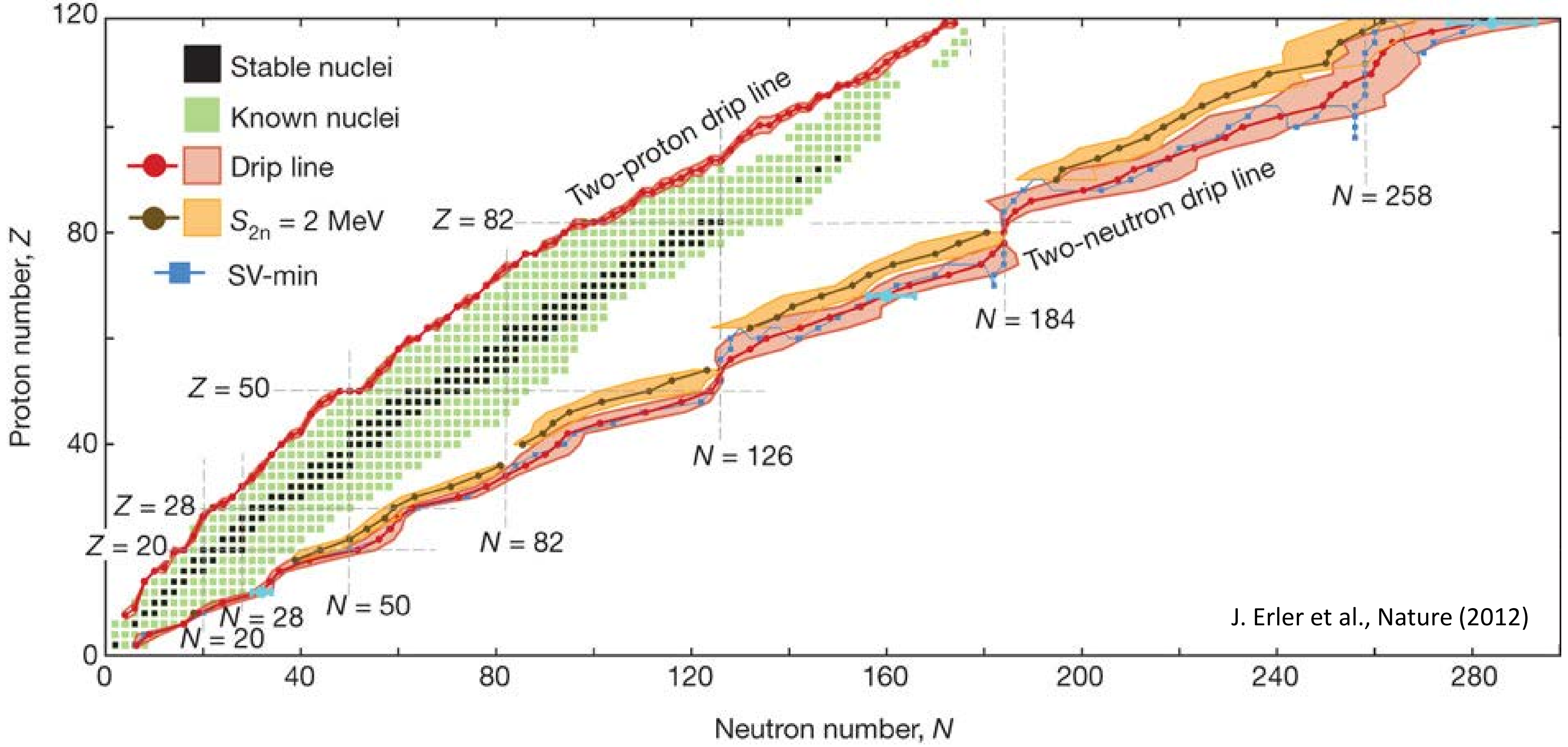
Deviation provided 1st evidence for "island of inversion"
[C. Thibault et al. PRC 1975]

See that some mass models do more poorly than it looks when looking only at mass predictions.



$$D_n(Z,A) = (-1)^{N+1}[S_n(Z,A+1) - S_n(Z,A)]$$

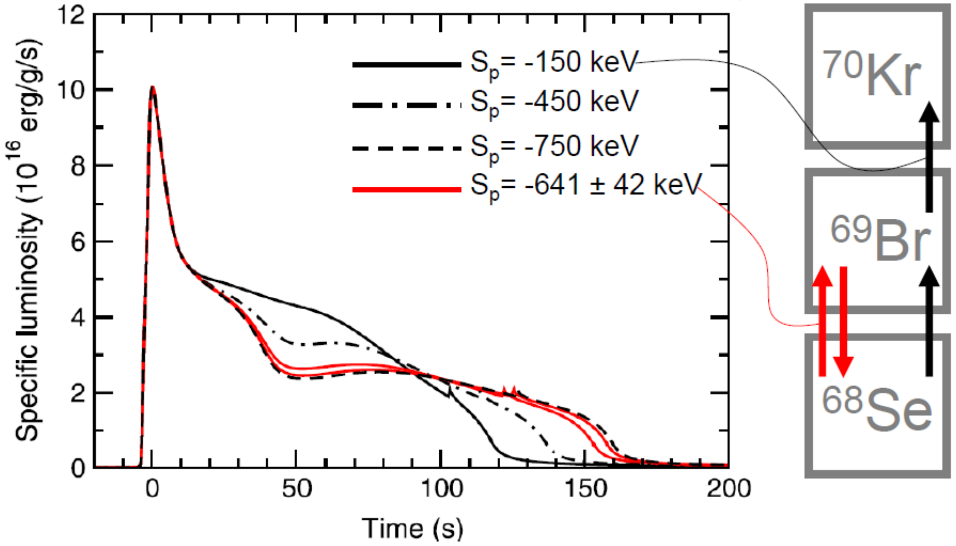
Nuclear masses define the nuclear landscape



Nuclear Masses in Astrophysics (Selected Examples)

