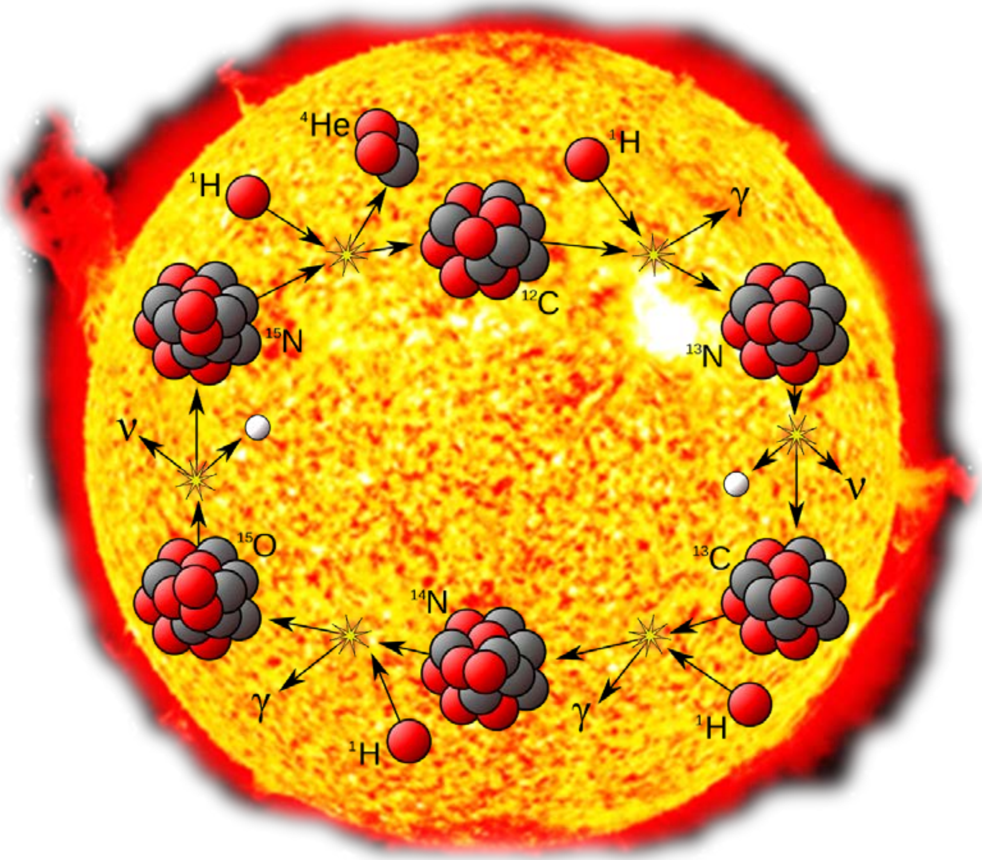
Lecture 20: Astrophysical Reaction Rates

- Evidence for nuclear reactions in space
- Thermonuclear reaction rates
- Non-resonant rates
- Resonant rates
- Reaction networks

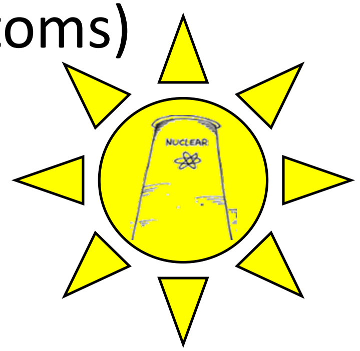


Birth of Nuclear Astrophysics

- Nuclear astrophysics was spawned in 1920 by a realization of Arthur Eddington (The Observatory (1920):
- Based on fossil evidence at the time, the Earth was known to be more than several hundred million years old and the sun presumably had to be at least as old
- A plausible explanation for the sun's power might be gravitational energy being converted into heat from the gaseous solar sphere collapsing, taking place on the Kelvin-Helmholtz timescale
  - $\tau_{KH} = \frac{Total\ Kinetic\ Energy}{Energy\ Loss\ Rate} = \frac{(Potential\ Energy)/2}{Luminosity} = \frac{GM^2}{2RL} \approx 16Myr$ for the sun

    from the virial theorem
- Instead, maybe it's just a lump of burning coal!
  - Typical chemical bond energies are ~eV
  - The sun has a mass of ~10^{30}kg and a nucleus is ~10^{-27}kg, it has ~10^{57} nuclei (or atoms)
  - Since the sun releases ~10^{39}MeV/s, chemical burning would last roughly

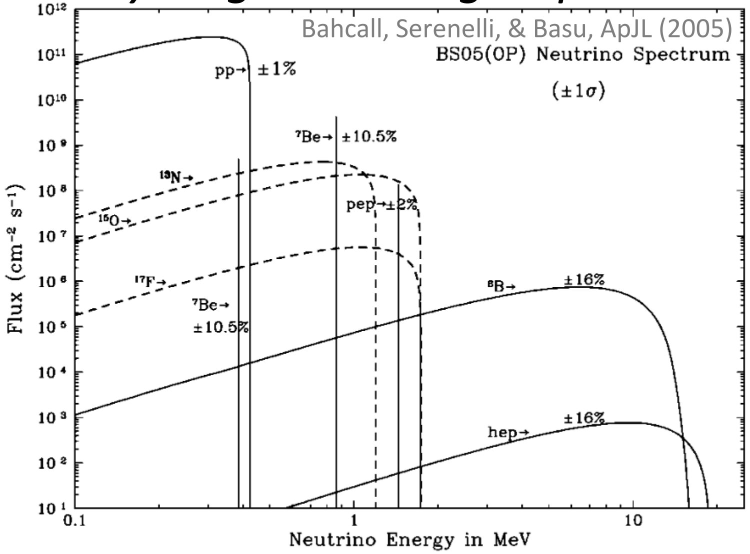
$$t_{burn} = \frac{Amount\ of\ Fuel}{Rate\ of\ Fuel\ Burning} \sim \frac{10^{57} atoms * 1eV/atom}{10^{39} MeV/s * 10^{6} eV/MeV} \sim 10^{12} s \sim 30kyr$$



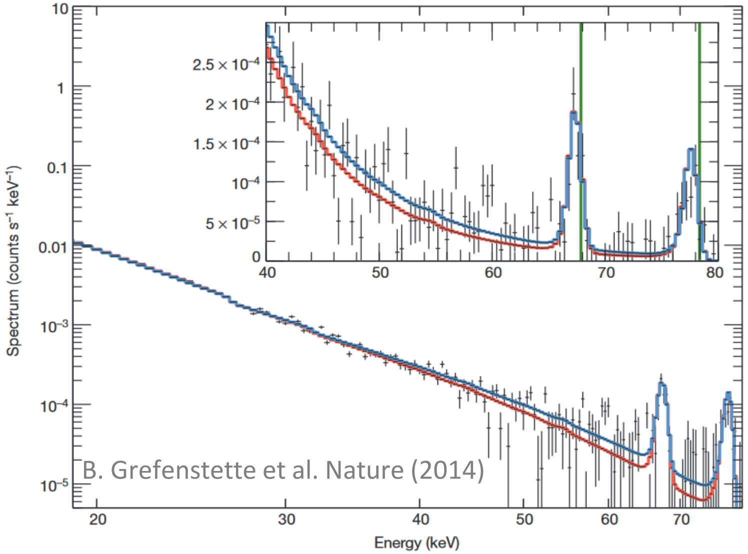
- Third time's the charm…let's look at nuclear energy
  - Aston measured a 32MeV discrepancy between 4 protons and 1 Helium nucleus (4p->α actually yields ~27MeV)
  - Multiplying our fuel amount by 10^6 (eV->MeV) results in $\sim 10Gyr$ burn time …which finally does the job

Evidence for recent nuclear reactions in space (selected examples)

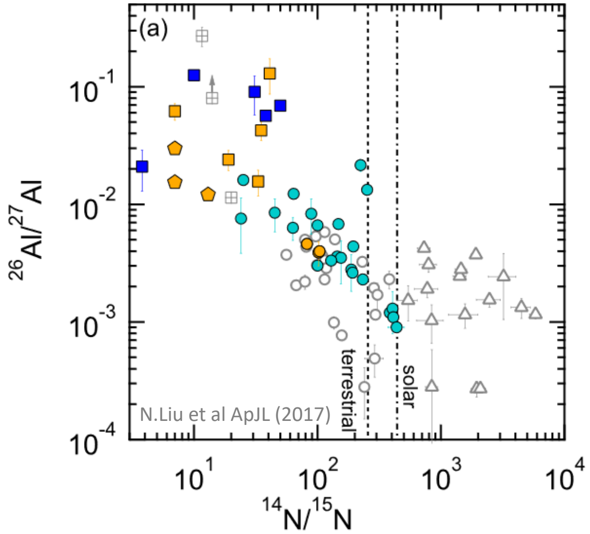
Solar ν attributable to hydrogen burning sequences



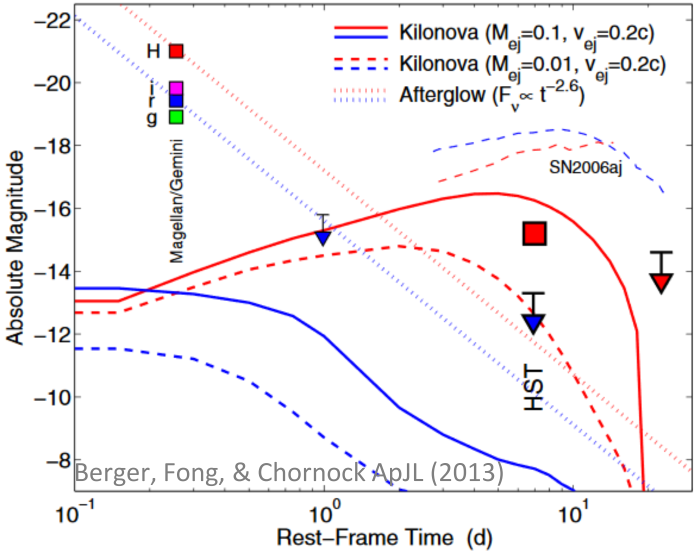
γ-rays of short-lived isotopes (e.g. ^{44}Ti) in supernova remnants



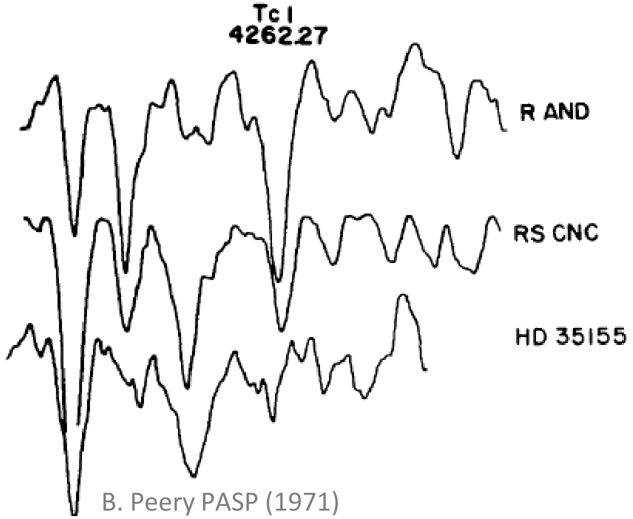
Anomalous isotopic ratios in meteoritic "pre-solar" dust grains



