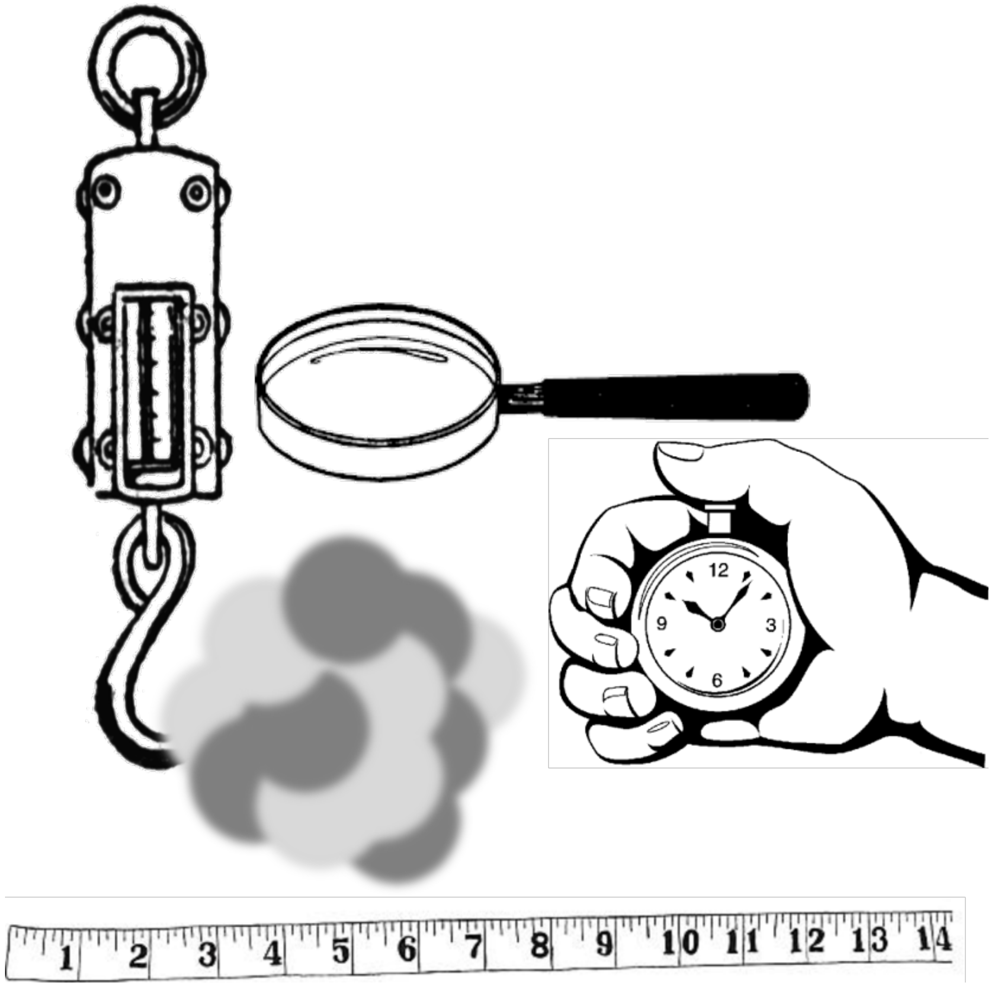
Lecture 1: *Nuclear Properties*

- General descriptors of nuclei
- Nuclear phenomenology
- The nuclear landscape



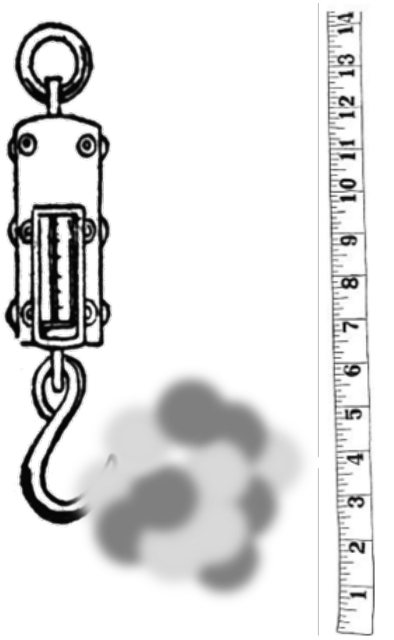
The Atomic Nucleus

- A conglomeration of *nucleons*, a.k.a. neutrons and protons
- Convention for referring to a particular configuration, a.k.a *nuclide*
  - ${}^{A}_{Z}(chemical\ symbol)_{N}$
  - A = # of nucleons, Z = # of protons, N = # of neutrons.
  - The chemical symbol is from the periodic table that corresponds to Z
  - Since A=Z+N and Z is indicated by the chemical symbol, more common notation is: ${}^{A}chemical\ symbol$
  - For example, the most common type of carbon (Z=6) has 6 neutrons: ${}^{12}C$ Conversationally, you would call this "carbon twelve"
- Nuclides with the same Z but different N are *isotopes* *(though this term is often used in lieu of nuclides)*
- Nuclides with the same N but different Z are *isotones*
- Nuclides with the same A are *isobars*
- Nuclides with nicknames:
  - ${}^{1}H$ = proton, ${}^{2}H$ = deuteron (d), ${}^{3}H$ = triton (t), ${}^{4}He$ = α, ${}^{3}He$ = helion *(rarely used)*

An "m" after the mass-number of a nucleus indicates an isomer, a long-lived excited state. E.g. ${}^{26m}Al$

How big is a nucleus?