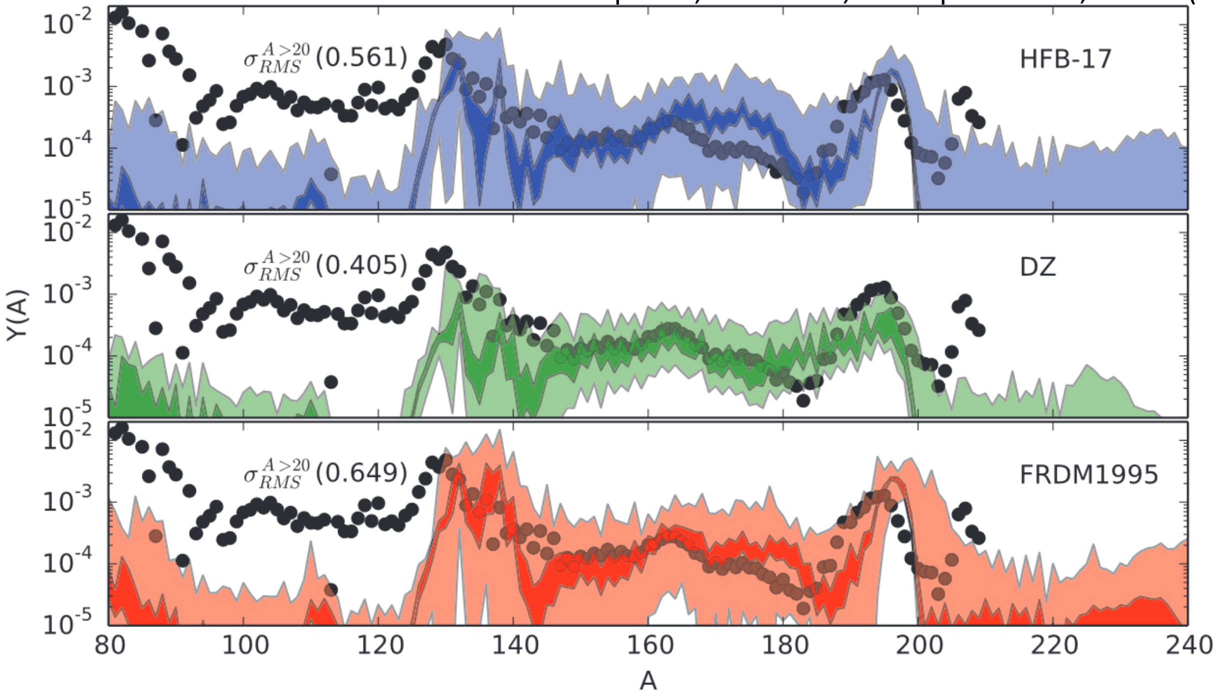
X-ray burst light curves

M. del Santo et al., Phys. Lett. B (2014)

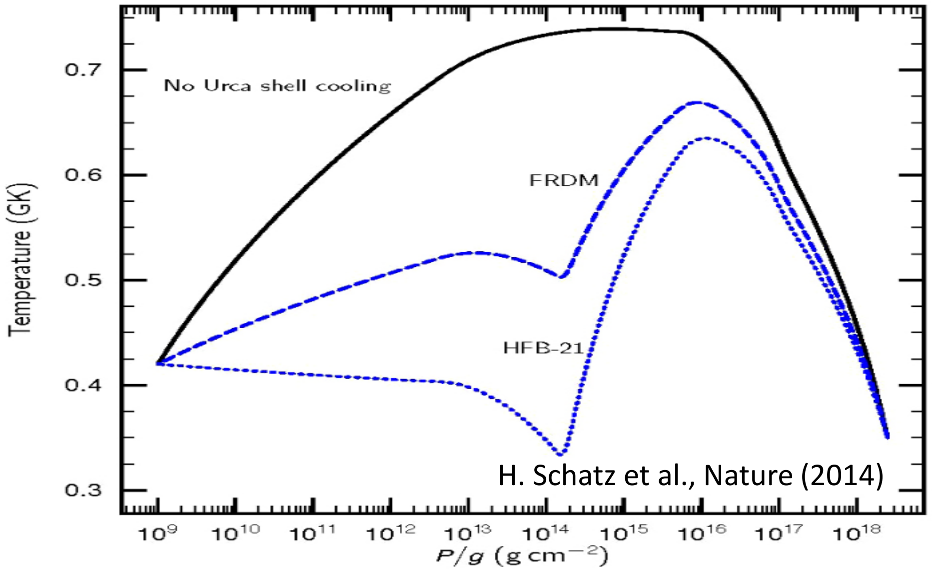


r-process abundance yields

M. Mumpower, R. Surman, & A. Aprahamian, EPJWC (2015)



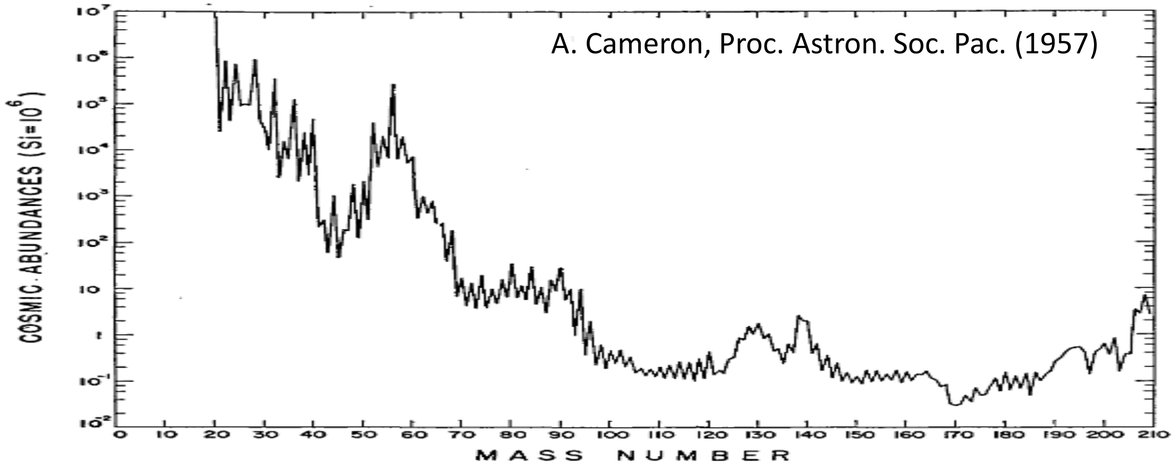
Neutron star crust temperature



H. Schatz et al., Nature (2014)

Cosmic Abundance Pattern

A. Cameron, Proc. Astron. Soc. Pac. (1957)



Charge & Matter Distributions

- The nucleus can't be a point object.
  - Nucleons are fermions: Pauli exclusion forbids putting several in the same place.

- The radial distribution of charge & matter are probed via scattering.
  - Electrons are best for the charge radius, because structure-less but charged
  - Neutrons are best for the matter radius, because not charged and simple-ish structure

Nuclear charge distribution

Review: R. Hofstadter, Rev. Mod. Phys. (1956)

- Scattering of point charged particles (with charges *e* and *Ze*) is well-described by the relativistic correction to Rutherford scattering, i.e. the Mott scattering cross section
  - $\sigma_{Mott}(\theta_{cm}) = \left(\frac{Ze^2}{2E_{cm}}\right)^2 \frac{\left(cos\left(^{\theta cm}/_2\right)\right)^2}{\left(sin\left(^{\theta cm}/_2\right)\right)^4}$

another notation for σ(θ) is dσ/dΩ(θ)

- It turns out, from considering scattering of a plane wave off of an extended object and solving for the outgoing spherical wave, one realizes a form factor needs to be included*:

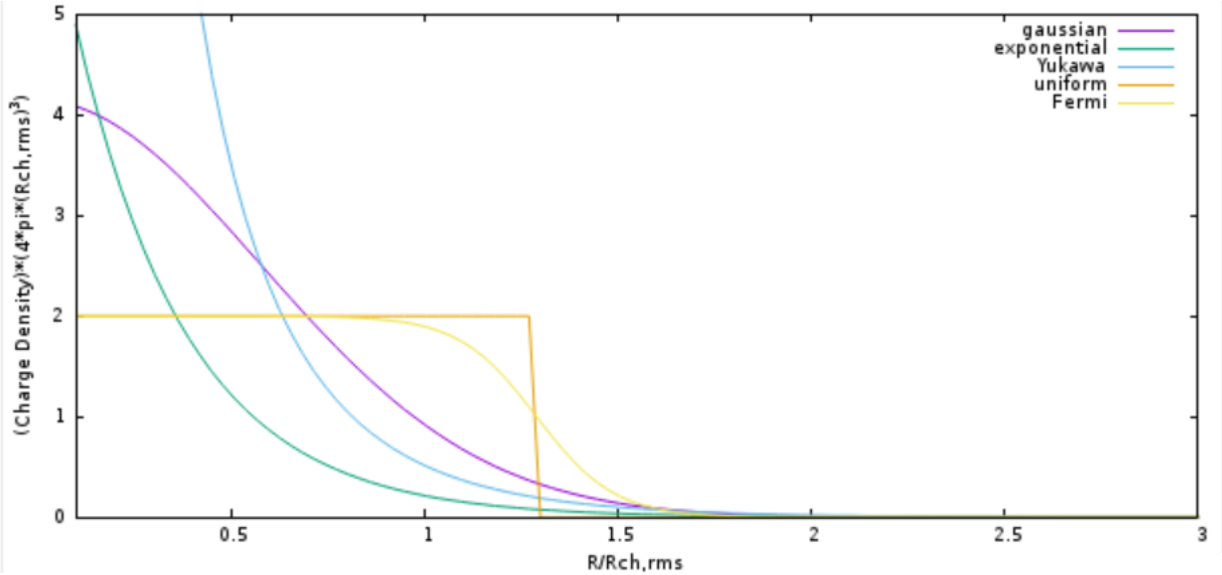
*It's there for a point object, but it's a delta function

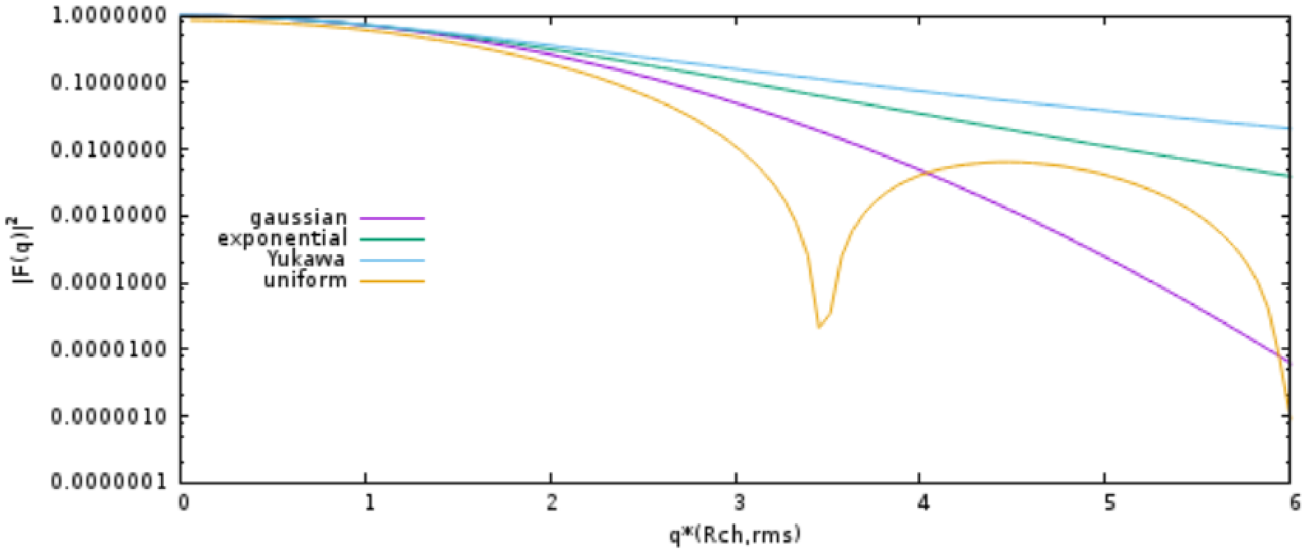
  - $\sigma_{scatt}(\theta_{CM}) = \sigma_{Mott}(\theta_{CM}) * \left|\int_{nuclear\ volume} \rho_{charge}(r) e^{i\boldsymbol{q}\cdot r}\right|^2$ where $\boldsymbol{q}$ is the momentum transfer
  - $\sigma_{scatt}(\theta_{CM}) = \sigma_{Mott}(\theta_{CM}) * |F(\boldsymbol{q}^2)|^2$
  - i.e. the form factor $F(\boldsymbol{q}^2)$ is the Fourier transform of the charge distribution

Nuclear charge distribution

Review: R. Hofstadter, Rev. Mod. Phys. (1956)

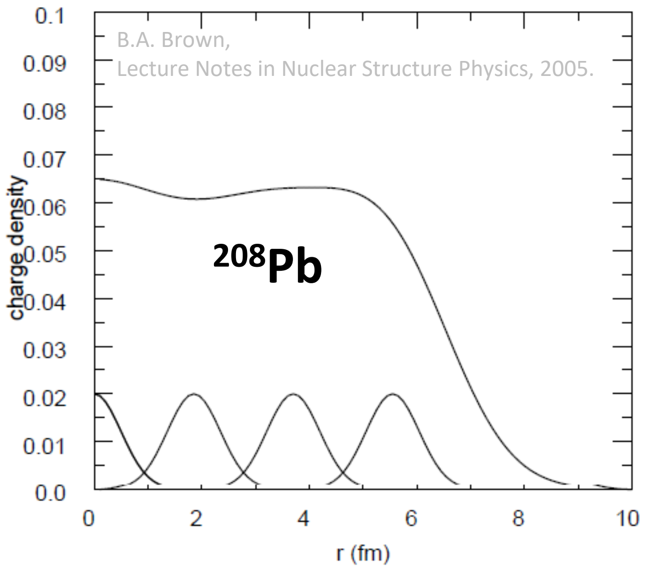
- Luckily, sharp people have solved for form factors corresponding to typical charge distributions
  See, e.g. Table 1 of R. Hofstadter, Rev. Mod. Phys. (1956)





