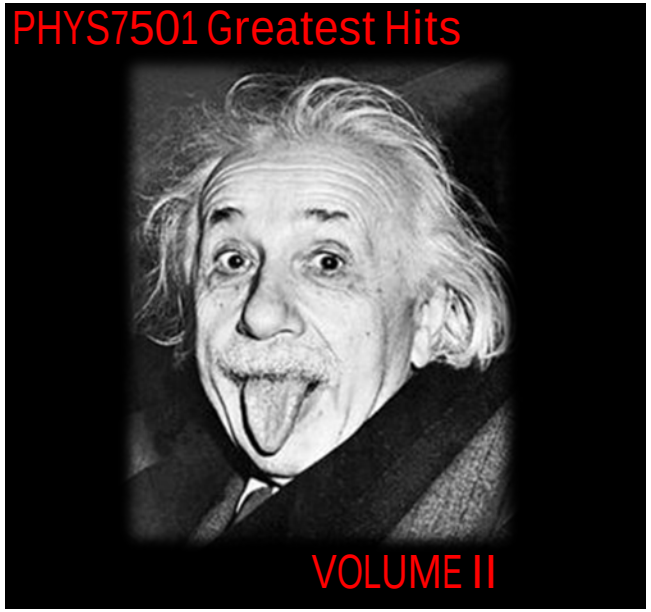
Lecture 24: Review of Reactions, Astrophysics, & Applications

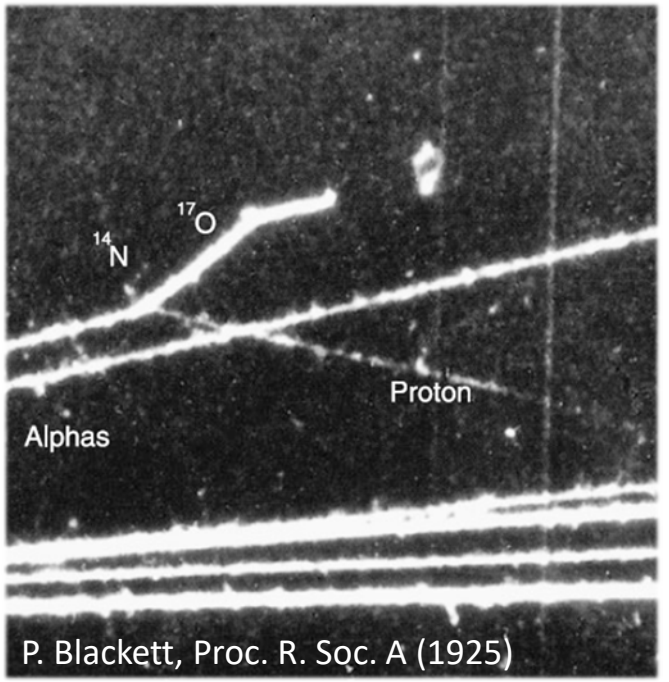
- Highlights from Lectures 14-23 of PHYS7501
- ***This doesn't necessarily cover everything that will be asked on the midterm, but it encompasses most of the main take-aways***



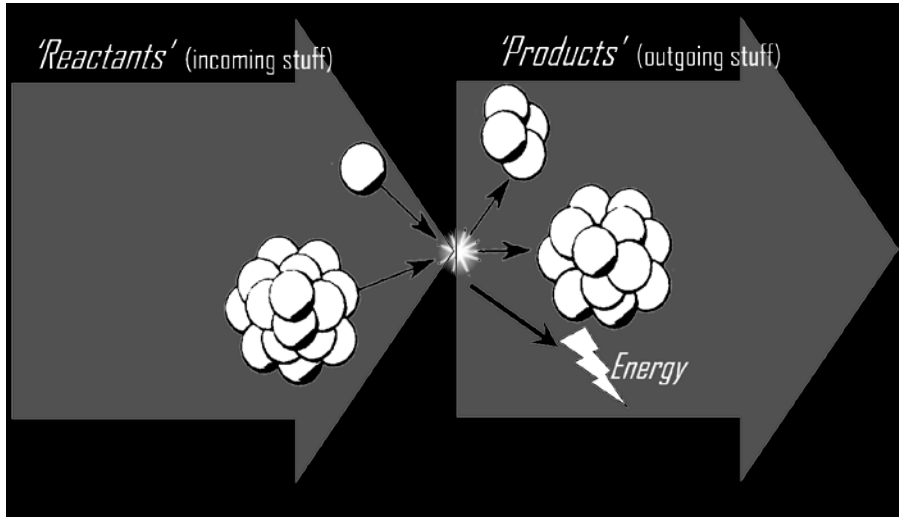
Defining a reaction

- A nuclear reaction consists of the interaction of two or more nuclei or nucleons that results in some final product
- The initial stuff is known as the reactants [projectile and target, in the lab] and the final stuff is known as the products [recoil and ejectile, in the lab]
- Several sets of products are often possible for a pair of reactants colliding at a given energy, including simple scattering
  - The ways of "decaying" from the nucleus briefly formed by the reaction are known as channels
- The modern notation for a reaction is always to put the lighter of the reactants and products on the inside of a pair of brackets, like A(b,c)D, where $M_A > M_b$ and $M_D > M_c$
- Nuclear reactions are governed by the strong force and so they conserve baryon number, nuclear charge, energy, linear momentum, angular momentum, and parity

1st nuclear fusion reaction observed in the lab

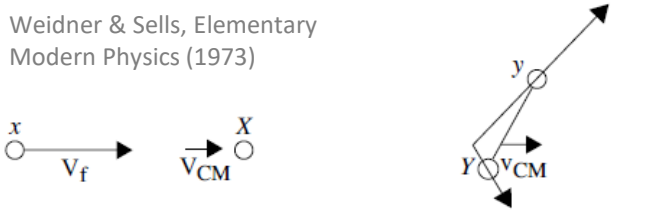


P. Blackett, Proc. R. Soc. A (1925)



Energy in the center of mass (CM) frame

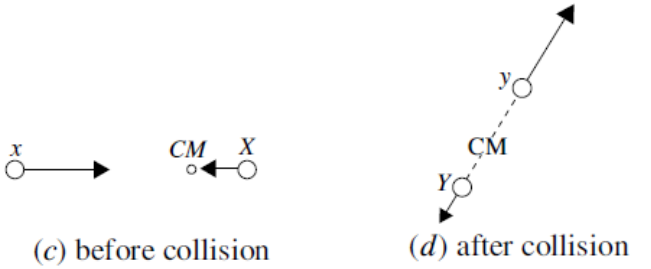
Weidner & Sells, Elementary Modern Physics (1973)



(a) before collision (b) after collision as seen in the laboratory

- For mathematical convenience *(and to keep the outsiders out!)* often the center-of-mass (CM) frame is employed
- For the energetic conversion between the two, consider the kinetic energy in the CM frame: $KE_{cm} = KE_{lab}\frac{A_t}{A_p+A_t}$

What does this imply about the choice of forward vs inverse kinematics measurements?



(c) before collision (d) after collision as seen from the center of mass

- So, the center of mass energy is always lower than the laboratory energy, but how much lower depends on which reactant is the beam and which is the target

For an accelerator with a limited minimum voltage (i.e. all of them), inverse kinematics will reach a lower center-of-mass energy.

- As an aside, consider an alternative way to specify the laboratory beam energy: MeV/u
- Let's look at an example reaction and use Energy/nucleon for the forward & inverse kinematics:
  - ^{96}Zr(α,n) at $E_{cm} = 7.68MeV$
  - For an α beam on a ^{96}Zr target, $E_{\alpha,lab} = \frac{96+4}{96}7.68MeV = 8MeV = \left(\frac{8}{4}\right) = 2MeV/u$
  - For a ^{96}Zr beam on an α target, $E_{^{96}Zr,lab} = \frac{96+4}{4}7.68MeV = 192MeV = \left(\frac{192}{96}\right) = 2MeV/u$
  - Hence, MeV/u is often a more useful way to speak about reaction energies

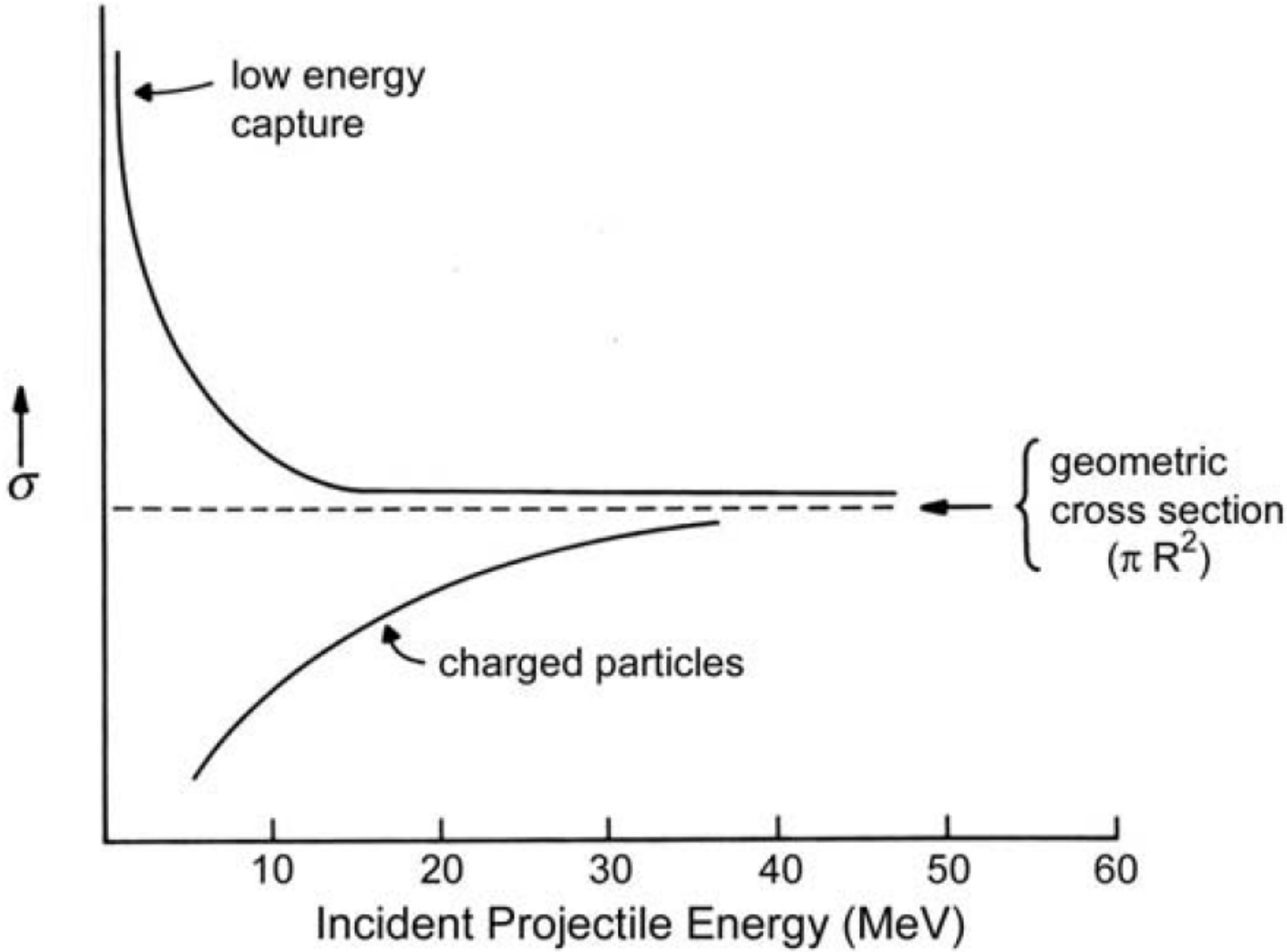
Reaction likelihood: The cross section σ

- To quantify the probability of a nuclear interaction occurring, we use the cross section
- The number of reactions that occur N when we impinge an ion beam of intensity I (in units of particles/second) for some time t on a target with areal density n_T (in units of atoms/cm^2) is scaled by a sort of reaction probability σ: $N = Itn_T\sigma$
- For the units to work: # = (s^{-1})*(s)*(atoms/cm^2), [σ] = cm^2, so we call this area the cross section.
- You can think of it as the overlap between the wave-like projectile and target, though it can be very different from the classical value from a physical overlap
- Practically, a reaction product detection efficiency ε needs to be included: $\sigma = \frac{N_{detected}}{Itn_T\varepsilon}$
- And the cross section and detection efficiency need not be uniformly distributed, so we should consider the cross section for particles detected at some angle θ by a detector with some solid-angle $\Omega = \frac{Detector\ Area}{(Target-Detector\ Distance)^2}$:

$$\sigma(\theta) = \frac{N_d}{I_b t_{meas} n_T \varepsilon_{det}(\theta)\Omega_{det}(\theta)}$$

Fun fact: Beam intensities are usually measured by the current those ions create on a beam-stopping device. To convert to I_{pps}, you need to use the fact that 1 unit of electric charge is 1.602×10^{-19}C and 1A=1C/s.

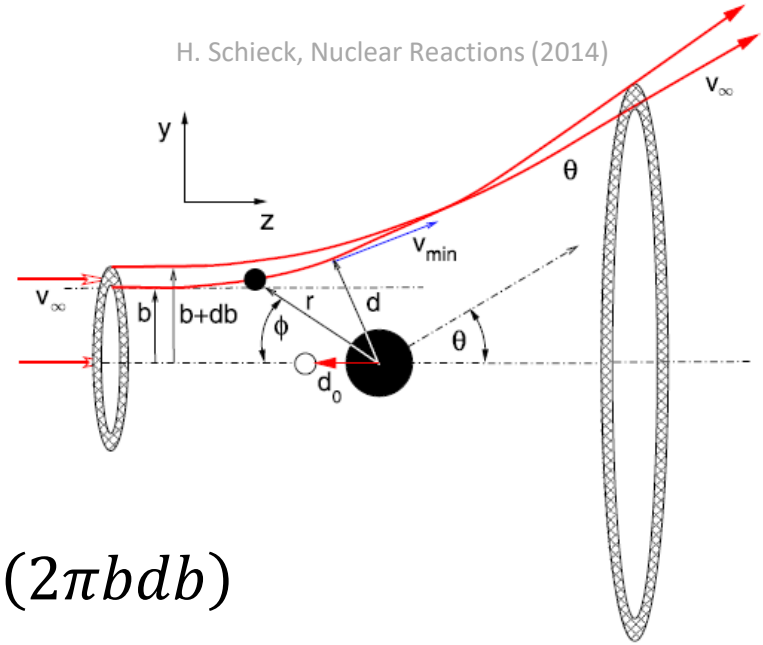
Cross sections from a semi-classical view: *Near Threshold*



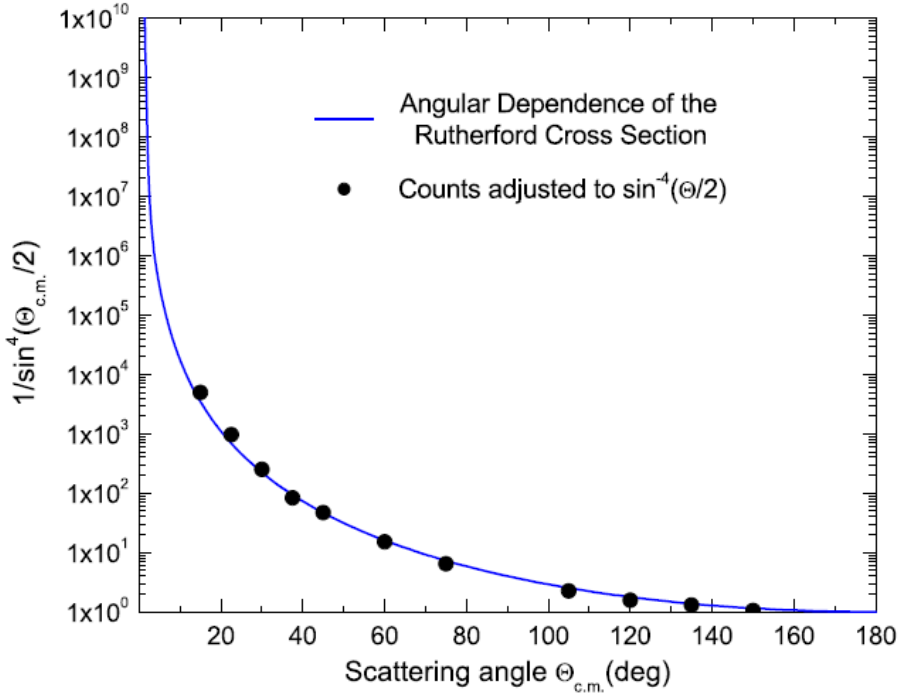
Ehmann & Vance, Radiochemistry and Nuclear Methods of Analysis (1991)

Rutherford (a.k.a. elastic Coulomb) scattering

H. Schieck, Nuclear Reactions (2014)



- Now that we know the scattering angle θ corresponding to an impact parameter b, we can solve for an experimentally useful property: *the angular distribution*
- Let's consider the intensity of particles arriving within a ring in with the impact parameter range $b + db$
  - $dI = \left(\frac{Intensity}{Unit\ Area}\right)(ring\ area) = F_0(\pi(b+db)^2 - \pi b^2) \approx F_0(2\pi b db)$
- By swapping in our previous result for $\theta(b)$, we get the intensity of particles scattered through a solid angle $d\Omega$ at angle θ
  - $dI = \frac{\pi}{4} I_0 \left(\frac{2Z_1 Z_2 \alpha\hbar c}{m_p v_i^2}\right)^2 \frac{\cos(\theta/2)}{\sin^3(\theta/2)} d\theta$
- The number scattered through $d\Omega = 2\pi\sin(\theta)d\theta$ sr at θ is:
  - $\frac{d\sigma}{d\Omega} = \frac{dI}{F_0}\frac{1}{d\Omega} = \left(\frac{Z_1 Z_2 \alpha\hbar c}{4KE_{cm}}\right)^2 \frac{1}{\sin^4(\theta_{cm}/2)}$

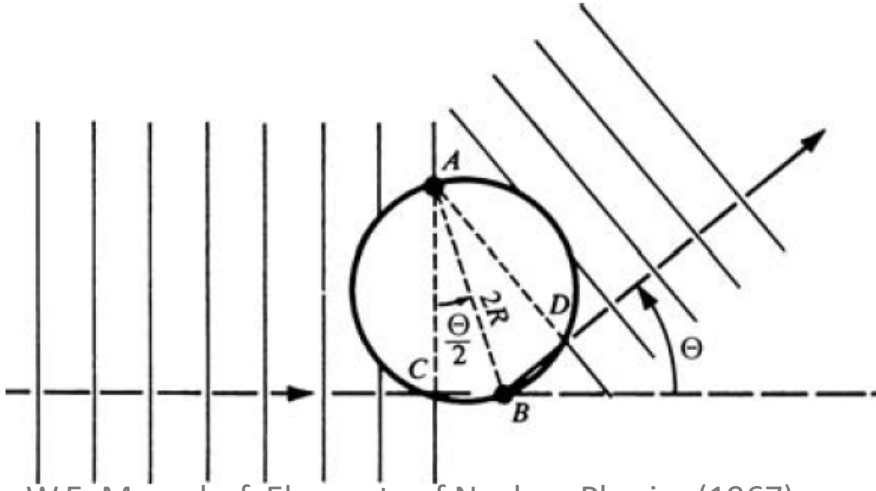


Rutherford scattering creates a background for all charged particle experiments, but is minimal at backward angles

Geiger & Marsden, Phil. Mag. (1913),
as reproduced by H. Schieck, Nuclear Reactions (2014)