The definition of u being based on ^{12}C means it is a valuable tool for high-precision mass measurements, e.g. C.Scheidenberger et al., Nuc. Phys. A 2002



Phenomenological estimates:

- A nucleus's mass is roughly: M(Z,A) = A*amu
  - 1amu = atomic mass unit = u = 931.494MeV/c^2 ≈1.66x10^{-24}g
  - The amu is defined such that M(^{12}C) ≡ 12u
- A nucleus's (charge) radius is roughly: R(Z,A) = (1.2fm)*$A^{1/3}$
  - fm = femtometer (a.k.a. *fermi*) = 10^{-15}m
  - The radius of a nucleon is often referred to as r_0=1.2fm
  - For the RMS radius, multiply by $\sqrt{3/5}$
- Therefore, an estimate for the nuclear density is:
  - $\rho=\frac{M}{V}=\frac{M}{\frac{4}{3}\pi R^3}=\frac{(1u)A}{\frac{4}{3}\pi(1.2fm)^3A}\approx 0.14\,nucleons/fm^3$
  - For fun, in terms of mass-density: $\rho=\frac{(1.66\times10^{-24}g)A}{\frac{4}{3}\pi(1.2fm)^3A}\approx 2.3\times\frac{10^{-25}g}{fm^3}$

…which doesn't sound like much, but this is 2x10^{14} g/cm^3 (the Great Pyramid of Giza is only ~10^{12} grams)

Since A cancels in the ρ expression, the nuclear density is independent of the nuclear size, much as a liquid's density is independent of the size of the drop.
Partly inspired by this property, some basic nuclear calculations are based on this liquid drop analogy.
(G. Gamow, Proc. Roy. Soc. A 1929,1930)

Nuclear Transmutation

- Rules for converting one nuclide (or nuclides) to another (or others)
  - *Charge conservation*: $\sum q_{before} = \sum q_{after}$ (q from protons + positrons + electrons)
  - *Baryon conservation*: $\sum A_{before} = \sum A_{after}$ (A from neutrons + protons)
  - *Lepton number conservation*: $[N_{leptons} - N_{anti-leptons}]_{before} = [N_{leptons} - N_{anti-leptons}]_{after}$

  * Transmutation likelihoods are impacted by energetics and spin/parity selection rules

The first nuclear reaction intentionally made in the laboratory was ^{14}N(α,p) in 1919. (E. Rutherford, Nature 1935).

The first measured radioactive decay was α decay from uranium. (H. Becquerel, Comptes Rendus 1896).

Two Types:

- Reactions
  - Multiple reactants create one or more products
  - Notation: A+b → c+D is written as *A(b,c)D*, where M(b)<M(A) and M(c)<M(D)
    - E.g. $^{12}C+\alpha \rightarrow {}^{16}O+\gamma$ is $^{12}C(\alpha,\gamma)^{16}O$ or even just $^{12}C(\alpha,\gamma)$ and is called “carbon-twelve alpha gamma”
- Decays
  - α, β^+, β^-, e^--capture, β-delayed γ/p/α/n emission, fission, cluster emission, prompt γ
  - Lose nucleons for all above except β decay, e^--capture, and prompt γ (following a reaction)

Nuclear Forces, mechanisms for binding & transmutation

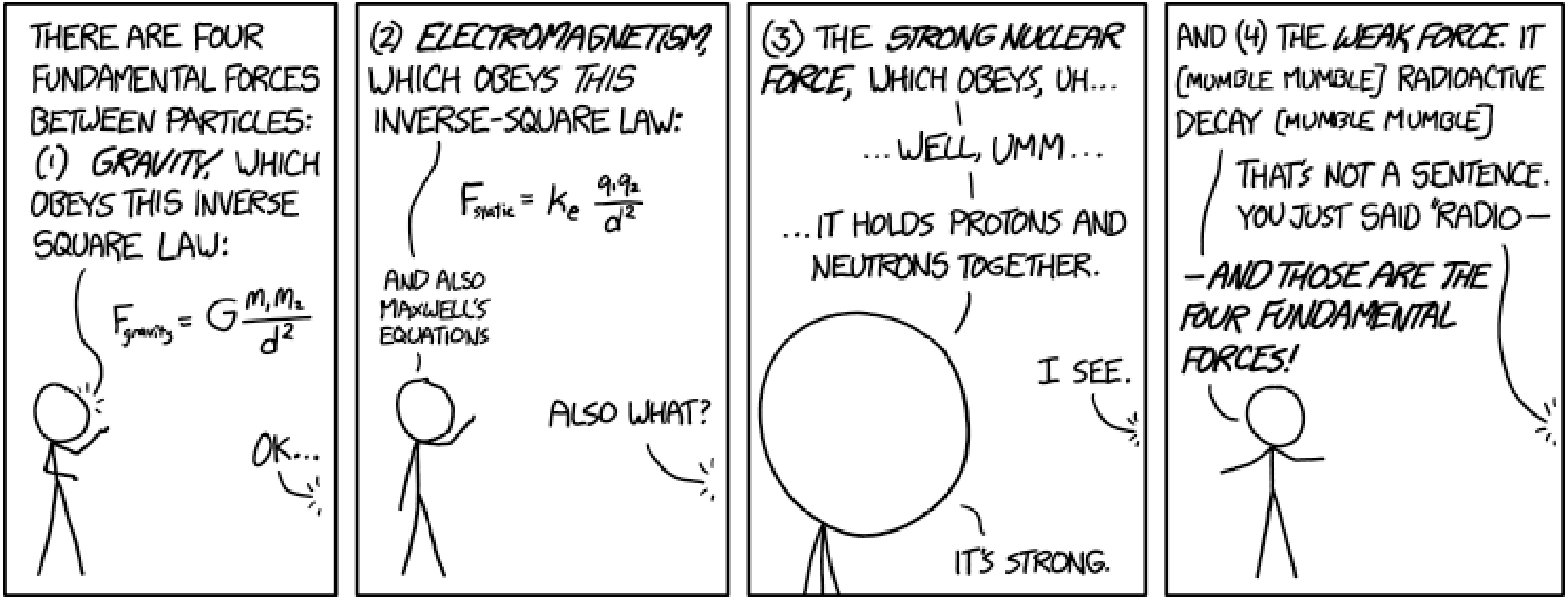
- Fundamental forces of nature:

Force	Range (m)	Relative strength	Force carrier, X
Gravitational	∞	10^{-38}	Graviton
Weak	10^{-18}	10^{-5}	$W^{\pm}$, Z^0
Electromagnetic	∞	$\alpha \approx 1/137$	Photon
Strong	10^{-15}	1	Gluon, Pion