- Comparing measured $\sigma_{scatt}(\theta)$ to calculations reveals a Fermi-like distribution:
  - $\rho_{ch,Fermi}(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{a}\right)}$
  - The associated form factor is absurdly cumbersome, so usually a sum of Gaussians is used instead, since they have more manageable $F(q)$:

Typical values are $R \sim 1.1 * A^{1/3}$fm, $a \sim 0.5$fm, $\rho_0 \sim 0.07$



Nuclear charge distribution

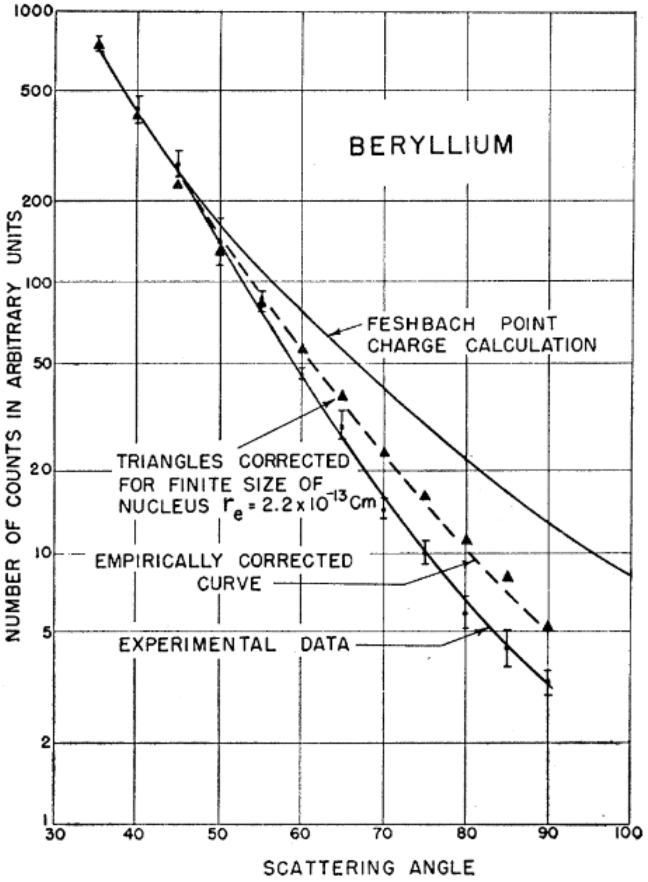
Review: R. Hofstadter, Rev. Mod. Phys. (1956)

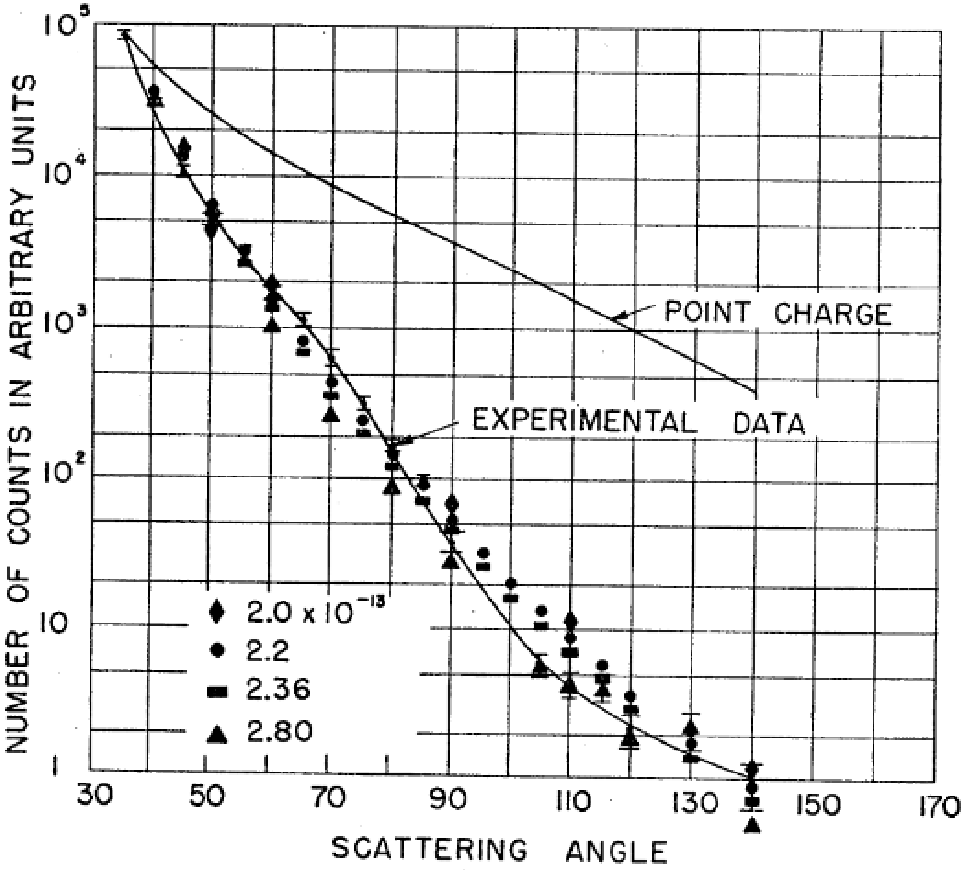
- Comparing measured $\sigma_{scatt}(\theta)$ to calculations w/ various $F(\boldsymbol{q})$ reveals a Fermi-like distribution

(R. Woods & D. Saxon, Phys. Rev. (1954))

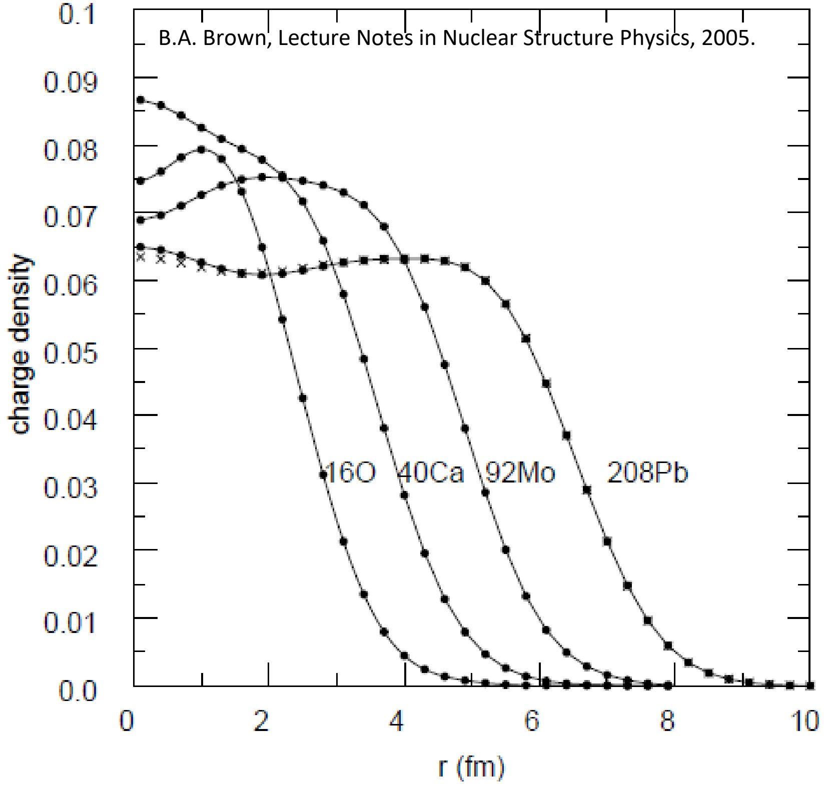
125MeV e⁻ on Be **125MeV e⁻ on Au**

R. Hofstadter, H. Fechter, & J. McIntyre, Phys. Rev. (1953)





Measured ρ_{ch} examples

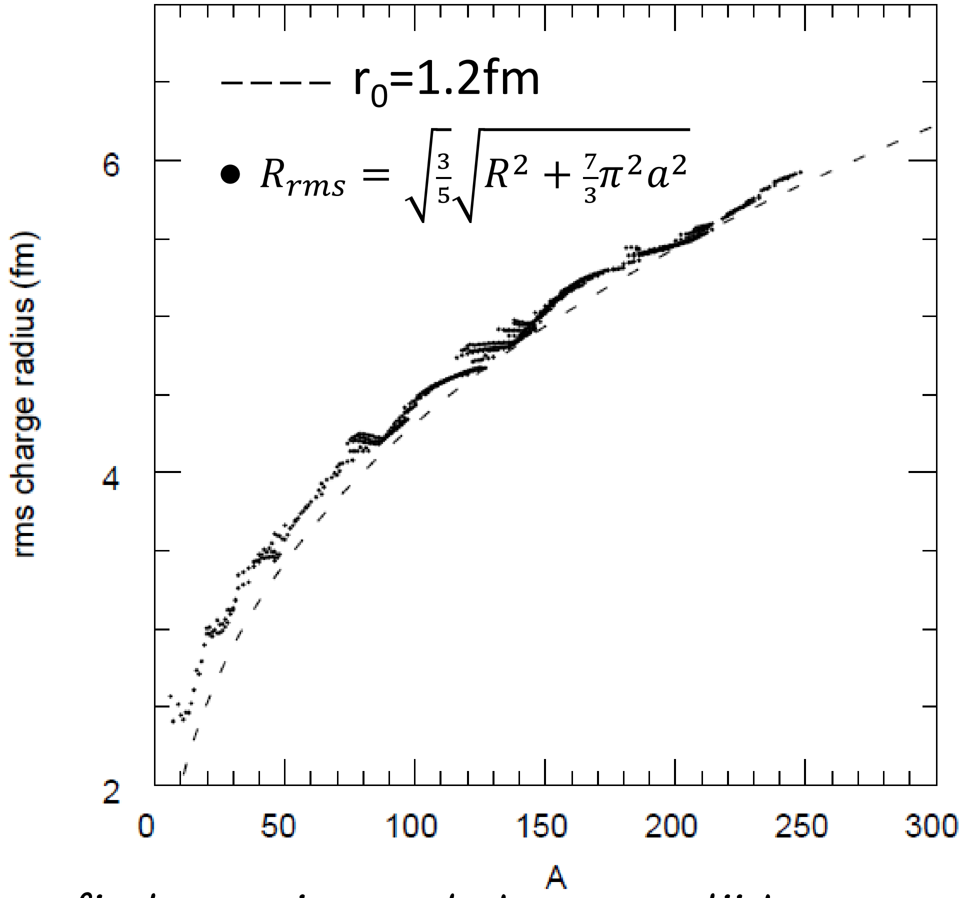


Many estimated distributions can be obtained at the Nuclear Charge Density Archive: http://faculty.virginia.edu/ncd/, based on the data of Atomic and Nuclear Data Tables, Volumes 14, 36 and 60