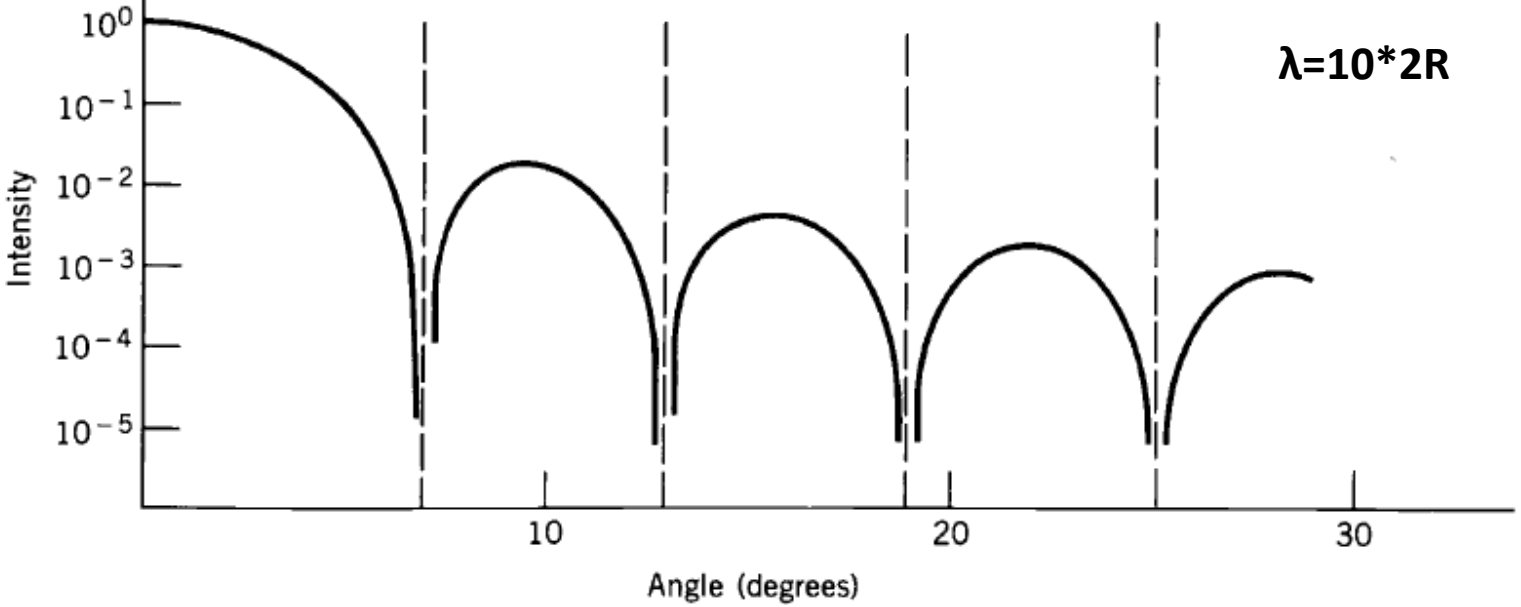
Elastic Nuclear Scattering

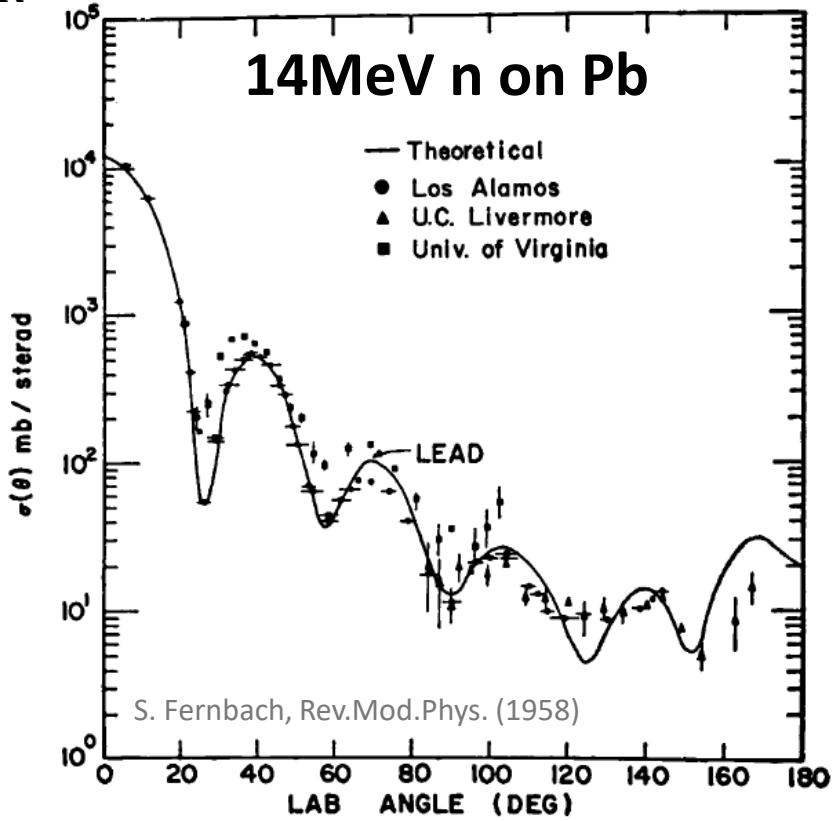
- Considering a projectile as a plane wave and a target nucleus a opaque disk, a creative person realizes this situation looks like diffraction of light off of an opaque disk
- The opticians among us recall that diffraction on a sharp edge results in a diffraction pattern with the first minimum at $\theta \approx \frac{\lambda}{2R}$ and succeeding minima at roughly equal spacing, with a decreasing maxima [like the $\mathrm{sinc}(\theta)$ function]



W.E. Meyerhof, Elements of Nuclear Physics (1967)

K.S. Krane, Introductory Nuclear Physics (1987)





S. Fernbach, Rev.Mod.Phys. (1958)

Surface interaction implications

- Since direct reactions only involve one or a few nucleons at the surface of the target, we can estimate typical projectile energies that lead to this case
  - By "seeing" the surface nucleon(s) and not the nucleus, this implies the de Broglie wavelength of the projectile is more nucleon-sized than nucleus-sized
    - $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$
    - So in general, higher projectile energies are going to be more prone to direct reactions
- Since the interaction is essentially only a quick grazing, we can estimate the reaction timescale as the nucleus crossing time
  - For example, ^{56}Fe(d,p) for a 15MeV incident deuteron
    - $v_d = \sqrt{\frac{2E}{m_d}} = \sqrt{\frac{2 \cdot 15MeV}{2 \cdot 931.5MeV}} c \approx 3 \times 10^7 fm/fs$
    - $R \approx 1.2A_T^{1/3} fm = 4.6fm$
    - $t_{crossing} = R/v = 4.6fm/(3 \times 10^7 fm/fs) \approx 10^{-22} s$
  - Practically speaking, this means there isn't time for momentum from the collision to be shared amongst the target nucleons and there are few opportunities for multiple scatters in the nucleus

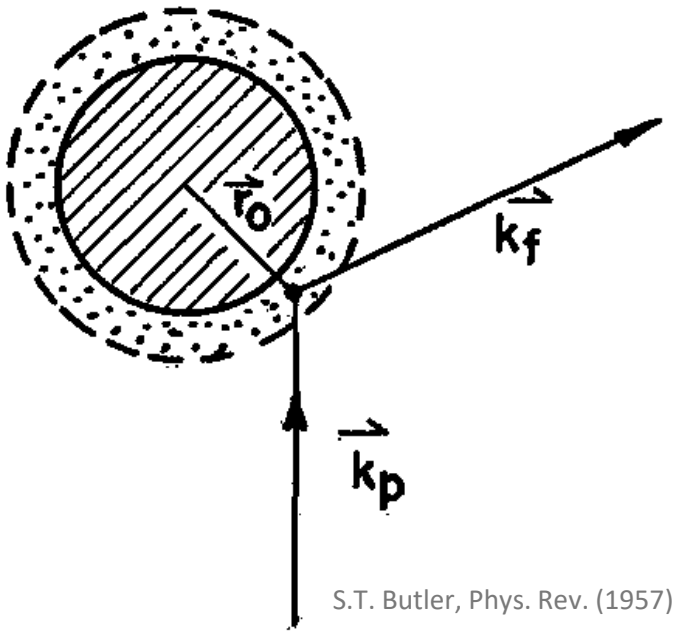
Note that the projectile need not be the lighter of the nuclides. Direct reactions happen just as well for heavy beams on light targets…which is actually how they're more commonly used in experiments today.

Other factors influencing the direct process

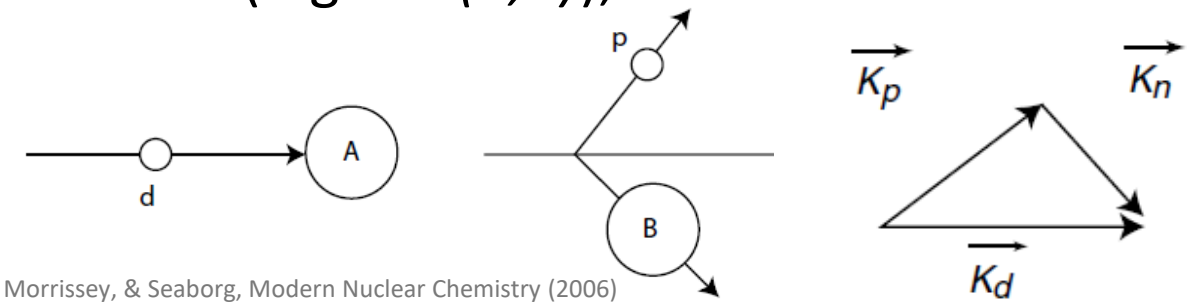
- The general characteristics of a particular reaction type allows one to estimate whether the direct reaction mechanism is important or not
- Consider a deuteron stripping reaction, *(d,p)*
  - For this case, (by definition) a charged particle needs to leave the nucleus
  - It is unlikely the charged particle is going to be able to “evaporate” out of a nucleus that has absorbed energy from a projectile and shared it among the nucleons (in a compound process), since the proton has to tunnel out of the Coulomb barrier
  - For the direct reaction, the emitted proton carries a larger portion of the reaction energy, and so tunneling out is less problematic.
  - Thus, the direct mechanism is favored for this case
- Consider America’s favorite reaction channel, *(α,n)*
  - For this case, a direct reaction mechanism would imply three nucleons are simultaneously transferred to the target
  - That process is as unlikely as it sounds, and so here the neutron evaporation via a compound process will be the primary reaction mechanism

Direct Reaction Angular distribution

- Due to the quick crossing time, there is little chance for many scattering-type events to happen for the projectile within the target
- As such, it is expected that the direct reaction products should be forward-peaked [i.e. along the beam direction], as we've seen for elastic scattering
- Consider the case where an incident projectile interacts with only the outer layer of a nucleus
  [where all deeper interactions correspond to a different reaction mechanism]
  without worrying about what the ejectile is
  [i.e. it could be the same particle as the projectile, or it could be something else]
- For a surface interaction, it's difficult to impart much momentum to the target, so generally low-lying excitations(including no excitation) will occur
- Considering a momentum triangle for the reaction (e.g. for *(d,n)*), it's clear that low-lying excitations imply forward-peaked reaction products



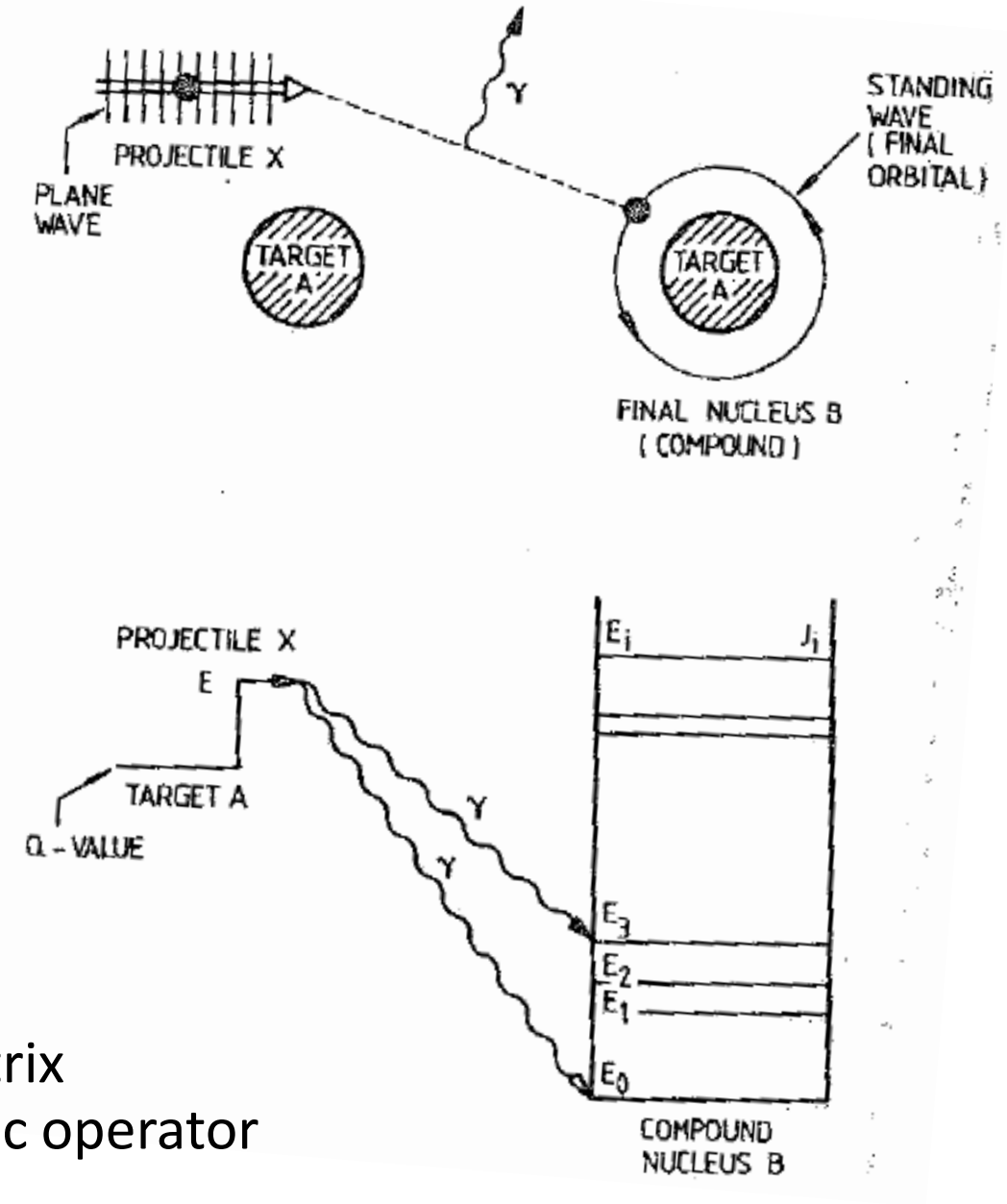
S.T. Butler, Phys. Rev. (1957)



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

Non-resonant reactions

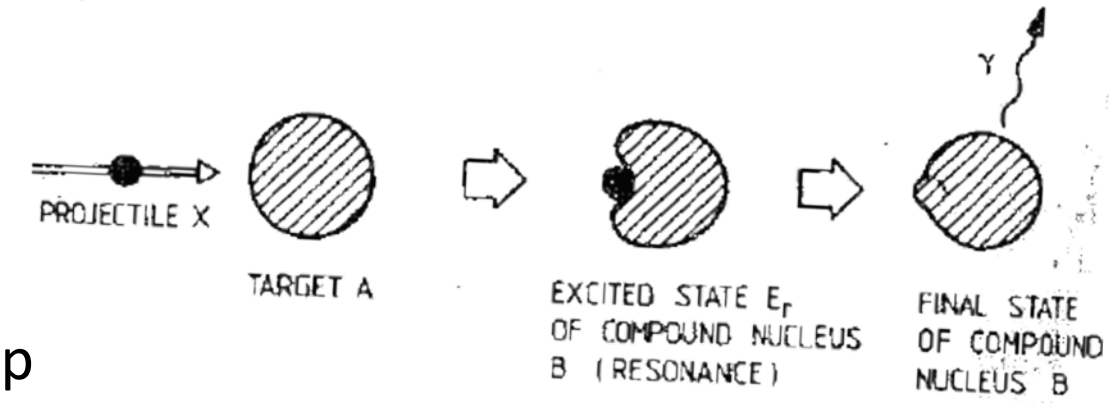
- Before we discuss resonant reactions, let's first consider a non-resonant reaction
- The non-resonant reaction is the process we discussed two lectures ago when we considered low-energy collisions with and without Coulomb effects included
- An example is the direct capture reaction shown in the figure on the right
  - The interaction of the plane-wave of the projectile with the potential of the target results in a standing-wave in the compound nucleus that is characterized by angular momentum l
  - The transition between the initial and final states is accomplished directly via photon emission, so the matrix element connecting these states is the electromagnetic operator $\sigma_{A(a,\gamma)B} \propto \left|< B\left|H_\gamma\right|A + x >\right|^2$, where semi-classical estimates can be obtained as we did previously

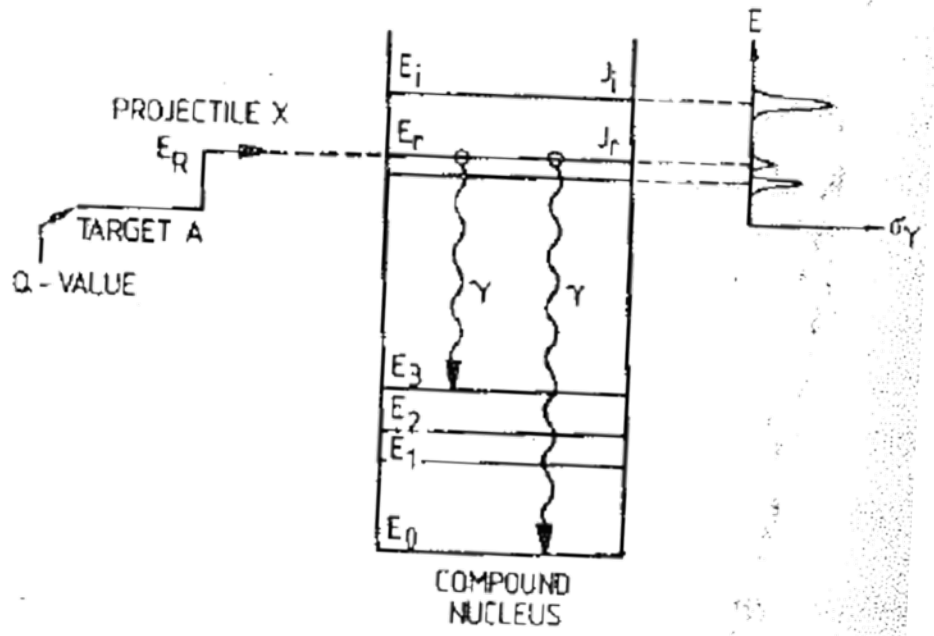


Rolfs & Rodney, Cauldrons in the Cosmos (1988)

Resonant reaction

- Now, like the good capitalists we are, we're going to add a middle-man
- If it so happens that the sum of the mass excesses of our reactants and their center-of-mass energy lines-up with an excited state in the reactant's compound nucleus $\{(^{A_{compound}}Z_{compound} = {}^{A_{target}+A_{projectile}}(Z_{product}+Z_{projectile})\}$, then capture can proceed into that state
- The excited state of the compound nucleus then decays in a second step, e.g. via γ-emission in the figure on the right
- This process requires $E_{cm} = E_B^* - Q_{A+x\rightarrow B+y}$
  - In the more standard notation, $E_R = E_r - Q$
  - This energy is referred to as the resonance energy
- This process, as we'll see, causes a strong enhancement in the cross section near the center-of-mass energy that fulfills the condition above





Rolfs & Rodney, Cauldrons in the Cosmos (1988)

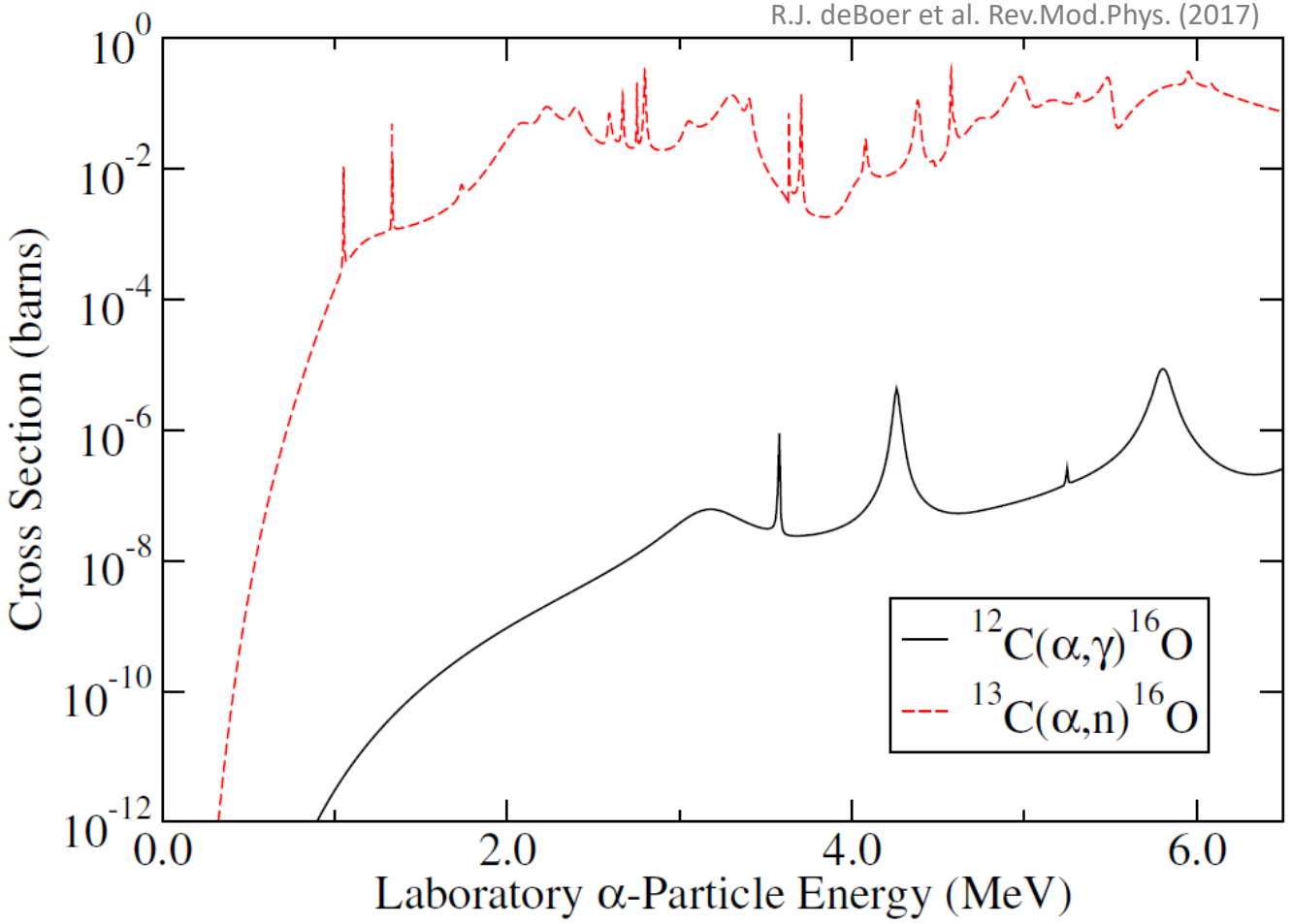
Breit-Wigner formula

- Our ho-hum Lorentzian, now becomes the bright and shiny Breit-Wigner formula,

$$\sigma_{BW,X(a,b)Y}(E) = \pi\left(\frac{\lambda}{\pi}\right)^2 \frac{2J+1}{(2J_a+1)(2J_X+1)} \frac{\Gamma_{aX}(E)\Gamma_{bY}(E)}{(E-E_R)^2+(\Gamma(E))^2/4}$$

