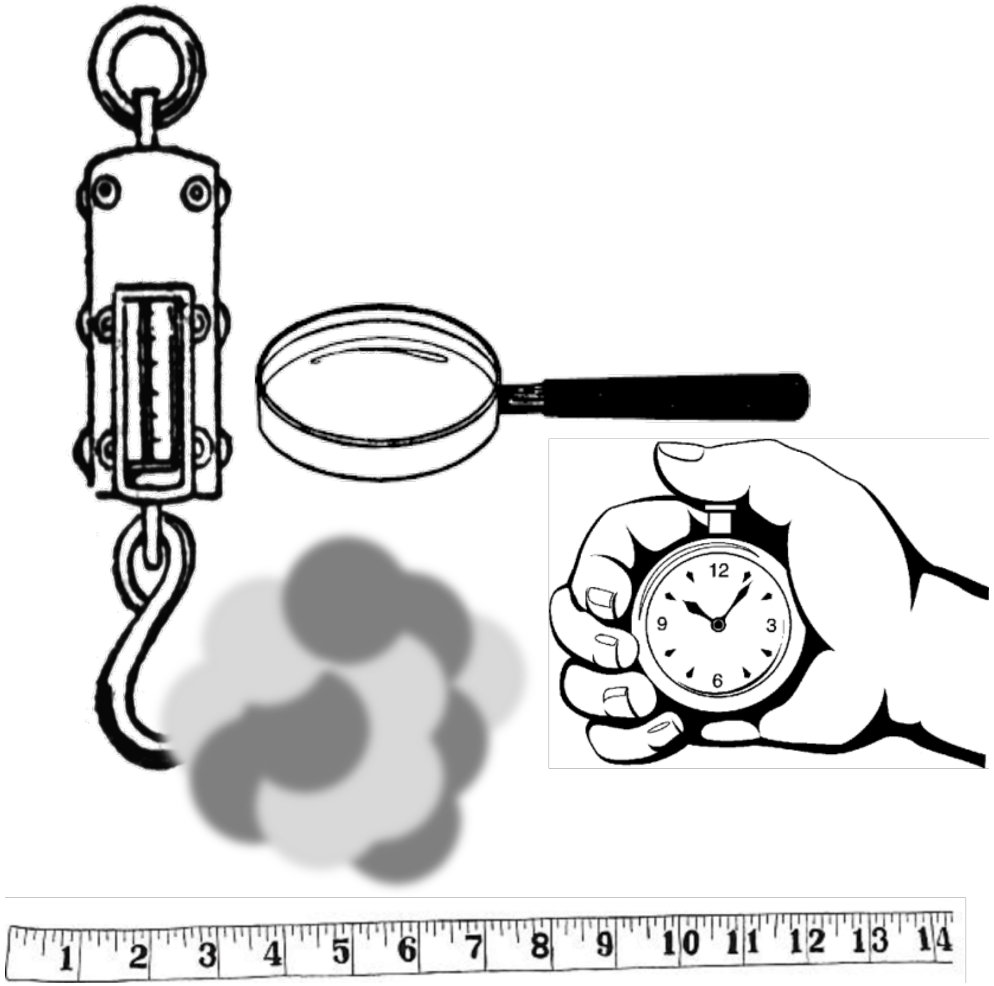
Lecture 1: *Nuclear Properties*

- General descriptors of nuclei
- Nuclear phenomenology
- The nuclear landscape



The Atomic Nucleus

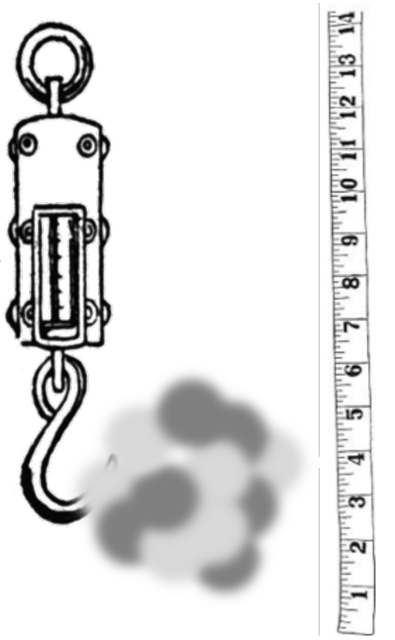
- A conglomeration of *nucleons*, a.k.a. neutrons and protons
- Convention for referring to a particular configuration, a.k.a *nuclide*
  - ${}^{A}_{Z}(chemical\ symbol)_{N}$
  - A = # of nucleons, Z = # of protons, N = # of neutrons.
  - The chemical symbol is from the periodic table that corresponds to Z
  - Since A=Z+N and Z is indicated by the chemical symbol, more common notation is: ${}^{A}chemical\ symbol$
  - For example, the most common type of carbon (Z=6) has 6 neutrons: ${}^{12}C$ Conversationally, you would call this "carbon twelve"

An "m" after the mass-number of a nucleus indicates an isomer, a long-lived excited state. E.g. ${}^{26m}Al$

- Nuclides with the same Z but different N are *isotopes* *(though this term is often used in lieu of nuclides)*
- Nuclides with the same N but different Z are *isotones*
- Nuclides with the same A are *isobars*
- Nuclides with nicknames:
  - ${}^{1}H$ = proton, ${}^{2}H$ = deuteron (d), ${}^{3}H$ = triton (t), ${}^{4}He$ = α, ${}^{3}He$ = helion *(rarely used)*

How big is a nucleus?

The definition of u being based on ^{12}C means it is a valuable tool for high-precision mass measurements, e.g. C.Scheidenberger et al., Nuc. Phys. A 2002



Phenomenological estimates:

- A nucleus's mass is roughly: M(Z,A) = A*amu
  - 1amu = atomic mass unit = u = 931.494MeV/c^2 ≈1.66x10^{-24}g
  - The amu is defined such that M(^{12}C) ≡ 12u
- A nucleus's (charge) radius is roughly: R(Z,A) = (1.2fm)*$A^{1/3}$
  - fm = femtometer (a.k.a. *fermi*) = 10^{-15}m
  - The radius of a nucleon is often referred to as r_0=1.2fm
  - For the RMS radius, multiply by $\sqrt{3/5}$
- Therefore, an estimate for the nuclear density is:
  - $n = \frac{\#}{V} = \frac{\#}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi(1.2fm)^3 A} \approx 0.14\, nucleons/fm^3$
  - For fun, in terms of mass-density: $\rho = \frac{(1.66\times10^{-24}g)A}{\frac{4}{3}\pi(1.2fm)^3 A} \approx 2.3 \times \frac{10^{-25}g}{fm^3}$

…which doesn't sound like much, but this is 2x10^{14} g/cm^3 (the Great Pyramid of Giza is only ~10^{12} grams)

Since A cancels in the ρ expression, the nuclear density is independent of the nuclear size, much as a liquid's density is independent of the size of the drop.
Partly inspired by this property, some basic nuclear calculations are based on this liquid drop analogy.
(G. Gamow, Proc. Roy. Soc. A 1929,1930)

Nuclear Transmutation

- Rules for converting one nuclide (or nuclides) to another (or others)
  - *Charge conservation*: $\sum q_{before} = \sum q_{after}$ (q from protons + positrons + electrons)
  - *Baryon conservation*: $\sum A_{before} = \sum A_{after}$ (A from neutrons + protons)
  - *Lepton number conservation*: $[N_{leptons} - N_{anti-leptons}]_{before} = [N_{leptons} - N_{anti-leptons}]_{after}$

  * Transmutation likelihoods are impacted by energetics and spin/parity selection rules

The first nuclear reaction intentionally made in the laboratory was $^{14}N(\alpha,p)$ in 1919. (E. Rutherford, Nature 1935). Though his student Blackett was the first to understand that they were seeing (α,p) and not α-capture to disintegration (P. Blackett, Proc. R. Soc. A (1925))

The first measured radioactive decay was α decay from uranium. (H. Becquerel, Comptes Rendus 1896).