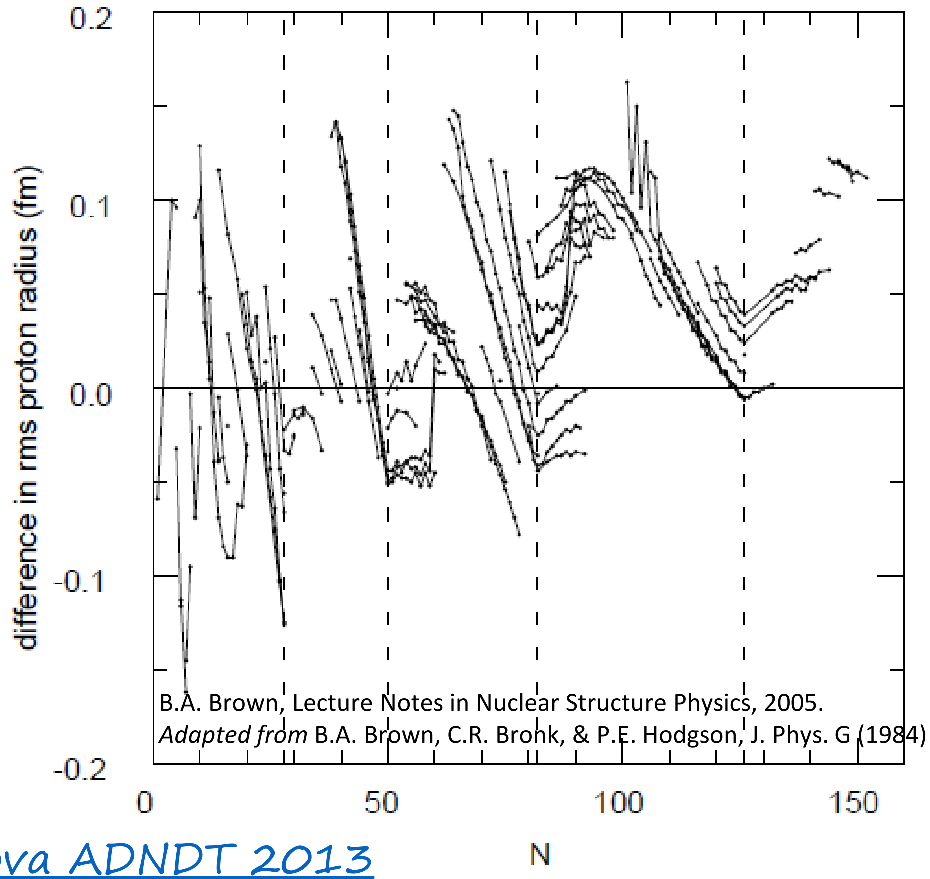
What about our handy relationship for R(A)!?

- Recall, an estimate for the nuclear radius was stated last time as: $R(A)=r_0A^{1/3}$
- Can compare RMS radius from Fermi distribution, $\{\rho_{ch,Fermi}(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{a}\right)}\}$, to R(A):

B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.
Adapted from B.A. Brown, C.R. Bronk, & P.E. Hodgson, J. Phys. G (1984)



...interesting trends in the residual



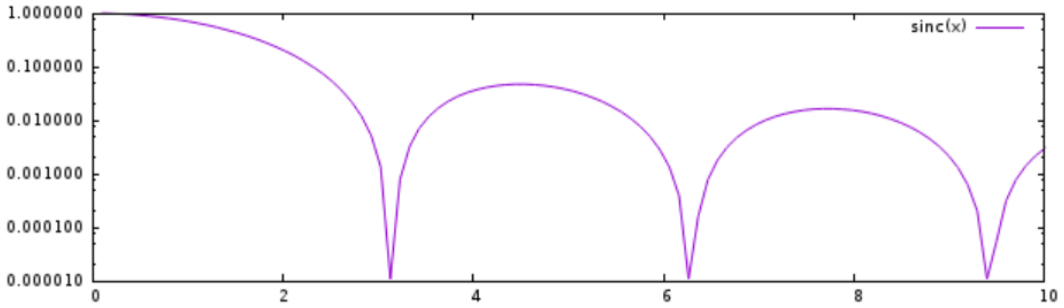
You can find experimental charge-radii here: [Angeli & Marinova ADNDT 2013]

Nuclear **matter** distribution

- The low-budget technique to obtain the matter density, $\rho_{matt}(r)$, would be:
  - $\rho_{matt}(r) = \rho_{ch}(r) * ({}^{A}/_{Z})$
  - Since $\rho_{ch}(0)$ steadily decreases with A, this leads to a near constant $\rho_{matt}(0)$ for all nuclides, which turns out to be $\rho_{matt}(0) \sim 0.16$nucleons/fm^3

A recent measurement of the neutron radius used neutrino(!) scattering (Cadeddu et al. PRL 2018)

- Those with more discerning tastes prefer actual data, which is generally* obtained via neutron scattering, but can be using any hadron
- The appreciable de Broglie wavelength of the probe (e.g. a neutron) and significant contributions from strong-force interactions, mean that the plane wave of the probe will undergo diffraction, as in optics
- A light probe (e.g. n, p, d) at appreciable energy (e.g. 10's of MeV), … is sort of like light passing by an absorbing disk*: Get Fraunhofer diffraction
  - $\sigma(\theta) \propto sinc^2(a\theta)$



**If you're a glutton for punishment, the mathematics for diffraction from an absorbing disk is here: [R.E. English & N. George, Applied Optics (1988)]*

Nuclear **matter** distribution

- The analogy is taken further with the “optical model”
- Like a photon, incoming nucleons can be scattered or absorbed by the scattering medium (i.e. the target)
- In optics, the imaginary part of the refractive index accounts for absorption of the incident light wave
- Here, the imaginary part of a complex potential (which describes the interaction between the projectile and target) corresponds to all inelastic reactions
  [As a refresher, elastic means kinetic energy is conserved. Everything else is inelastic]
- An important feature of the optical model is that the main property of relevance for the target is the nuclear size, so an interaction potential for similar target and a similar projectile & energy should do a decent job
- This allows for “global optical models”,
  e.g. Perey & Perey, Phys. Rev. 1966
    and
    Koning & Delaroche, Nuc. Phys. A 2003