- Each resonance adds a sharp spike onto the non-resonant cross section
- This has some major implications:
  1. If we just want to make a reaction happen, it's best to pick an energy on a resonance
  2. If we don't want a reaction to happen (e.g. background), we had better avoid the resonance energy
  3. If we're considering an environment with an energy distribution (e.g. a star), the resonant rate is mostly what matters
  4. Since σ_{BW} has such a strong energy dependence, we can use it to measure energy-loss and therefore target thickness

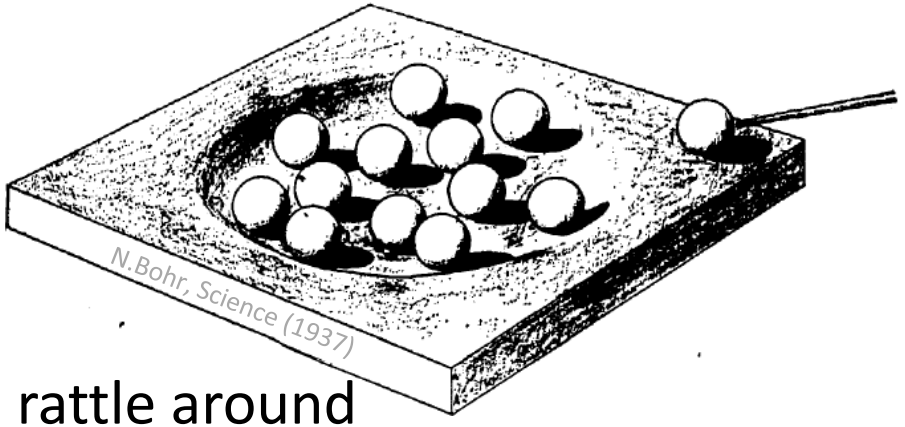
R.J. deBoer et al. Rev.Mod.Phys. (2017)



J^{π} considerations

- Resonant reactions are due to the strong interaction, so spin is conserved
- Therefore, for a spin J_1 particle impinging on a spin J_2 target, bringing in an orbital angular momentum l, can only populate excited states for a limited range of spins J
  - For example, nucleon capture on an even-A nucleus can only populate states with $\left|l-\frac{1}{2}\right| \leq J \leq \left|l+\frac{1}{2}\right|$
- Similarly, the parity is constrained by $\pi(J) = \pi_1\pi_2(-1)^l$
  - If $\pi_1 = \pi_2 = +1$, then $\pi(J) = (-1)^l$,
    i.e. the parity of the resonance is determined by the orbital angular momentum of the reaction channel
  - Such a resonant state is said to have “natural parity”.
  - If $\pi(J) \neq (-1)^l$, then that resonant state has “unnatural parity”

Statistical Reaction Semi-classical picture



- Consider the case where a projectile fuses with the target, sharing its energy amongst many nucleons in the nucleus, like a billiard ball entering a well and causing several others to rattle around
- The nucleon energies will be distributed statistically and they will scatter with each other until one nucleon happens to pick-up enough of the energy to escape the nucleus (In the analogy, one billiard ball can climb out of the well)
- Adopting this qualitative picture, we expect a few things to result
  - The de-excitation of the compound nucleus is akin to evaporation, meaning the ejectile energy distribution should have a Maxwell-Boltzmann character
  - The multiple collisions occurring with the nucleus erases any signatures left by the initial reaction, so
    - The ejectiles should be isotropic (in the center of mass frame, since momentum still has to be conserved)
    - The de-excitation characteristics for a given compound nucleus excited state energy (e.g. the ejectiles and their energy distributions) shouldn't depend on how the compound nucleus was created

This is termed "amnesia" or "the independence hypothesis"

Hauser-Feshbach formalism

- Now, by swapping-in the transmission coefficients T, which we can get using the optical model, for the average resonance widths Γ, which we generally don't know, we get the Hauser-Feshbach cross section

$$\sigma^{HF}_{X(a,b)Y} = \sum_J \left\langle \sigma^J_{X(a,b)Y}(E) \right\rangle = \pi \left(\frac{\lambda}{\pi}\right)^2 \sum_J \frac{2J+1}{(2J_a+1)(2J_X+1)} W_{ab} \frac{T_{aX} T_{bY}}{\sum_{chan} T_{chan}}$$

- The full sum requires taking into account angular momentum conservation, parity conservation, and energy conservation (to determine which outgoing channels are possible)
- For exit channels, we need to take into account the number of discrete states that are available for such a decay. Naturally, more final states leads to a higher probability for that type of decay. As such, really T_{bY} is $\sum_{n_b} T_{bY}$ and T_{chan} is $\sum_{n_{chan}} T_{chan}$.
- So, the key ingredients to calculating the Hauser-Feshbach cross section are
  - The transmission coefficient for the entrance channel — *… from an optical potential*
  - The transmission coefficient for all exit channels — *… from optical potentials*
  - The number of available levels (and their energies) for all exit channels — *… from level density models and spin-cutoff parameters*

**For γ-rays, instead of an OMP, need a γ-strength function (γSF)*

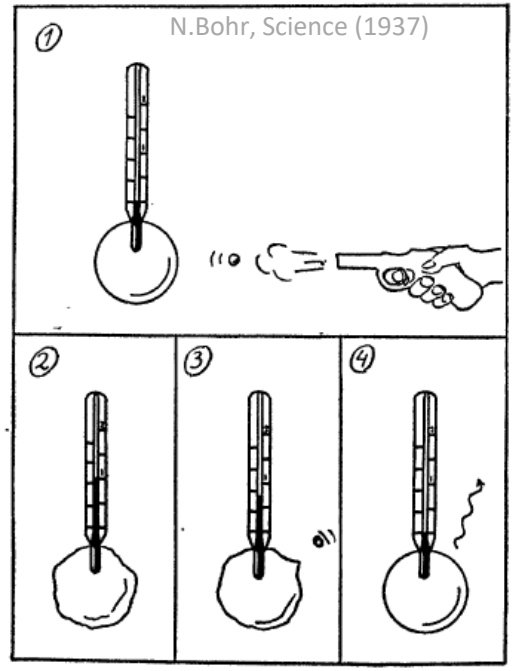
See the Scattering and Alpha-Decay lectures for T-coeffs from OMPs, the Nuclear Structure 3 for level-densities and spin-cutoff parameters, and the Gamma-decay for gamma-strength functions

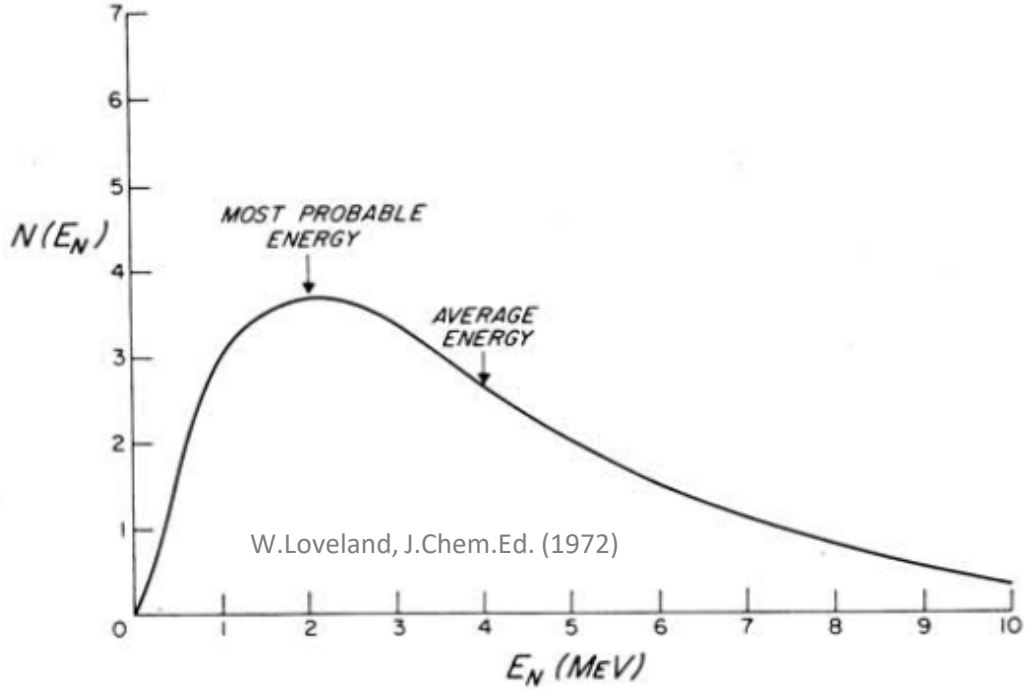
Ejectile energy distribution

- Putting those pieces together, $P_{C\to b+R}(\varepsilon) = \sigma_{b+R}\frac{\sqrt{2\varepsilon/m}}{V_R}\frac{\rho_R(E_R^*)}{\rho_C(E_C^*)}\rho_b(\varepsilon)$,

  becomes $P_{C\to b+R}(\varepsilon) \propto \varepsilon\frac{\exp\left(\frac{E_R^*}{T_R}\right)}{\exp\left(\frac{E_C^*}{T_C}\right)} = \varepsilon\frac{\exp\left(\frac{E_C^*-S_{b,C}-\varepsilon}{T_R}\right)}{\exp\left(\frac{E_C^*}{T_C}\right)} \propto \varepsilon\exp(-\varepsilon/T_R)$

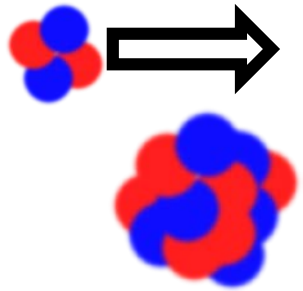
- Thus, we've arrived at the promised Maxwell-Boltzmann distribution for the ejectile energy distribution, confirming the picture of a "heated" nucleus "evaporating" nucleons to "cool"
- The distribution is more commonly written as $N_b(\varepsilon) = \varepsilon\exp(-\varepsilon/T_R)$, since the other factors wind up being ≈ 1
- To arrive at useful numbers, recall the nuclear temperature is related to excitation energy by $T \approx \sqrt{E^*/a}$ where (empirically) $a \approx \frac{A}{8}MeV^{-1}$



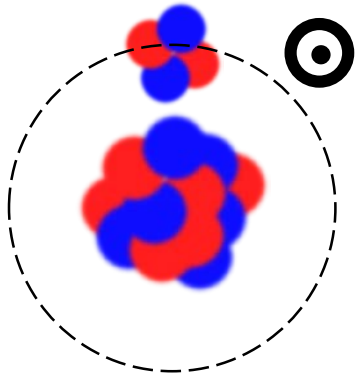


Ejectile **angular** distribution

- If we take the independence hypothesis to heart, then we would expect the compound nucleus to have lost all information about how it was formed
- As such, in the center-of-mass frame we expect an isotropic emission of ejectiles
  (in the lab there will be a bias toward forward angles from momentum conservation)
  … however, that picture is a bit too naïve



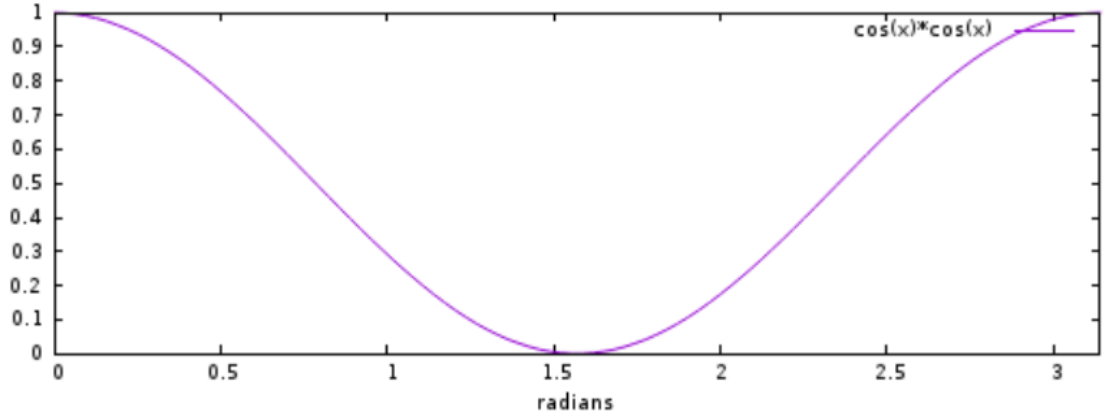
- Consider the classical picture of a projectile bringing in some angular momentum $\vec{l}$
- In order to rid the angular momentum from the system,
  the best scenario would be for the ejectile to be emitted perpendicular to $\vec{l}$
- By considering all trajectories corresponding to $\left|\vec{l}\right|$, we realize that ejectiles will be preferentially emitted at forward and backward angles,
  since these angles are perpendicular to all $\vec{l}$ for a given $\left|\vec{l}\right|$,
  and symmetric about $\theta_{cm} = 90°$



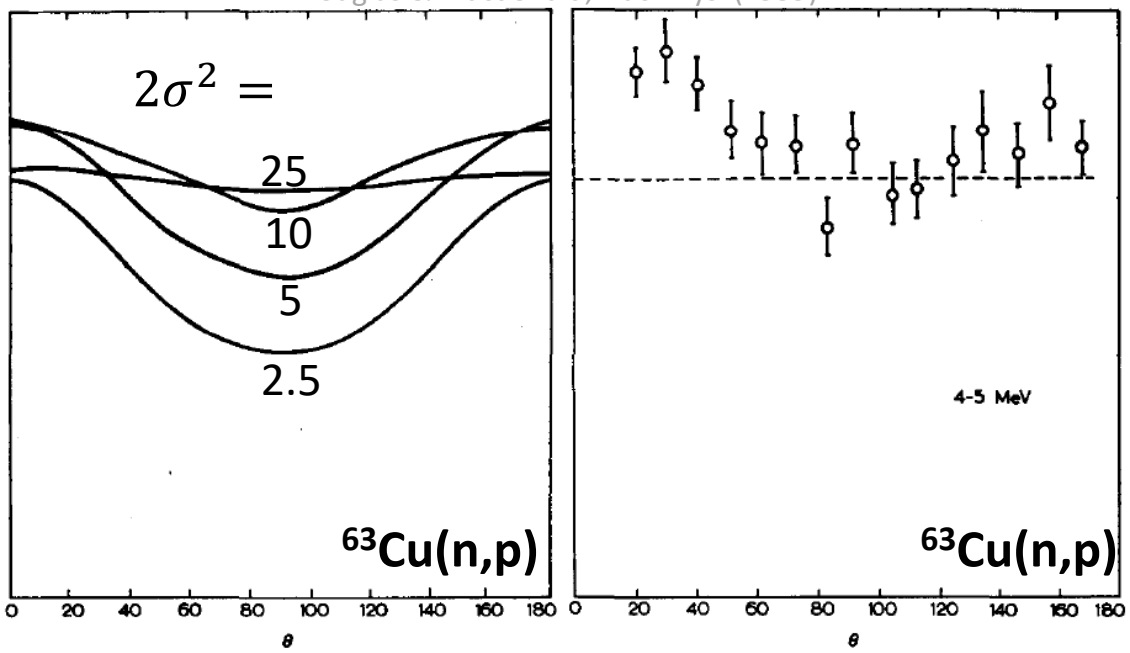
- As we might anticipate from our qualitative picture,
  the anisotropy is only appreciable for heavy projectiles and/or large incident energies
  (*larger incident energies will face more competition with forward-peaked direct processes)

Ejectile angular distribution

- For a slightly more quantitative analysis, consider the fact that the scattered wave goes like: $\psi_{sc} \propto \sum_{l=0}^{\infty} P_l(\cos(\theta))$
- To satisfy the symmetry about $90°$, only even-l enter in the sum
- Since we expect high l to be suppressed by the centrifugal barrier, to first order the anisotropy will be similar to the $l = 2$ Legendre polynomial: $\propto \cos^2(\theta)$
- Each l is also weighted (among other factors) by the final density of levels with J that can be accessed by angular momentum transfer l
- Recall that the level density is the state density weighted by the spin-distribution
  $\rho(E^*, J) \approx \rho(E^*) \frac{2J+1}{2\sigma^3\sqrt{2\pi}} \exp\left(\frac{-J(J+1)}{2\sigma^2}\right)$,
  where σ is the spin-cutoff parameter
- Smaller σ will result in a narrower spin distribution, and therefore a more anisotropic angular distribution (since fewer l can participate)

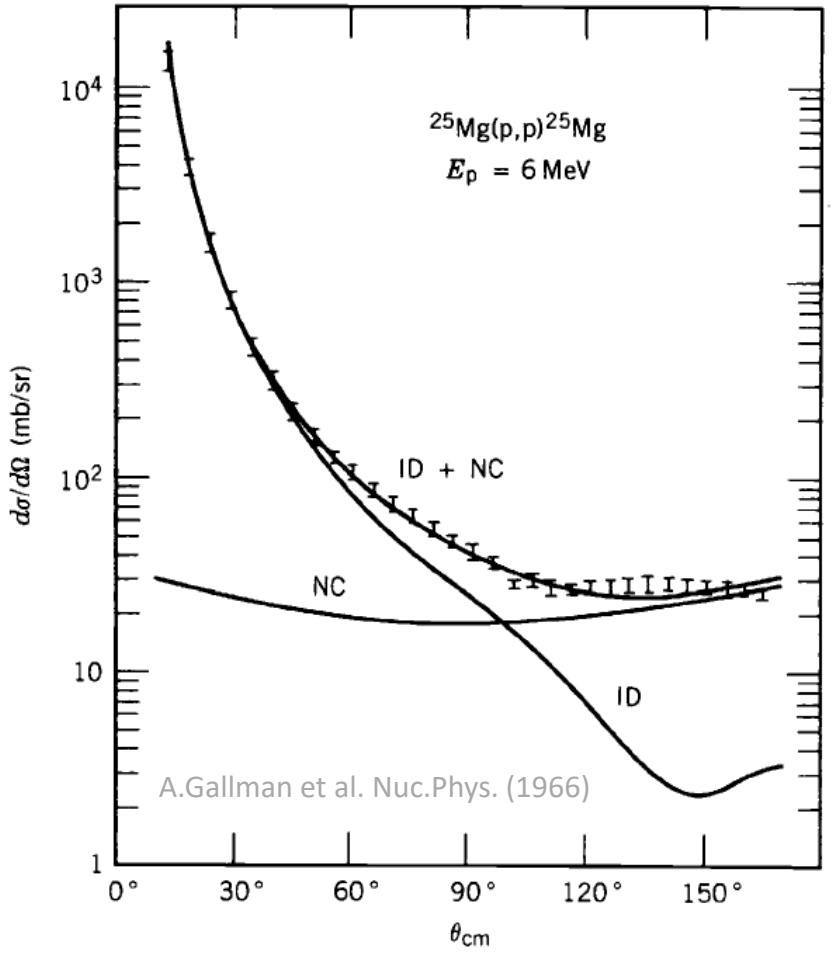


Douglas & Macdonald, Nuc.Phys. (1959)



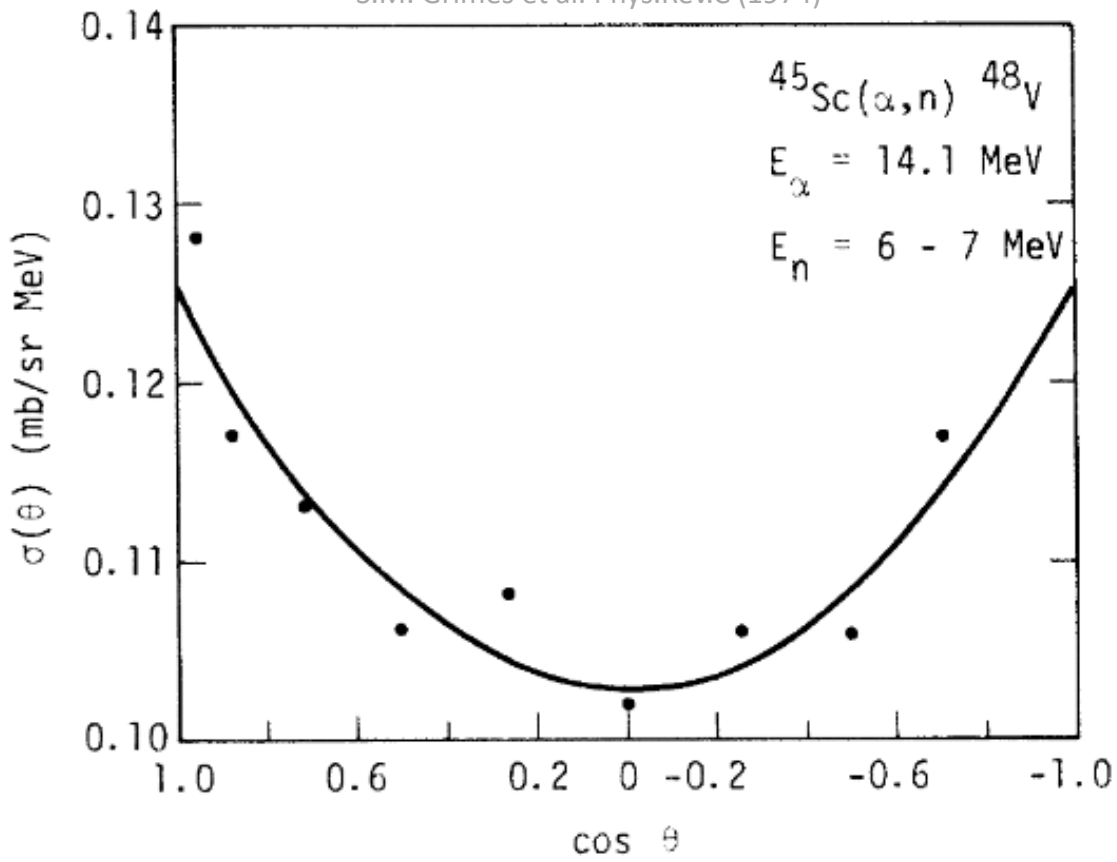
Ejectile angular distribution, experimental considerations