Two Types:

- Reactions
  - Multiple reactants create one or more products
  - Notation: A+b → c+D is written as *A(b,c)D*, where M(b)<M(A) and M(c)<M(D)
    - E.g. $^{12}C+\alpha \rightarrow {}^{16}O+\gamma$ is $^{12}C(\alpha,\gamma)^{16}O$ or even just $^{12}C(\alpha,\gamma)$ and is called "carbon-twelve alpha gamma"
- Decays
  - α, β^+, β^-, e^--capture, β-delayed γ/p/α/n emission, fission, cluster emission, prompt γ
  - Lose nucleons for all above except β decay, e^--capture, and prompt γ (following a reaction)

Nuclear Forces, mechanisms for binding & transmutation

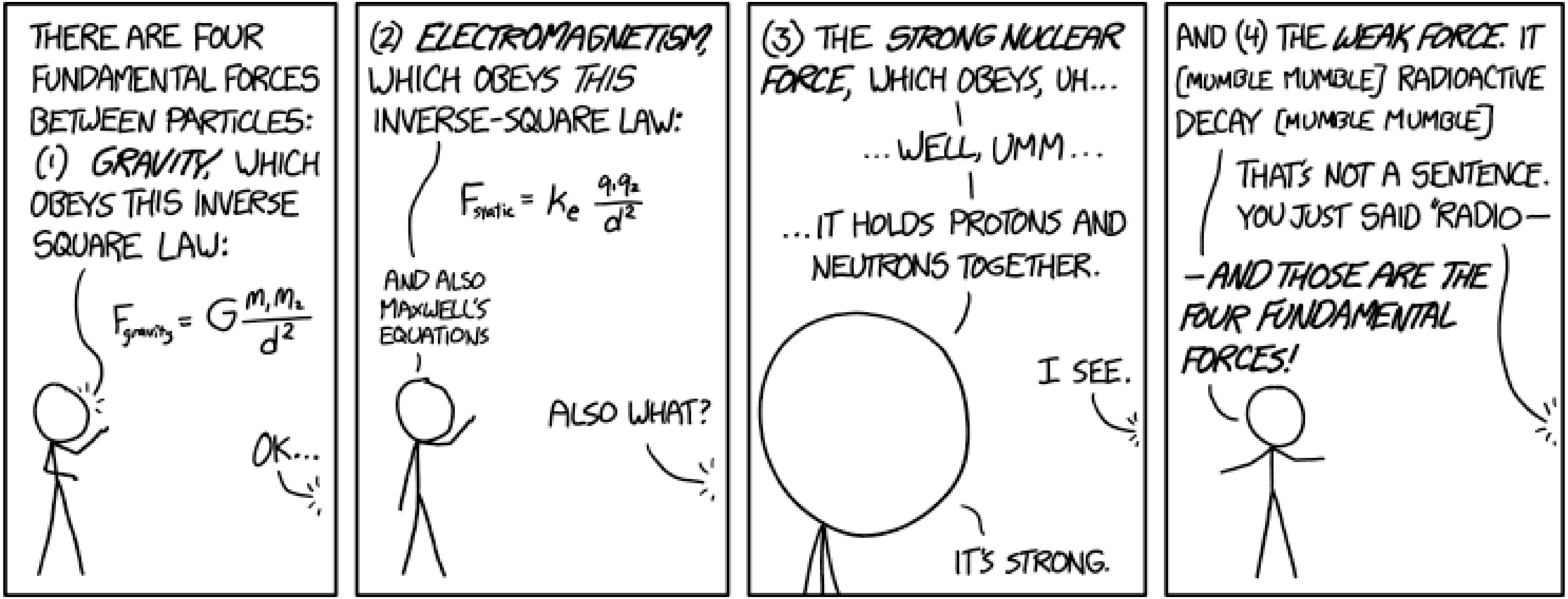
- Fundamental forces of nature:

Force	Range (m)	Relative strength	Force carrier, X
Gravitational	∞	10^{-38}	Graviton
Weak	10^{-18}	10^{-5}	$W^{\pm}$, Z^0
Electromagnetic	∞	$\alpha \approx 1/137$	Photon
Strong	10^{-15}	1	Gluon, Pion

- The range comes from the uncertainty principle:
  - When converting reactant(s) to product(s), energy is spent on the mass of the force carrier: $\Delta E \geq M_X c^2$
  - Violating energy conservation like this is only allowed due to the uncertainty principle: $\Delta E \Delta t \geq \hbar/2$
  - And therefore the force carrier must be reabsorbed before: $t \leq \hbar/(2\Delta E)$
  - Since a particle can't be moving faster than the speed of light, c, the furthest range a force can be mediated is: $r \leq tc = \hbar c/(2\Delta E) = \hbar c/(2M_X c^2) = \hbar/(2M_X c)$
  - Thus, since factors of 2 are for chumps, the range of an interaction is defined as: $\mathbf{R \equiv \hbar/(M_X c)}$
  - Since the graviton and photon are massless, $R = \infty$.
  - For the weak force, $M_X \sim 100\text{GeV}/c^2$, so $R_{weak} \sim 10^{-3}$ fm.
  - Similarly, since $M_\pi \sim 100\text{MeV}/c^2$, $R_{strong} \sim 1$fm.

*The trick for these calculations is that $\hbar c \approx 197$MeV*fm.*

Note force range relative to nucleon size.



Randall Munroe, xkcd.com

Nuclear Landscape (known nuclei)

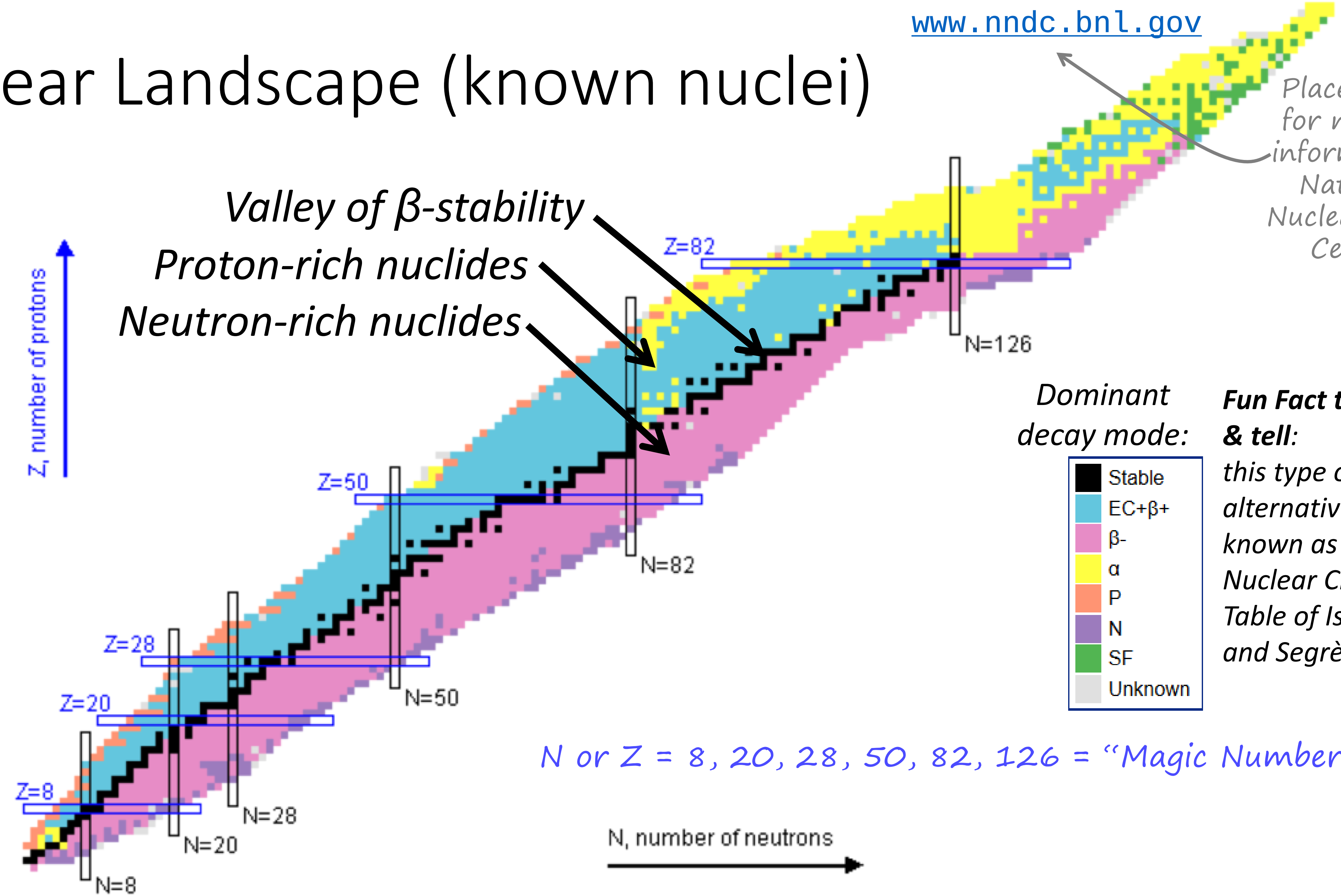
www.nndc.bnl.gov

Place to go for nuclear information: National Nuclear Data Center

Valley of β-stability

Proton-rich nuclides

Neutron-rich nuclides



Dominant decay mode:

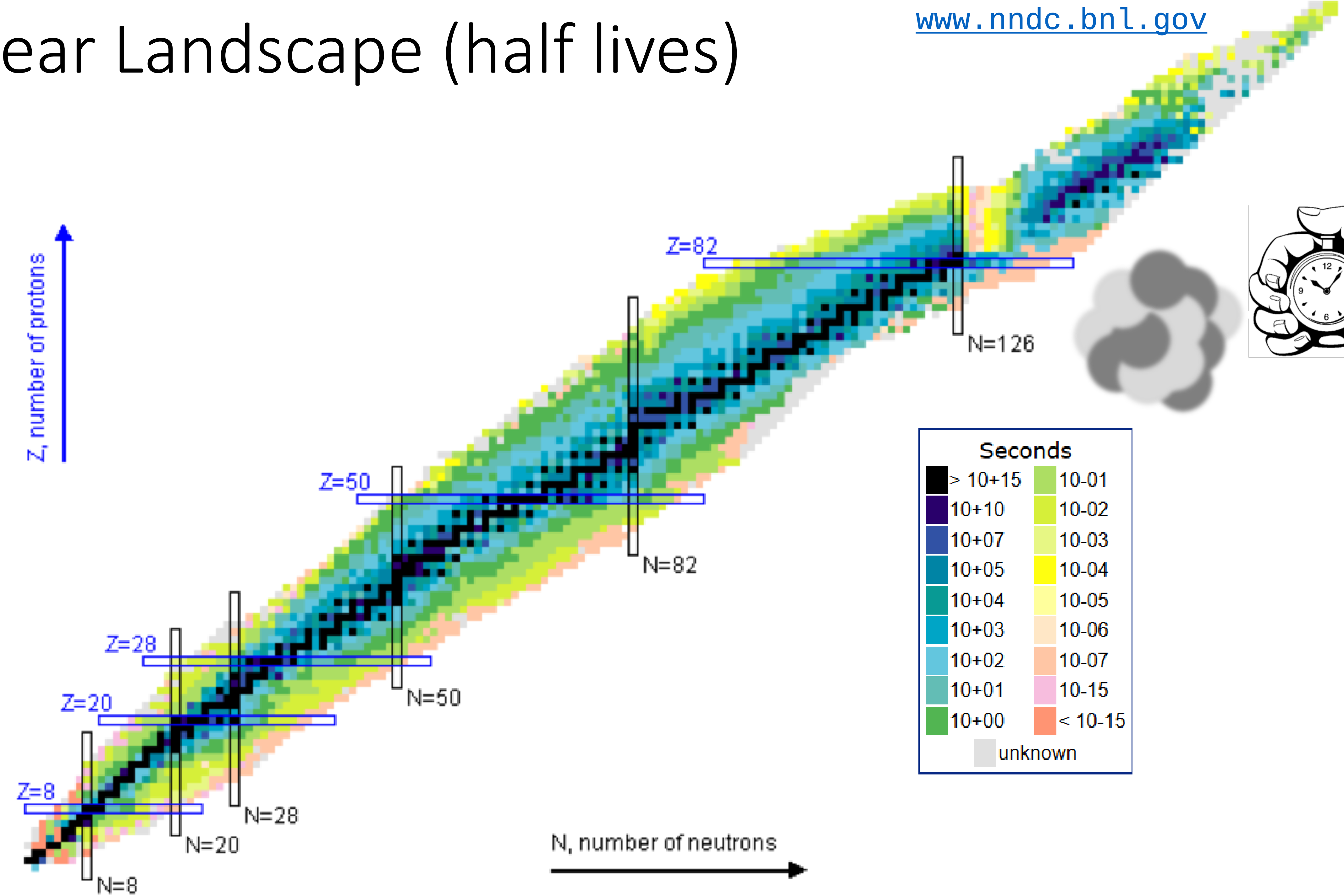
■	Stable
■	EC+β+
■	β-
■	α
■	P
■	N
■	SF
■	Unknown

Fun Fact to know & tell*:* *this type of plot is alternatively known as the Nuclear Chart, Table of Isotopes, and Segrè Plot*

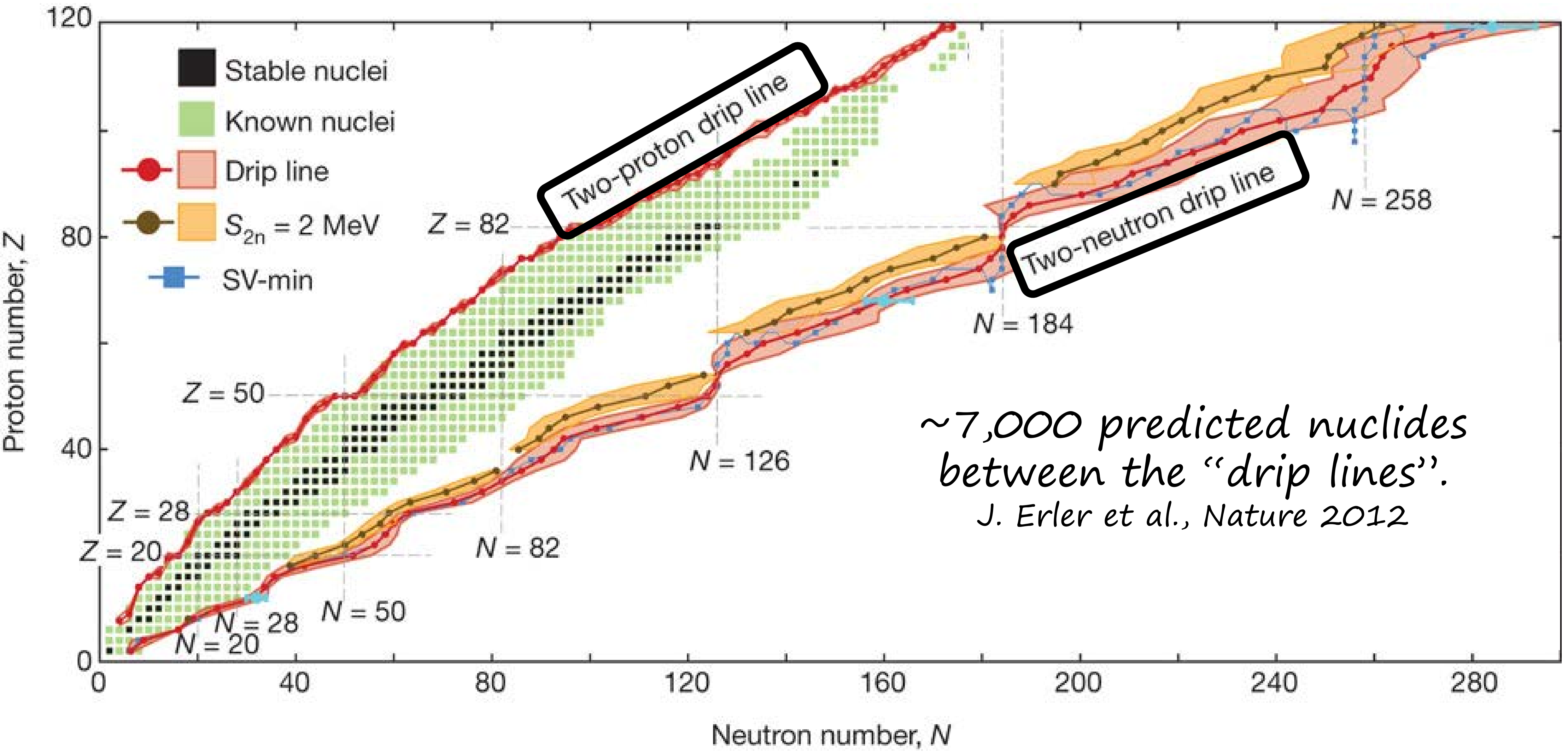
N or Z = 8, 20, 28, 50, 82, 126 = "Magic Numbers"