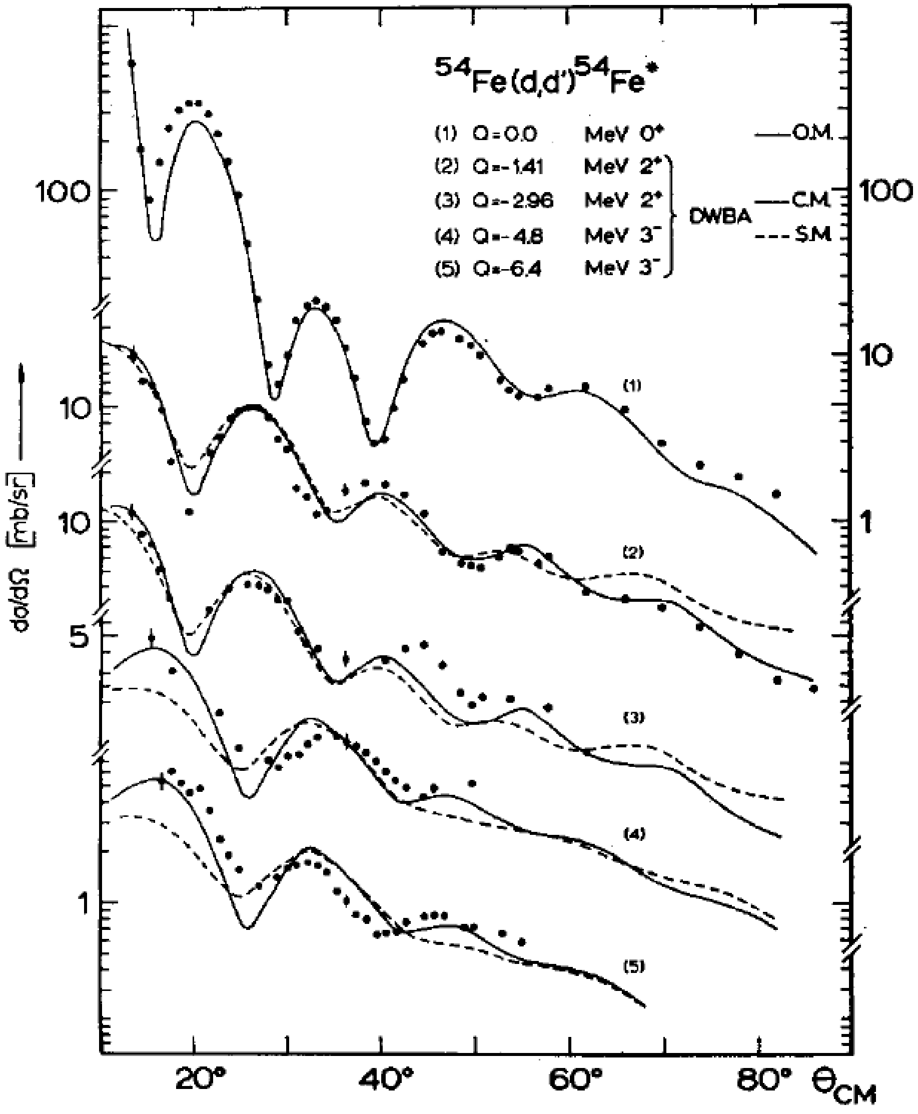
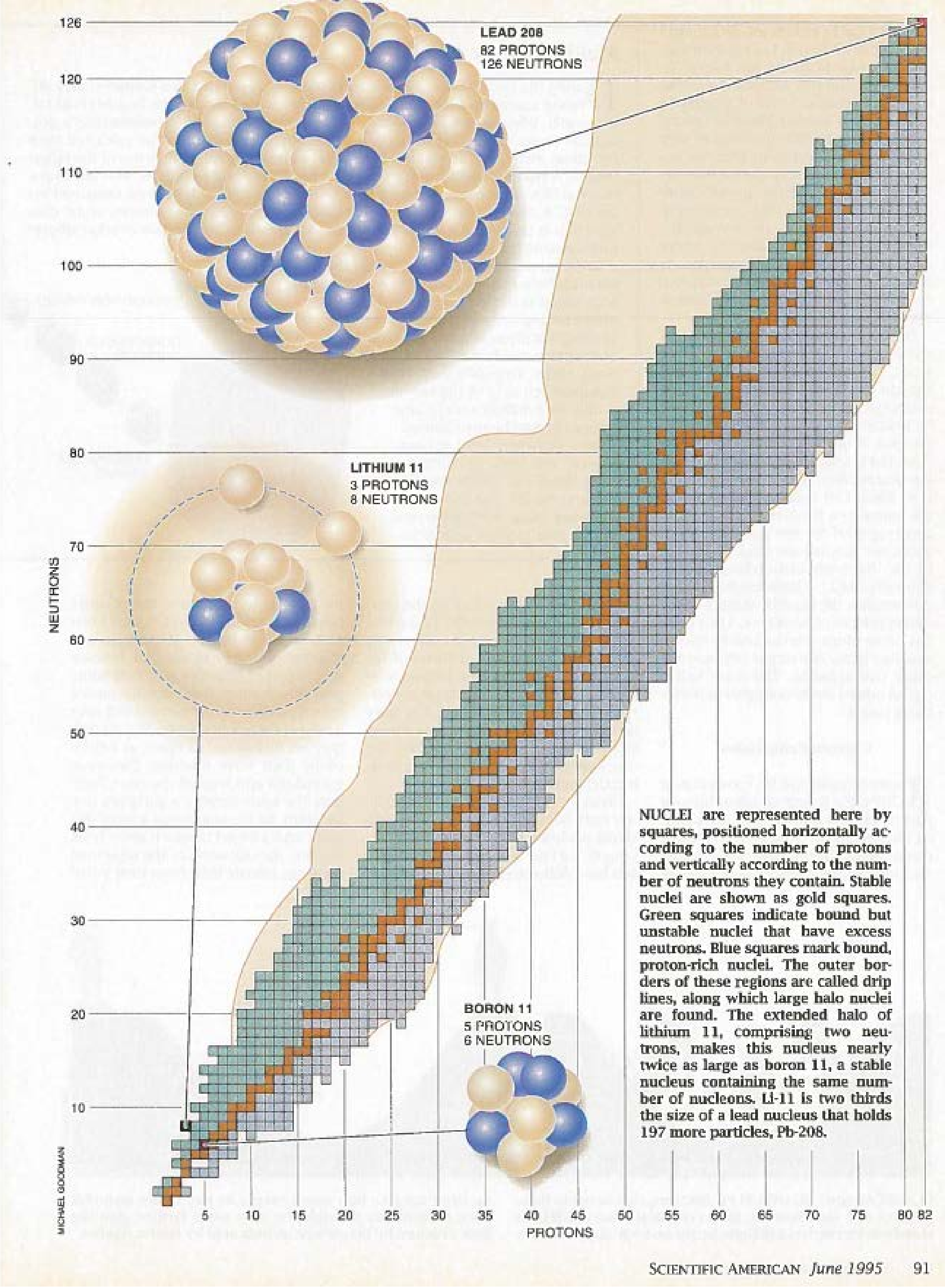


Fig. 8. Angular distributions of the elastically (1) and inelastically (2, 3, 4, 5) scattered deuterons from ^{54}Fe and optical-model fit (O.M.), one-phonon collective-model (C.M.) and shell-model (S.M.) calculations.

F. Hinterberger et al. Nuc. Phys. A (1968)

Exceptions to the rule: Halo nuclei

- Some nuclei exhibit radii far larger than expected from the $r_0A^{1/3}$ estimate
- Their large radius is due to 1, 2, or 4 loosely bound nucleons
- The small binding energy of these valence nucleons corresponds to a low tunneling barrier
- From Heisenberg's uncertainty principle, the nucleon(s) can exist in the classically forbidden region beyond the barrier for a rather long time, since $\hbar \propto \Delta E \Delta t$ and ΔE is small
- Examples:
  - 1n: ^{11}Be, ^{19}C
  - 1p: ^{8}B, ^{26}P
  - 2n: ^{6}He, ^{11}Li, ^{17}B, ^{22}C
  - 2p: ^{17}Ne, ^{27}S
  - 4n: ^{8}He, ^{14}Be, ^{19}B



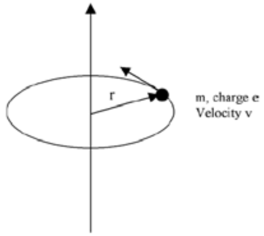
S. Austin & G. Bertsch, Scientific American (1995)

Electric & Magnetic Moments

In general, a moment is a distance multiplied by a physical quantity. For distributions you integrate the quantity's distribution with respect to distance.

- The nuclear *magnetic* moments describe the distribution of electric currents in the nucleus
  - Causes nuclei to align along an external magnetic field, which can be exploited using NMR, MRI, etc.
- The nuclear *electric* moments describe the distribution of electric charges in the nucleus
  - Used as a measure of the nuclear shape

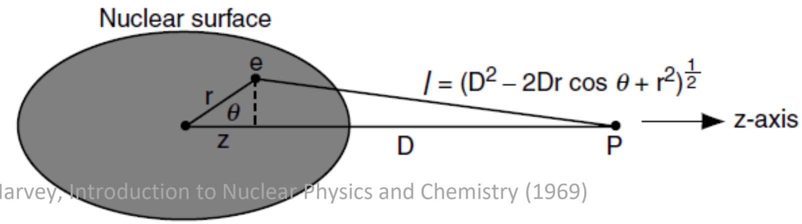
Nuclear magnetic dipole moment



- For a classical charge, *e*, orbiting in a circle with radius *r* at velocity *v*, the magnetic dipole moment is: $|\mu| = (Circle\ Area) * (Current) = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$
- Since angular momentum of the charge with mass *m* is $l = mvr$:
  - $|\mu| = \frac{el}{2m}$, where m would be the proton mass for an orbiting proton
- In quantum mechanics, the analogous circular orbit is given by the z-projection of l, m_l:
  - $\mu = \frac{e}{2m} m_l \hbar$ ….defining the nuclear magneton to be $|\mu_N| \equiv \frac{e\hbar}{2m_p}$, then: $\mu = m_l \mu_N$
  - $\mu_N = 3.15 \times 10^{-12} \frac{\mathrm{eV}}{\mathrm{gauss}} = 0.105\ e\mathrm{fm}$
- …but that relationship turns out to not quite be true, so we include a fudge-factor, called the g-factor: $\mu = g_l m_l \mu_N$
- On top of that, there is also a contribution from the intrinsic spin angular momentum and we usually just quantify μ in terms of magnetons : $\mu = g_l m_l + g_s m_s$
- For the free proton and free neutron, $\mu = g_s \frac{1}{2}$, where $g_{s,p} = 5.58$, $g_{s,n} = -3.83$
  - …if they were structure-less, one would expect $g_{s,p} = 2$ and $g_{s,n} = 0$

Want more μ? Go wild with [N. Stone's Table of Nuclear Magnetic Dipole and Electric Quadrupole Moments]