- The statistical nuclear reaction mechanism will compete with direct reactions, so backward angles are where one looks for information about the compound nucleus
- Similarly, lower bombarding energies and channels that would require multiple nucleon-transfer for a direct reaction are more promising for statistical nuclear reaction signatures



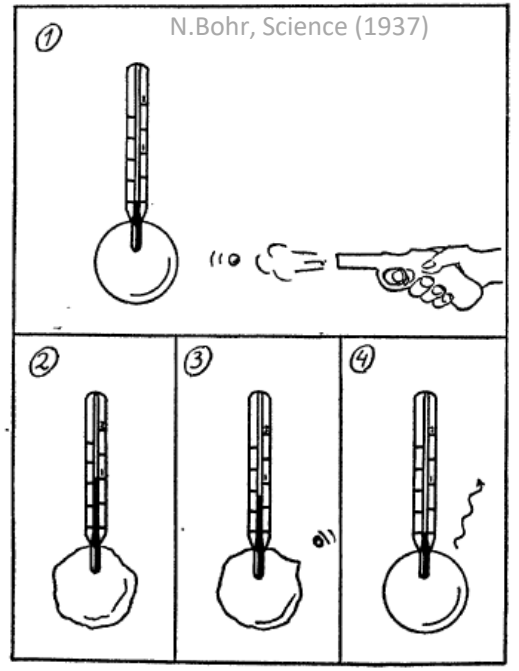
A.Gallman et al. Nuc.Phys. (1966)

S.M. Grimes et al. Phys.Rev.C (1974)

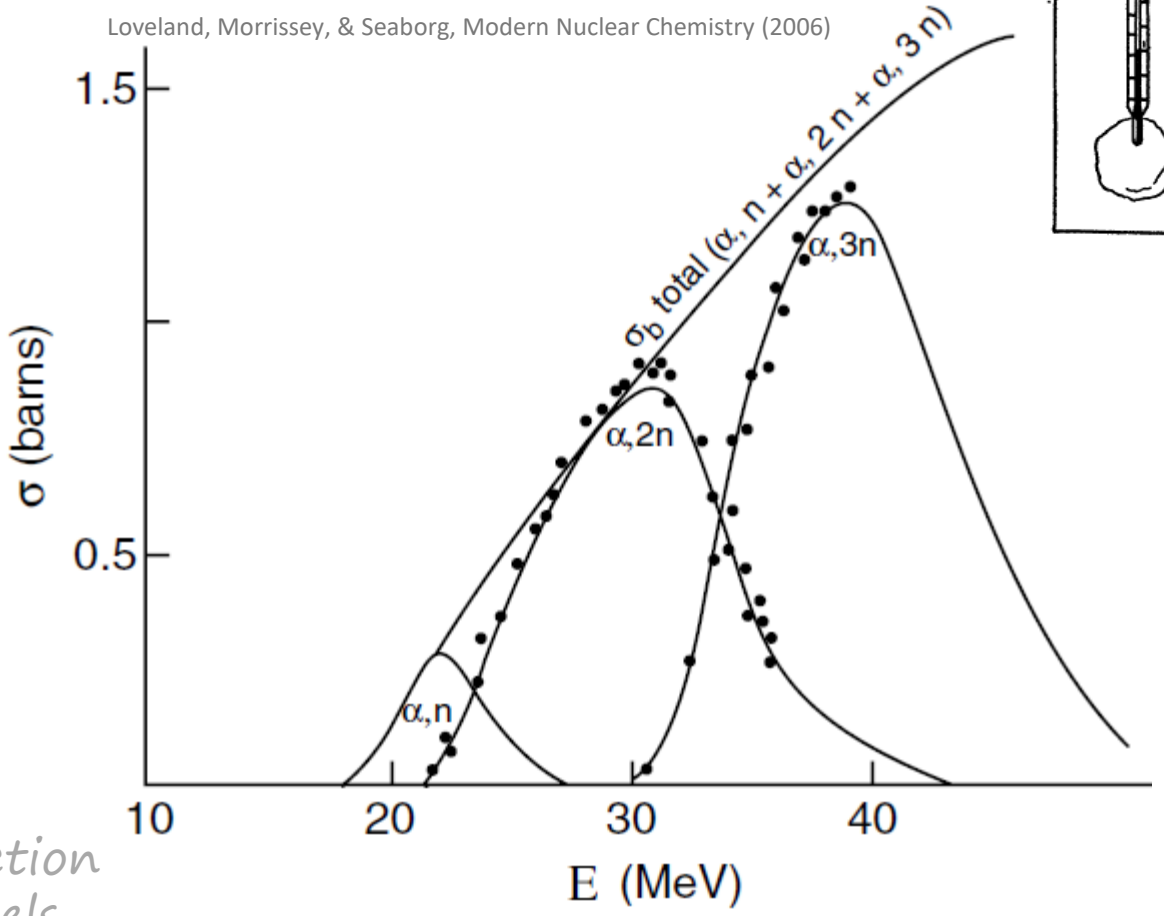


Inclusive cross section

- In the evaporation picture, wouldn't it be more effective to "cool off" by "boiling" off more than one nucleon?
- Indeed! That's exactly what happens, once it's energetically favorable
- This sort of energy behavior is characteristic for cross sections from evaporation processes
- For the case on the right, the sum over all neutron-emitting channels is written as (α, xn) and is called the "inclusive cross section"



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)



Fun fact to know & tell:
In the α-process of core-collapse supernovae, only the (α,xn) cross section is relevant, not the individual channels.
Which is to say that sometimes the inclusive cross section is the only thing that matters.

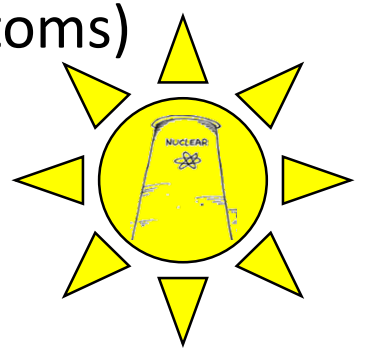
Birth of Nuclear Astrophysics

- Nuclear astrophysics was spawned in 1920 by a realization of Arthur Eddington (The Observatory (1920)):
- Based on fossil evidence at the time, the Earth was known to be more than several hundred million years old and the sun presumably had to be at least as old
- A plausible explanation for the sun's power might be gravitational energy being converted into heat from the gaseous solar sphere collapsing, taking place on the Kelvin-Helmholtz timescale
  - $\tau_{KH} = \frac{Total\ Kinetic\ Energy}{Energy\ Loss\ Rate} = \frac{(Potential\ Energy)/2}{Luminosity} = \frac{GM^2}{2RL} \approx 16Myr$ for the sun

from the virial theorem

- Instead, maybe it's just a lump of burning coal!
  - Typical chemical bond energies are ~eV
  - The sun has a mass of ~10^{30}kg and a nucleus is ~10^{-27}kg, it has ~10^{57} nuclei (or atoms)
  - Since the sun releases ~10^{39}MeV/s, chemical burning would last roughly

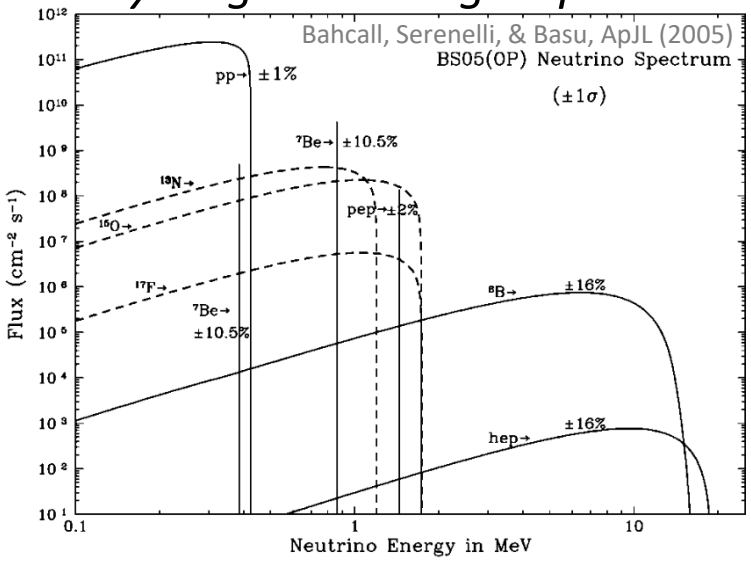
$$t_{burn} = \frac{Amount\ of\ Fuel}{Rate\ of\ Fuel\ Burning} \sim \frac{10^{57} atoms * 1eV/atom}{10^{39} MeV/s * 10^{6} eV/MeV} \sim 10^{12} s \sim 30kyr$$



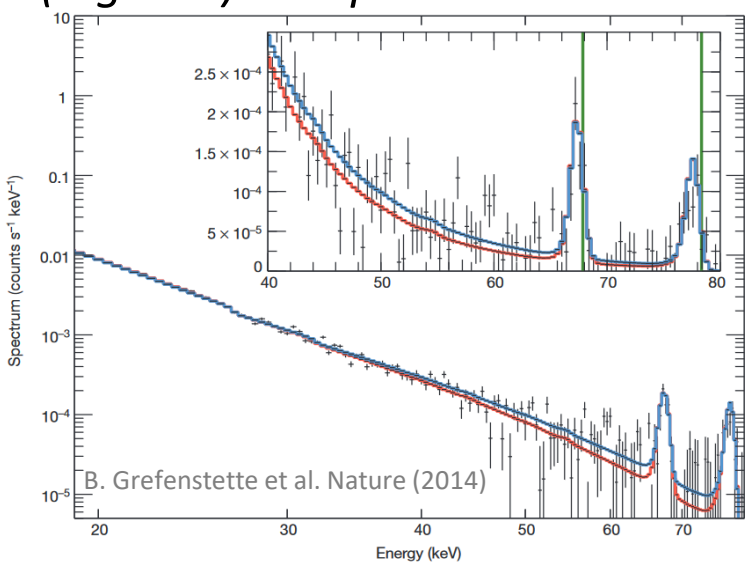
- Third time's the charm…let's look at nuclear energy
  - Aston measured a 32MeV discrepancy between 4 protons and 1 Helium nucleus (4p->α actually yields ~27MeV)
  - Multiplying our fuel amount by 10^6 (eV->MeV) results in $\sim 10Gyr$ burn time …which finally does the job

Evidence for recent nuclear reactions in space (selected examples)

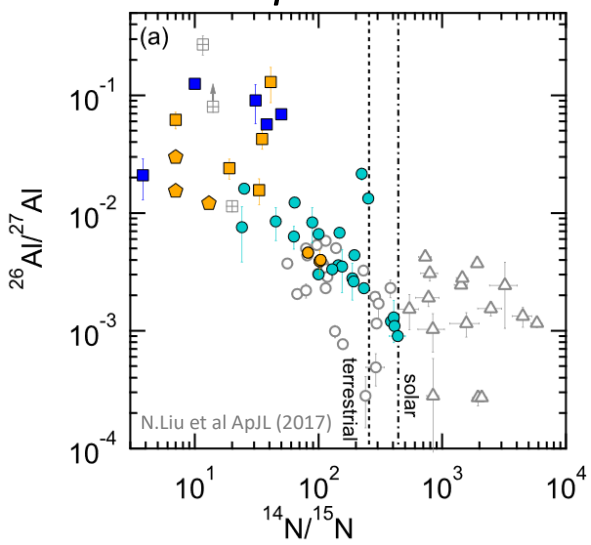
Solar ν attributable to hydrogen burning sequences



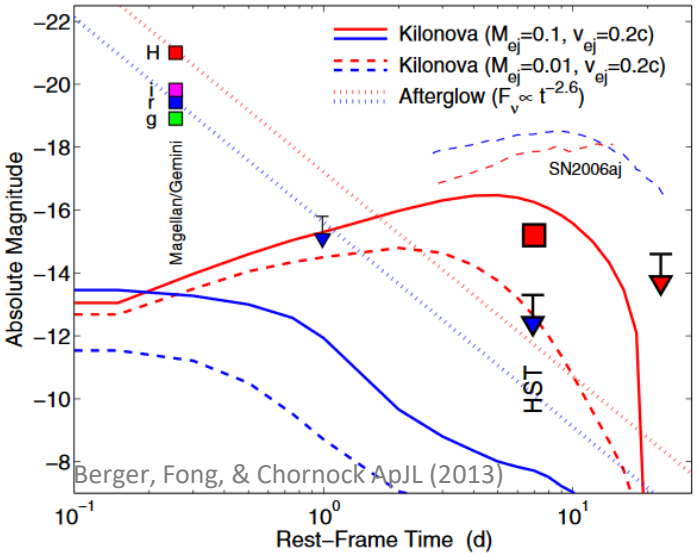
γ-rays of short-lived isotopes (e.g. ^{44}Ti) in supernova remnants



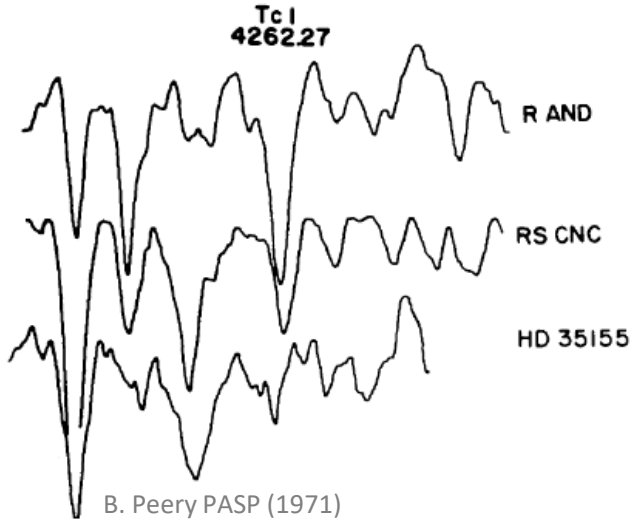
Anomalous isotopic ratios in meteoritic "pre-solar" dust grains



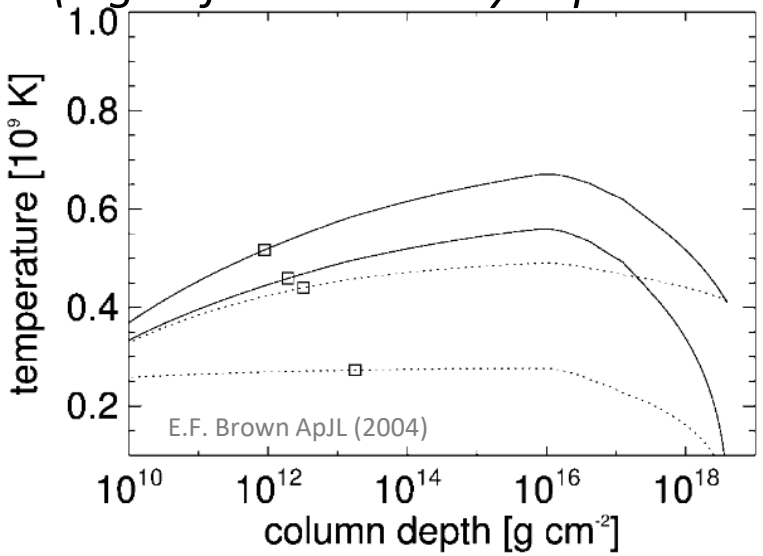
Neutron star merger afterglow associated with radioactive decay



Radioactive elements (e.g. Tc) in stellar spectra



Match in energetics (e.g. C-fusion in X-ray superbursts)



Thermonuclear rates and the Maxwell-Boltzmann distribution

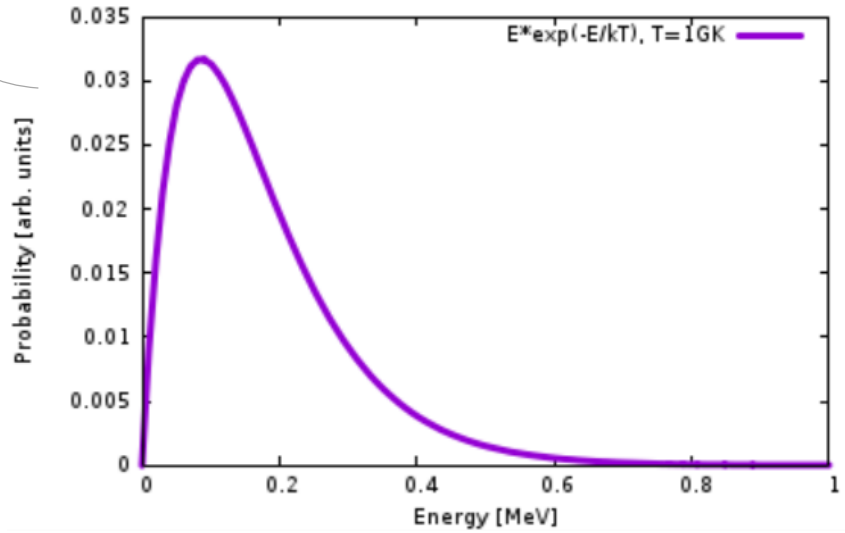
- Inserting $P_{MB}(v)$ into $\langle \sigma v \rangle_{12} = \int_0^\infty P_{MB}(v)\sigma_{12}(v)v\,dv$ yields

$$\langle \sigma v \rangle_{12} = 4\pi \left(\frac{\mu}{2\pi k_B T}\right)^{3/2} \int_0^\infty \sigma_{12}(v) v \left(v^2 \exp\left(-\frac{\mu v^2}{2k_B T}\right)\right) dv$$

- Noting the center of mass energy $E = \frac{1}{2}\mu v^2$ and so $dE = \mu v\,dv$ (i.e. $v^2 \rightarrow \frac{2}{\mu}E$, $dv \rightarrow \frac{1}{\mu v}dE$), we finally arrive at a useful equation for the astrophysical reaction rate

$$\langle \sigma v \rangle_{12} = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sigma_{12}(E) E \exp\left(-\frac{E}{k_B T}\right) dE$$

- Note that this is the general formula for classical gases. We'll go over special cases (for classical gases) in a bit.
- As an aside, personally I find k_B hard to remember, but I find it easier to remember that $11.6045 * E_{MeV} = T_9$, where E_{MeV} is energy in MeV and T_9 is temperature in GK



Reverse (a.k.a. inverse) rates

- Making the substitution that $\left(\frac{\lambda_{ij}}{2\pi}\right)^2 = \frac{\hbar^2}{p^2} = \frac{\hbar^2}{2\mu_{ij}E_{ij}}$ and noting $\mu_{ij} = \frac{m_i m_j}{m_i + m_j}$, taking the ratio of the forward and reverse cross sections yields

$$\frac{\sigma_{12}}{\sigma_{34}} = \frac{A_3 A_4}{A_1 A_2}\frac{E_{34}}{E_{12}}\frac{(2J_3+1)(2J_4+1)}{(2J_1+1)(2J_2+1)}\frac{(1+\delta_{12})}{(1+\delta_{34})}$$

  where A_i can be used instead of m_i since the units cancel

- For the rates, recall $\langle\sigma v\rangle_{12} = \sqrt{\frac{8}{\pi\mu_{12}}}\frac{1}{(k_B T)^{3/2}}\int_0^\infty \sigma_{12}(E_{12})E_{12}\exp\left(-\frac{E_{12}}{k_B T}\right)$,

$$\text{so } \langle\sigma v\rangle_{34} = \sqrt{\frac{8}{\pi\mu_{34}}}\frac{1}{(k_B T)^{3/2}}\int_0^\infty \sigma_{34}(E_{34})E_{34}\exp\left(-\frac{E_{34}}{k_B T}\right)$$