Nuclear Landscape (half lives)

www.nndc.bnl.gov

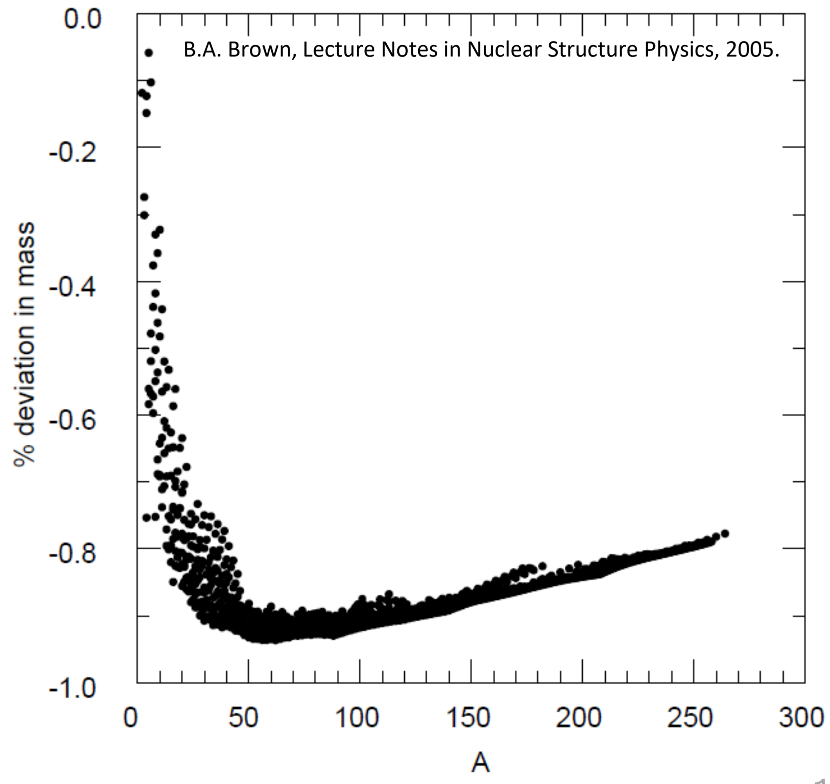


Nuclear Landscape (predicted nuclei)



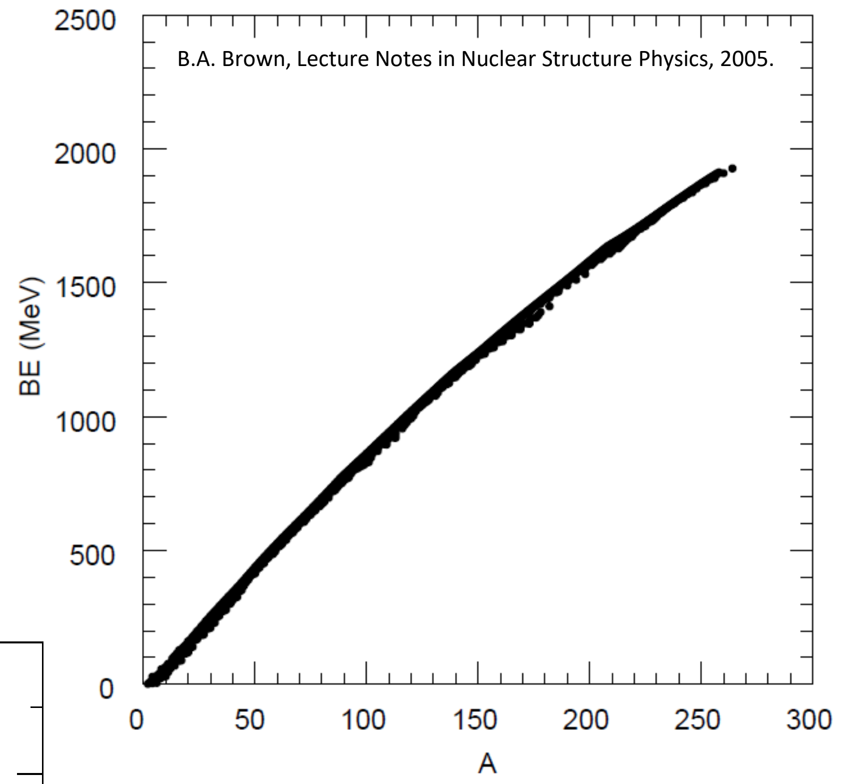
Nuclear Binding Energy

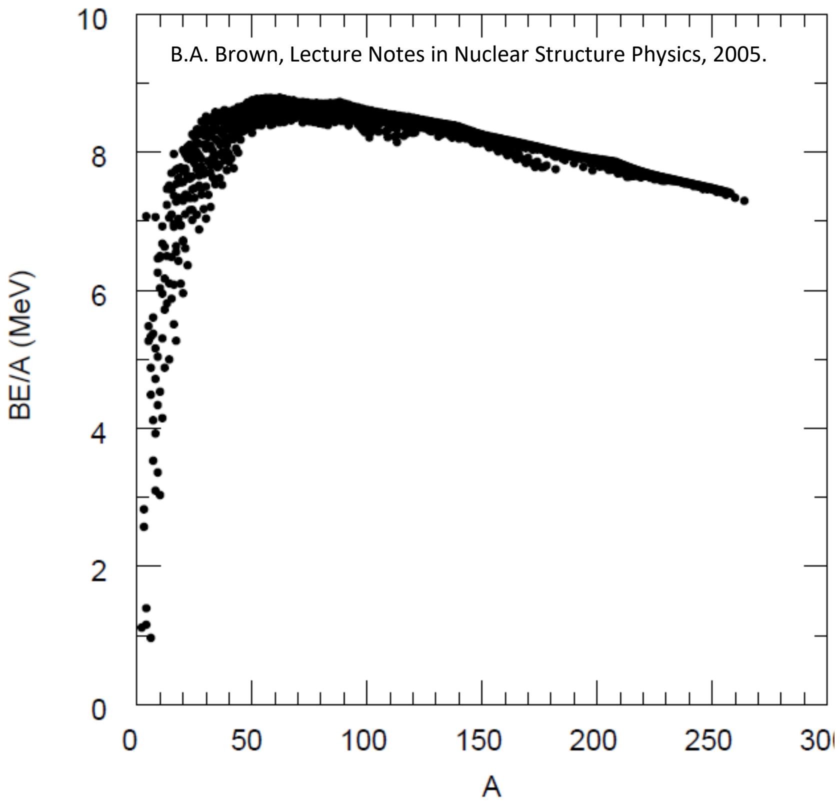
- Early mass spectrometry showed that nuclear masses are (nearly) integer multiples of the hydrogen atom mass. (Coined the "whole number rule" (F. Aston, Nature 1919))
- However, high-precision work showed deviations on the sub-% level (F. Aston Nature 1924)
- This deviation is known as the "mass defect", whereby the nuclear mass is a bit less than the sum of the constituent nucleon masses:
- Which is of course a result of Einstein's postulate, $E=mc^2$
- Nucleons within a nucleus are bound together, and this binding requires energy.
- The binding energy, BE, is paid-for via reduced mass.
- Indeed, the lower-energy state of a nucleus is the only reason nucleons cluster together at all.

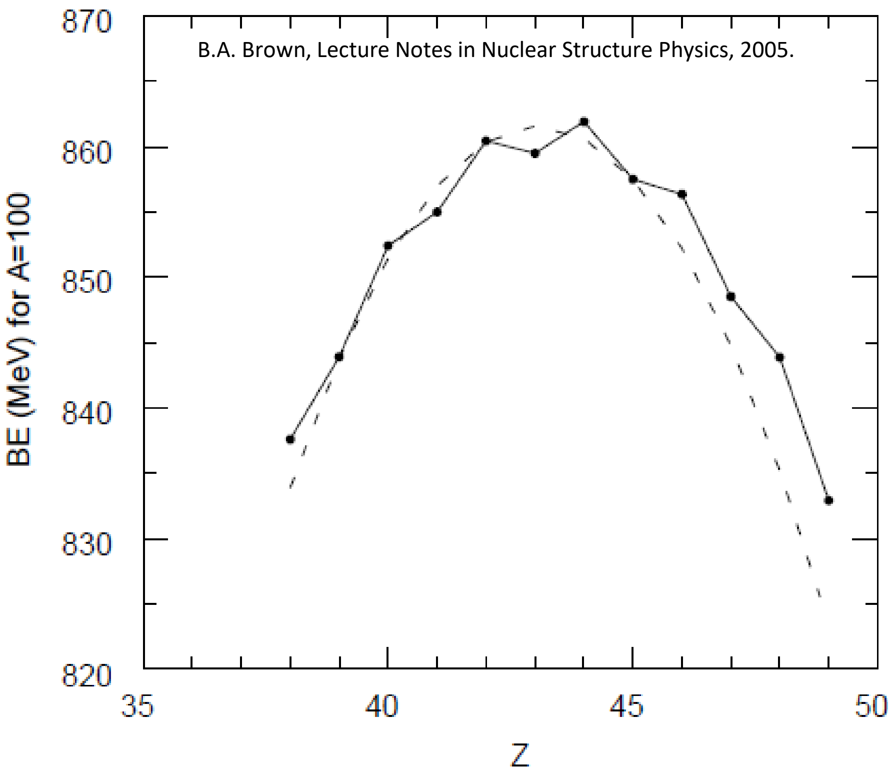


Nuclear Binding Energy

- The binding energy increases nearly linearly with A:
- It's not a surprise that more nucleons would lead to more binding, since nucleons will be prone to attraction via the strong force.
- As such, a more interesting quantity is BE/A