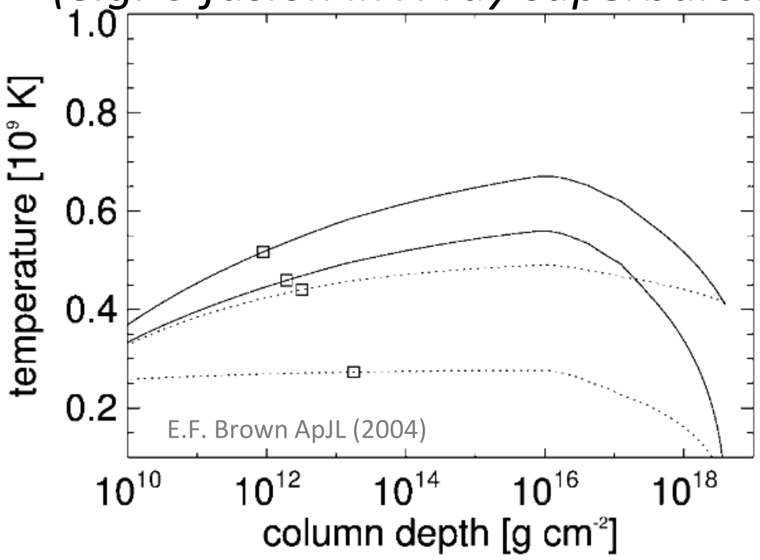
Neutron star merger afterglow associated with radioactive decay



Radioactive elements (e.g. Tc) in stellar spectra



Match in energetics (e.g. C-fusion in X-ray superbursts)



Reaction rate

- The way nuclei influence and are influenced by astrophysical environments is through energy generation and element transmutation ("nucleosynthesis").
  The importance of a pair of nuclides is going to be related to the rate at which they react.
- We can figure out what that reaction rate is by considering a simple example to a gas with two species, each with their own number density, inside a volume at some temperature
- For simplicity, we'll consider one species as the projectile and the other as the target
  (though it doesn't matter since all of our calculations will be in the center of mass)
- The reaction rate between species 1 and 2 will be:
  $Rate = (Flux\ of\ 1) * (Number\ Density\ of\ 2) * (Interaction\ Probability\ for\ 1+2)$
  - 1's flux is the number of 1 atoms per area per time: $F_1 = n_1 v$
    where n_1 is the 1 number density and v is the relative velocity
  - The interaction probability is provided by the cross section $\sigma_{12}(v)$
- So, $r(v) = F_1 n_2 \sigma_{12}(v) = n_1 n_2 \sigma_{12}(v) v$, is the reaction rate for velocity v

Thermonuclear reaction rate

- Of course, nature isn't so neat and orderly. Any environment will have a finite temperature and therefore any nuclei will have a velocity distribution $P(v)$ that's defined by the temperature.
- To get an average rate given some relative velocity distribution, we need to do an average

$$\langle r \rangle = \int_0^\infty P(v) n_1 n_2 \sigma_{12}(v) v \, dv = n_1 n_2 \int_0^\infty P(v) \sigma_{12}(v) v \, dv = n_1 n_2 \langle \sigma v \rangle_{12}$$

(If the reaction is endothermic then the integral lower-limit is the reaction threshold)

- To avoid double-counting of particles, we have to make a slight modification for identical nuclei: $\langle r \rangle = \frac{n_1 n_2 \langle \sigma v \rangle_{12}}{1+\delta_{12}}$, where δ_{12} is the Kronecker delta
- Determining the reaction rate between a pair of particles $\langle \sigma v \rangle$ is the nuclear physicist's task
- The number densities are determined be calculating the effects of all reaction rates as a function of time in a reaction network to see how many nuclei are created/destroyed
- It's more common to deal with matter densities and get the number density by taking into account the fraction of mass species i contributes: $n_i = \rho N_A \frac{X_i}{m_{mole,i}} = \rho N_A Y_i$ where N_A is Avogadro's number, $m_{mol,i}$ is the molar mass (a.k.a. mass in amu), and X_i (≤ 1) and Y_i are the mass and mole fractions, respectively

Similarly, the reduced reaction rate $N_A\langle \sigma v \rangle$ is often used

Thermonuclear rates and the Maxwell-Boltzmann distribution

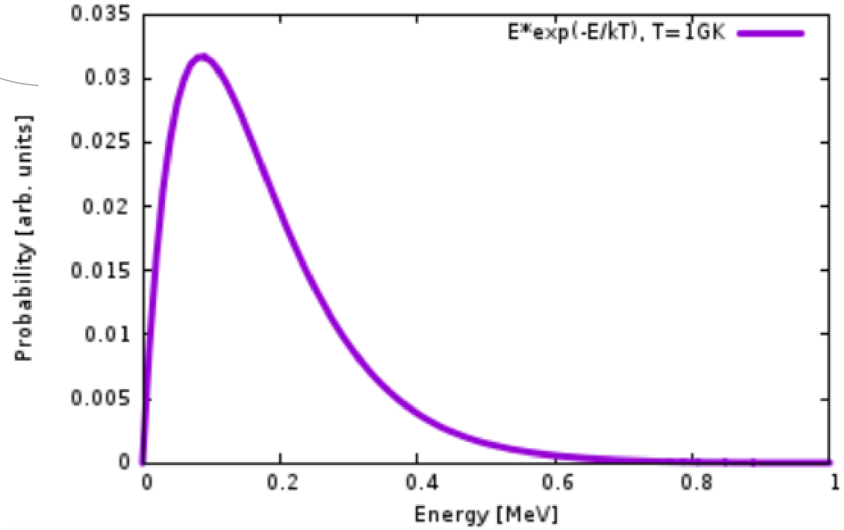
- Typically, the environment we're considering is a classical gas in thermodynamic equilibrium, so the velocity distribution is described by the Maxwell-Boltzmann distribution $P_{MB}(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right)$, where k_B is the Boltzmann constant and the pre-factor is for normalization (i.e. $\int_0^\infty P_{MB}(v)\, dv = 1$)
- Both species interacting in an environment will have this distribution, so taking velocities for 1 and 2 in the lab-frame, $\langle\sigma v\rangle_{12} = \int_0^\infty \int_0^\infty P_{MB}(v_1) P_{MB}(v_2)\, \sigma_{12}(v) v dv_1 dv_2$
- This is less-than desirable, since we would rather be working in terms of relative velocity only, so we consider the fact that $v_i = v_{cm} \pm v$
- This means the double integral can be re-stated in terms of integrating over v and v_{cm}: $\langle\sigma v\rangle_{12} = \int_0^\infty \int_0^\infty P_{MB}(v_{cm}) P_{MB}(v)\, \sigma_{12}(v) v dv_{cm} dv$
- Since the cross section only depends on the relative velocity and $\int_0^\infty P_{MB}(v_{cm})\, dv_{cm} = 1$,

$$\langle\sigma v\rangle_{12} = \int_0^\infty P_{MB}(v) \sigma_{12}(v) v\, dv$$

  where we should keep in mind m in P_{MB} refers to the reduced mass μ

Thermonuclear rates and the Maxwell-Boltzmann distribution

- Inserting $P_{MB}(v)$ into $\langle\sigma v\rangle_{12} = \int_0^\infty P_{MB}(v)\sigma_{12}(v)v\,dv$ yields
$$\langle\sigma v\rangle_{12} = 4\pi\left(\frac{\mu}{2\pi k_B T}\right)^{3/2}\int_0^\infty \sigma_{12}(v)v\left(v^2\exp\left(-\frac{\mu v^2}{2k_B T}\right)\right)dv$$
- Noting the center of mass energy $E = \frac{1}{2}\mu v^2$ and so $dE = \mu v\,dv$ (i.e. $v^2 \rightarrow \frac{2}{\mu}E$, $dv \rightarrow \frac{1}{\mu v}dE$), we finally arrive at a useful equation for the astrophysical reaction rate
$$\langle\sigma v\rangle_{12} = \sqrt{\frac{8}{\pi\mu}}\frac{1}{(k_B T)^{3/2}}\int_0^\infty \sigma_{12}(E)E\exp\left(-\frac{E}{k_B T}\right)dE$$
- Note that this is the general formula for classical gases. We'll go over special cases (for classical gases) in a bit.
- As an aside, personally I find k_B hard to remember, but I find it easier to remember that $11.6045 * E_{MeV} = T_9$, where E_{MeV} is energy in MeV and T_9 is temperature in GK



Reverse (a.k.a. inverse) rates

- Even when a particular reaction direction is exothermic ($Q > 0$), for a finite temperature, some fraction of particles will have energies with $E \geq Q$, and that fraction will grow with T
- Recall that the cross section for an entrance channel through a compound nuclear state is the product of the effective geometric area of the projectile ($\left(\frac{\lambda_{12}}{2\pi}\right)^2$), the number of sub-states that can be populated ($2J + 1$), the probability of being in a particular sub-state for each of the reactants ($\frac{1}{(2J_1+1)}\frac{1}{(2J_2+1)}$), a factor of 2 for identical particles ($1 + \delta_{12}$), and the matrix elements for forming the compound nucleus C through $1 + 2$ and decaying via $3 + 4$:

$$\sigma_{12\rightarrow 34} = \pi\left(\frac{\lambda_{12}}{2\pi}\right)^2 \frac{2J+1}{(2J_1+1)(2J_2+1)}(1+\delta_{12})|\langle 3+4|H_2|C\rangle\langle C|H_1|1+2\rangle|^2$$

- The reverse process would then be

$$\sigma_{34\rightarrow 12} = \pi\left(\frac{\lambda_{34}}{2\pi}\right)^2 \frac{2J+1}{(2J_3+1)(2J_4+1)}(1+\delta_{34})|\langle 1+2|H_1|C\rangle\langle C|H_2|3+4\rangle|^2$$

- I.e. they're identical except for the statistical and geometric factors out front

Reverse (a.k.a. inverse) rates

- Making the substitution that $\left(\frac{\lambda_{ij}}{2\pi}\right)^2 = \frac{\hbar^2}{p^2} = \frac{\hbar^2}{2\mu_{ij}E_{ij}}$ and noting $\mu_{ij} = \frac{m_i m_j}{m_i + m_j}$, taking the ratio of the forward and reverse cross sections yields
$$\frac{\sigma_{12}}{\sigma_{34}} = \frac{A_3 A_4}{A_1 A_2}\frac{E_{34}}{E_{12}}\frac{(2J_3+1)(2J_4+1)}{(2J_1+1)(2J_2+1)}\frac{(1+\delta_{12})}{(1+\delta_{34})}$$
where A_i can be used instead of m_i since the units cancel