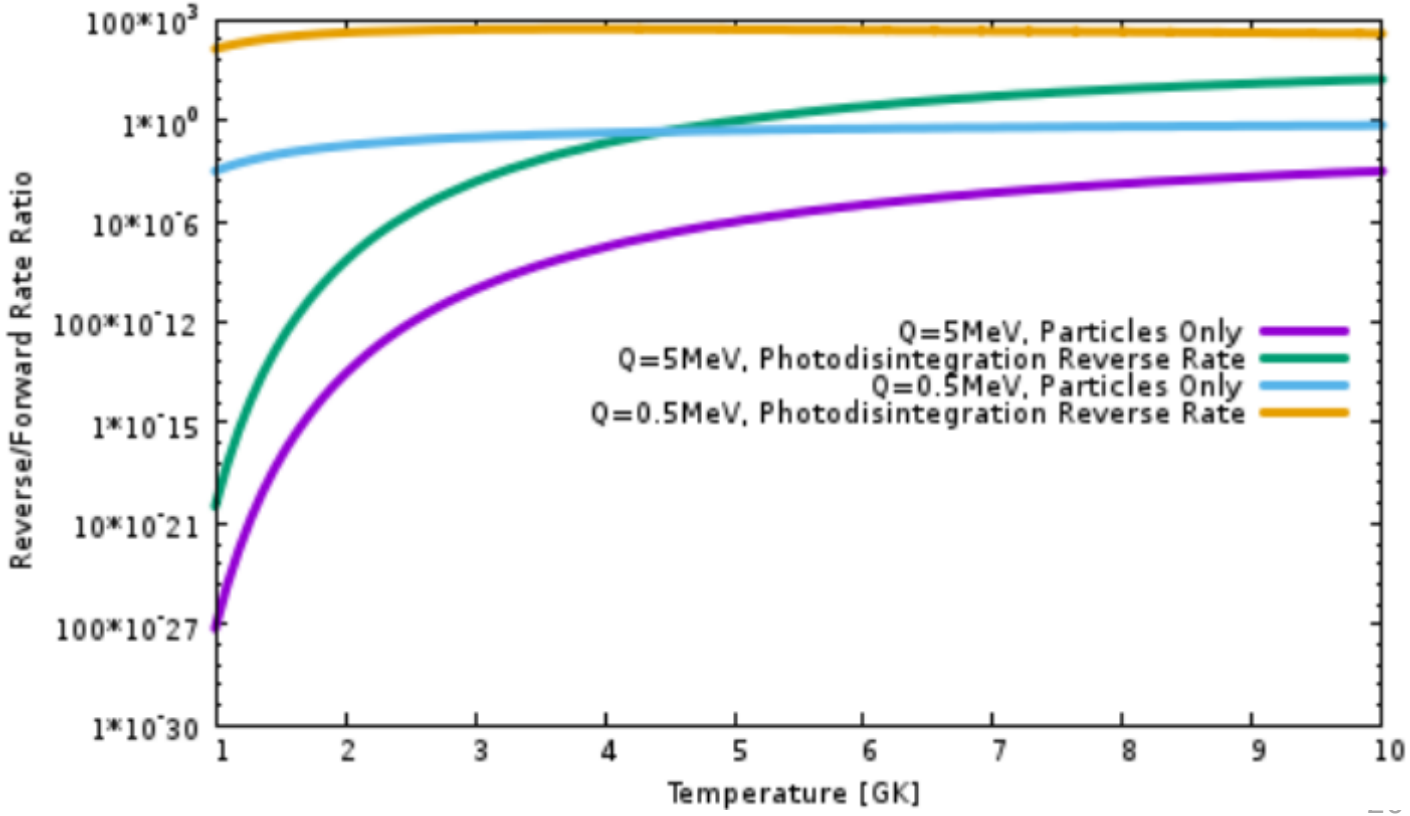
- Since $E_{34} = E_{12} + Q_{12}$ (if $Q_{12} > 0$), when we take the ratio $\langle\sigma v\rangle_{34}/\langle\sigma v\rangle_{12}$, the integrands mostly cancel except for the energy-independent part:

$$\frac{\langle\sigma v\rangle_{34}}{\langle\sigma v\rangle_{12}} = \frac{(2J_1+1)(2J_2+1)}{(2J_3+1)(2J_4+1)}\frac{(1+\delta_{34})}{(1+\delta_{12})}\left(\frac{\mu_{12}}{\mu_{34}}\right)^{3/2}\exp\left(-\frac{Q_{12}}{k_B T}\right)$$

- The exponential dominates, so $\frac{\langle\sigma v\rangle_{34}}{\langle\sigma v\rangle_{12}} \approx \exp\left(-\frac{Q_{12}}{k_B T}\right)$ gives the correct order of magnitude

Reverse (a.k.a. inverse) rates: photodisintegration

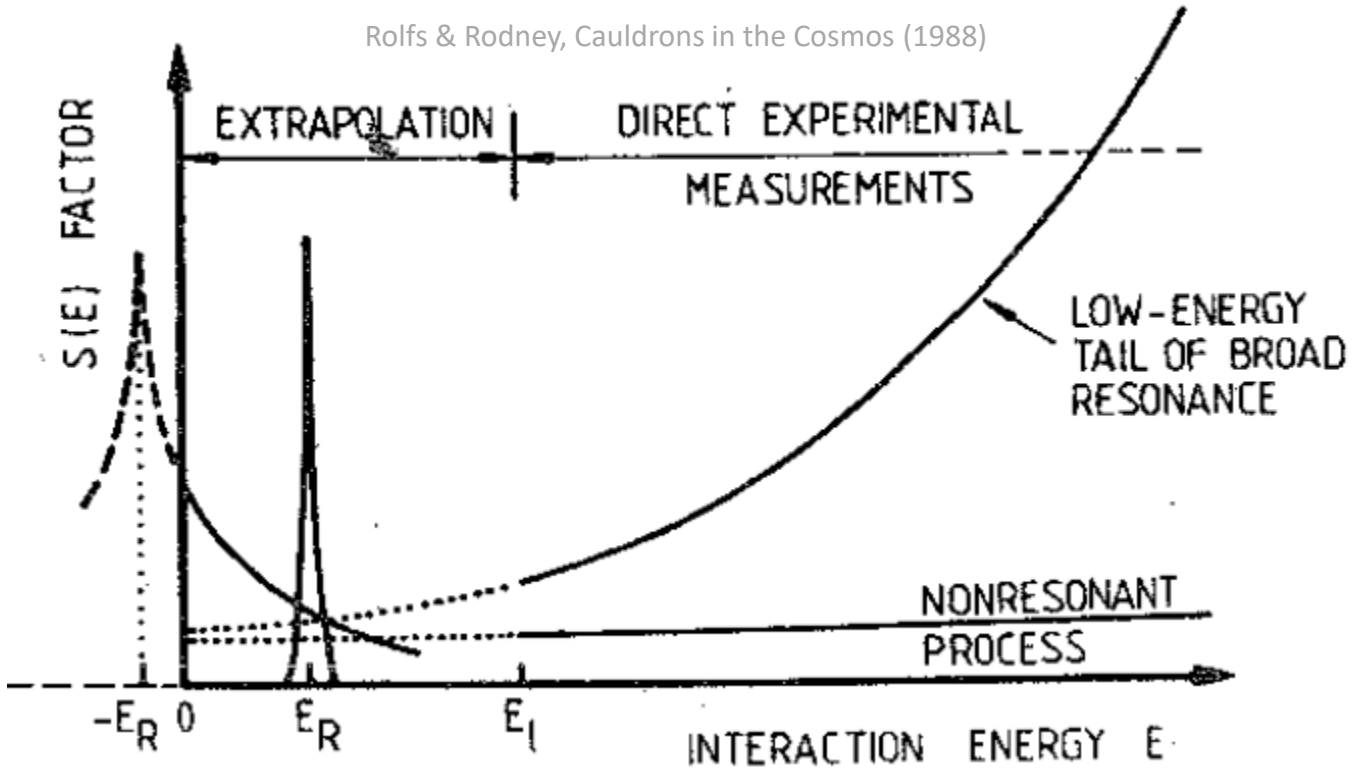
- For photodisintegration, we have to take into account the fact that the photons follow a Planck distribution and not a Maxwell-Boltzmann distribution
- So the reverse rate becomes: $\frac{\langle \sigma v \rangle_{3\gamma}}{\langle \sigma v \rangle_{12}} \approx \left(\frac{\mu_{12} c^2}{k_B T}\right)^{3/2} \exp\left(-\frac{Q_{12}}{k_B T}\right)$
- This extra factor is actually a huge deal because of that extra temperature dependence
- In fact, for low Q-value reactions, the photodisintegration rate is dominant



Aside: S-factors, the nuclear physics nuggets of $\langle \sigma v \rangle$

- Often it's useful to remove the trivial energy dependence from the cross section, in particular for charged-particle reaction rates
- The idea is that $S(E)$ contains all of the interesting physics
- Since the energy dependence is different for different types of reactions, e.g. direct capture of a neutron as compared to direct capture of a charged particle, the factorization that is done to get $S(E)$ depends on the reaction type

Rolfs & Rodney, Cauldrons in the Cosmos (1988)



Thermonuclear rate: **Direct neutron-capture**

- Recall that at low energies, those of interest for nuclear astrophysics, the neutron-capture cross section is described by the $1/v$ law: $\sigma_{n\,cap} \propto \frac{1}{v_n}$
- As such, it's clear that $\langle \sigma v \rangle_{n\,cap} \approx constant$
- The cross section (and rate) can be characterized by the S-factor at thermal energies and any deviation from $1/v$ behavior is accounted for by the local derivative(s) of the S-factor:

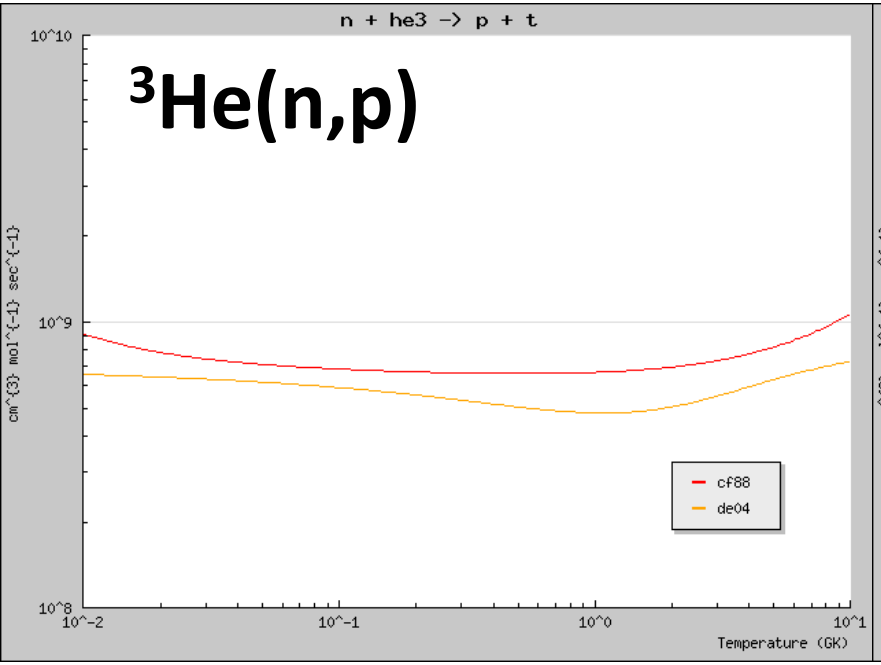
$$\sigma(E) = \sqrt{\frac{\mu}{2E}}\left(S(0) + \dot{S}(0)E^{½} + \tfrac{1}{2}\ddot{S}(0)E + \cdots\right)$$

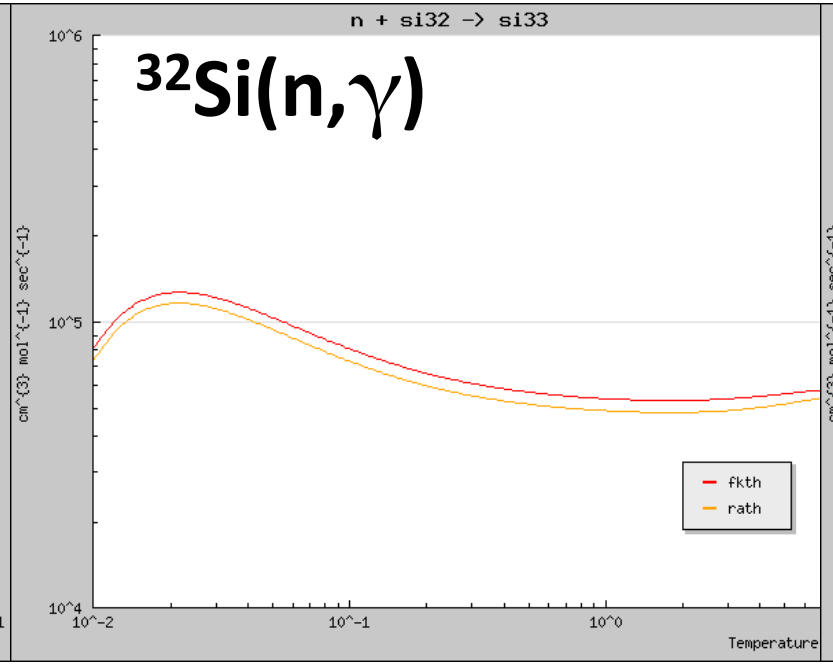
- $S(0)$ is generally the S-factor at thermal energy ($v_{th} = 2.2 \times 10^5 \frac{cm}{s} = E_L = 2.53 \times 10^{-8} MeV$): $S(0) = \sigma_{th} v_{th} = 2.2 \times 10^{-19} \sigma_{th} \frac{cm^3}{s}$, where σ_{th} is the thermal cross section in barns
- $\dot{S}(0)$ and $\ddot{S}(0)$ are fit to cross section data near thermal energies
- Employing this $\sigma(E)$ in the general equation $\langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE$, results in

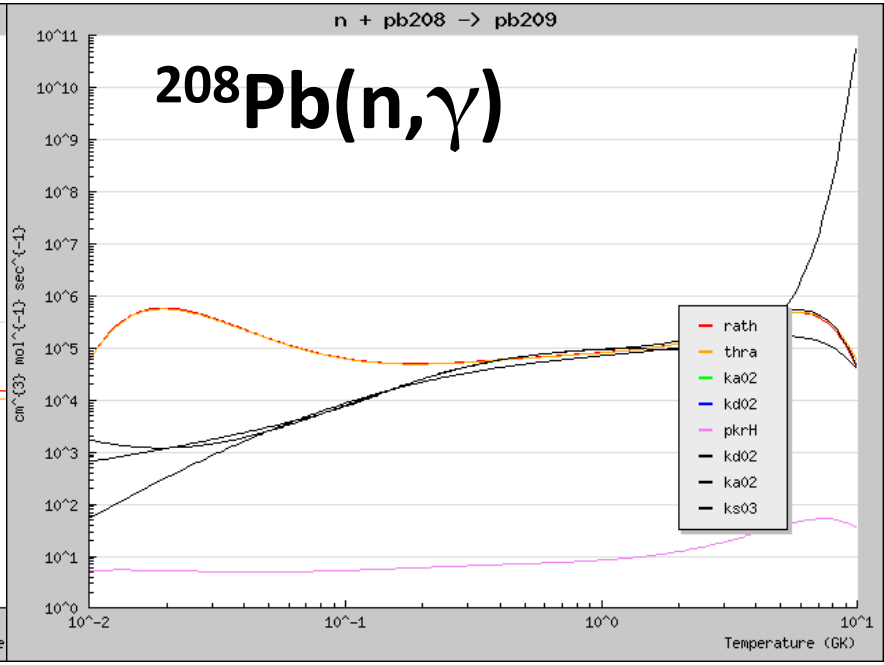
$$\langle \sigma v \rangle_{n\,cap} = S(0) + \sqrt{\frac{4}{\pi}} \dot{S}(0)(k_B T)^{1/2} + \tfrac{3}{4}\ddot{S}(0) k_B T + \cdots$$

Thermonuclear rate: Direct neutron-capture

Examples (from the REACLIB database):







Clear that the main feature is $\langle \sigma v \rangle_{n\,cap} \approx constant$ over 3 orders of magnitude in T

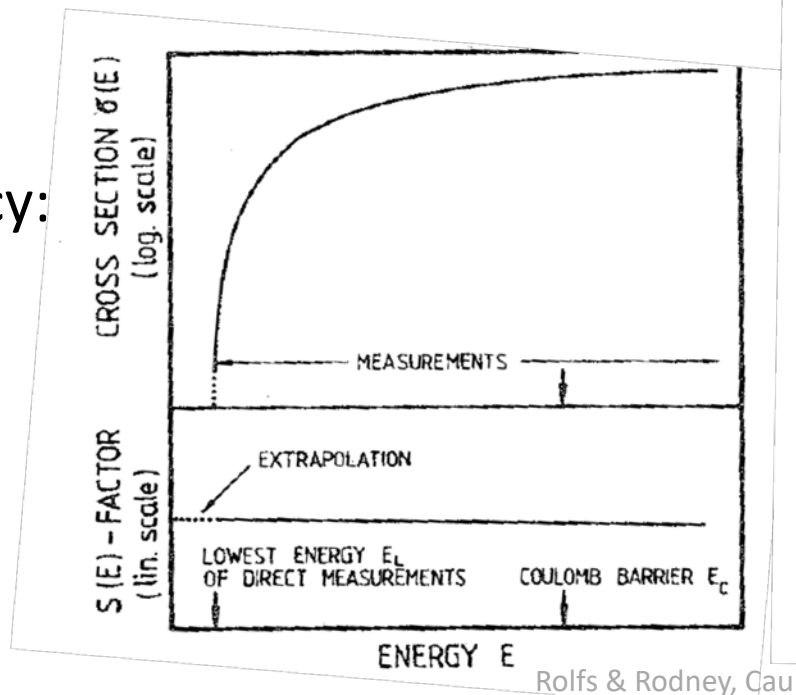
Thermonuclear rate: **Direct charged particle-capture**

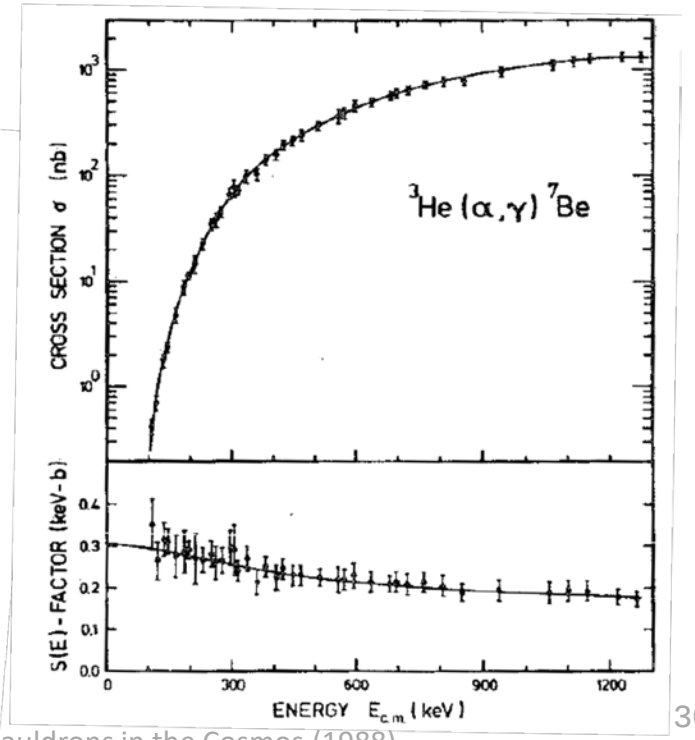
- Recall that the cross section for charged-particle capture depends on the effective geometric area of the projectile, described by $\lambda_{deBroglie}$, and the probability of the charged projectile tunneling through the Coulomb barrier of the target
- When discussing α decay, we showed $P_{tunnel}(E) = \exp(-2\pi\eta)$,

  where the factor with the Sommerfeld parameter is $2\pi\eta = \pi \frac{e^2}{\hbar c} Z_1 Z_2 \sqrt{\frac{2\mu c^2}{E}}$
- For E in MeV and A in atomic mass units (1u = 931.5MeV/c²):

  $2\pi\eta = 0.989 Z_1 Z_2 \sqrt{\frac{1}{E} \frac{A_1 A_2}{(A_1 + A_2)}}$
- Removing the trivial energy dependency:

  $\sigma_{ch.cap}(E) = \frac{1}{E} \exp(-2\pi\eta)\, S(E)$





Rolfs & Rodney, Cauldrons in the Cosmos (1988)

Thermonuclear rate: Direct charged particle-capture

- Employing $\sigma(E) = \frac{1}{E}\exp(-2\pi\eta)\,S(E)$ in $\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu}}\frac{1}{(k_BT)^{3/2}}\int_0^\infty \sigma(E)E\exp\left(-\frac{E}{k_BT}\right)dE$ gives:

  $= \sqrt{\frac{8}{\pi\mu}}\frac{1}{(k_BT)^{3/2}}\int_0^\infty S(E)\exp\left(-\frac{E}{k_BT} - 2\pi\eta\right)dE$

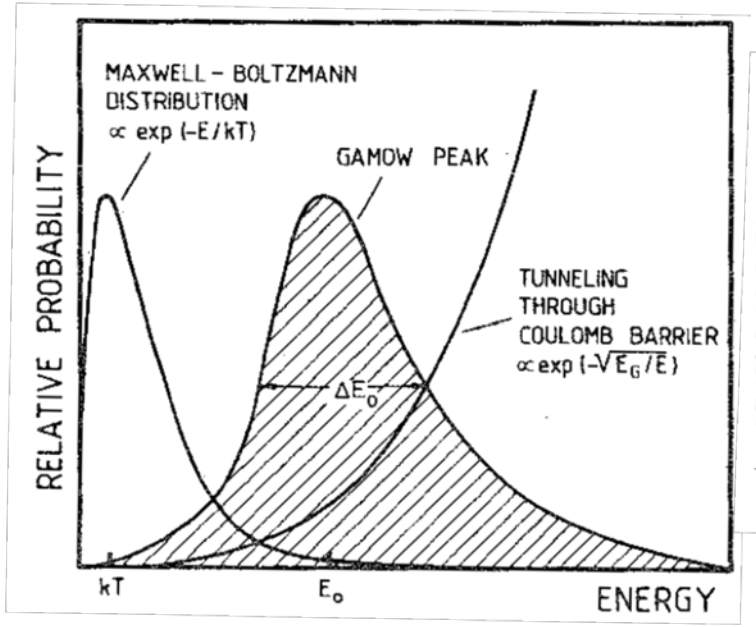
- The integrand is a product of the probability of a charged-particle pair having energy E (from the Maxwell-Boltzmann distribution) and the tunneling probability for that energy
- The integrand maximum (found by solving for the derivative being equal to zero) is:

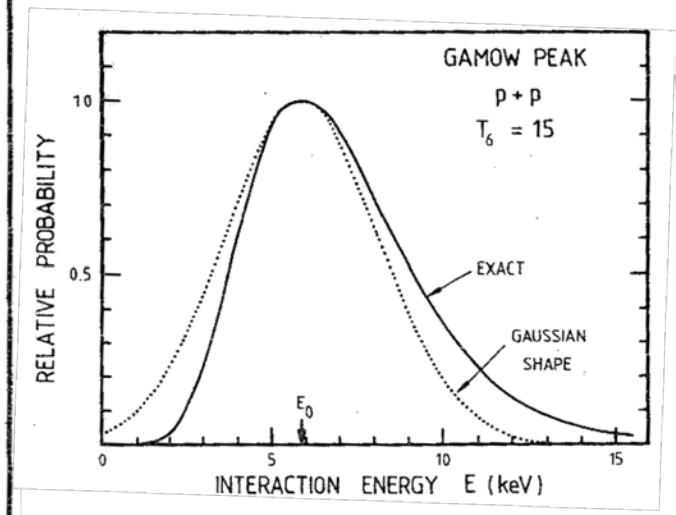
  $E_G = 0.122\left(Z_1^2Z_2^2\frac{A_1A_2}{(A_1+A_2)}T_9^2\right)^{1/3}$ MeV

- Approximating the integrand as a Gaussian results in a distribution with the $1/e$ width

  $\Delta_G = 4\sqrt{\frac{E_Gk_BT}{3}}$ MeV

- E_G is the Gamow peak and Δ_G is the width of the Gamow window, which is roughly the energy range for which we care about a charged particle reaction rate for some T





Rolfs & Rodney, Cauldrons in the Cosmos (1988)

Note: Don't be too naïve when using the Gamow window estimate.
It's based on a roughly constant S(E), so the true window of interest could be different (T.Rauscher PRC 2010).