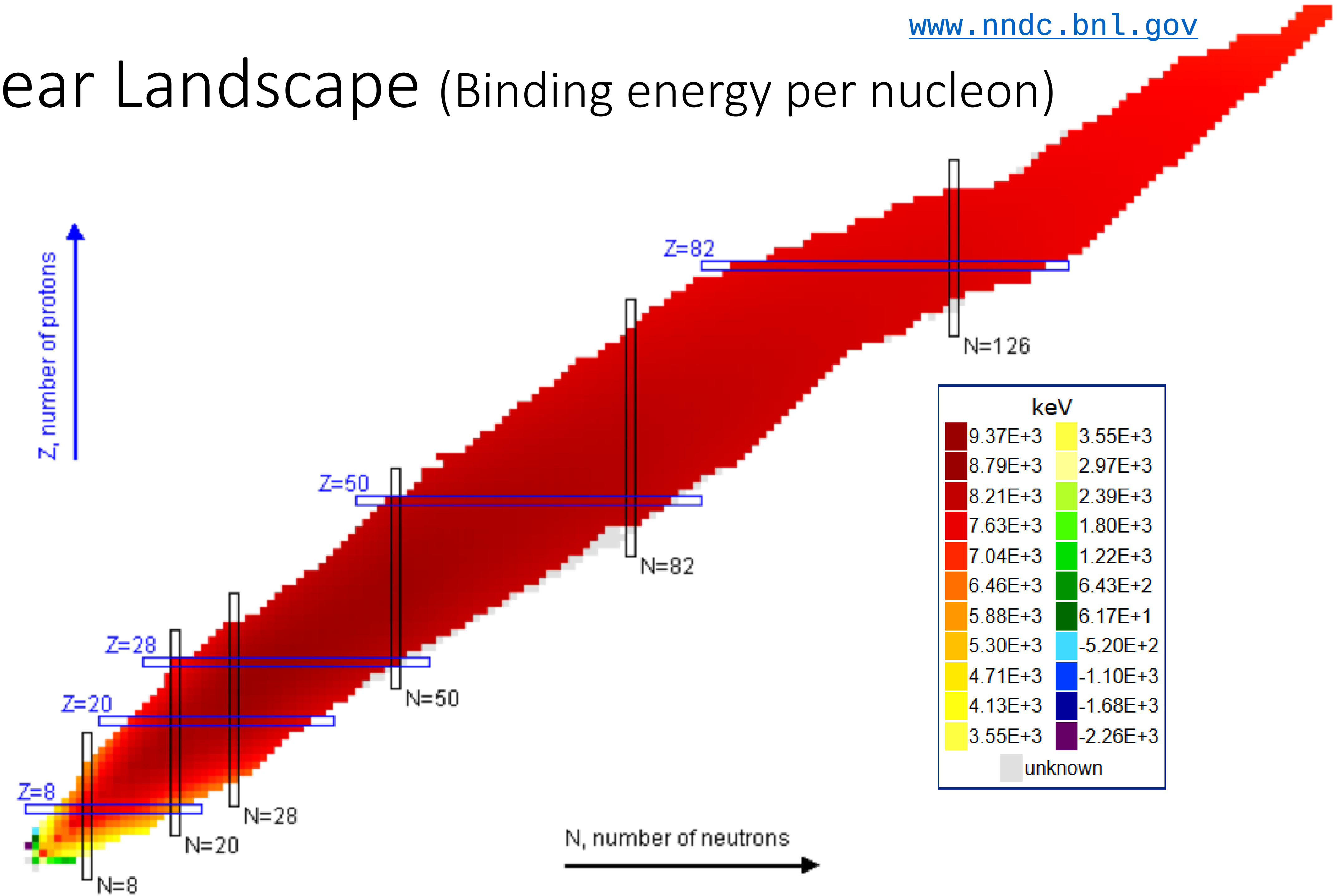


What are the key features you notice on the two left plots?

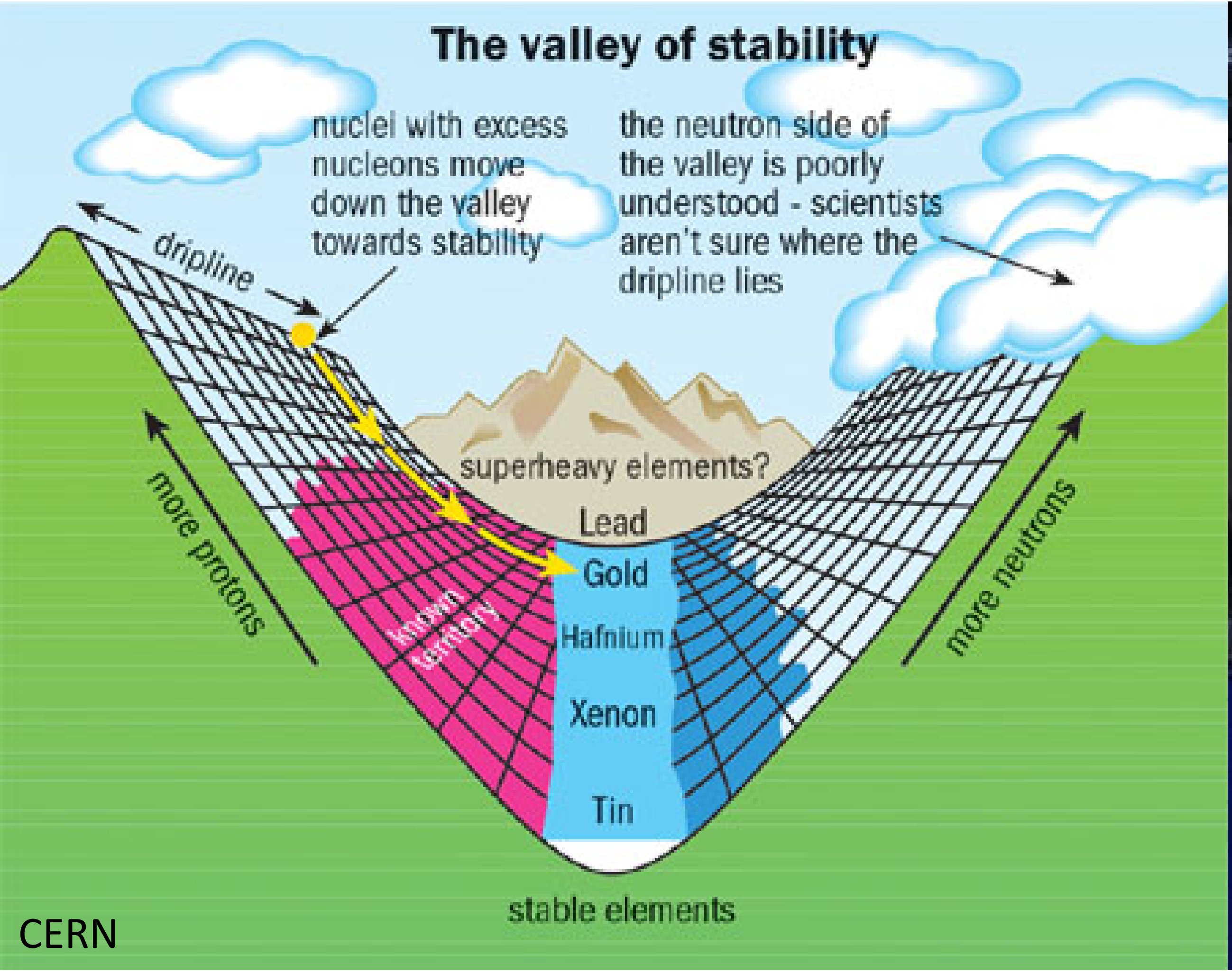
What to they imply about
- *stability to decay?*
- *fusion?*