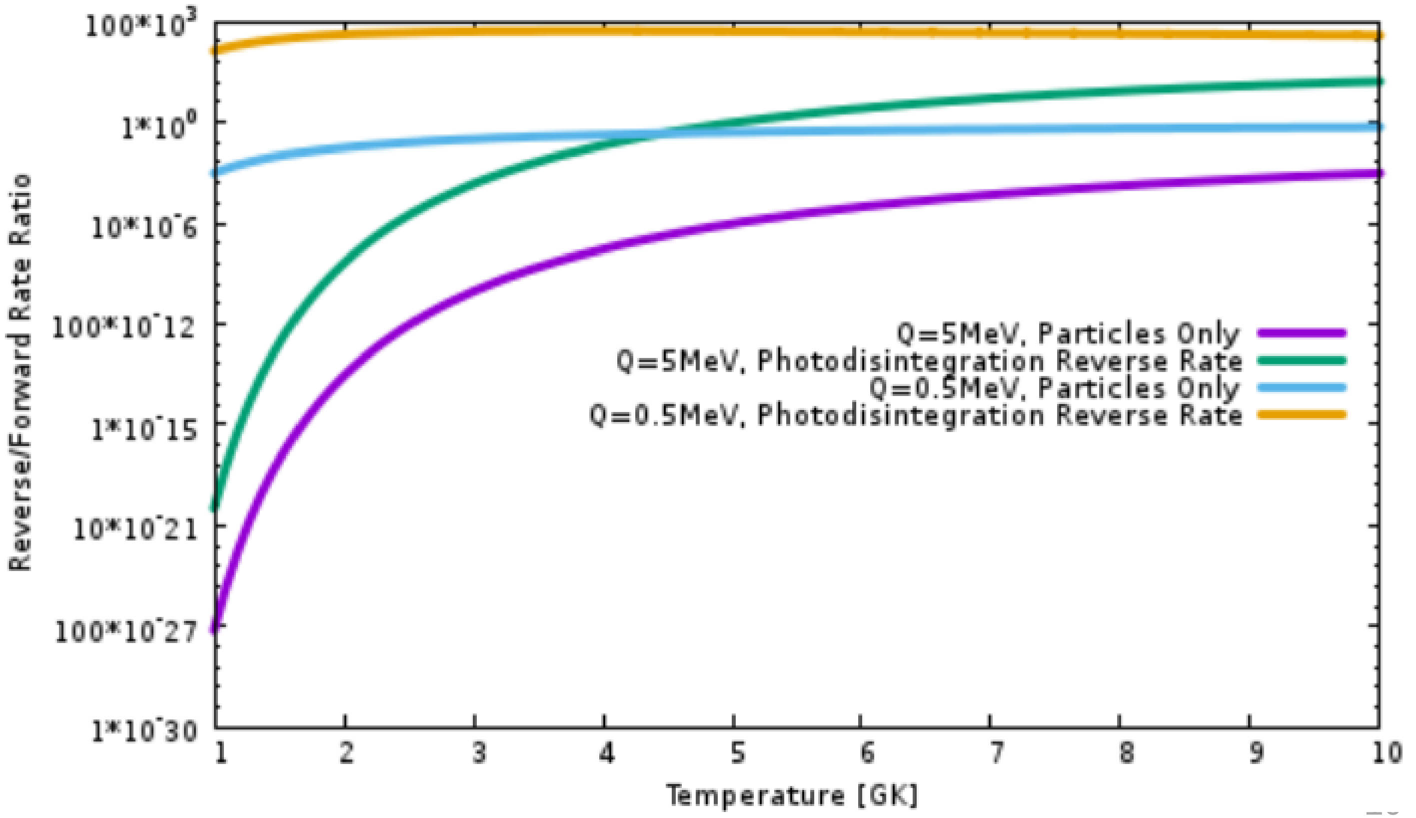
- For the rates, recall $\langle\sigma v\rangle_{12} = \sqrt{\frac{8}{\pi\mu_{12}}}\frac{1}{(k_BT)^{3/2}}\int_0^\infty \sigma_{12}(E_{12})E_{12}\exp\left(-\frac{E_{12}}{k_BT}\right)$,
so $\langle\sigma v\rangle_{34} = \sqrt{\frac{8}{\pi\mu_{34}}}\frac{1}{(k_BT)^{3/2}}\int_0^\infty \sigma_{34}(E_{34})E_{34}\exp\left(-\frac{E_{34}}{k_BT}\right)$

- Since $E_{34} = E_{12} + Q_{12}$ (if $Q_{12} > 0$), when we take the ratio $\langle\sigma v\rangle_{34}/\langle\sigma v\rangle_{12}$, the integrands mostly cancel except for the energy-independent part:
$$\frac{\langle\sigma v\rangle_{34}}{\langle\sigma v\rangle_{12}} = \frac{(2J_1+1)(2J_2+1)}{(2J_3+1)(2J_4+1)}\frac{(1+\delta_{34})}{(1+\delta_{12})}\left(\frac{\mu_{12}}{\mu_{34}}\right)^{3/2}\exp\left(-\frac{Q_{12}}{k_BT}\right)$$

- The exponential dominates, so $\frac{\langle\sigma v\rangle_{34}}{\langle\sigma v\rangle_{12}} \approx \exp\left(-\frac{Q_{12}}{k_BT}\right)$ gives the correct order of magnitude

Reverse (a.k.a. inverse) rates: photodisintegration

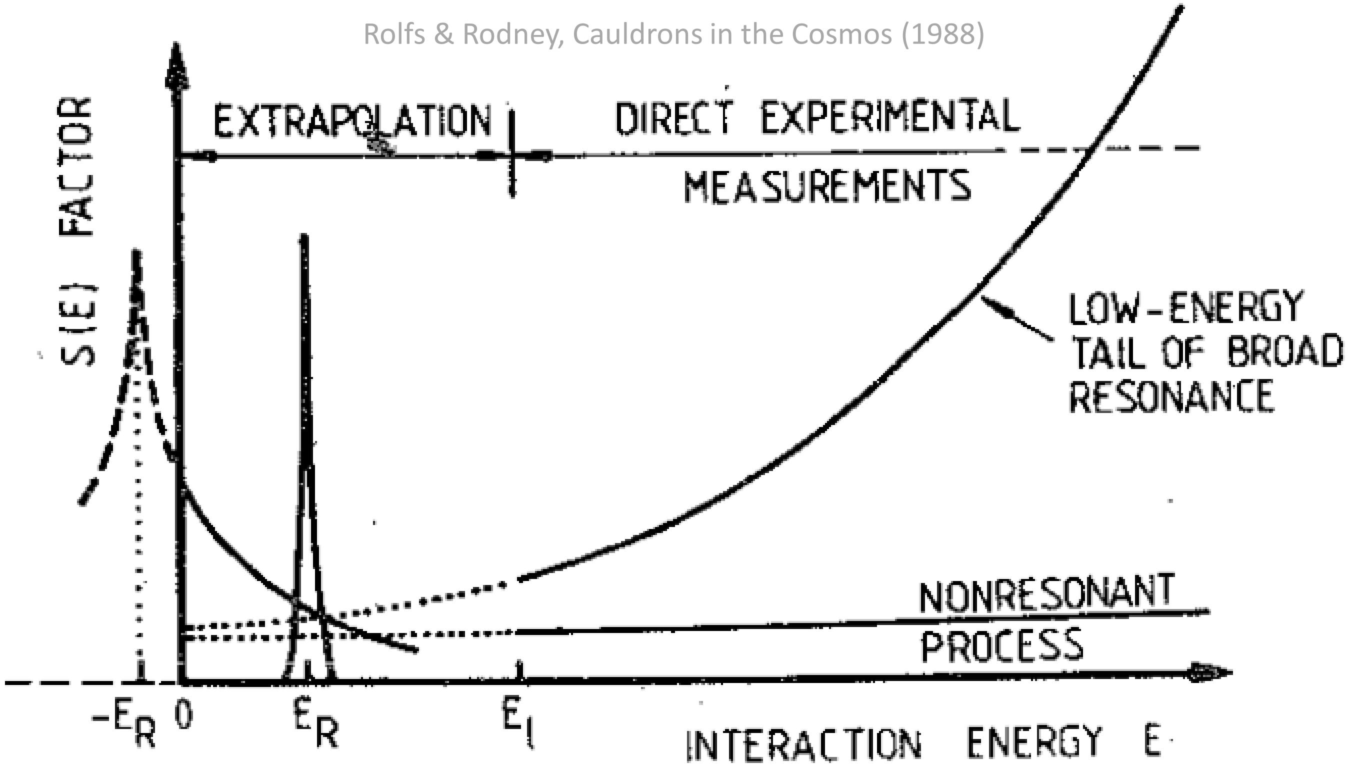
- For photodisintegration, we have to take into account the fact that the photons follow a Planck distribution and not a Maxwell-Boltzmann distribution
- So the reverse rate becomes: $\frac{\langle \sigma v \rangle_{3\gamma}}{\langle \sigma v \rangle_{12}} \approx \left(\frac{\mu_{12} c^2}{k_B T} \right)^{3/2} \exp\left(-\frac{Q_{12}}{k_B T} \right)$
- This extra factor is actually a huge deal because of that extra temperature dependence
- In fact, for low Q-value reactions, the photodisintegration rate is dominant



Aside: S-factors, the nuclear physics nuggets of $\langle\sigma v\rangle$

- Often it's useful to remove the trivial energy dependence from the cross section, in particular for charged-particle reaction rates
- The idea is that $S(E)$ contains all of the interesting physics
- Since the energy dependence is different for different types of reactions, e.g. direct capture of a neutron as compared to direct capture of a charged particle, the factorization that is done to get $S(E)$ depends on the reaction type

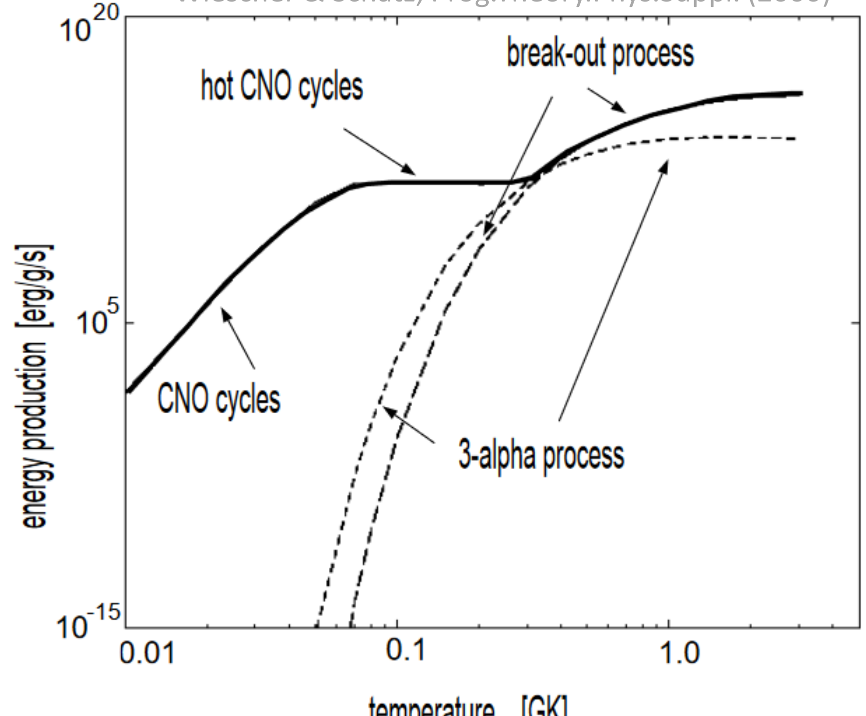
Rolfs & Rodney, Cauldrons in the Cosmos (1988)