- The range comes from the uncertainty principle:
  - When converting reactant(s) to product(s), energy is spent on the mass of the force carrier: $\Delta E \geq M_X c^2$
  - Violating energy conservation like this is only allowed due to the uncertainty principle: $\Delta E \Delta t \geq \hbar/2$
  - And therefore the force carrier must be reabsorbed before: $t \leq \hbar/(2\Delta E)$
  - Since a particle can't be moving faster than the speed of light, c, the furthest range a force can be mediated is: $r \leq tc = \hbar c/(2\Delta E) = \hbar c/(2M_X c^2) = \hbar/(2M_X c)$
  - Thus, since factors of 2 are for chumps, the range of an interaction is defined as: $\mathbf{R \equiv \hbar/(M_X c)}$
  - Since the graviton and photon are massless, R = ∞.
  - For the weak force, $M_X \sim 100\text{GeV}/c^2$, so $R_{weak} \sim 10^{-3}$ fm.
  - Similarly, since $M_\pi \sim 100\text{MeV}/c^2$, $R_{strong} \sim 1\text{fm}$.

The trick for these calculations is that $\hbar c \approx 197\text{MeV*fm}$.

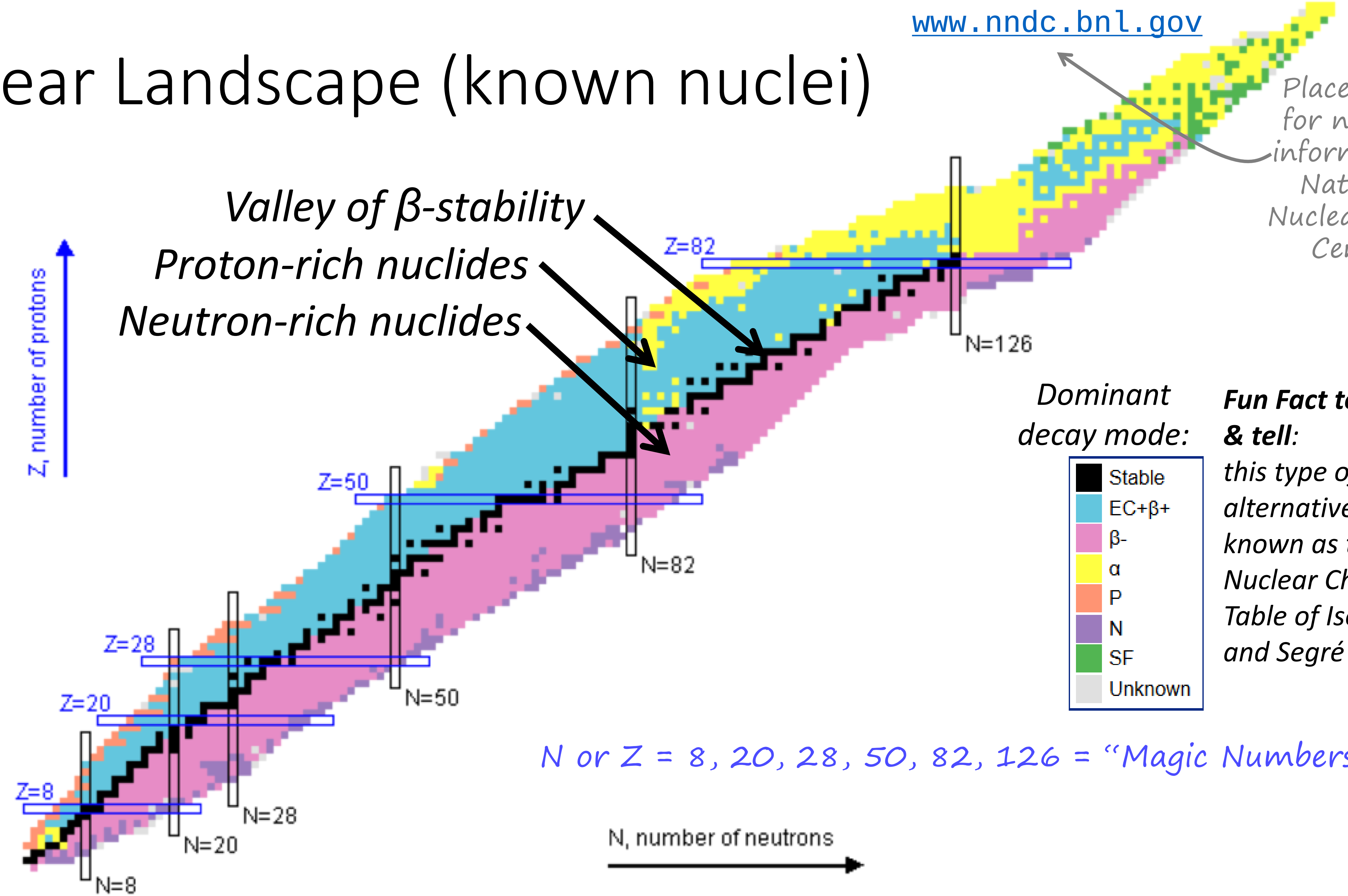
Note force range relative to nucleon size.



Randall Munroe, xkcd.com

Nuclear Landscape (known nuclei)