Thermonuclear rate: **Narrow resonance(s)**

- Recall the Breit-Wigner form we found for the resonant reaction cross section

$$\sigma_{BW,X(a,b)Y}(E) = \pi\left(\frac{\lambda}{\pi}\right)^2 \frac{2J+1}{(2J_a+1)(2J_X+1)} \frac{\Gamma_{aX}(E)\Gamma_{bY}(E)}{(E-E_R)^2+(\Gamma(E))^2/4}$$

- Employing this in the general form $\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE$, we realize that the contributions of the integrand are pretty negligible outside of E_R

- So, we make the approximation $\langle\sigma v\rangle_{res} = \sqrt{\frac{8}{\pi\mu}} \frac{E_R}{(k_B T)^{3/2}} \exp\left(-\frac{E_R}{k_B T}\right) \int_0^\infty \sigma_{BW}(E)\, dE$

- Noting that λ changes little over the resonance $\lambda \to \lambda_R$, writing the statistical factor as ω, and noting the widths Γ are essentially constant over the resonance we find

$$\int_0^\infty \sigma_{BW}(E)\, dE \approx \pi\left(\frac{\lambda_R}{\pi}\right)^2 \Gamma_{aX}\Gamma_{bY} \int_0^\infty \frac{1}{(E-E_R)^2+(\Gamma)^2/4} dE = 2\lambda_R^2 \omega \frac{\Gamma_{aX}\Gamma_{bY}}{\Gamma}$$

- For obfuscation purposes, we substitute in γ for $\frac{\Gamma_{aX}\Gamma_{bY}}{\Gamma}$ and call $\omega\gamma$ the resonance strength

- If we know the cross section at the peak of the resonance, we can make the approximation that the integral is half the width times the height: $\int_0^\infty \sigma_{BW}(E)\, dE \approx \pi\Gamma\sigma(E_R)$

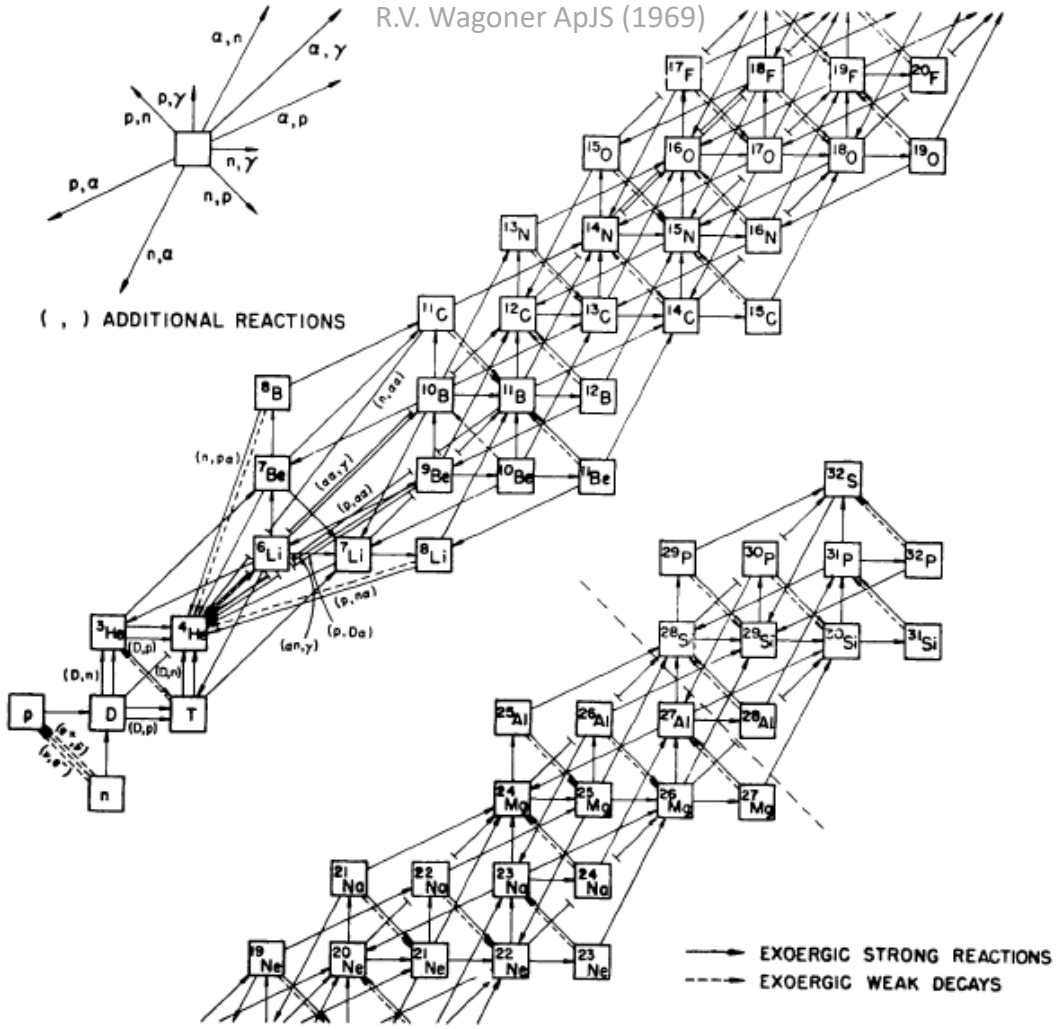
- The resonant rate becomes: $\langle\sigma v\rangle_{res} = \left(\frac{2\pi}{\mu k_B T}\right)^{3/2} \hbar^2 (\omega\gamma)_R \exp\left(-\frac{E_R}{k_B T}\right)$

Nuclear reaction networks

- Astrophysical environments typically contain many nuclei, each of which could in principle interact with the other
  (in practice usually only the photons and light projectiles matter)
- To evaluate what happens, we need to solve a reaction network
- The basic idea is to see how the abundance changes for each isotope at each step in time based on the production and destruction via all mechanisms and often to include the energy generation from said reactions
- For each species i,

$$\frac{dY_i}{dt} = \left[\sum_{j,k} Y_j Y_k \rho N_A \langle \sigma v \rangle_{jk \to i} + \sum_{l} \lambda_{l \to i} Y_l\right] - \left[\sum_{m} Y_i Y_m \rho N_A \langle \sigma v \rangle_{im \to any} + \sum_{n} \lambda_{i \to n} Y_l\right]$$

Production Destruction

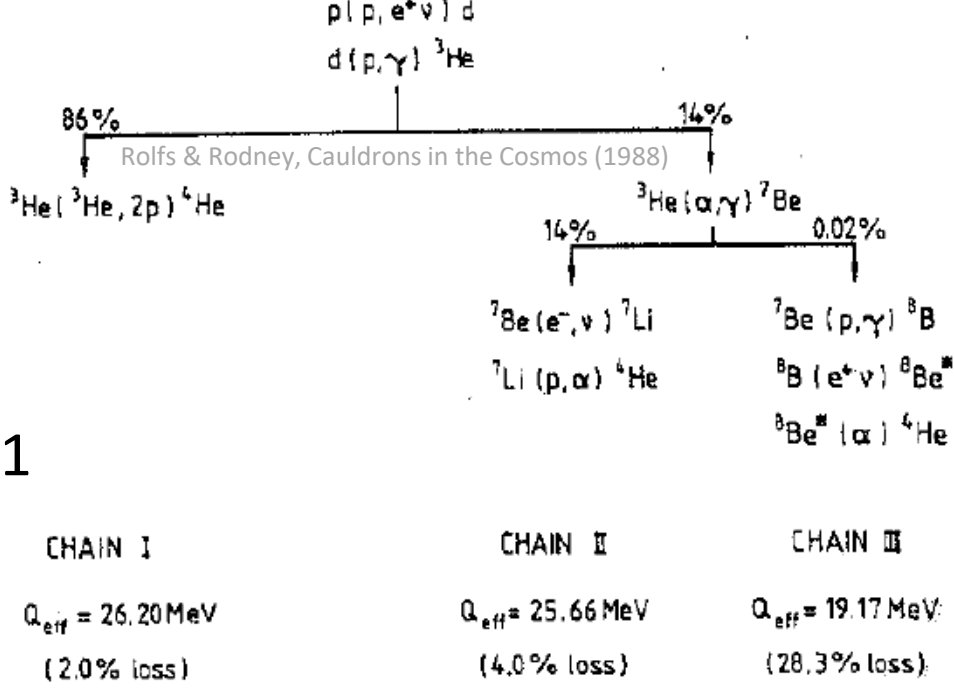


Stars' first ignition: The pp-chain

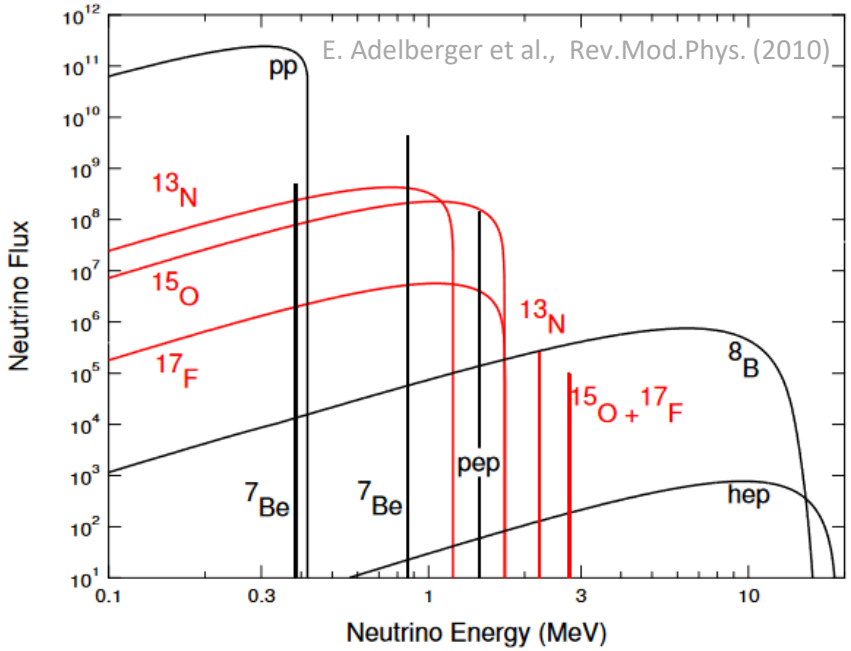
- For stars as or less massive than the sun, or primordial stars containing only H & He, nuclear burning begins in earnest with the pp-chain(s) [this is preceded by deuterium burning, but the energy generation is negligible in comparison]
- The end result of the pp-chains is to convert 4 protons in to 1 helium nucleus, releasing a total of ~27MeV in the process
- The first step (either $p + p \rightarrow d + e^+ + \nu$ or $p + e^- + p \rightarrow d + \nu$) requires a weak interaction and so the cross section is roughly $10^{20}\times$ smaller than a typical nuclear cross section and as such unmeasurable

(This is a good thing! Otherwise the sun probably would have burnt out by now.)

- The low temperature involved makes measuring these cross sections at relevant energies exceedingly difficult, meaning extrapolations of the S-factors are required and electron-screening effects must be taken into account
- The pp-chain rates are of particular interest, since the relative neutrino yields determine the pp-chain branching and therefore the internal temperature of the sun



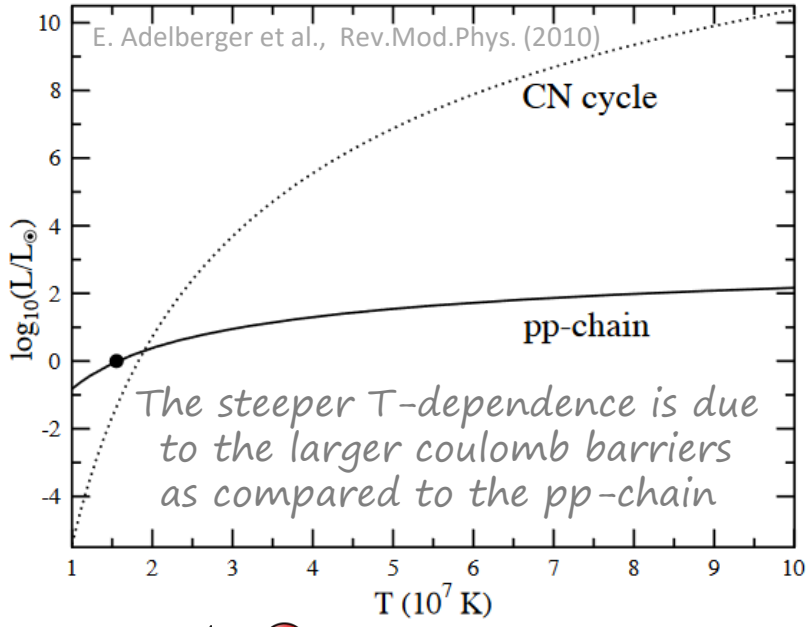
Rolfs & Rodney, Cauldrons in the Cosmos (1988)

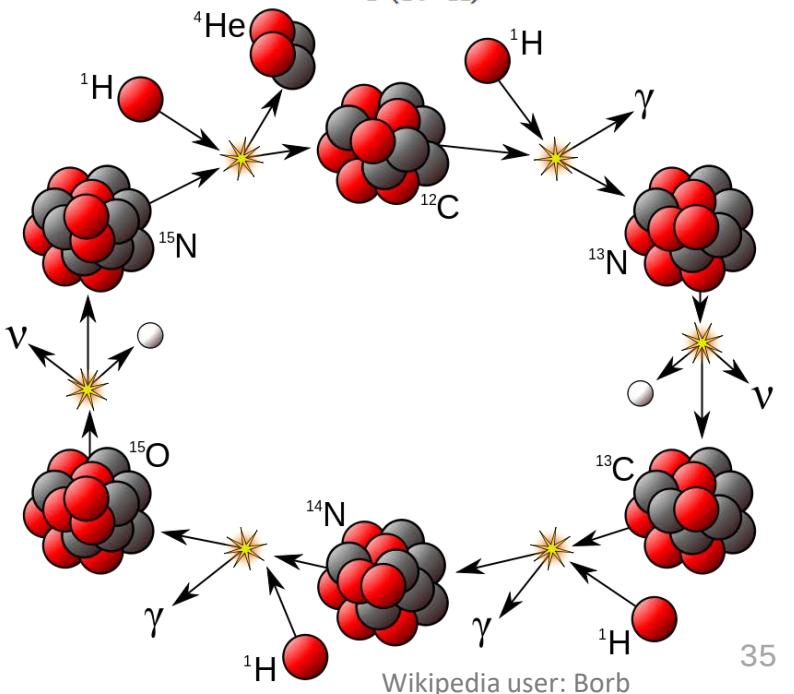


E. Adelberger et al., Rev.Mod.Phys. (2010)

It's getting hot in here, so start the CNO cycle

- For stars just a bit more massive than the sun, more radiation pressure is required to oppose gravitational contraction than the pp-chains can provide. As such, the core contracts until the central temperature is sustains a robust CNO cycle.
- The CNO cycle is another way to convert four protons into one helium nucleus. In this sequence, ^{12}C acts as a catalyst.
- At the modest core temperatures of the sun, $^{14}N(p,\gamma)$ is the rate-limiting step in the sequence
  $^{12}C(p,\gamma)^{13}N(\beta)^{13}C(p,\gamma)\mathbf{^{14}N(p,\gamma)}^{15}O(\beta)^{15}N(p,\alpha)^{12}C$
  meaning the equilibrium abundance will be concentrated in ^{14}N
- Weaker branches exist to other nuclei, which are the other CNO cycles (identified by different numbers)
- The rates of the CNO cycles play a key role in astrophysics, since they determine how long it takes for massive stars should take to burn through their core hydrogen and therefore provide the age for a globular cluster based on which mass stars (inferred by luminosity) have turned-off the main sequence (the line in the HR diagram [luminosity vs temperature] for core H-burning) yet

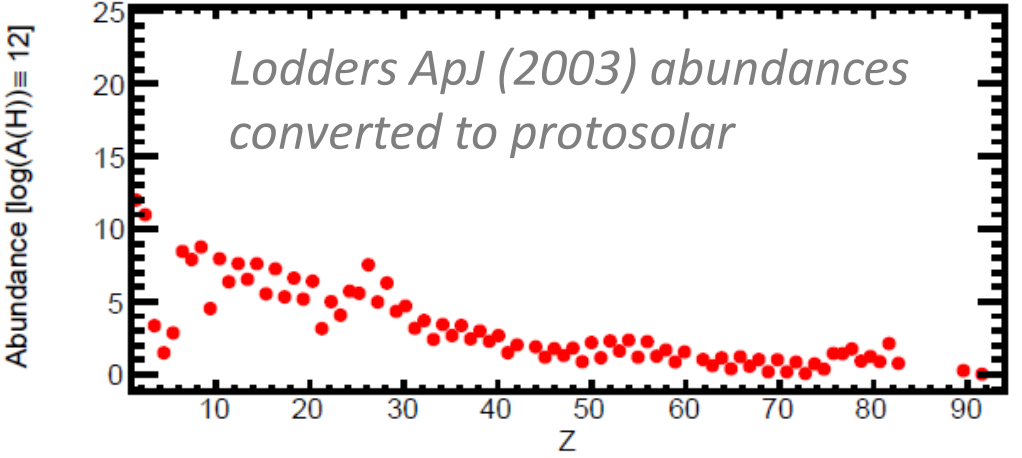


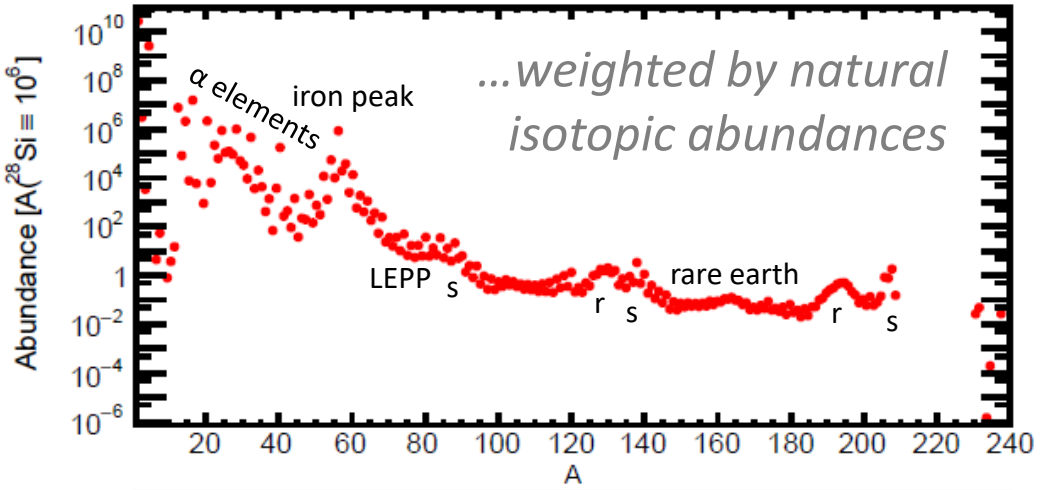


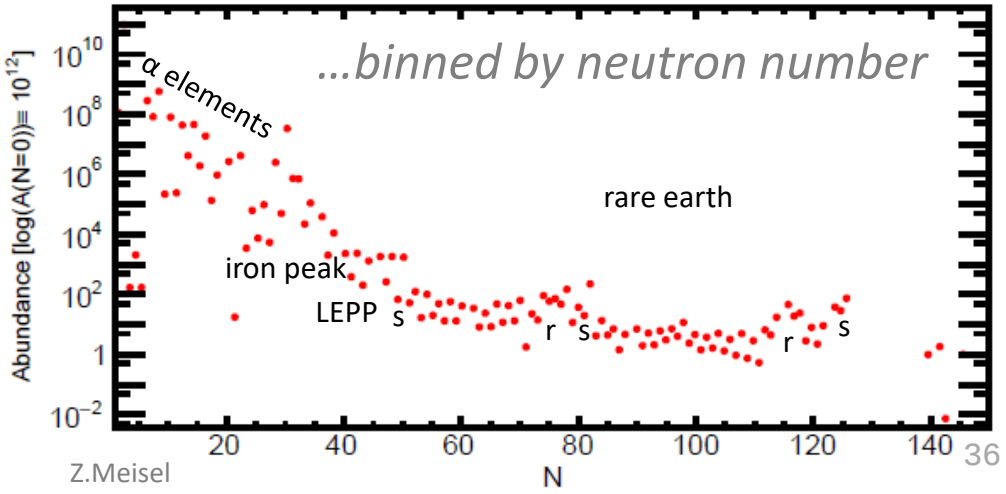
Wikipedia user: Borb

Interlude, the case for neutron-capture processes

- Looking at the solar system abundances in terms of A, we see some pretty interesting patterns
- The α elements and iron peak are consistent with our picture for massive star evolution, but we see several other features beyond iron
- Focusing on the peaks labeled *s* and *r*, we see that the *s* peaks are located at neutron magic numbers 50, 82, 126, with the *r* peaks located just below (at least for 82 & 126)
- Logically, it follows that the processes making the *s* and *r* processes somehow involve sequences of neutron capture reactions that pass through these magic numbers on the nuclear chart. This is because one feature of magic N nuclei is low neutron-capture cross sections relative to neighboring nuclei