Aside-er: Energy release in a reaction

- How much energy does a nuclear reaction rate release into the environment?
- This is as simple as it sounds, where the rate of energy release is just the product of the energy release per reaction times the reaction rate: $\epsilon_{12} = Q_{12} r_{12}$
- Given our pre-agreed upon units, $[\epsilon] = \frac{\mathrm{MeV}}{\mathrm{cm^3 s}}$,
  …though some people prefer energy released per gram: $\tilde{\epsilon}_{12} = \frac{Q_{12} r_{12}}{\rho}$ (units MeV/(g.s))
- To be on the safe side,
  one needs to keep in mind the reverse rate,
  since we saw it can be significant
  at high temperatures:
  $\epsilon_{12,net} = \epsilon_{12} + \epsilon_{34} = Q_{12}(r_{12} - r_{34})$

Wiescher & Schatz, Prog.Theory.Phys.Suppl. (2000)



Thermonuclear rate: **Direct neutron-capture**

- Recall that at low energies, those of interest for nuclear astrophysics, the neutron-capture cross section is described by the $1/v$ law: $\sigma_{n\ cap} \propto \frac{1}{v_n}$
- As such, it's clear that $\langle \sigma v \rangle_{n\ cap} \approx constant$
- The cross section (and rate) can be characterized by the S-factor at thermal energies and any deviation from $1/v$ behavior is accounted for by the local derivative(s) of the S-factor:

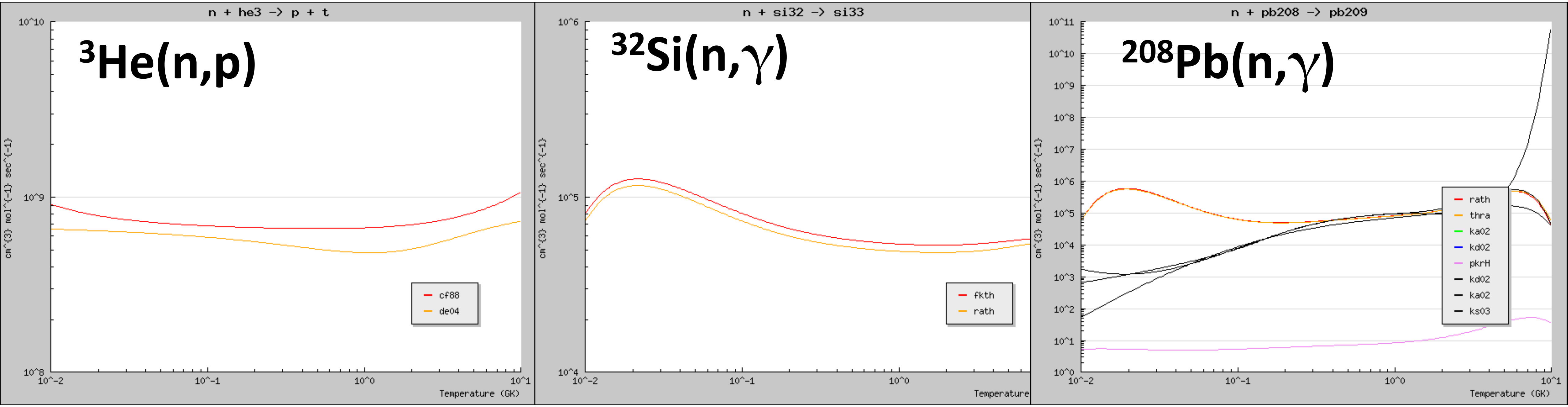
$$\sigma(E) = \sqrt{\frac{\mu}{2E}}\left(S(0) + \dot{S}(0)E^{1/2} + \tfrac{1}{2}\ddot{S}(0)E + \cdots\right)$$

- $S(0)$ is generally the S-factor at thermal energy ($v_{th} = 2.2 \times 10^5 \frac{cm}{s} = E_L = 2.53 \times 10^{-8} MeV$): $S(0) = \sigma_{th} v_{th} = 2.2 \times 10^{-19} \sigma_{th} \frac{cm^3}{s}$, where σ_{th} is the thermal cross section in barns
- $\dot{S}(0)$ and $\ddot{S}(0)$ are fit to cross section data near thermal energies
- Employing this $\sigma(E)$ in the general equation $\langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE$, results in

$$\langle \sigma v \rangle_{n\ cap} = S(0) + \sqrt{\tfrac{4}{\pi}}\dot{S}(0)(k_B T)^{1/2} + \tfrac{3}{4}\ddot{S}(0) k_B T + \cdots$$

Thermonuclear rate: Direct neutron-capture

Examples (from the JINA-CEE REACLIB database):



Clear that the main feature is $\langle\sigma v\rangle_{n\ cap} \approx constant$ over 3 orders of magnitude in T

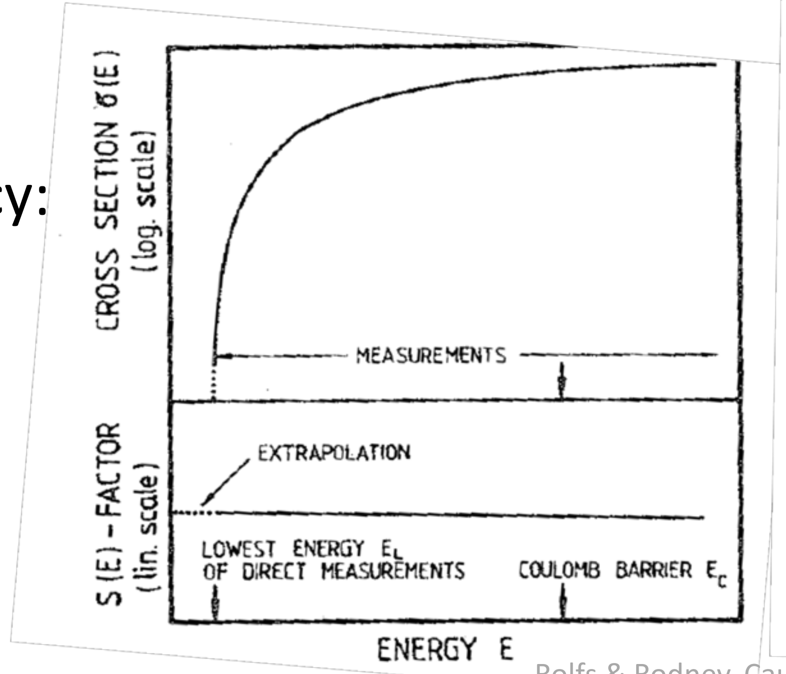
Thermonuclear rate: **Direct charged particle-capture**

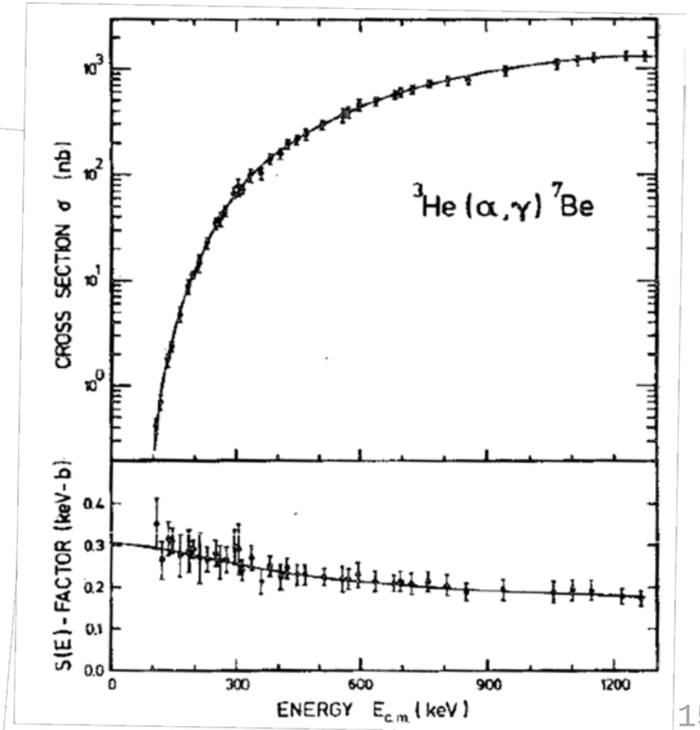
- Recall that the cross section for charged-particle capture depends on the effective geometric area of the projectile, described by $\lambda_{deBroglie}$, and the probability of the charged projectile tunneling through the Coulomb barrier of the target
- When discussing α decay, we showed $P_{tunnel}(E) = \exp(-2\pi\eta)$,

  where the factor with the Sommerfeld parameter is $2\pi\eta = 2\pi \frac{e^2}{\hbar c} Z_1 Z_2 \sqrt{\frac{\mu c^2}{2E}}$
- For E in MeV and A in atomic mass units (1u = 931.5MeV/c^2):

  $$2\pi\eta = 0.989 Z_1 Z_2 \sqrt{\frac{1}{E} \frac{A_1 A_2}{(A_1+A_2)}}$$
- Removing the trivial energy dependency:

  $$\sigma_{ch.cap}(E) = \frac{1}{E} \exp(-2\pi\eta)\, S(E)$$

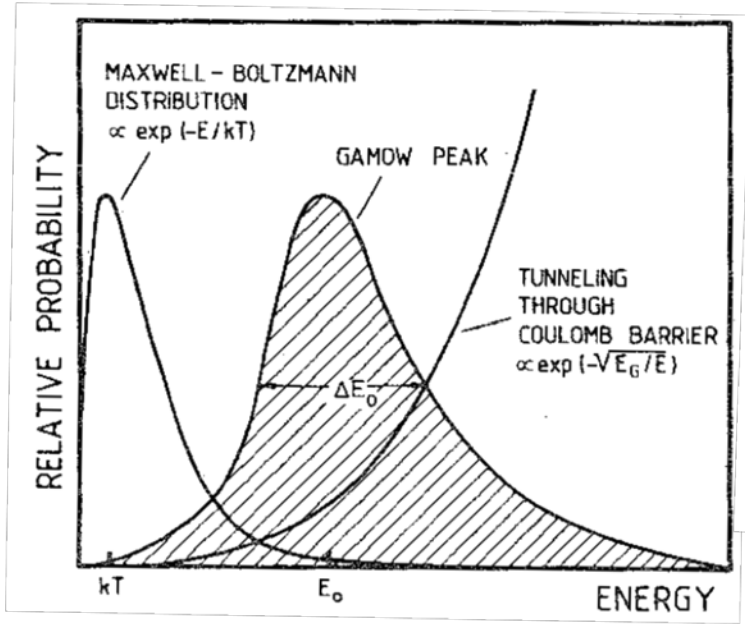


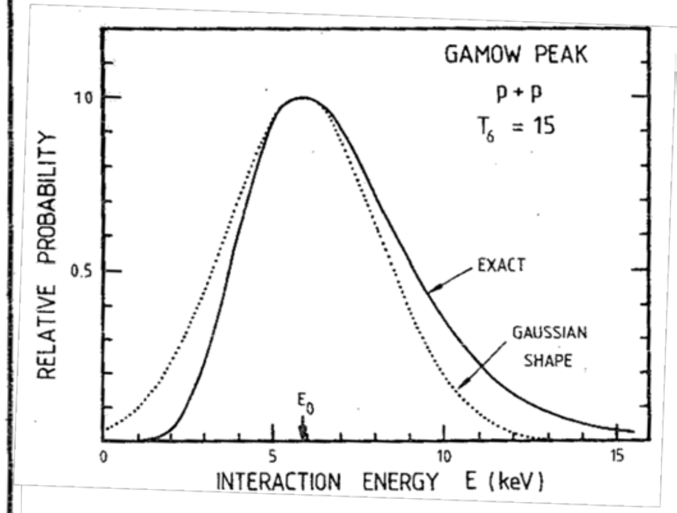


Rolfs & Rodney, Cauldrons in the Cosmos (1988)

Thermonuclear rate: Direct charged particle-capture

- Employing $\sigma(E)=\frac{1}{E}\exp(-2\pi\eta)\,S(E)$ in $\langle\sigma v\rangle=\sqrt{\frac{8}{\pi\mu}}\frac{1}{(k_BT)^{3/2}}\int_0^\infty \sigma(E)E\exp\left(-\frac{E}{k_BT}\right)dE$ gives:
$$=\sqrt{\frac{8}{\pi\mu}}\frac{1}{(k_BT)^{3/2}}\int_0^\infty S(E)\exp\left(-\frac{E}{k_BT}-2\pi\eta\right)dE$$
- The integrand is a product of the probability of a charged-particle pair having energy E (from the Maxwell-Boltzmann distribution) and the tunneling probability for that energy
- The integrand maximum (found by solving for the derivative being equal to zero) is:
$$E_G=0.122\left(Z_1^2Z_2^2\frac{A_1A_2}{(A_1+A_2)}T_9^2\right)^{1/3}\ \text{MeV}$$
- Approximating the integrand as a Gaussian results in a distribution with the $1/e$ width
$$\Delta_G=4\sqrt{\frac{E_Gk_BT}{3}}\ \text{MeV}$$
- E_G is the Gamow peak and Δ_G is the width of the Gamow window, which is roughly the energy range for which we care about a charged particle reaction rate for some T





Rolfs & Rodney, Cauldrons in the Cosmos (1988)

Note: Don't be too naïve when using the Gamow window estimate.
It's based on a roughly constant S(E), so the true window of interest could be different ([T.Rauscher PRC 2010]).

Thermonuclear rate: Direct charged particle-capture

- The S factor is again represented with a Taylor expansion $S(E) = S(0) + \dot{S}(0)E + \frac{1}{2}\ddot{S}(E)E^2 + \cdots$
- As with neutron-capture, it is determined experimentally. Here, by dividing the trivial energy dependence from the measured cross section as a function of energy
  [Or doing something equivalent with a fancy R-matrix fit, e.g. A.Kontos et al PRC2013)]
- The energy dependent terms of $S(E)$ winds up making the reaction rate integral have several temperature-dependent factors, which make for a rather complicated form:
  (See C. Iliadis, Nuclear Physics of Stars *or* [Fowler, Caughlan, & Zimmerman, Annu.Rev.Astron.&Astro. (1967)])

$$N_A\langle\sigma v\rangle = \frac{C_1}{T_9^{2/3}} e^{-C_2/T_9^{1/3}} \left(1 + C_3 T_9^{1/3} + C_4 T_9^{2/3} + C_5 T_9 + C_6 T_9^{4/3} + C_7 T_9^{5/3}\right) \qquad (\mathrm{cm^3 mol^{-1} s^{-1}})$$

$$C_1 = 7.8324\times10^9 \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1}\right)^{1/6} S(0)\sqrt{\frac{M_0+M_1}{M_0 M_1}}$$

$$C_2 = 4.2475 \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1}\right)^{1/3}$$

$$C_3 = 9.810\times10^{-2} \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1}\right)^{-1/3}$$

$$C_4 = 0.1220\frac{S'(0)}{S(0)} \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1}\right)^{1/3}$$

$$C_5 = 8.377\times10^{-2}\frac{S'(0)}{S(0)}$$

$$C_6 = 7.442\times10^{-3}\frac{S''(0)}{S(0)} \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1}\right)^{2/3}$$

$$C_7 = 1.299\times10^{-2}\frac{S''(0)}{S(0)} \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1}\right)^{1/3}$$

- This is the inspiration for the functional form of parameterized reaction rates used in databases, e.g. JINA REACLIB (R. Cyburt et al. ApJS (2010))

Thermonuclear rate: **Narrow resonance(s)**

- Recall the Breit-Wigner form we found for the resonant reaction cross section
$$\sigma_{BW,X(a,b)Y}(E) = \pi\left(\frac{\lambda}{\pi}\right)^2 \frac{2J+1}{(2J_a+1)(2J_X+1)} \frac{\Gamma_{aX}(E)\Gamma_{bY}(E)}{(E-E_R)^2+(\Gamma(E))^2/4}$$
- Employing this in the general form $\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_BT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_BT}\right) dE$, we realize that the contributions of the integrand are pretty negligible outside of E_R
- So, we make the approximation $\langle\sigma v\rangle_{res} = \sqrt{\frac{8}{\pi\mu}} \frac{E_R}{(k_BT)^{3/2}} \exp\left(-\frac{E_R}{k_BT}\right) \int_0^\infty \sigma_{BW}(E)\, dE$
- Noting that λ changes little over the resonance $\lambda \rightarrow \lambda_R$, writing the statistical factor as ω, and noting the widths Γ are essentially constant over the resonance we find
$$\int_0^\infty \sigma_{BW}(E)\, dE \approx \pi\left(\frac{\lambda_R}{\pi}\right)^2 \Gamma_{aX}\Gamma_{bY} \int_0^\infty \frac{1}{(E-E_R)^2+(\Gamma)^2/4} dE = 2\lambda_R^2\omega \frac{\Gamma_{aX}\Gamma_{bY}}{\Gamma}$$
- For obfuscation purposes, we substitute in γ for $\frac{\Gamma_{aX}\Gamma_{bY}}{\Gamma}$ and call $\omega\gamma$ the resonance strength
- If we know the cross section at the peak of the resonance, we can make the approximation that the integral is half the width times the height: $\int_0^\infty \sigma_{BW}(E)\, dE \approx \pi\Gamma\sigma(E_R)$
- The resonant rate becomes: $\langle\sigma v\rangle_{res} = \left(\frac{2\pi}{\mu k_BT}\right)^{3/2} \hbar^2(\omega\gamma)_R \exp\left(-\frac{E_R}{k_BT}\right)$

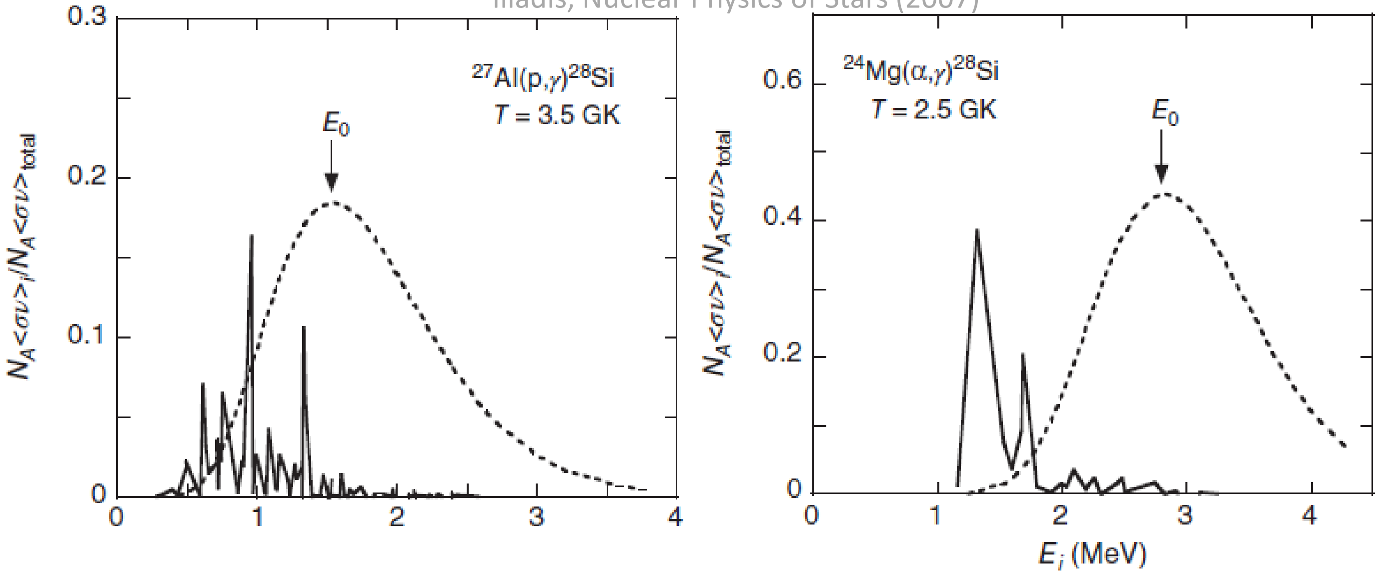
Thermonuclear rate: Narrow resonance(s)

- For a reaction with several resonances, the total rate is just the sum. Using units of MeV for $E_{R,i}$ and $(\omega\gamma)_{R,i}$ and amu for A_i,

$$N_A\langle\sigma v\rangle_{N\,res.} = \frac{1.54\times10^{11}}{\left(\frac{A_1A_2}{A_1+A_2}T_9\right)^{3/2}}\sum_{i=1}^{N}(\omega\gamma)_{R,i}\exp\left(-\frac{11.6045E_{R,i}}{T_9}\right)\frac{\mathrm{cm}^3}{\mathrm{mol\ s}}$$