www.nndc.bnl.gov



Valley of β-stability

Proton-rich nuclides

Neutron-rich nuclides

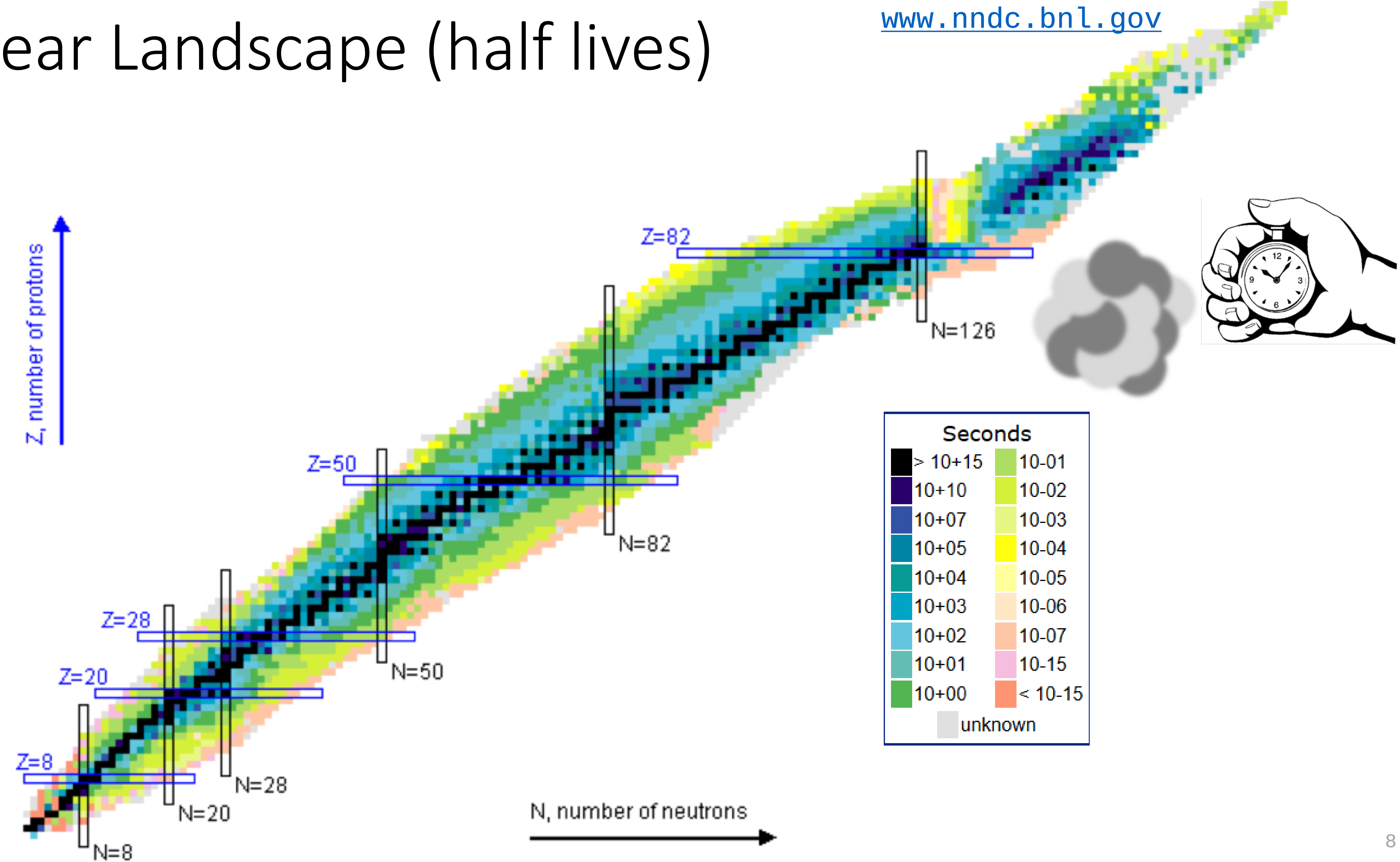
Place to go for nuclear information: National Nuclear Data Center

Dominant decay mode:

***Fun Fact to know & tell**: this type of plot is alternatively known as the Nuclear Chart, Table of Isotopes, and Segré Chart*

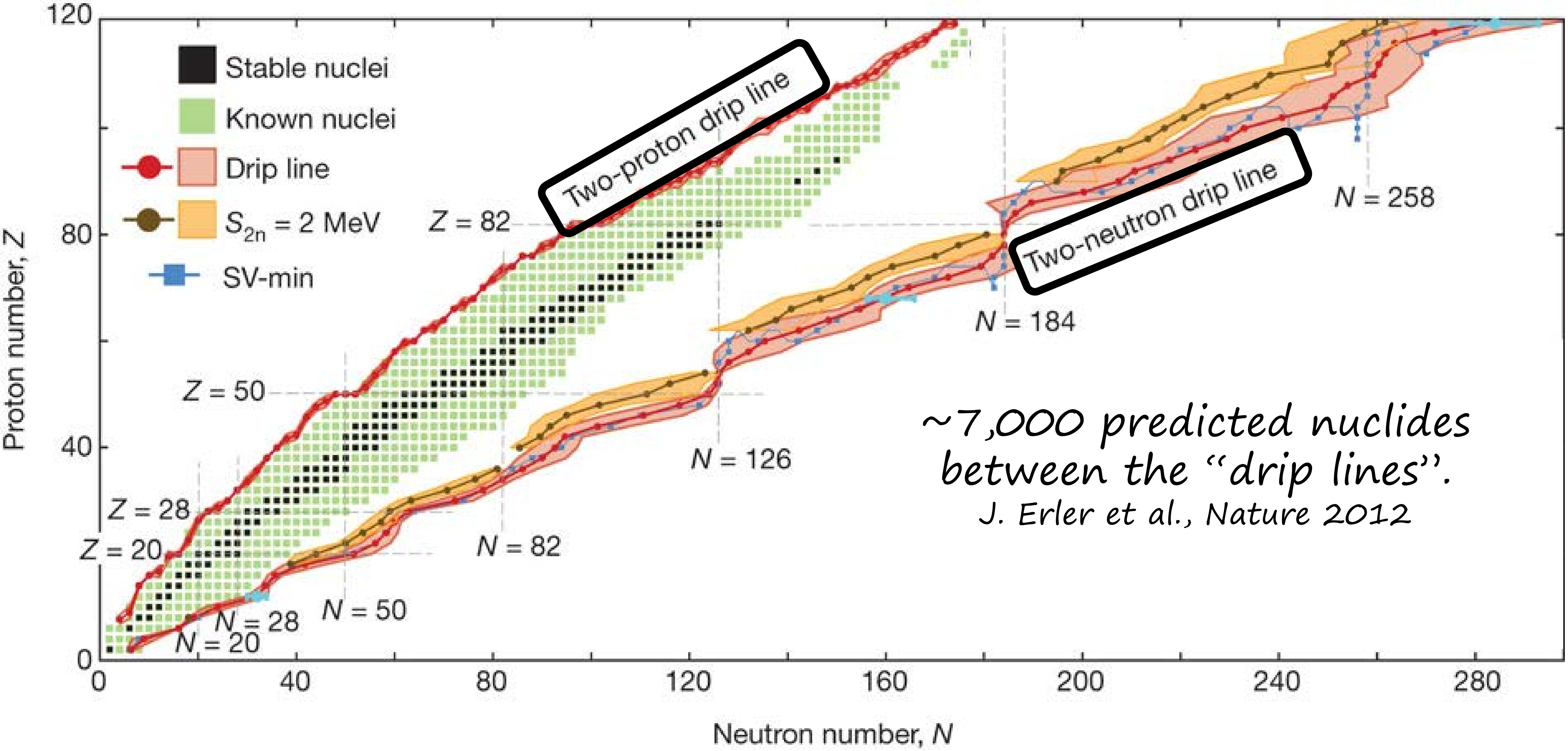
N or Z = 8, 20, 28, 50, 82, 126 = “Magic Numbers