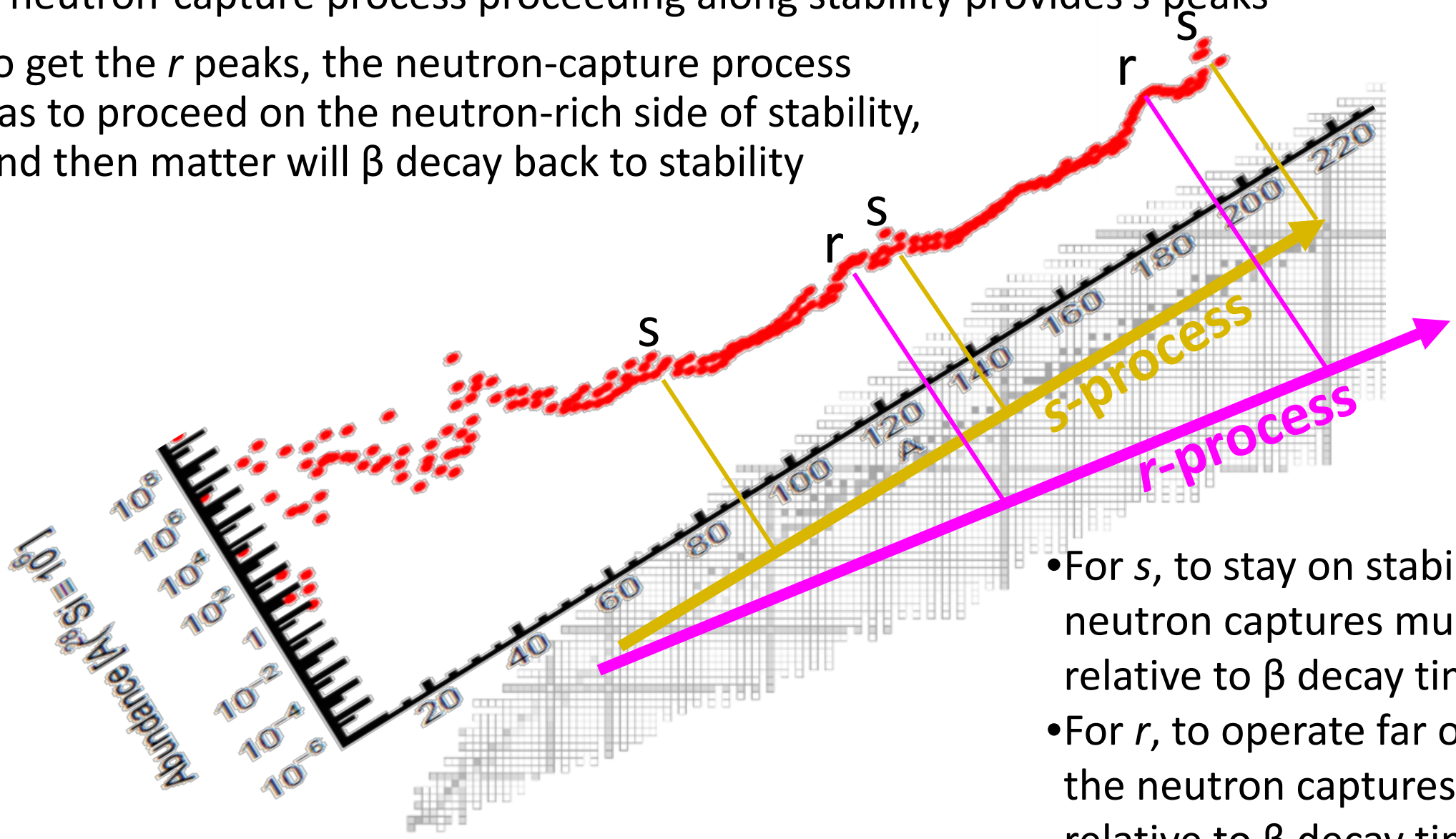






We can see where on the nuclear chart the *s* & *r* processes must have occurred by considering where in terms of *Z* pile-up at a magic *N* must have occurred to give the observed peak in *A*

- A neutron-capture process proceeding along stability provides *s* peaks
- To get the *r* peaks, the neutron-capture process has to proceed on the neutron-rich side of stability, and then matter will β decay back to stability



- For *s*, to stay on stability, the neutron captures must be ***s***low relative to β decay timescales
- For *r*, to operate far off stability, the neutron captures must be ***r***apid relative to β decay timescales

Slow and steady, the *s-process*

- For the *s*-process, we need an environment where nuclei can bathe for a long time in a moderate neutron density ($n_n \sim 10^{7-10}\mathrm{cm}^{-3}$). Let's consider the reaction flow for this case.
- For isotope (Z,A), the abundance change is the sum of production & destruction mechanisms, $\frac{dN_{Z,A}}{dt}(t) = n_n(t)n_{Z,A-1}(t)\langle\sigma v\rangle_{Z,A-1} - n_n(t)n_{Z,A}(t)\langle\sigma v\rangle_{Z,A} - \lambda_{\beta:Z,A}n_{Z,A}(t)$
- We'll ignore species with fast β-decays, since they'll just form the more stable isobar more or less instantly, so we can set the last term to zero
- Also, recall $\langle\sigma v\rangle_{n\ cap} \sim constant$, and anyhow we'll assume a constant temperature, so $\langle\sigma v\rangle_i \rightarrow \sigma_i v_T$, where v_T is the thermal velocity for the environment temperature
- As such, our abundance change equation is now $\frac{dN_{Z,A}}{dt}(t) = n_n(t)v_T\left(\sigma_{Z,A-1}n_{Z,A-1} - \sigma_{Z,A}n_{Z,A}\right)$
- Note that $n_n(t)v_T$ is the neutron flux. Integrating this over time gives the neutron irradiation $\tau = \int_0^t n_n(t)v_T\,dt = v_T\int_0^t n_n(t)\,dt$, which is referred to as the neutron exposure
- Re-casting our abundance change in terms of τ, $\frac{dN_{Z,A}}{d\tau} = \sigma_{Z,A-1}n_{Z,A-1} - \sigma_{Z,A}n_{Z,A}$
- In equilibrium, $\frac{dN_{Z,A}}{d\tau} = 0$, so $\sigma_{Z,A-1}n_{Z,A-1} = \sigma_{Z,A}n_{Z,A} = constant$
- So, we can get *s*-process relative abundances based solely on neutron-capture cross sections!

Life in the fast lane, the (hot) *r*-process

- For the *r*-process, a huge neutron density ($n_n \gtrsim 10^{20}\mathrm{cm}^{-3}$, as you'll show in the group activity) is needed in order for neutron capture lifetimes to be much shorter than the ~ms β-decay lifetimes of the extremely neutron-rich nuclei involved
- Neutron-capture will proceed along an isotopic chain until it competes with the photodisintegration rate: $\langle\sigma v\rangle_{3\gamma}/\langle\sigma v\rangle_{12} \approx 1$
- Recalling that $\frac{\langle\sigma v\rangle_{3\gamma}}{\langle\sigma v\rangle_{12}} \approx \left(\frac{\mu_{12}c^2}{k_BT}\right)^{3/2} \exp\left(-\frac{Q_{12}}{k_BT}\right)$, observing $\tau_{n,\gamma} = \frac{1}{n_n\langle\sigma v\rangle_{n,\gamma}}$ and $\tau_{\gamma,} = \frac{1}{n_\gamma\langle\sigma v\rangle_{\gamma,n}}$, and remembering (*how could we ever forget!?*) the Planck distribution $n_\gamma = \frac{8\pi^4}{13c^3h^3}(k_BT)^3$,

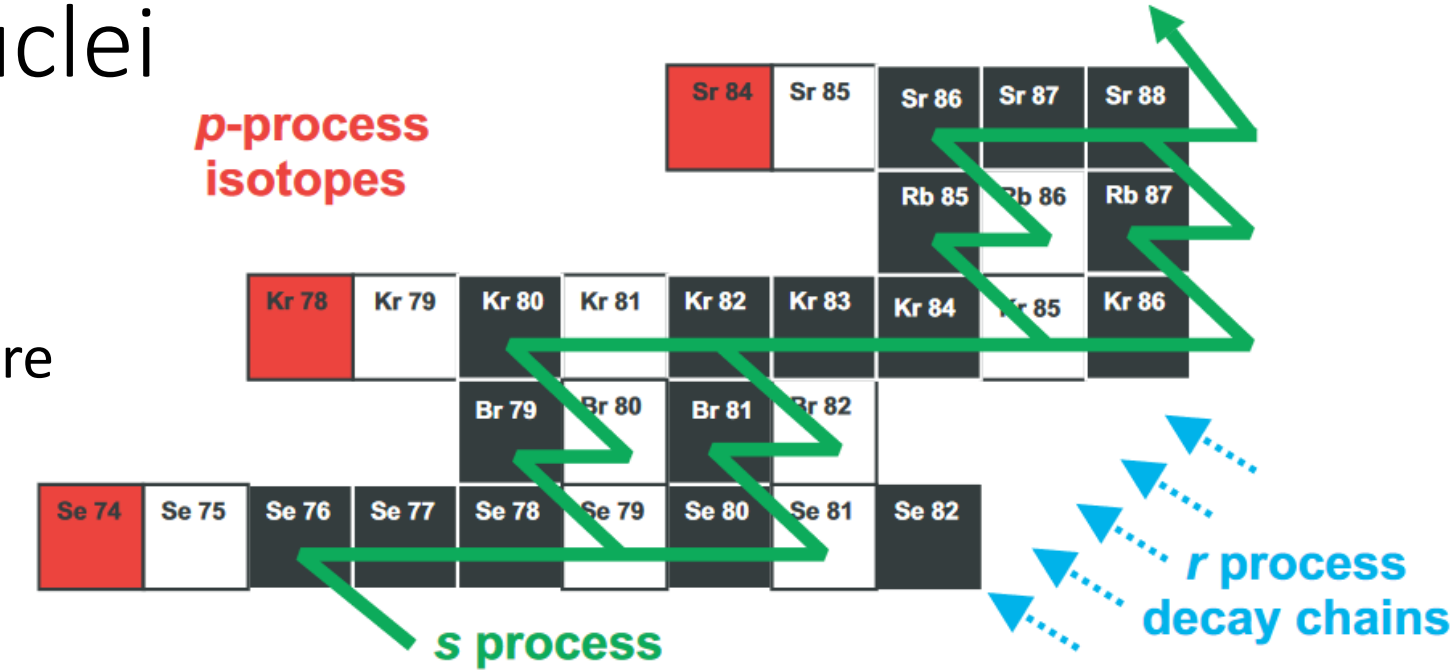
$$\frac{1}{\tau_{\gamma,n}} = \lambda_{\gamma,n} \propto (k_BT)^{3/2}\frac{1}{n_n}\exp\left(-\frac{Q_{12}}{k_BT}\right)\lambda_{n,\gamma}$$

Because we're dealing with (n,γ), Q=S_n

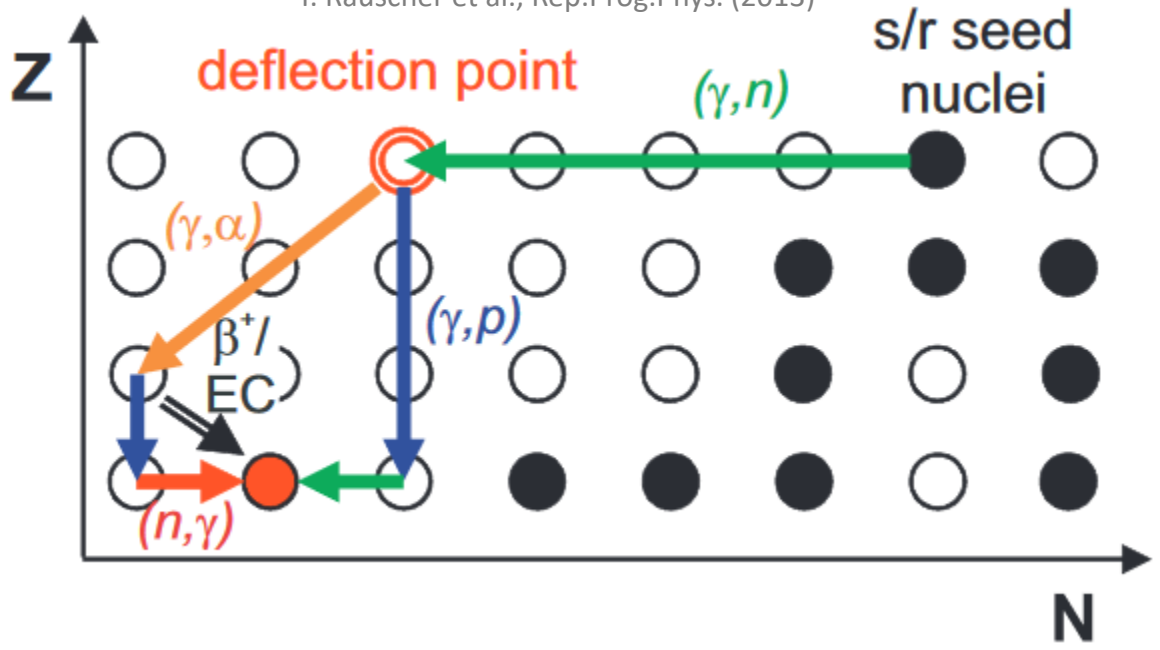
- This means that the *r*-process follows a path with constant neutron-separation energy S_n, where the S_n of the path is determined by n_n and T
  (For example, when taking into account all of the proper constants, $S_n \approx 2\mathrm{MeV}$ for $T = 1\mathrm{GK}, n_n = 10^{24}\mathrm{cm}^{-3}$)
- The *r*-process will stall at the isotope of an element with the path's S_n and wait for β-decay.
- Clearly, this implies *r*-process abundances will pile-up at nuclei with long half-lives. Of course, the magic *N* nuclei have relatively long half-lives (and small neutron-capture cross sections), so abundances will pile-up there, resulting in the characteristic *r*-process peaks

Who ordered these? p-nuclei

- Five minutes ago, when we still had our youth, we were pretty into the idea of neutron-capture producing most of our elements. But how would neutron-capture produce these?
- These nuclides (35 of 'em) on the proton-rich side of stability need some other explanation
- The vp-process is a possibility for the low-A ones, but it certainly won't work for A≳70
- Generally, the favored *p*-nuclide production site is the *p*-process in the outer shells of massive stars during core-collapse supernovae. This process comprises photodisintegration reactions of heavy nuclides and some capture reactions of the photodisintegration products

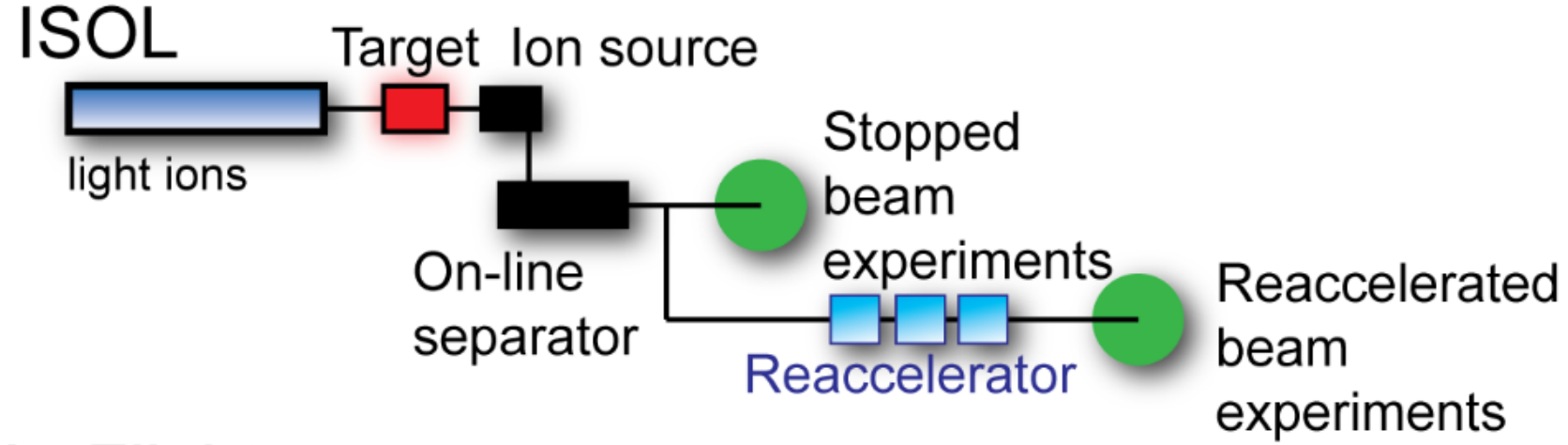


T. Rauscher et al., Rep.Prog.Phys. (2013)

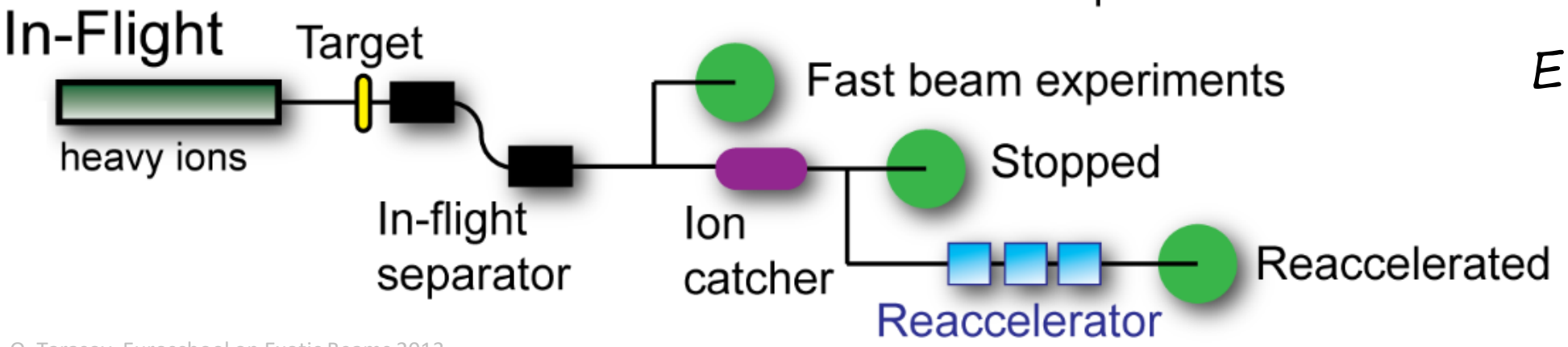


Radioactive Ion Beams (RIBs)

- For a long enough lived isotope, e.g. ^{7}Be, it's possible to directly produce an ion beam using a traditional ion source, like we just discussed. However, this only works for special cases.
- RIB production is typically done either *in-flight*, with isotope separation online (*ISOL*), or by capturing and re-accelerating fission fragments (e.g. CARIBU @ Argonne National Lab)