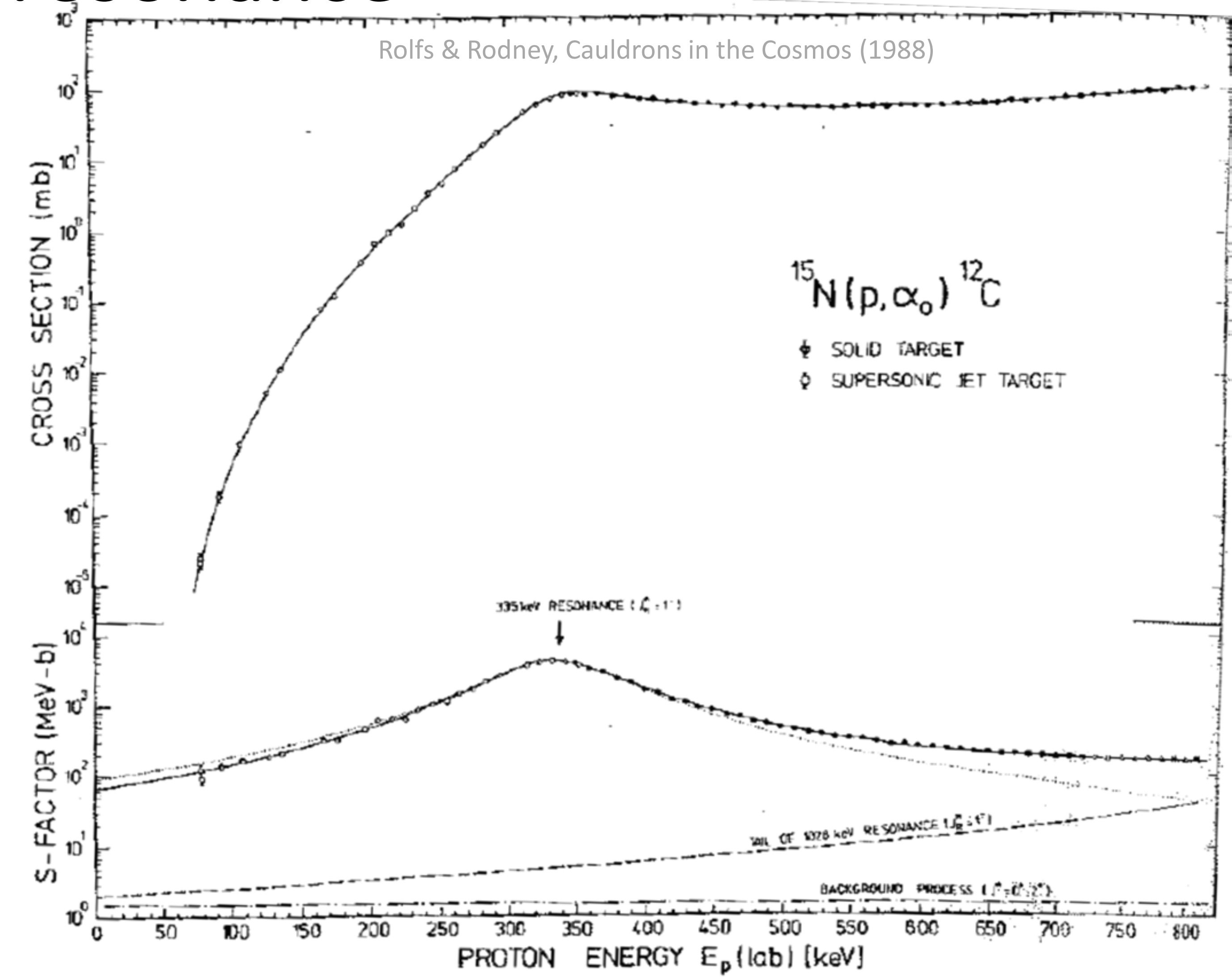
- But how do I know which resonances matter?
- You might guess the ones in the Gamow window …but it depends on the relative widths for the exit and entrance channels.
- For high temperatures, particle emission makes the Gamow Window concept especially dubious
- Nonetheless, the GW is good first guess as to which resonances are likely important

Iliadis, Nuclear Physics of Stars (2007)



Thermonuclear rate: **Broad resonance**

- In the past we've specified any resonance with $\Gamma/E_R \gtrsim 10\%$ as "broad"
- For such cases, we obviously need to take into account the energy dependence of the widths when performing the integral over energy.
  See the resonant reactions lecture for a discussion on these energy dependencies.
- Broad resonances are a pain because they make extrapolation of the reaction rate at very low energies (via the S-factor) a sketchy enterprise
- One approach to represent these rates analytically is as a non-resonant contribution (for $E \ll E_R$) plus an analytic contribution, resulting in: $\langle\sigma v\rangle_{broad\ res} = \langle\sigma v\rangle_{direct} + a_1 T^b \exp\left(-\frac{a_2}{k_B T}\right)$,
  where a_i and b are fit to the rate calculated at several T by integrating over the cross section

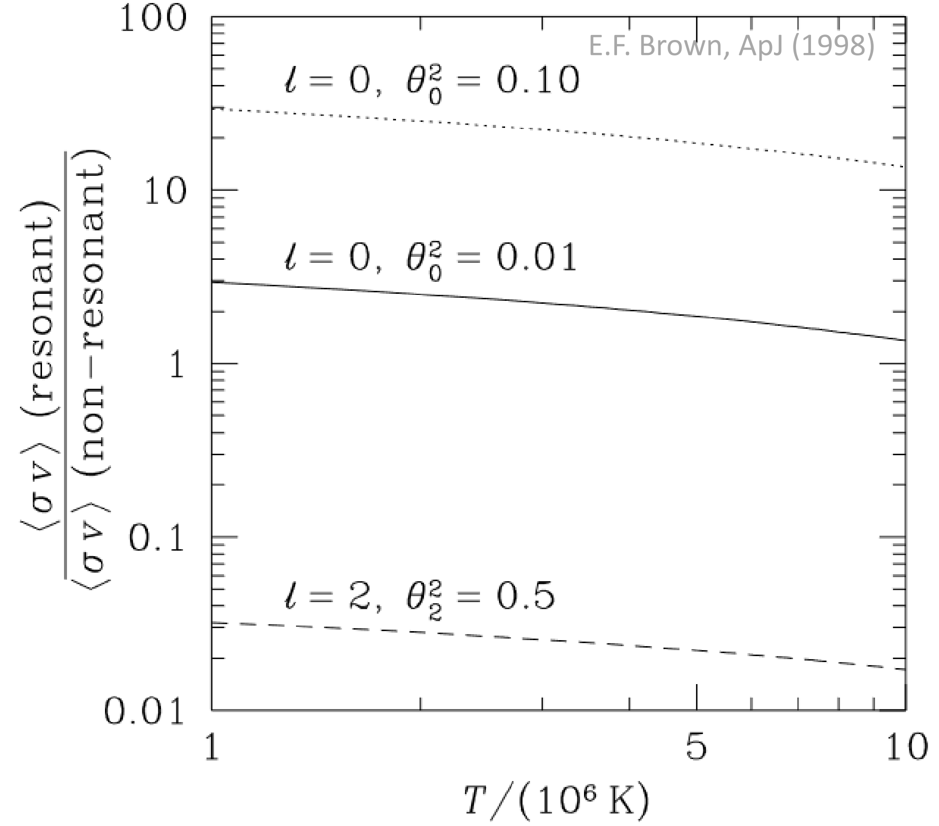


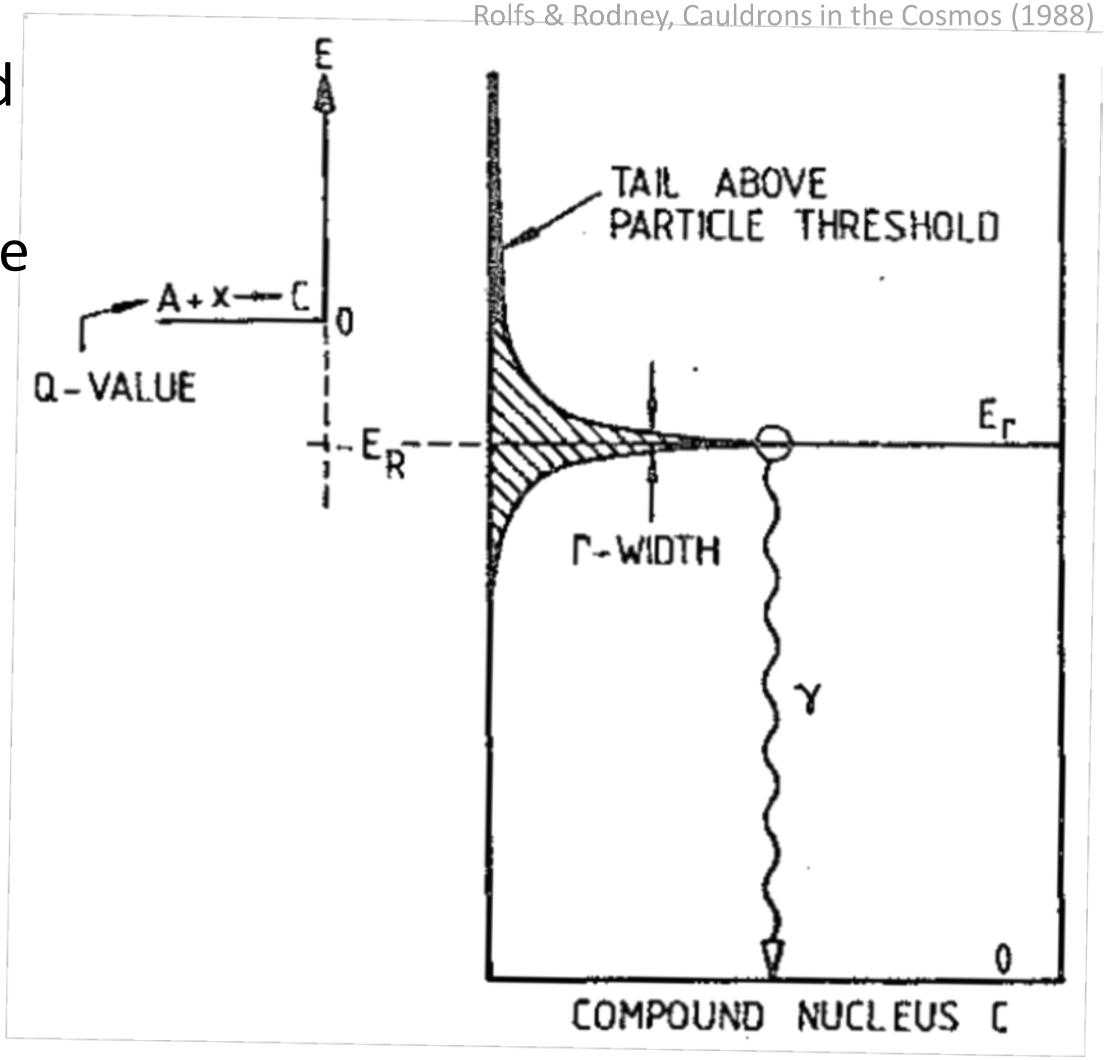
Rolfs & Rodney, Cauldrons in the Cosmos (1988)