Nuclear Landscape (half lives)



www.nndc.bnl.gov

Nuclear Landscape (predicted nuclei)

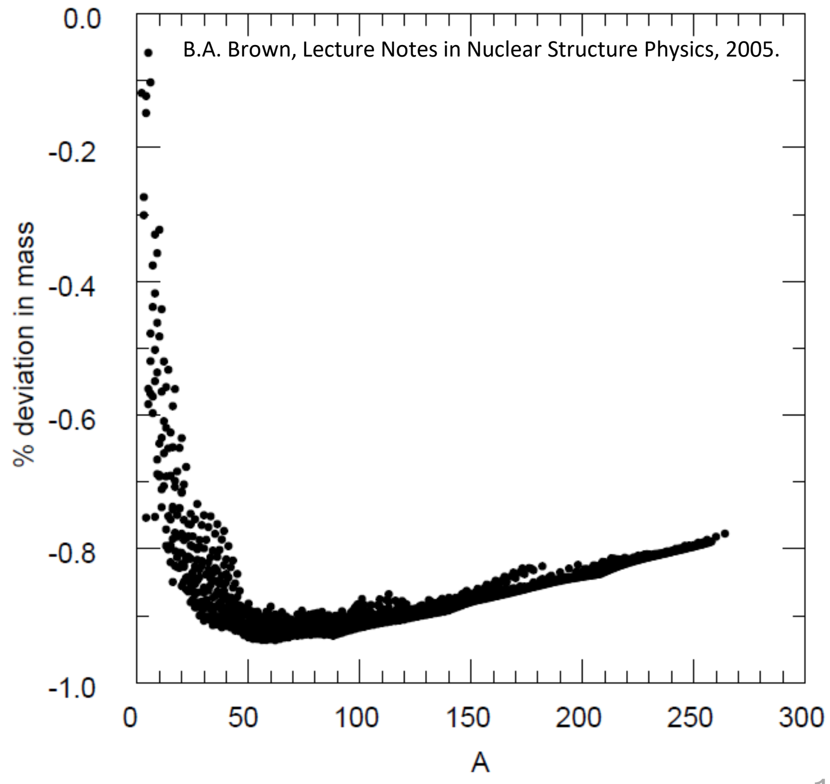


~7,000 predicted nuclides between the "drip lines".

J. Erler et al., Nature 2012

Nuclear Binding Energy

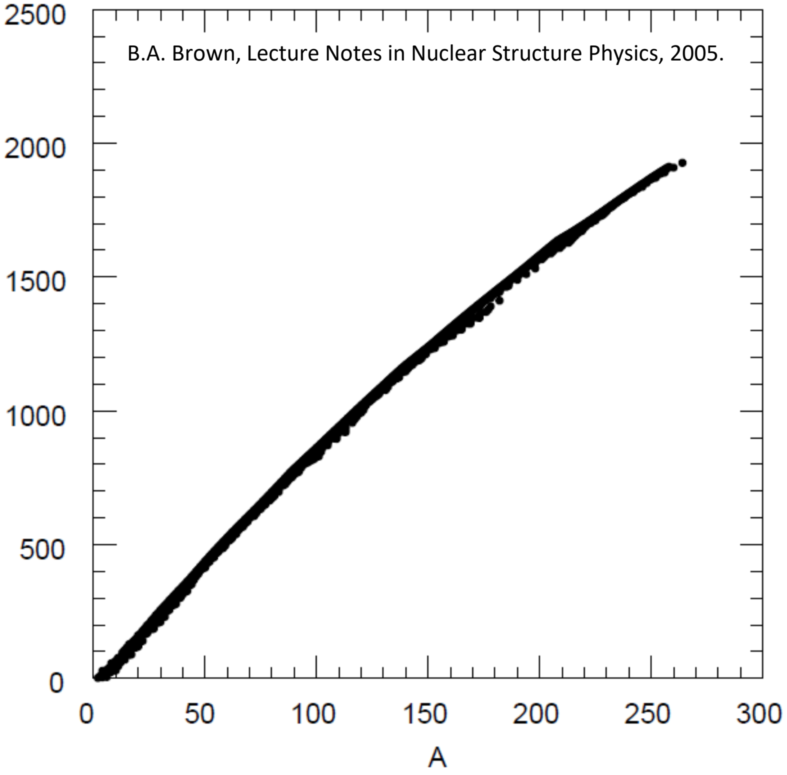
- Early mass spectrometry showed that nuclear masses are (nearly) integer multiples of the hydrogen atom mass. (Coined the "whole number rule" (F. Aston, Nature 1919))
- However, high-precision work showed deviations on the sub-% level (F. Aston Nature 1924)
- This deviation is known as the "mass defect", whereby the nuclear mass is a bit less than the sum of the constituent nucleon masses:
- Which is of course a result of Einstein's postulate, $E=mc^2$
- Nucleons within a nucleus are bound together, and this binding requires energy.
- The binding energy, BE, is paid-for via reduced mass.
- Indeed, the lower-energy state of a nucleus is the only reason nucleons cluster together at all.