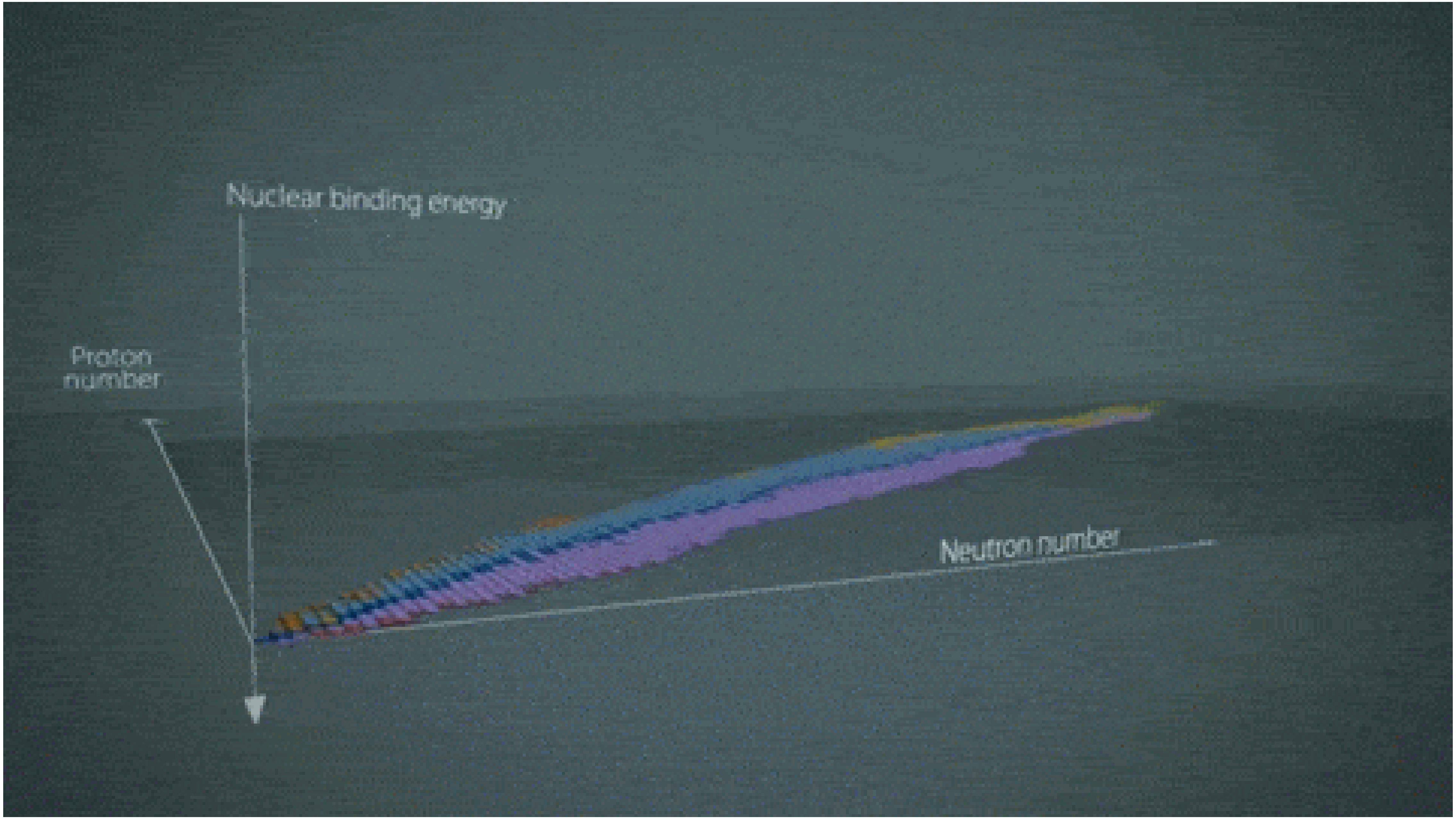
Why do nuclei other than the peak BE/A form at all?

Nuclear Landscape (Binding energy per nucleon)

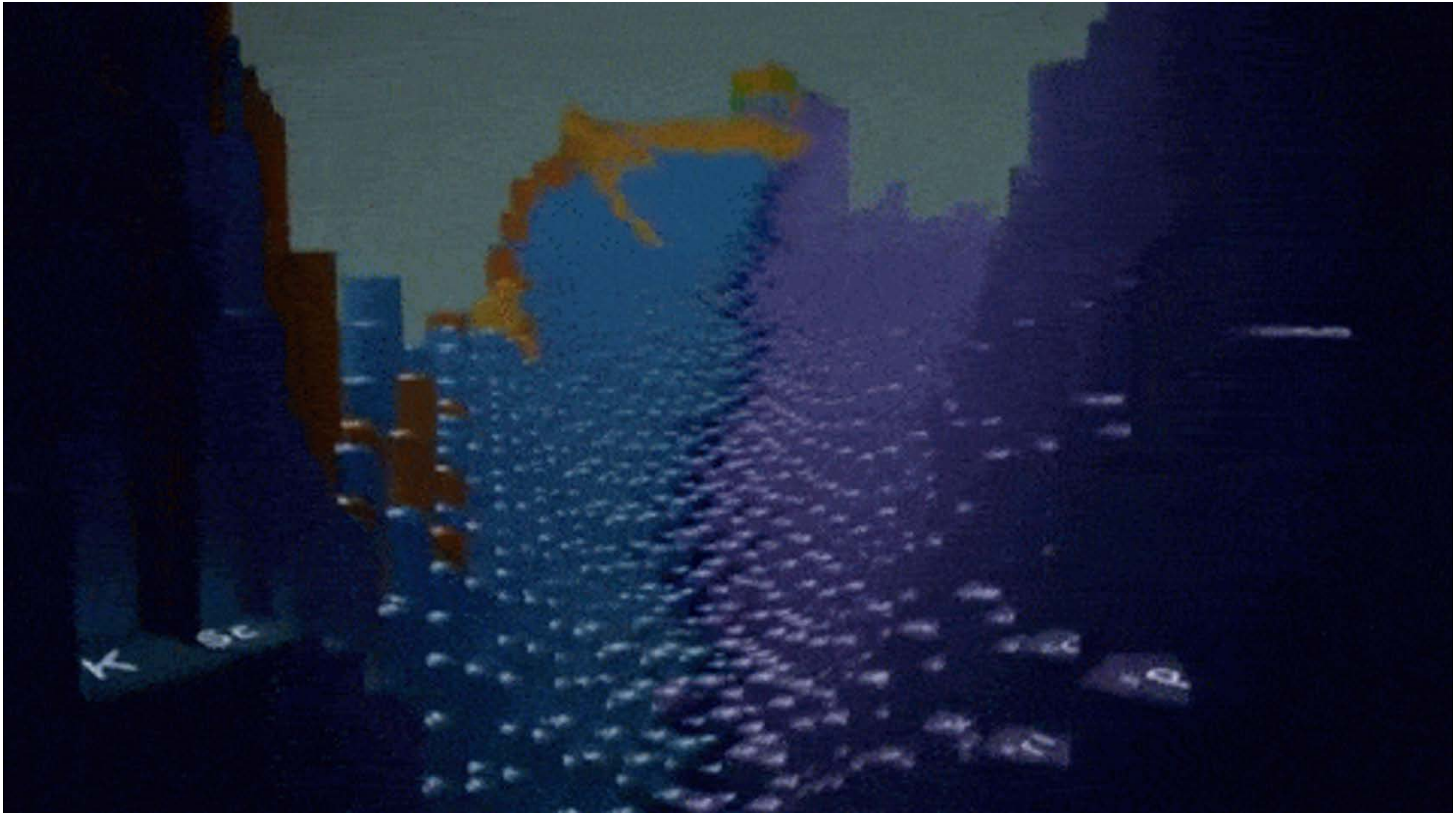


Binding Energy per nucleon: sort of a valley or half-pipe





F. Legrand & F. Durillon, https://www.youtube.com/watch?v=UTOp_2ZVZmM



F. Legrand & F. Durillon, https://www.youtube.com/watch?v=UTOp_2ZVZmM

Aside on nuclear masses, atomic mass excess

- Rather than binding energy, "atomic mass excess" is more commonly used.
- The conversion can be accomplished with the following equations:
  1. $BE(Z,A) \equiv Z*(m_p + m_e) + N*m_n - M(Z,N)$
  2. $ME(Z,A) \equiv M(Z,N) - (Z+N)*m_u$ *ME is is just the mass-defect with a minus sign*

  here M(Z,N) is the mass of a nucleus with Z protons and N neutrons,

  m_p = 938.272000 MeV/c^2 is the proton mass

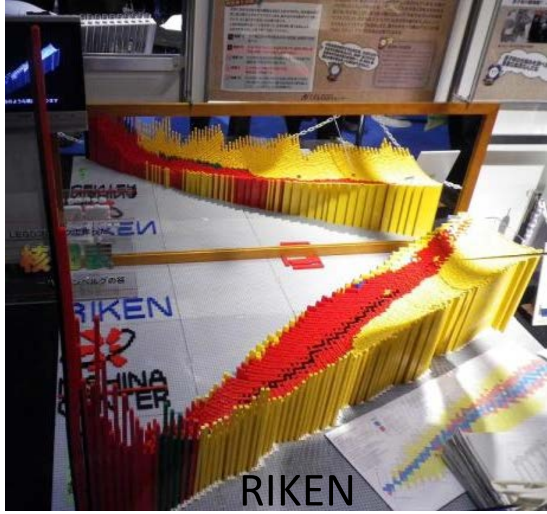
  m_e = 0.51100 MeV/c^2 is the electron mass