E.g. SPIRAL@GANIL, ISOLDE@CERN, ISAC@TRIUMF



E.g. GSI/FAIR, NSCL/FRIB, RIBF@Riken, GANIL

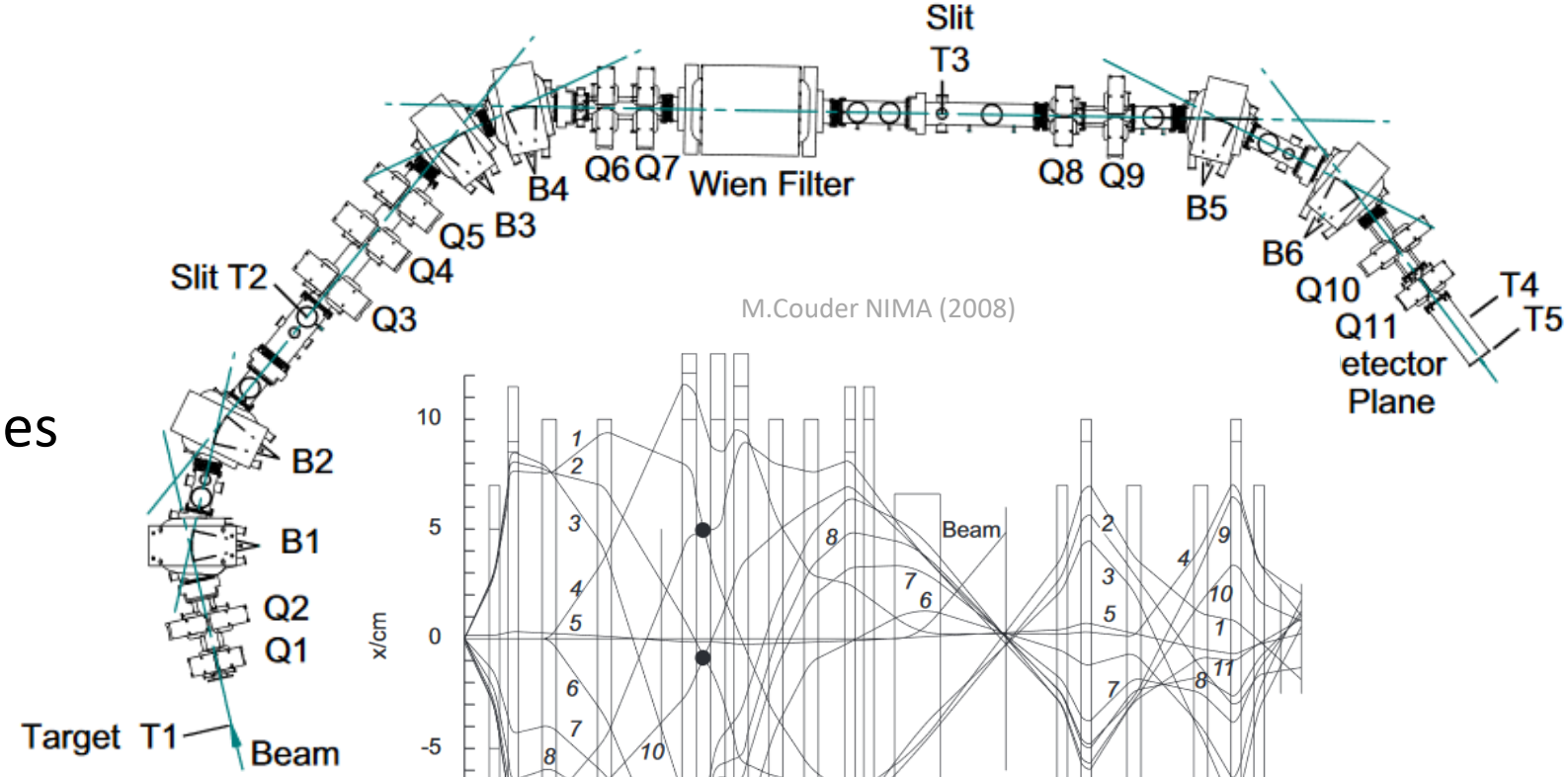
Beam Transport

- Ion transport consists of steering charged ions with dipoles and focusing with higher-order multipoles
- For low energies (10's of keV/nucleon), electrostatic elements are used, so magnetic steering/focusing elements are far more prevalent
- By setting the Lorentz force equal to the centrifugal force, it becomes clear dipoles can be used for ion separation:

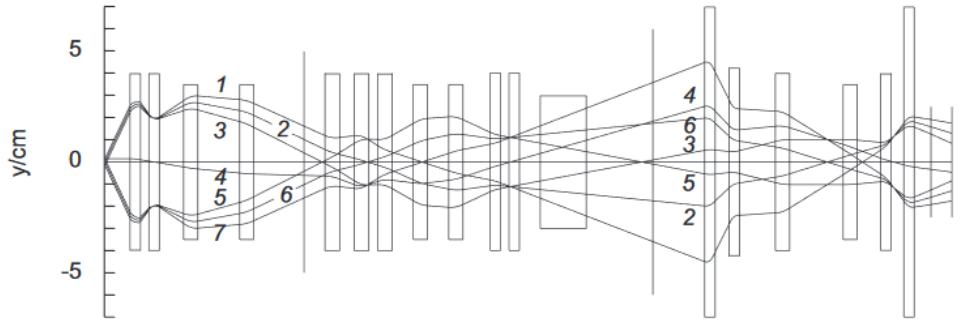
$$F_c = F_L$$

$$\frac{mv^2}{r} = qvB$$

$$\frac{mv}{r} = \frac{p}{q} = Br \equiv B\rho = \text{"}magnetic\ rigidity\text{"}$$



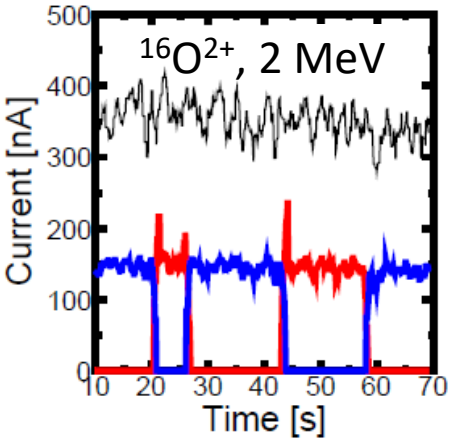
M.Couder NIMA (2008)



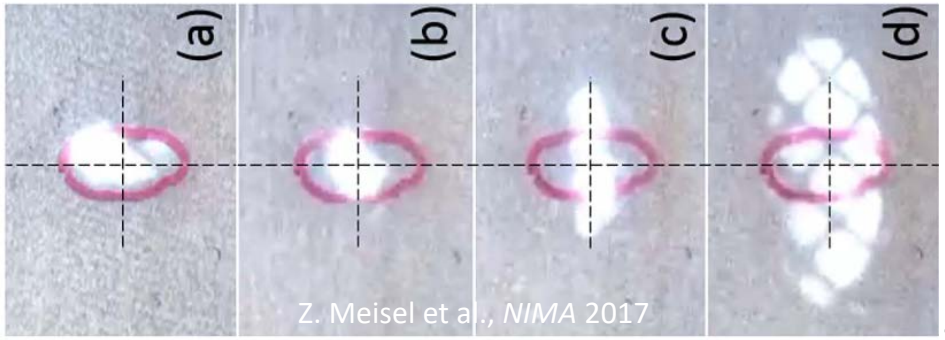
Beam detection: *Simplest techniques (cups & viewers)*

- Since ions are charged particles, these can be collected, resulting in an electric current
- Beam currents are typically recorded using a Faraday Cup
  - The cup is electrically isolated, so deposited charges can be read out
  - Suppression electrodes are located upstream of the back of the cup to prevent escaping electrons from altering current readings
  - These only work for relatively large beam intensities, since $1e = 1.602 \times 10^{-19} C$ means $1pA$ requires $\sim 6x10^{6} pps$

Z. Meisel et al., *NIMA* 2017



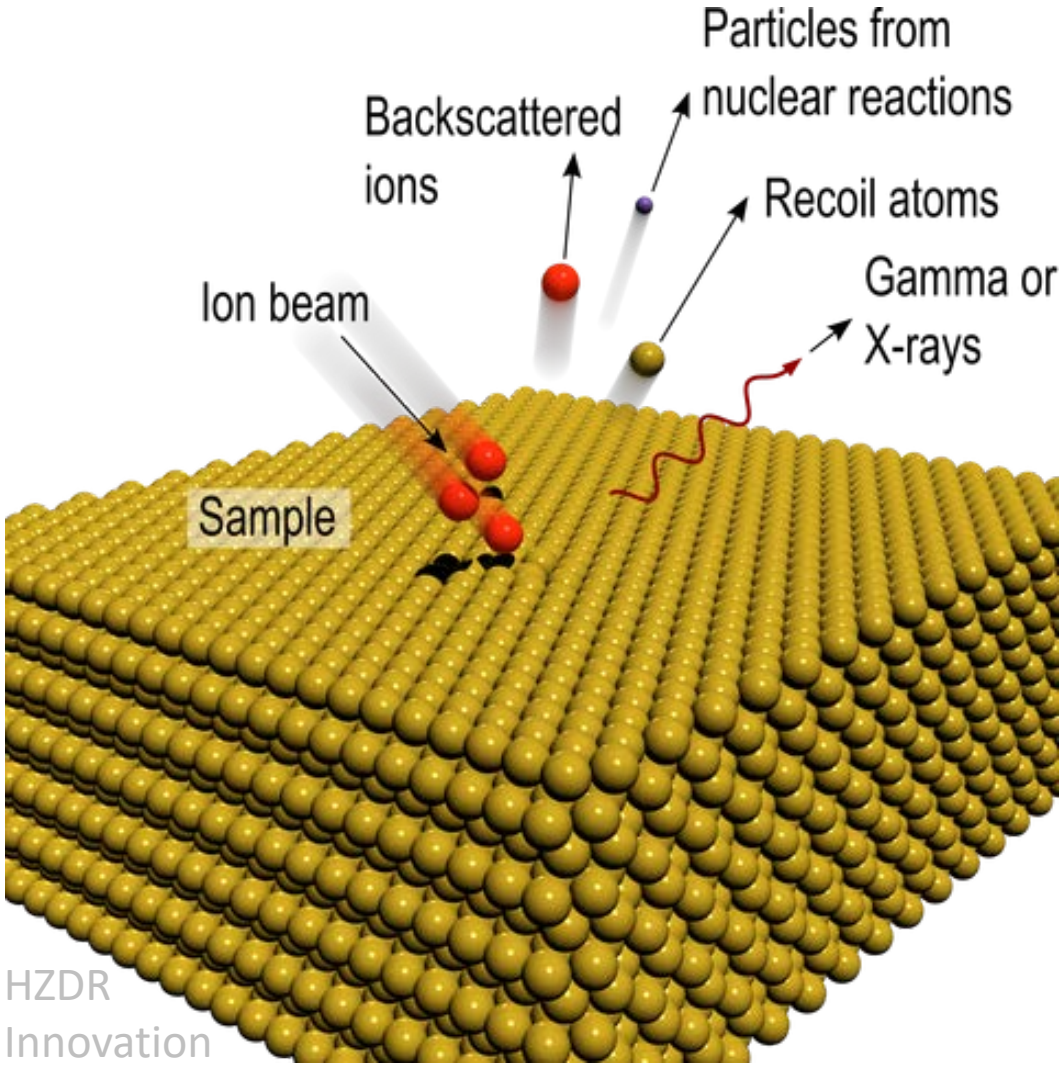
- Other relatively high-intensity beam detecting devices include beam viewers, which typically rely on fluorescence or phosphorescence of a material
  - These are also limited to the $\sim pA$ to $\sim nA$ regime



Z. Meisel et al., *NIMA* 2017

Material analysis

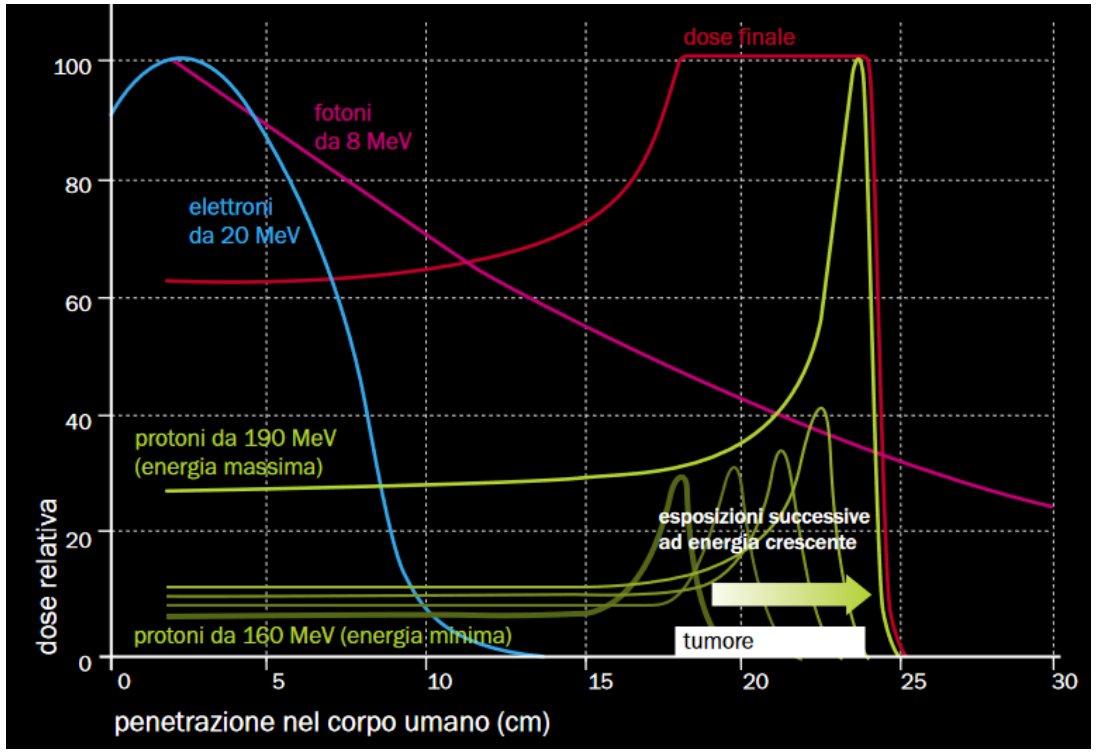
- Material analysis consists of measuring a nuclear reaction between an ion beam (typically protons or alphas) and a sample that's under study
- The reaction products are analyzed to determine the chemical (and often isotopic) composition
- Typical reaction probes are scattering (Rutherford Backscattering and Elastic Recoil Detection), prompt photon emission (Particle Induced X-ray and Gamma Emission), and delayed photon emission (Activation)
- Distinct advantages of ion beam based materials analysis techniques are that they're non-destructive and high-precision



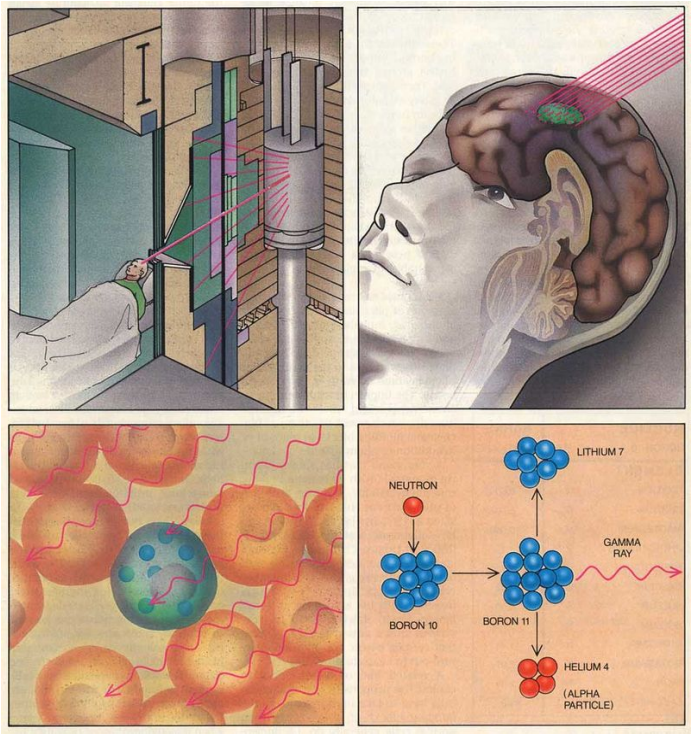
HZDR Innovation

Nuclear Medicine: *Therapy with radiation*

- All radiation types will have some sort of interaction with matter and, in the process, it will deposit energy. The energies associated with nuclear interactions are far larger than molecular binding energies and so nuclear energy deposition can be used to destroy unwanted cells
- X-ray and electron therapies are well suited to treating large cancerous regions, especially those located near the edge of the body, because their energy deposition is spread out
- Ions have a much narrower stopping region, known as the Bragg peak, and so are better suited for localized tumors
- Charged particles can be generate from within, e.g. via Boron Neutron Capture Therapy (BNCT), where tumors are boron doped and the patient is placed near a large neutron source, leading to $^{10}B(n,\alpha)$, or by implanting a radioactive source with low energy decay products



U. Amaldi, Asimmetrie (April 2008)



Barth, Soloway, & Fairchild, Sci.Am. (1990)

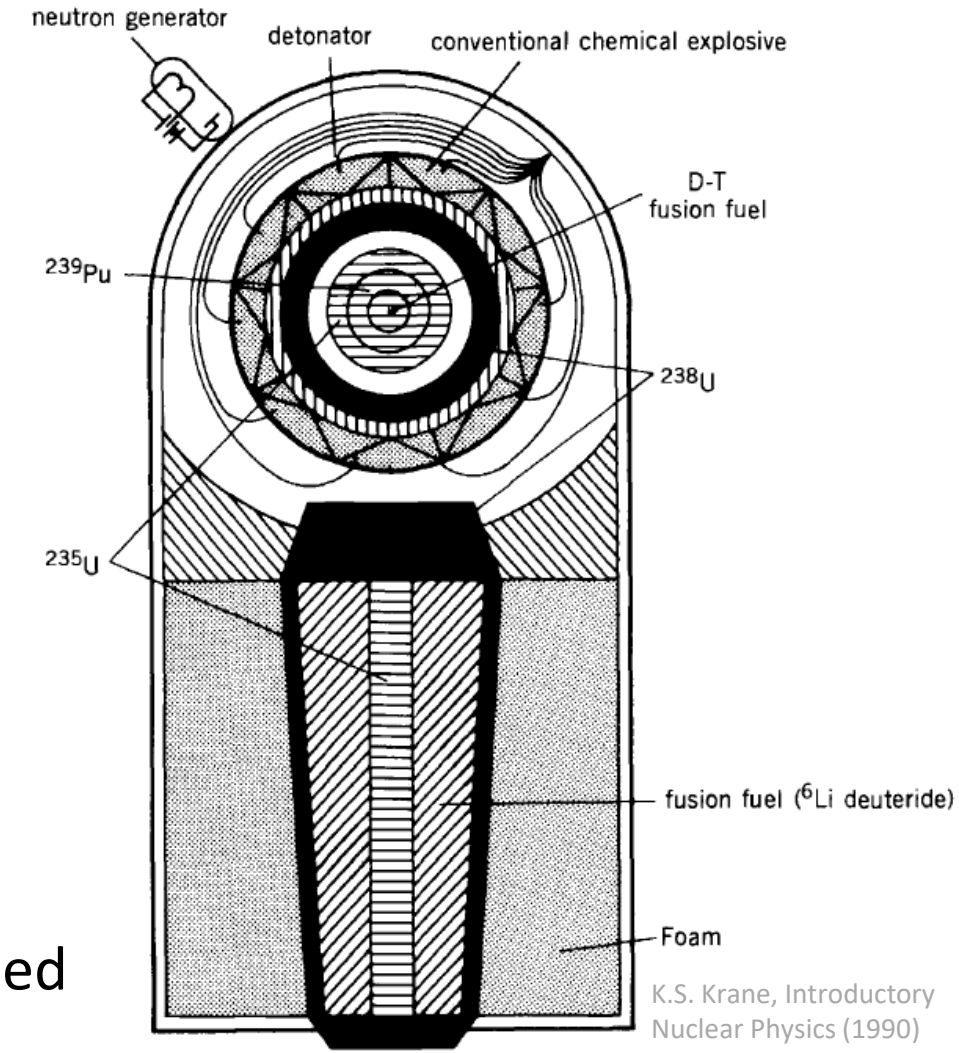
Nuclear Energy *by fission: simple estimate of criticality*

- The critical radius for a sphere of fissile material can be estimated by equating the neutron production rate from fission P_N and the neutron loss rate through the sphere's boundary L_N
- P_N is the product of the number of neutrons in the sphere $n_n V$ and the number of neutrons produced per fission ν, divided by the time between fission events (corrected for the loss of 1 neutron captured)
  - $P_N = n_n V \frac{\nu-1}{\tau} = n_n \frac{4}{3}\pi R^3 \frac{\nu-1}{\tau}$
    - The neutron mean-free path between fissions is $\lambda_f = \frac{1}{n\sigma_{n,f}}$, where n is the number density of the fissile nuclei and $\sigma_{n,f}$ is that fuel's fission cross section for a typical neutron energy
    - Taking a typical neutron energy from fission as $\sim MeV$, it turns out the typical time between fissions will be $\tau_f \approx \frac{\lambda_f}{v} \sim 10^{-8}$s, where v is the average neutron energy
- L_N can be estimated as the number of neutrons within λ_f of R, which will escape in time τ_f
  - $L_N = \left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R-\lambda_f)^3\right] n_n v \approx 4\pi R^2 \lambda_f$
- $P_N = L_N$ leads to the result that $R_{critical} = \frac{3}{\nu-1}\lambda_f$
- For ^{235}U: $E_n \sim 5MeV$, $\sigma_{n,f}(E_n) \sim 1b$, $\nu \sim 3$, $n \sim 10^{23} cm^{-3}$: $R_c \sim 15cm$, i.e. $M_{sphere} \sim 270kg$

The actual value determined with Monte Carlo (and verified empirically) is $M_c(^{235}U) \approx 52kg$. Because of similar ν and λ, all fissile nuclei with appreciable $\sigma_{n,f}$ require this order of mass.

Nuclear Weapons: *Warheads*

- The nuclear reactor concept is to have a nice slow burn that is controllable and outputs energy at a rate that can be easily used and stored …but what if we burn all the fuel as fast as possible?
- The favored approach to this problem is to use conventional explosives to rapidly combine two pieces of sub-critical fissile material so that they become supercritical and a chain reaction ensues.
  Once the chain reaction ensues, it becomes hot enough to initiate fusion of deuterium+tritium fuel located nearby, boosting the chain reaction (with neutrons from d(t,n)α)
  and also releasing a large amount of energy in its own right
  (^{6}Li provides a local fuel creation via ^{6}Li(n,t)α)
- A major design challenge is to ensure the material goes supercritical when you want it to and not before.
  Thus materials with large (α,n) cross sections need to be minimized (also implying (α,n) cross sections on stable nuclei should be well known)
- The amount of fuel required to go supercritical can be reduced from our previous estimate by surrounding the fuel with "tamping" material that reflects neutrons and helps contain the explosion



K.S. Krane, Introductory Nuclear Physics (1990)