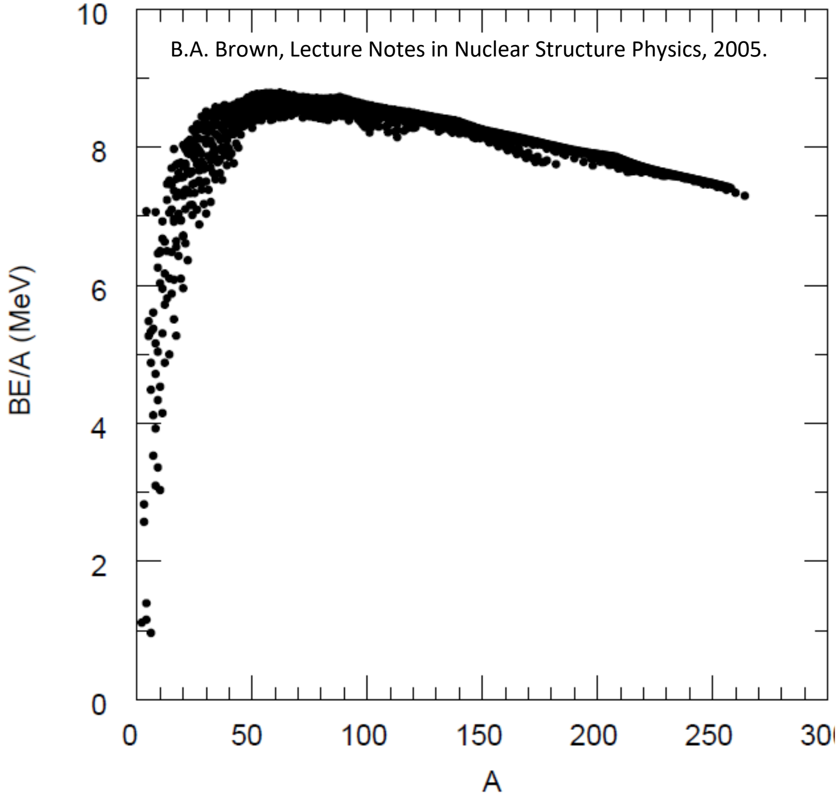


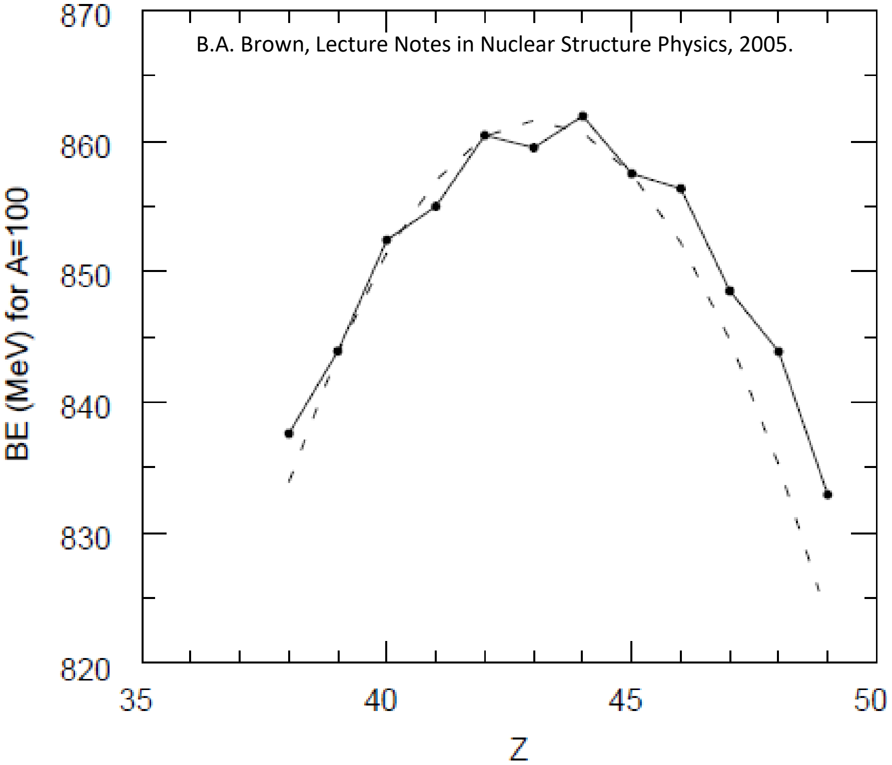
B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.

Nuclear Binding Energy

- The binding energy increases nearly linearly with A:
- It’s not a surprise that more nucleons would lead to more binding, since nucleons will be prone to attraction via the strong force.
- As such, a more interesting quantity is BE/A


B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.


B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.


B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.