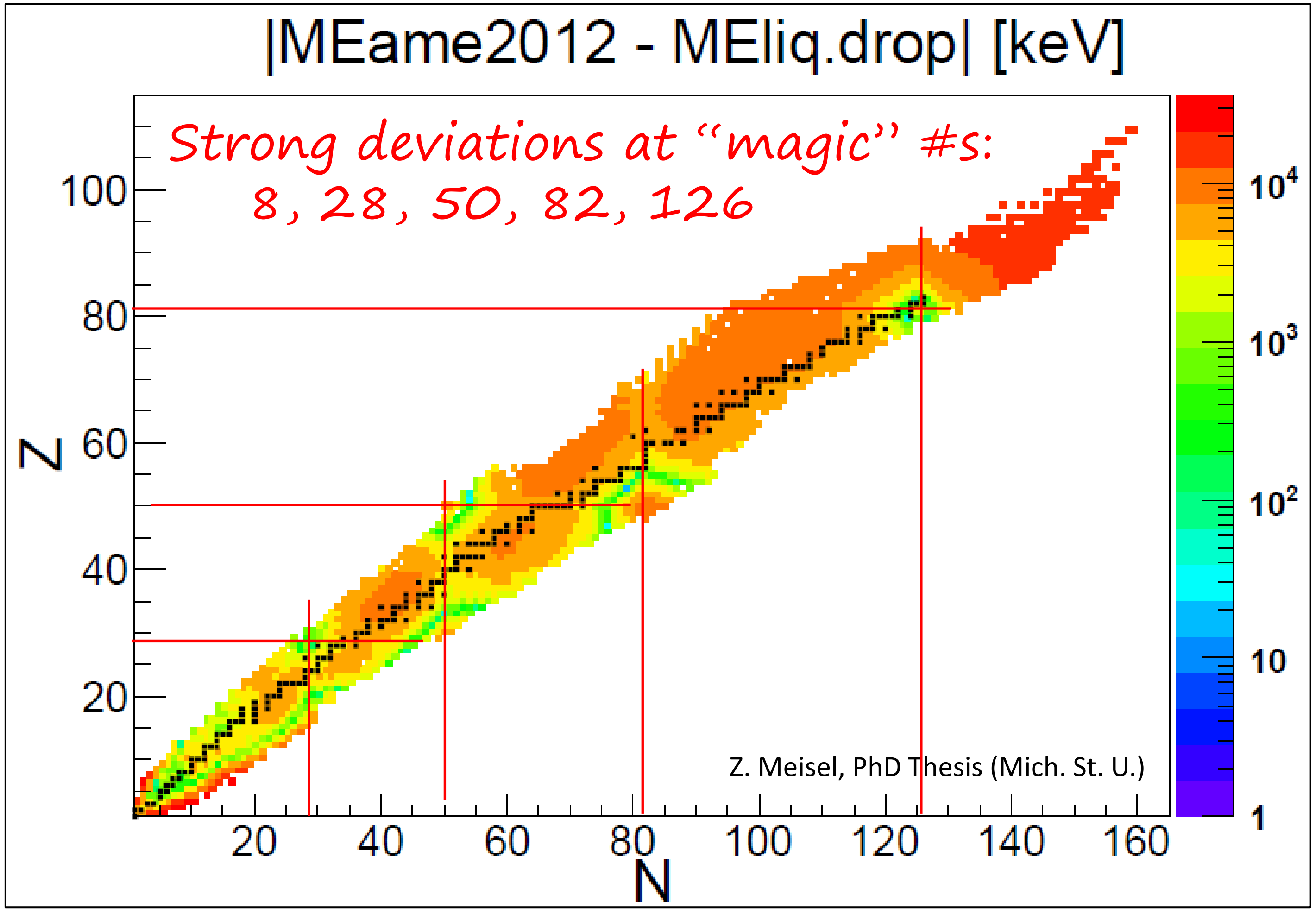
  m_n = 939.56533 MeV/c^2 is the neutron mass

  m_u = 931.494013 MeV/c^2 is the atomic mass unit
- Periodic evaluations are released for all known atomic masses, evaluated from a variety of experimental data by a group of noble souls.
  This is referred to as the "Atomic Mass Evaluation" or "AME".
  The latest is from 2020. (W. J. Huang et al. Ch. Phys. C 2021)

Binding Energy & Magic Numbers

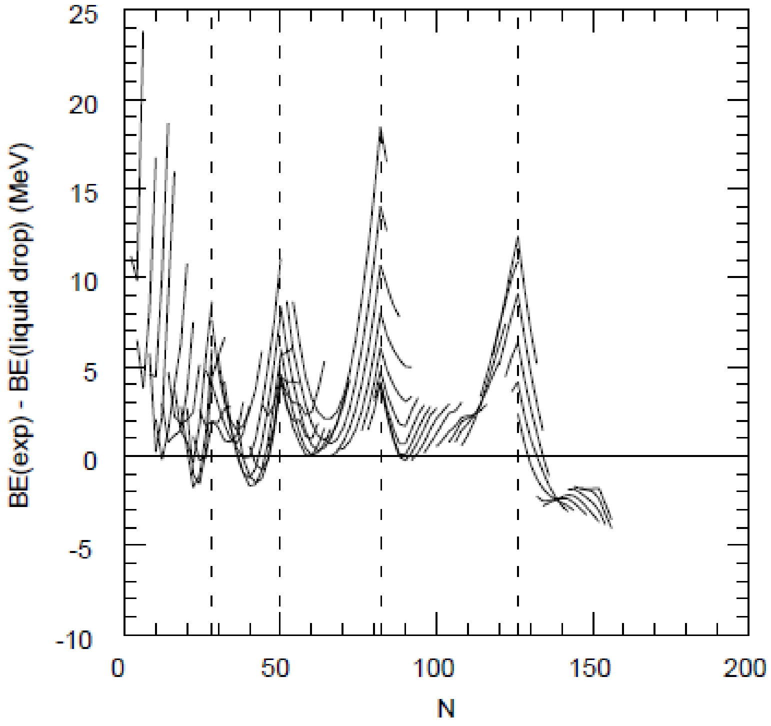
- Binding energy per nucleon looks like a pretty smooth surface, what happens if we fit a smooth function to it & plot the residual? (we'll worry about what this smooth function is next time)





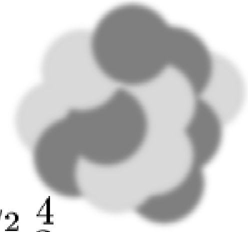
Even easier to see in 1D:

(here even-even nuclei only)

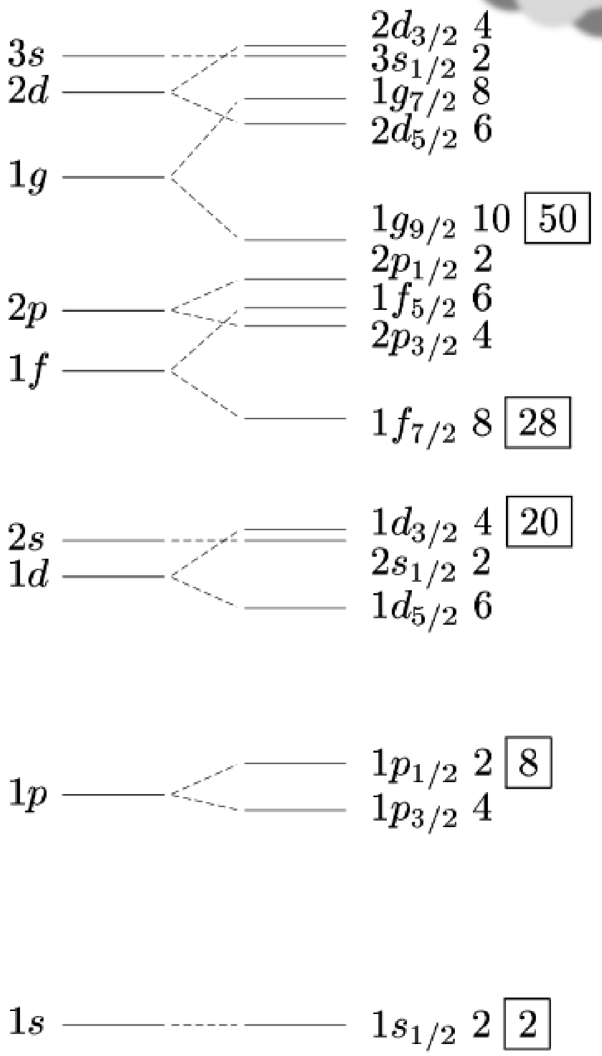


B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.