Nuclear electric quadrupole moment

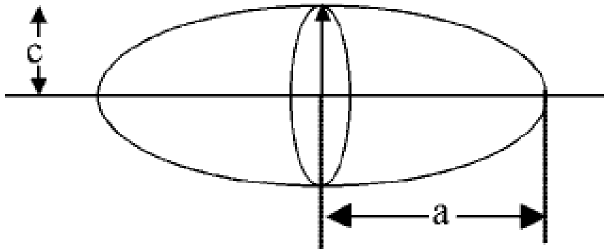


B. G. Harvey, Introduction to Nuclear Physics and Chemistry (1969)

- The nucleus is a charged volume, V, with some shape
- The potential at some point a distance l away from the nucleus due to a small slice of the nucleus that has some charge density $\rho(r,\theta,\varphi)$, where r, θ, φ are w.r.t. the nuclear centroid, is:
  - $d\phi = \frac{\rho dV}{l} = \frac{\rho dV}{\sqrt{D^2+r^2-2Dr\cos\theta}} = \frac{\rho dV}{D}\left(1 - 2Dr\cos\theta + \frac{r^2}{D^2}\right)^{-½}$
- Someone clever noticed a Taylor expansion of the quantity in parentheses yields a multipole expansion:
  - $d\phi = \frac{\rho dV}{D}\left(1 + \frac{2\cos\theta}{2}\frac{r}{D} + \frac{1}{8}(3(2\cos\theta)^2 - 4)\frac{r^2}{D^2} + \cdots\right)$
  - $\quad = \frac{\rho dV}{D}\left(1 + \frac{r}{D}\cos\theta + \left(\frac{3}{2}(\cos\theta)^2 - \frac{1}{2}\right)\frac{r^2}{D^2} + \cdots\right)$
- allowing us to spot the Legendre Polynomials, $P_1(\cos\theta) = \cos\theta$, $P_2(\cos\theta) = \frac{3}{2}(\cos\theta)^2 - \frac{1}{2}$:
  - $d\phi = \frac{\rho dV}{D}\left(1 + \frac{r}{D}P_1(\cos\theta) + \frac{r^2}{D^2}P_2(\cos\theta) + \cdots\right)$
- When we integrate over the nuclear volume to get the full potential, all odd Legendre polynomials will drop-out, so:
  - $\phi = \frac{1}{D}\underbrace{\iiint \rho(r,\theta,\varphi)\, r^2 dr \sin\theta\, d\theta d\varphi}_{\text{Nuclear charge, } Ze} + \frac{1}{D^3}\underbrace{\iiint r^2\rho(r,\theta,\varphi) r^2 dr \sin\theta\, d\theta d\varphi}_{\text{Electric quadrupole moment, } Q} + \cdots$

Nuclear electric quadrupole moment, Q

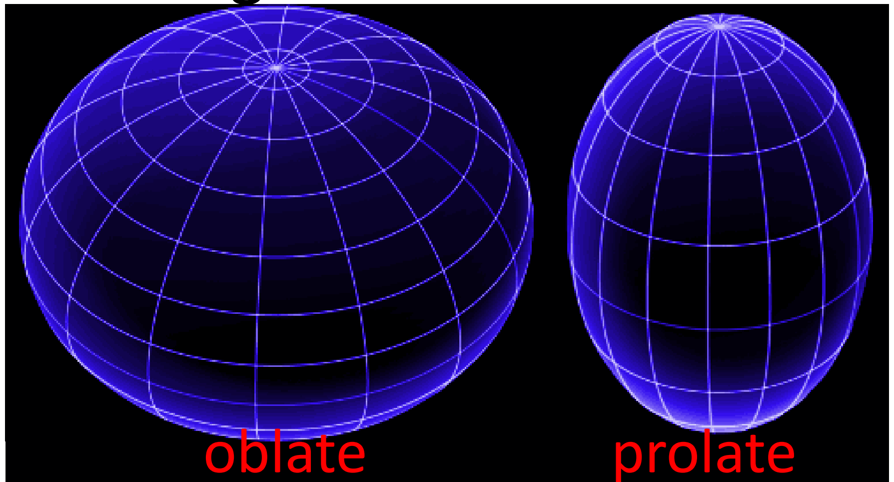
- $Q = \iiint r^2 \rho(r,\theta,\varphi) r^2 dr \sin\theta \, d\theta d\varphi$
- Note that if ρ is spherically symmetric, the integral over $d\theta$ will make $Q = 0$
- So, any non-zero Q indicates a non-spherical nuclear shape
- Choosing an ellipsoid model for the nucleus and evaluating Q above yields (it turns out)
  - $Q = \frac{2}{5} Ze(a^2 - c^2)$



Loveland, Morrissey, Seaborg, Modern Nuclear Chemistry (2006)

Q is generally provided in units of e

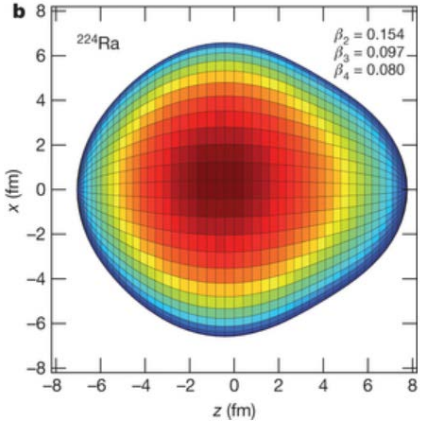
- With a measurement of Q, we can solve for a and c using the formula for the radius of an ellipsoid:
  - $R^2 = \frac{1}{2}(a^2 + c^2) = \left(r_0 A^{1/3}\right)^2$
- If $a > c$, $Q > 0$ and the nucleus is "prolate"
- If $a < c$, $Q < 0$ and the nucleus is "oblate"



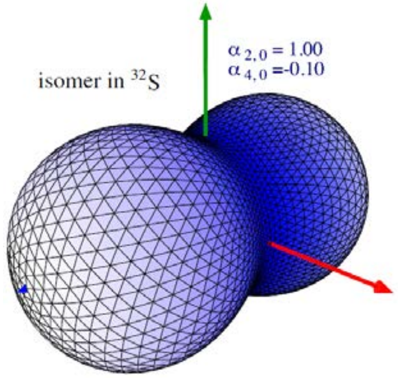
Nuclear electric quadrupole moment, Q

- Q/e is commonly reported, which has units of area
- A convenient unit for such small areas is the barn:
    - 1barn = 10^{-24} cm^2

higher-order deformation is also possible:

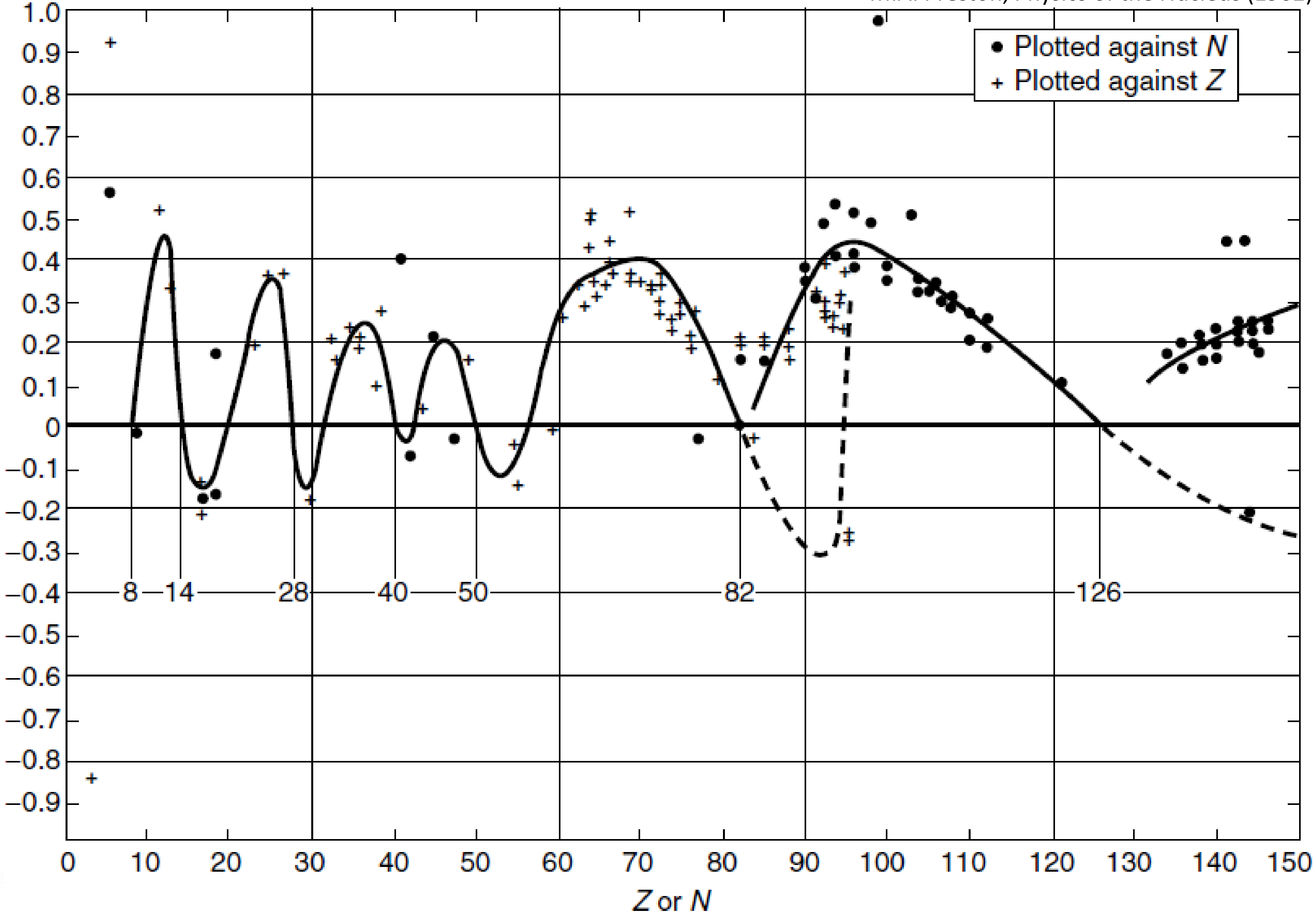


L. Gaffney et al. Nature (2013)



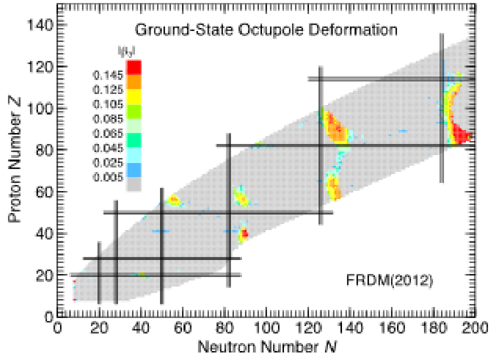
A. Gaamouci et al. PRC (2021)