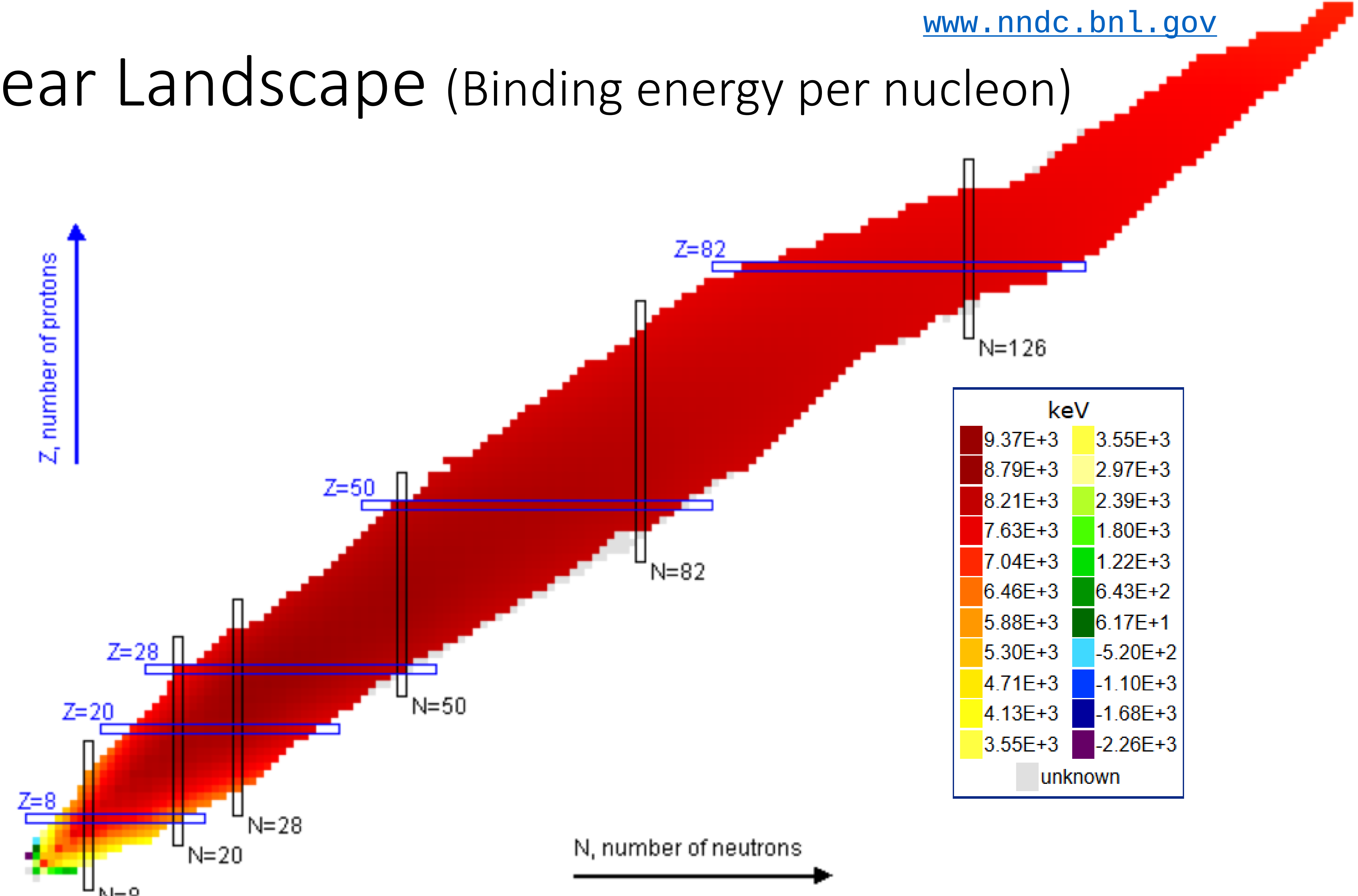
What are the key features you notice on the two left plots?

What to they imply about
- stability to decay?
- fusion?

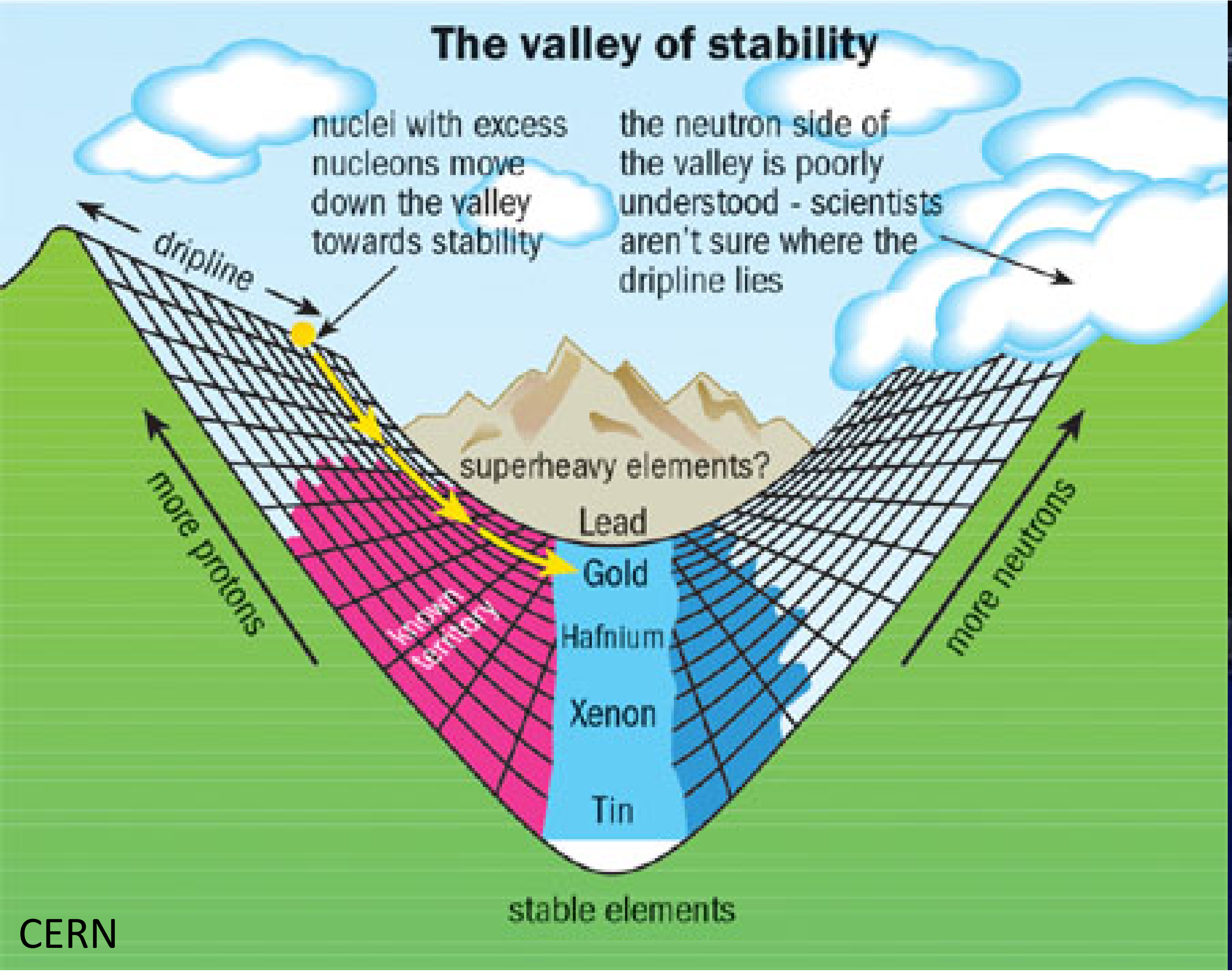
Why do nuclei other than the peak BE/A form at all?