Magic Numbers and Nuclear Structure

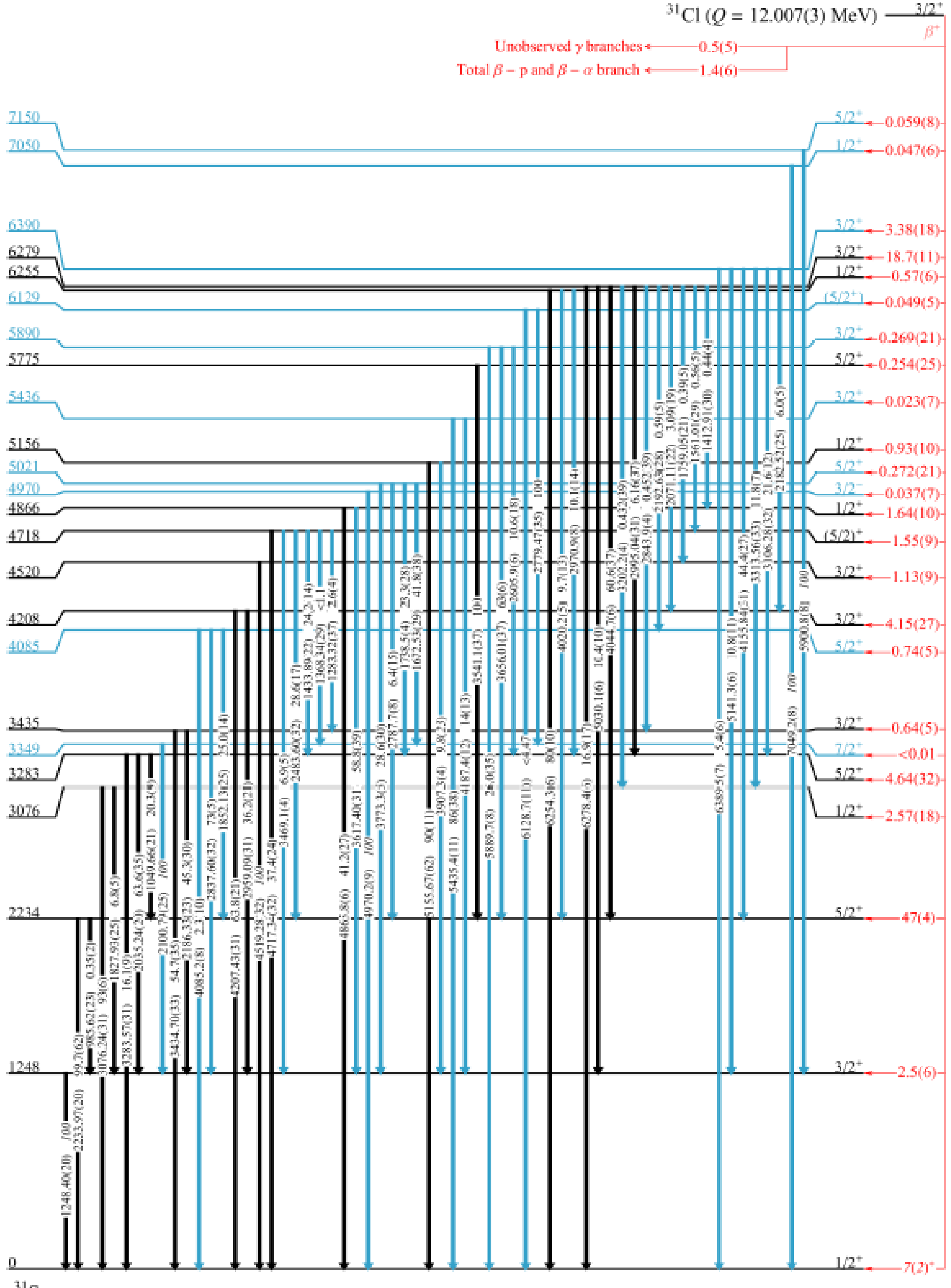


- Nucleons can only be combined in discrete configurations, signified as a “state”
- The lowest-energy state is the “ground state”.
- A nucleus populated in a higher-lying state, an “excited state” , will often γ-decay to de-excite to the ground state (though it may decay to a state of another nucleus).
- In the shell model (which we’ll get to in later weeks), the properties of a state are calculated by considering neutrons and protons filling orbitals, like electrons for an atom.
- These are called single-particle levels:
- Note the gaps at the magic numbers.
  (We’ll talk about how those come to be later)
- Exciting nucleons (or pairs) into other levels gives rise to excited states.



Nuclear Excited States

- The example on the right is for ^{31}S, where the levels shown were populated by β-decay from ^{31}Cl
- Note that the density of levels increases with excitation energy pretty dramatically.
- The only reason it doesn't keep increasing with excitation energy here is because, as we'll learn, this particular mechanism isn't suited for populating the higher lying states.
- Properties of the states include not only the excitation energy, but also
    - Width: related to time to decay via γ or particle emission
    - Spin: intrinsic angular momentum
    - Parity: related to symmetry of wave function



M. Bennett et al. PRC 2017

Timeline of Selected Major Early Developments in Nuclear Physics

- 1896: Radioactivity discovered (uranium) (H. Becquerel, Comptes Rendus)
- 1905: Mass-energy equivalence proposed (A. Einstien, Annalen der Physik)
- 1911: Nuclear model of atom proposed (E. Rutherford, Phil. Mag.)
- 1913: Mass spectrometry invented (proton discovered) (J. Thomson, Proc. Phys. Soc. London)
- 1919: Isotope existence discovered (F. Aston, Nature)
- 1920: Nuclear transmutation proposed to power the sun (A. Eddington, The Observer)
- 1928: α-decay theory & barrier penetration (1st application of QM to real world problem) (G. Gamow, Z. Phys.)
- 1930: Liquid drop model of nucleus proposed (G. Gamow, Proc. Roy. Soc. A)
- 1932: Neutron discovered (J. Chadwick, Nature)
- 1932: Nuclei proposed to be interacting nucleons (based on neutron discovery) (W. Heisenberg, Z. Phys.)
- 1932: Nuclear transmutation using a particle accelerator ($^{7}Li(p,\alpha)\alpha$) (J. Cockroft & E. Walton, Proc. Phys. Soc. London)
- 1934: Theory of β decay (rejected from Nature & caused Fermi to switch to nuclear experiment) (E. Fermi, Z. Phys.)
- 1935: Semi-empirical mass formula developed (based on liquid drop) (C. Wiezsäcker, Z. Phys)
- 1936: Fusion theory (based on liquid drop) (N. Bohr, Nature)
- 1939: Fission proposed (based on liquid drop) (N. Bohr & J. Wheeler, Phys. Rev.)
- 1939: Fission measured (O. Hahn & F. Strassmann, Die Naturwissenschaften)
- 1939: Fission theory (L. Meitner & O. Frisch, Nature)
- 1949: Magic number explanation (M. Goeppert-Mayer, Phys. Rev.)
- 1952: Compound nuclear reaction formalism (W. Hauser & H. Feshbach, Phys. Rev.)
- 1957: Comprehensive theory of nucleosynthesis (M. Burbidge, G. Burbidge, W. Fowler, F. Hoyle, Rev Mod Phys.)
- 1957: Parity violation discovered (C. Wu et al., Phys. Rev)

Further Reading

- Chapters 1 & 2: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapters 1 & 2: Nuclear & Particle Physics (B.R. Martin)
- Chapter 1: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapters 1 & 2: Handbook of Nuclear Chemistry (free from campus VPN)