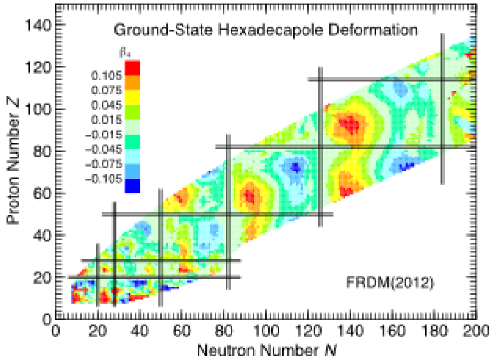
M.A. Preston, Physics of the Nucleus (1962)

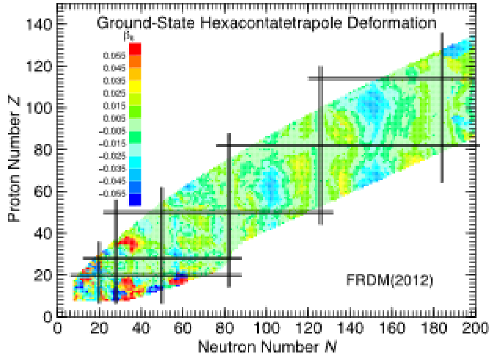


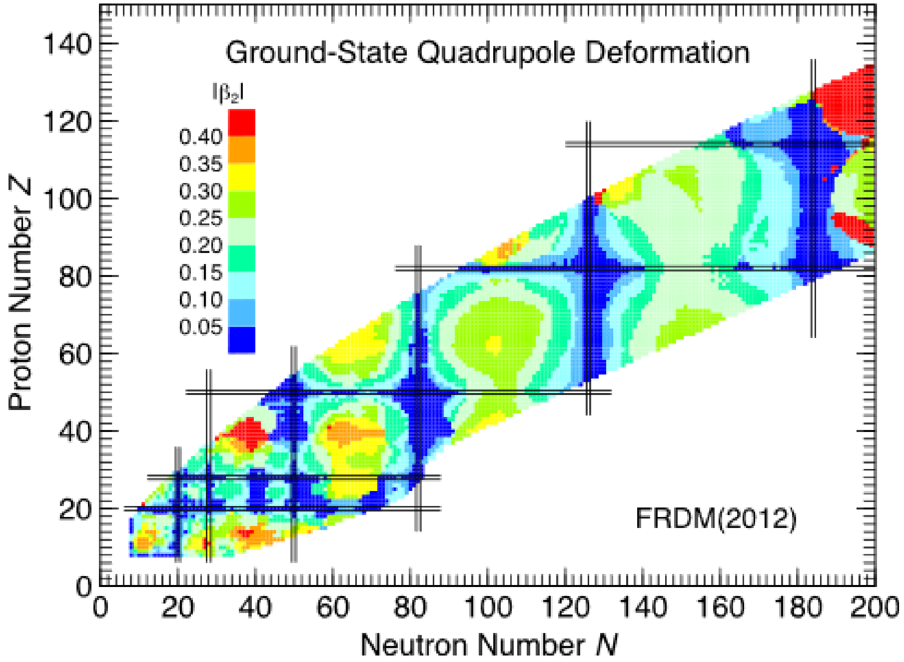
Q and deformation, β

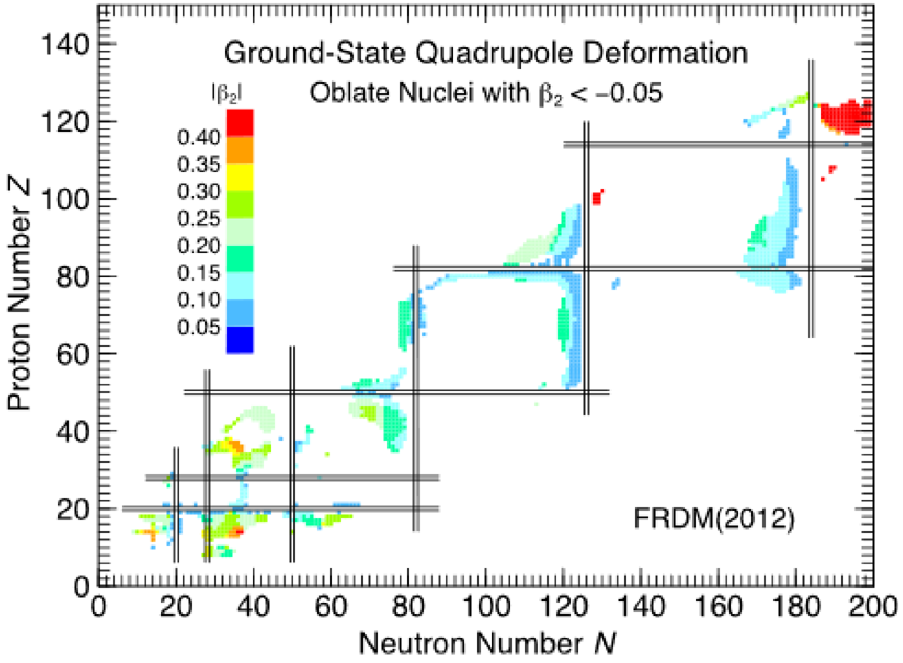
- For an ellipsoid, radius can be represented as an expansion in spherical harmonics
  - $R(\theta,\varphi) = R_{sph}\left(1+\sum_{\lambda=2}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda,\mu}Y_{\lambda,\mu}(\theta,\varphi)\right)$
  - where R_{sph} is the radius of sphere with the same volume, $r_0A^{1/3}$
  - λ is the multipole (1 is neglected because that's C.O.M. motion), and μ is the z-projection
  - …for an axially symmetric nucleus (e.g. prolate & oblate), $\mu = 0$
    - $\beta_2 \equiv \alpha_{2,0} = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{(a-c)}{R_{sph}}$ …which is a commonly used metric for deformation









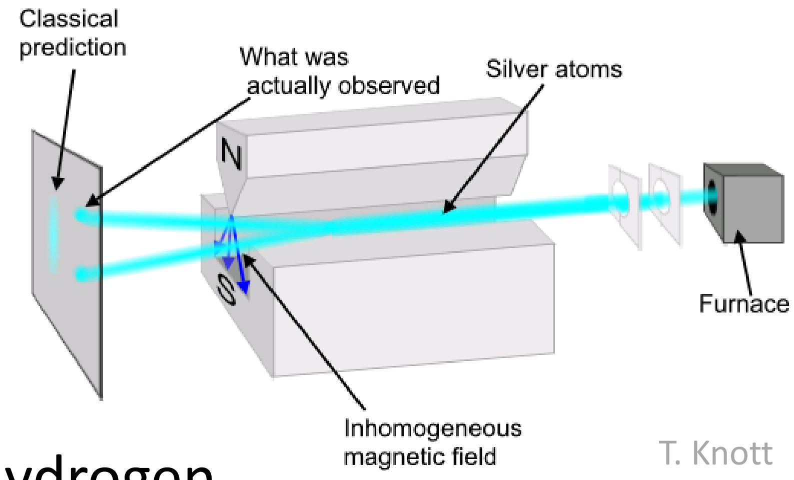


[P. Möller et al. ADNDT (2016)]

Spin

- The quantized nature of atomic observables was noted early on in quantum mechanics (e.g. x-ray energies from electrons changing orbitals)
- Stern & Gerlach set-out to prove this quantization by taking advantage of the fact that an inhomogeneous magnetic field exerts a force on a magnetic dipole (W. Gerlach & G. Stern, Z.Phys. (1922)):
  - $\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$
  - Upon sending neutral silver atoms through their apparatus, they found a split beam
  - Since $\mu = g_s m_s$, this demonstrated there were two spin projections and thus spin=1/2.
  - A follow-up (Phipps & Taylor, Phys. Rev. (1927)) demonstrated spin=1/2 for Hydrogen

Luckily the stable isotopes of ^{107}Ag and ^{109}Ag both have spin-1/2 ground-states …otherwise I feel like this experiment would have generated quite a bit of confusion!



T. Knott

- Neutrons & protons each have intrinsic spin of ½
- Thus, nuclei have intrinsic spins which are half-integer multiples, where the exact value depends on the nucleus's structure.
- Nucleons pair, when possible, in a nucleus, cancelling spins. Thus unpaired nucleons determine a nucleus's spin.

For example, ALL even-Z,even-N nuclides have a ground-state spin of zero.

Parity, π

- The quantum mechanical state a nucleus is in is described by a wave function, which will either be even or odd
- ***Even*** wave-functions are symmetric about the origin (e.g. cosine) and thus, upon flipping the spatial coordinate and spin:
  - $\psi(r, s) = \psi(-r, -s)$
  - This is known as *positive parity*
- ***Odd*** wave-functions are antisymmetric about the origin (e.g. sine) and thus, upon flipping the special coordinate and spin
  - $\psi(r, s) = -\psi(-r, -s)$
  - This is known as *negative parity*
- For a spherically symmetric potential (i.e. $V(r, \theta, \varphi) = V(r)$), the parity of a particle is given by its orbital angular momentum:
  - $\pi = (-1)^{\ell}$
- For a state of several nucleons, $\pi = \prod_i \pi_i$ …as with spin, it's the unpaired nucleons that matter
- Parity is conserved for strong & electromagnetic interactions

When referring to a state of a nucleus, we typically denote both the spin & parity by: J^{π}

Isospin

- Empirically, the nuclear force between nucleons appears to be charge-independent, i.e. it doesn't matter if you're dealing with neutrons or protons. For instance:
    - Masses of isobars differ only by the different Coulomb energy and the n-p mass difference
    - Nuclear charge radii are well predicted by considering A, not Z and N separately
- Then, one can consider the neutron and proton as two states of the same particle, the nucleon