Thermonuclear rate: **Subthreshold resonance**

- Broad resonances lurking below the reaction threshold can contribute to the rate as well
- This can cause a huge boost over the non-resonant rate

Contributions from a hypothetical sub-threshold resonance with various strengths for ^{9}Be(p,α):



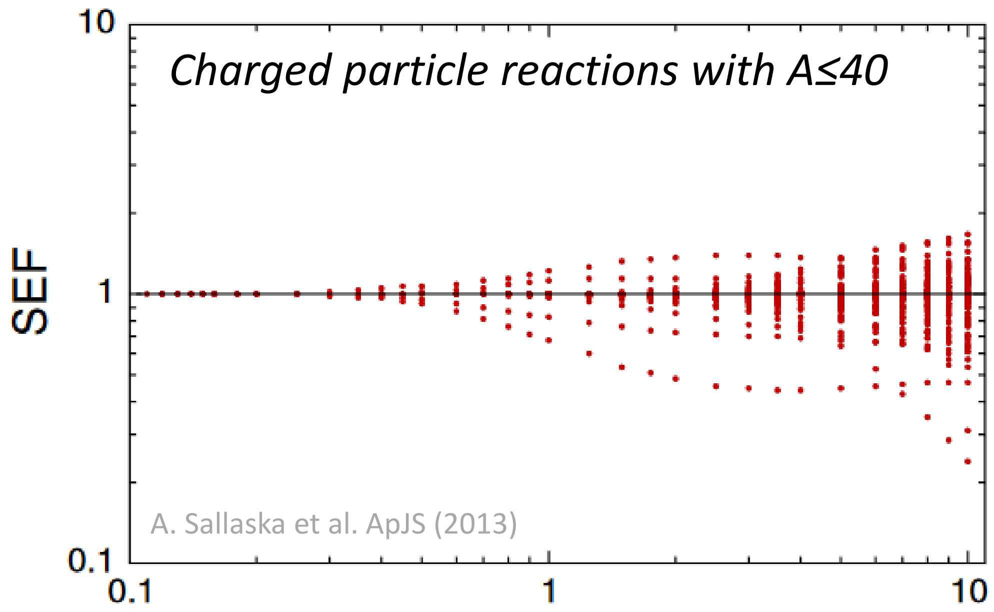


Aside: Stellar Enhancement Factors (SEFs)

- It's important to note that the rate calculated using the laboratory cross section is only for the ground-state of the target
- However, in a hot environment, excited states in the target will be thermally populated
- The stellar enhancement factor (SEF) is the ratio of the stellar rate (which is what we want) to the rate determined from the laboratory measurement.
- The SEF is the ratio of the

$$SEF = \frac{Stellar\ Rate}{Laboratory\ Rate} = \frac{\sum_{target\ states}(Fraction\ of\ Target\ Species\ in\ an\ Excited\ State)(Rate\ for\ that\ State)}{(Laboratory\ Rate)}$$

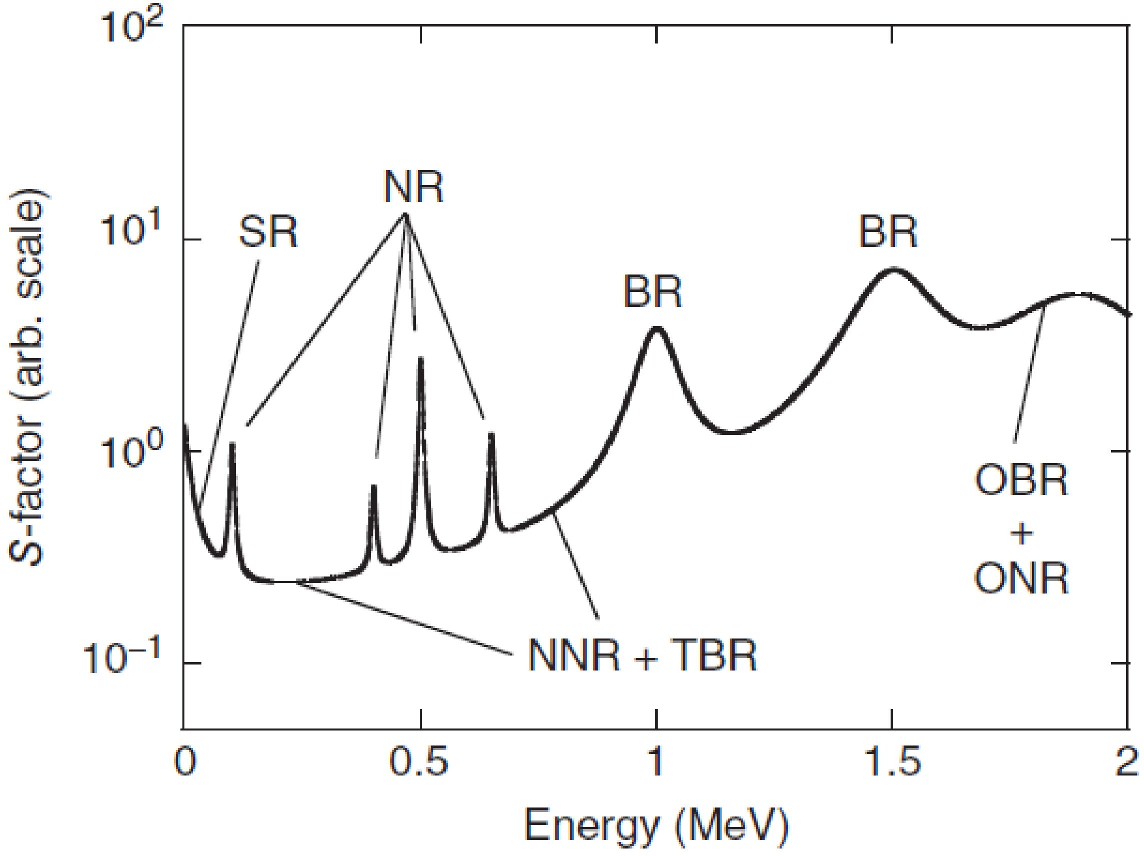
- The excited state populations are determined by the partition function, and they're typically estimated using the statistical model (See e.g. A. Sallaska et al. ApJS 2013)
- A commonly used set of partition functions used to calculate SEFs comes from Rauscher & Thielemann ADNDT 2000

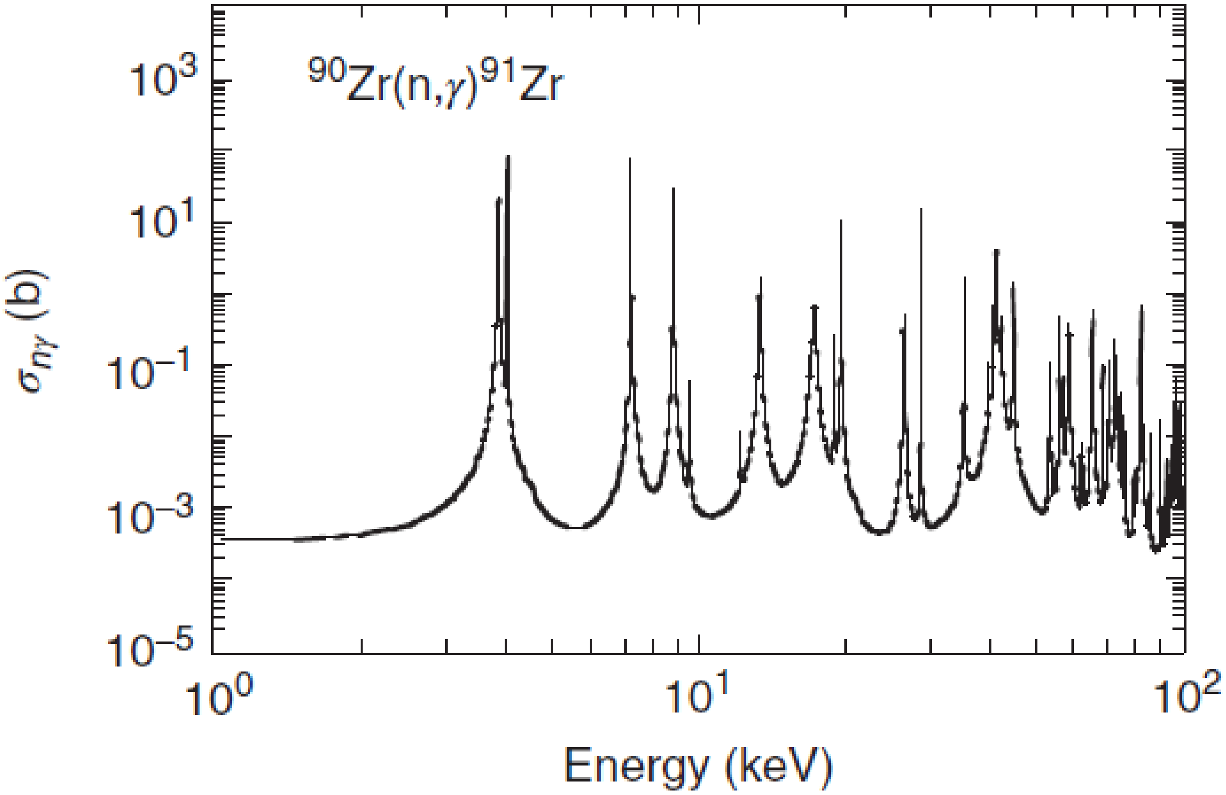


Total thermonuclear reaction rate

- Any and all of the aforementioned rate types can contribute to the overall reaction rate

Iliadis, Nuclear Physics of Stars (2007)





Where can I get thermonuclear reaction rates?

JINA-CEE

REACLIB database

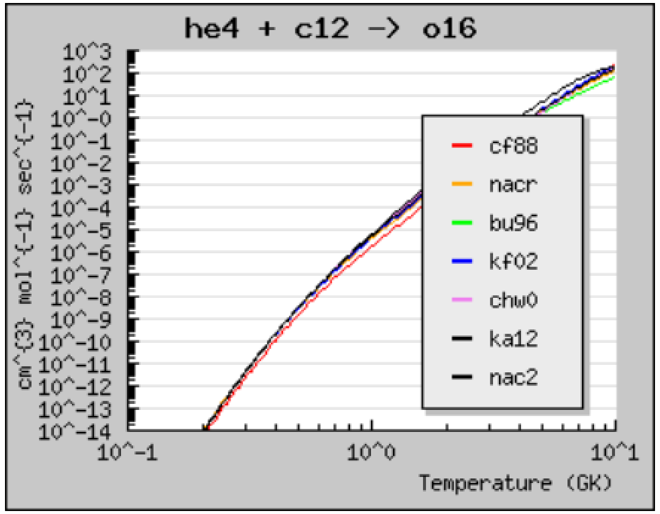
https://groups.nscl.msu.edu/jina/reaclib/db/

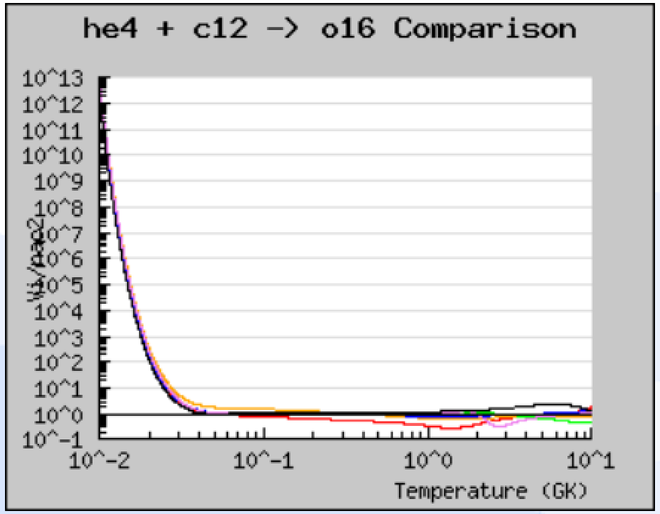
- The Joint Institute for Nuclear Astrophysics hosts REACLIB, which contains thermonuclear reaction rates in parameterized and tabular form
- Rates are based on published experimentally-constrained and otherwise purely theoretical rates
- REACLIB is a standard in the field, e.g. it is employed as the default in the MESA stellar modeling software

Rate Details: he4 + c12 → o16

Rate:	he4 + c12 -> o16 nac2(v9)
Q (MeV):	7.16192
SEF:	None
Fits Parameters	View
Data Points	View
Version	9 of 9
Label	nac2
Recommended	Yes
Future	No
Popular Categories	Light
Files	View
Last Modified:	2016-03-12 01:22:04 by schatz
Comments:	
Reverse Rate	
Reaclib1 Format Download	

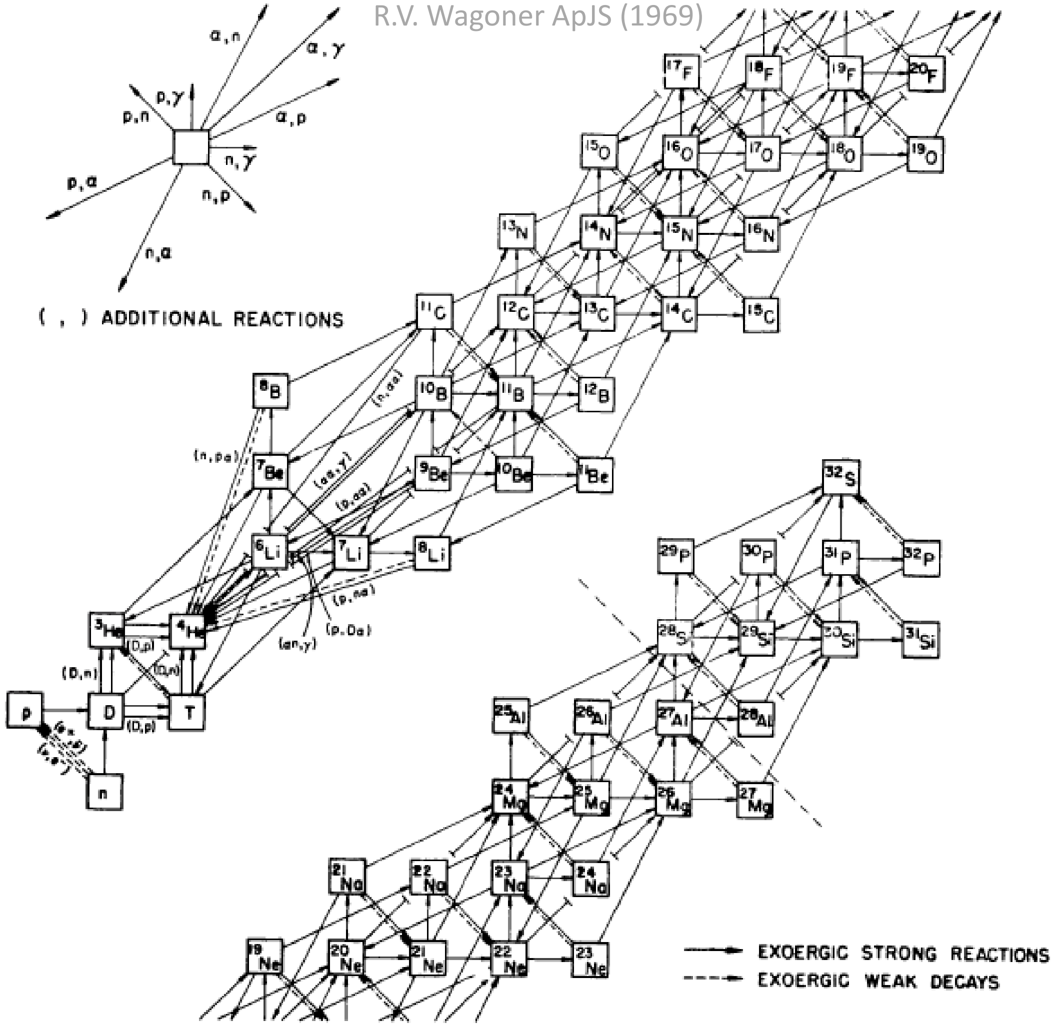
Displayed Versions: ☑ cf88 ☑ nacr ☑ bu96 ☑ kf02 ☑ chw0 ☑ ka12 ☑ ☑ ☑ nac2





Nuclear reaction networks

R.V. Wagoner ApJS (1969)

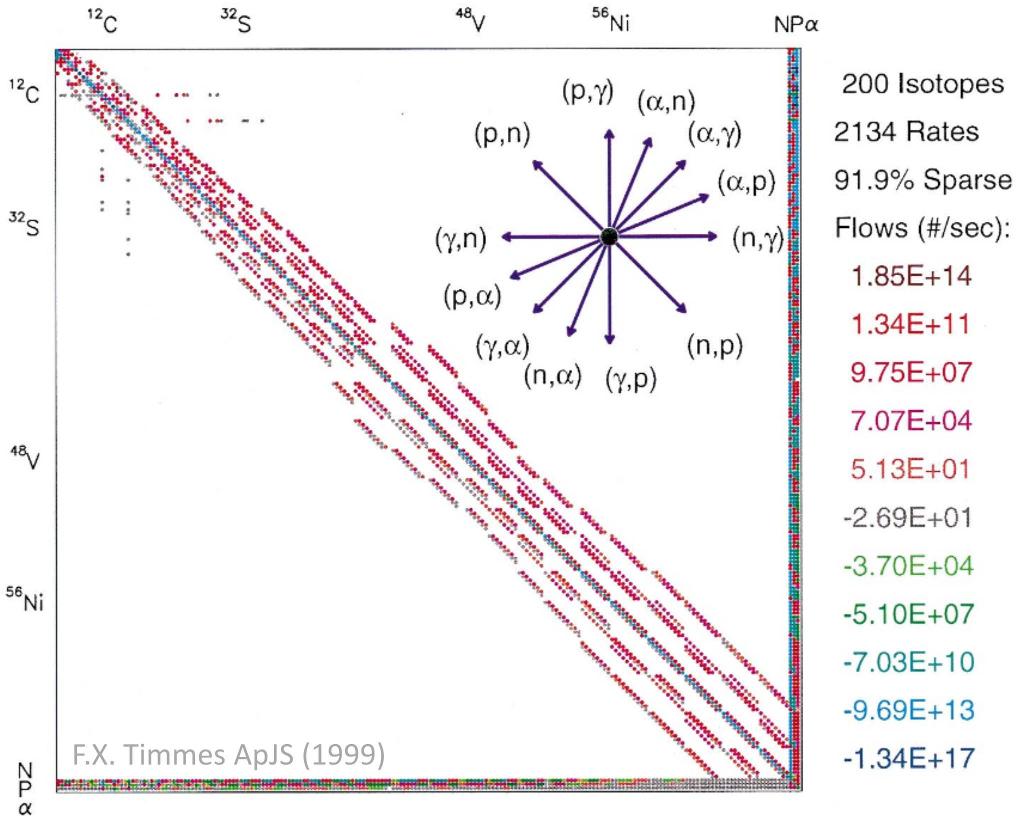


- Astrophysical environments typically contain many nuclei, each of which could in principle interact with the other
  (in practice usually only the photons and light projectiles matter)
- To evaluate what happens, we need to solve a reaction network
- The basic idea is to see how the abundance changes for each isotope at each step in time based on the production and destruction via all mechanisms and often to include the energy generation from said reactions
- For each species i,

$$\frac{dY_i}{dt} = \left[\sum_{j,k} Y_j Y_k \rho N_A \langle\sigma v\rangle_{jk\rightarrow i} + \sum_l \lambda_{l\rightarrow i} Y_l\right] - \left[\sum_m Y_i Y_m \rho N_A \langle\sigma v\rangle_{im\rightarrow any} + \sum_n \lambda_{i\rightarrow n} Y_l\right]$$

Production *Destruction*

Nuclear reaction networks



- You might consider solving the problem explicitly, stepping through time updating each Y_i
- The issue is that the ordinary differential equations $\frac{dY_i}{dt} = f(\vec{Y})$we're trying to solve are very "stiff", i.e. they're very sensitive to changes (because reaction rates have steep temperature dependencies) so we would need super tiny time-steps
- Instead, an implicit approach is needed
- We note that the change in abundance ΔY_i is equal to the change in abundance at the next time-step times the time-step $f(\acute{Y}_i(t+\Delta t))\Delta t$
- The change in abundance at the next timestep is equal to the change at the present time plus the sum of the change in the change due to other species' abundances changing
  $f\left(\vec{Y}_i(t+\Delta t)\right) = f\left(\vec{Y}_i(t)\right) + \sum_j \frac{\partial f(\vec{Y}_i(t))}{Y_j}\Delta Y_j$ …where $\frac{\partial f(\vec{Y}_i(t))}{Y_j}$ are the elements of the Jacobian matrix
- Solving for the change in all abundances results in $\Delta\vec{Y} = \vec{f}(t)\left(\frac{\tilde{1}}{\Delta t} - \tilde{J}\right)^{-1}$
- Inverting the matrix (the ~ bits) is where most of the computational cost comes in

Further Reading

- Chapter 12: Modern Nuclear Chemistry (Loveland, Morrissey, & Seaborg)
- Chapter 3: Nuclear Physics of Stars (C. Iliadis)
- Chapters 3 & 4: Cauldrons in the Cosmos (C. Rolfs & W. Rodney)
- Lecture Materials on Nuclear Astrophysics (H. Schatz)
- Chapters 10 & 11: Stellar Astrophysics (E.F. Brown)
- "Thermonuclear Reaction Rates", Fowler, Caughlan, & Zimmerman, Annu.Rev.Astron.&Astro. (1967)
- R.V. Wagoner, Astrophys. J. Suppl. Ser. (1969)