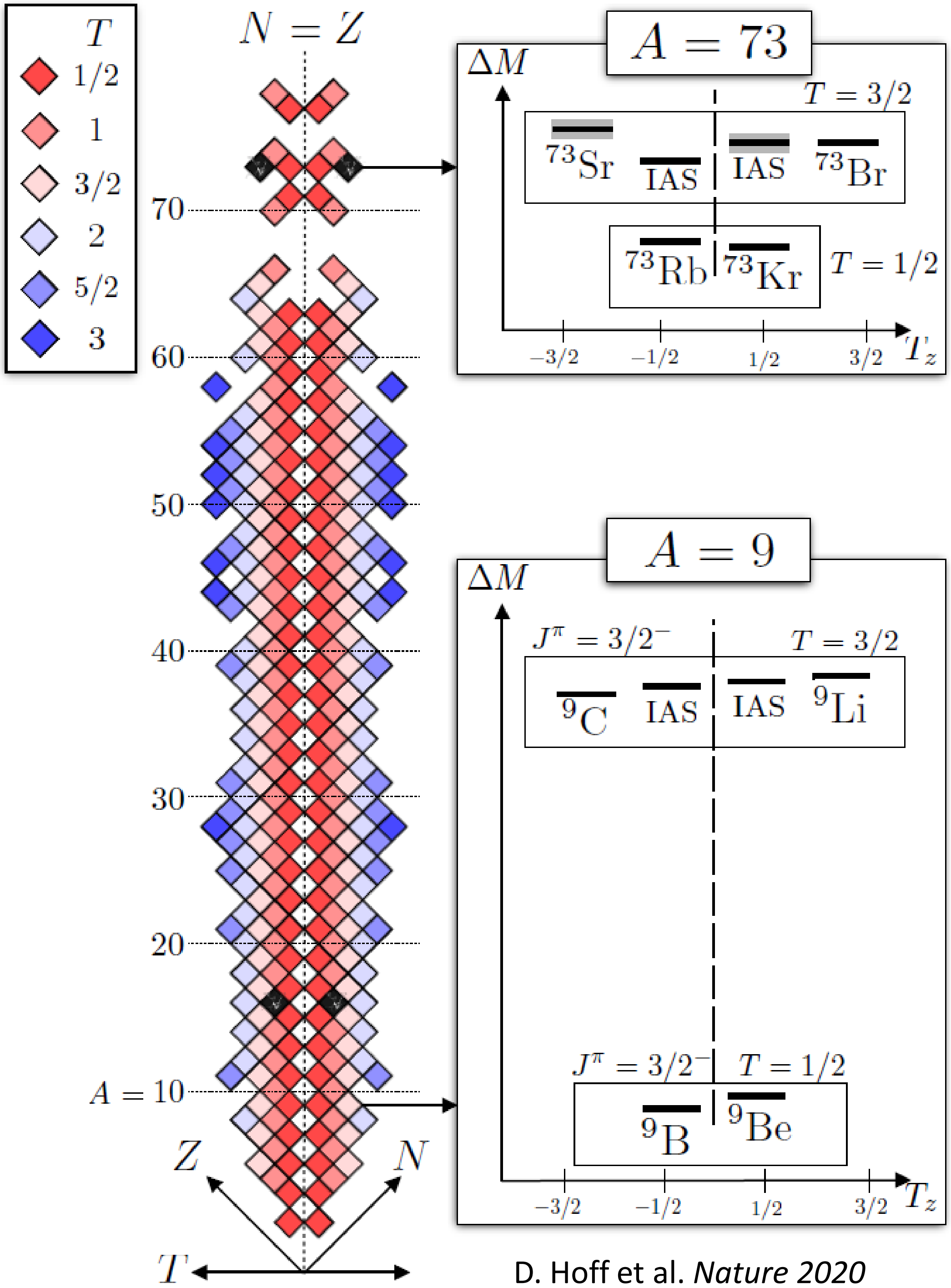
(W. Heisenberg, Z. Phys. A (1932))

- Since the mechanics have already been developed for something in two quantized states (spin-1/2 particles), the associated machinery can be hijacked and assign nuclear states the quantity *isospin*, T
    - The nucleon has $T = 1/2$ with two projections, $T_z = +\frac{1}{2}$ (proton) and $T_z = -\frac{1}{2}$ (neutron)
- For nucleus, $T_z = \frac{Z-N}{2}$ and $|T_z| \leq T \leq \frac{A}{2}$. Usually, for the ground-state $T = T_z$
- Isospin is conserved (approximately) in strong interactions

*just to keep life interesting, sometimes the opposite convention is used, where $T_{z,proton}$ =-1/2, $T_{z,neutron}$=1/2 and $T_{z,nucleus}$ = (N-Z)/2

Isospin symmetry

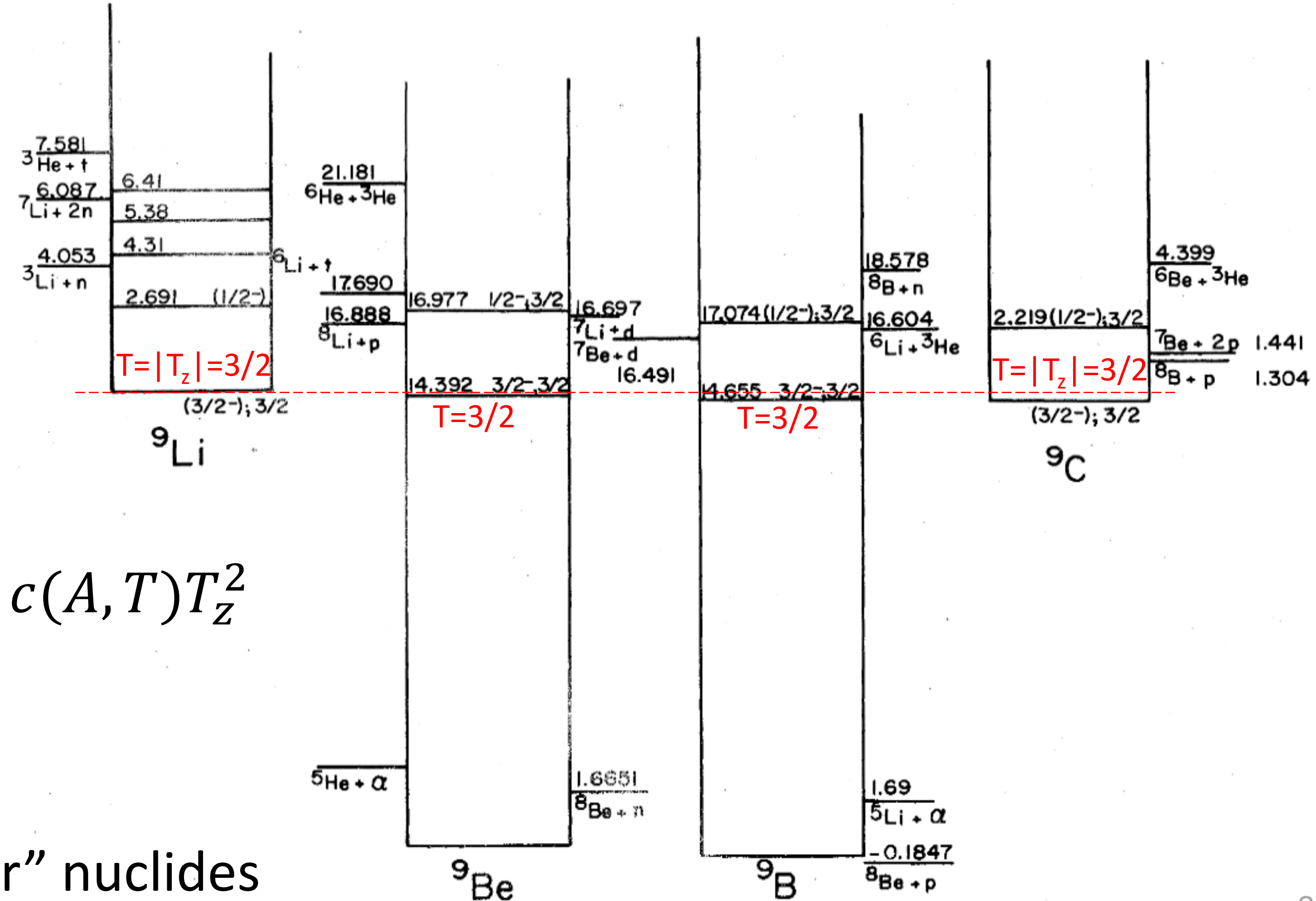
- As an example what isospin gets you, the ground-state spin-parity is the same for “mirror nuclei” (swap *N* & *Z*) except for 2 cases (as of 2021)
- This works for low-ish lying excited states too
  - This can be useful for astrophysical reaction rates, using the structure of the less-exotic nucleus in the pair to inform the structure of the more-exotic nucleus in the pair



D. Hoff et al. *Nature 2020*

Isospin symmetry

- A state with isospin T is a part of an isospin multiplet with 2T+1 members, which each are described by the same (or very similar) wave function but have different nuclear charge
- This isospin symmetry implies the members of the multiplet should have the same energy and, corrected for coulomb effects, indeed they do:
- The $T>T_z$ states in the multiplet are termed "isobaric analogue states" and they are favored when transitioning from a ground-state in the mulitplet with $T=T_z$
- The isobaric mass multiplet equation (IMME) describes the binding energies of the states which are members of the multiplet:
  - $ME(T,T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2$
- This fact is taken advantage of to make predictions about exotic nuclides which are difficult to measure
- Nuclides with N & Z switched are "mirror" nuclides



W. Benenson & E. Kashy, Rev. Mod. Phys. (1979)

Further Reading

- Review of nuclear masses: D. Lunney, J. Pearson, C. Thibault, Rev. Mod. Phys. (2003)
- Review of charge distributions from electron scattering: R. Hofstadter, Rev. Mod. Phys. (1956)
- Chapter 2,5: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapters 2,7: Nuclear & Particle Physics (B.R. Martin)
- Chapters 1-3,10: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapter 4, Example 4.4: Introduction to Quantum Mechanics, D. Griffiths (2005)