www.nndc.bnl.gov

Nuclear Landscape (Binding energy per nucleon)

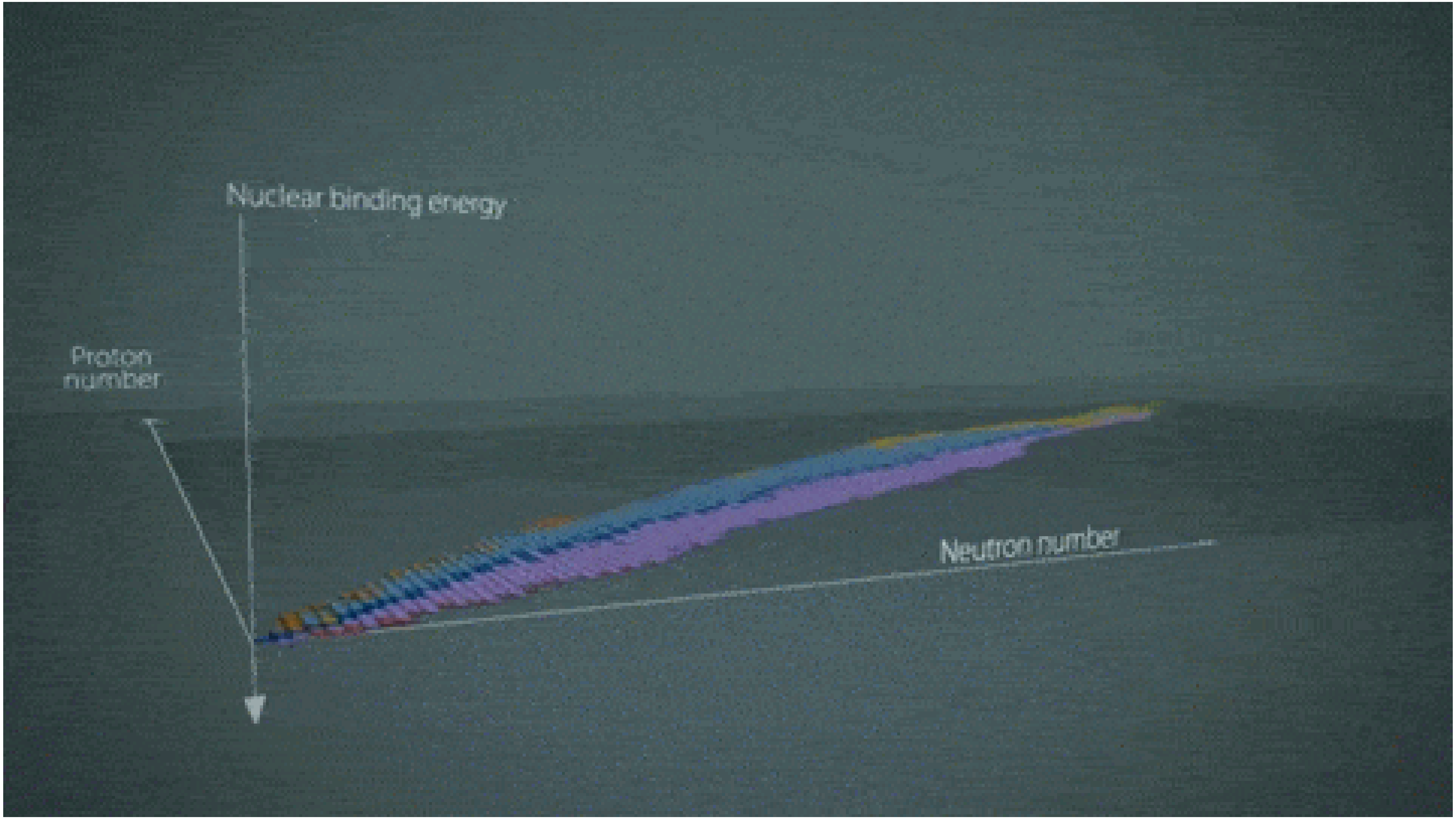


Binding Energy per nucleon: sort of a valley or half-pipe



CERN

RIKEN



F. Legrand & F. Durillon, https://www.youtube.com/watch?v=UTOp_2ZVZmM

Aside on nuclear masses, atomic mass excess

- Rather than binding energy, “atomic mass excess” is more commonly used.
- The conversion can be accomplished with the following equations:
  1. $BE(Z,A) \equiv Z*(m_p + m_e) + N*m_n - M(Z,N)$
  2. $ME(Z,A) \equiv M(Z,N) - (Z+N)*m_u$

  here M(Z,N) is the mass of a nucleus with Z protons and N neutrons,

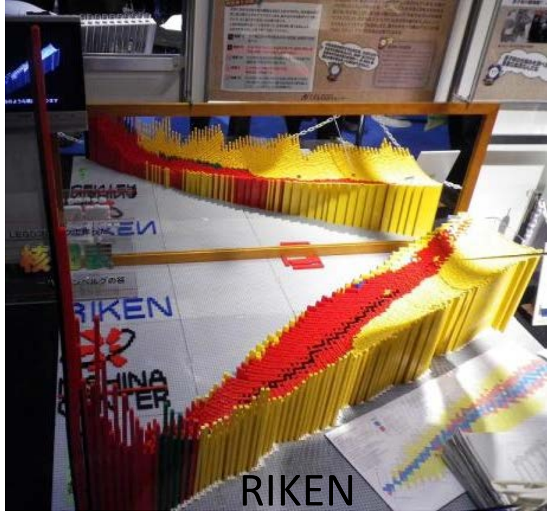
  m_p = 938.272000 MeV/c^2 is the proton mass

  m_e = 0.51100 MeV/c^2 is the electron mass

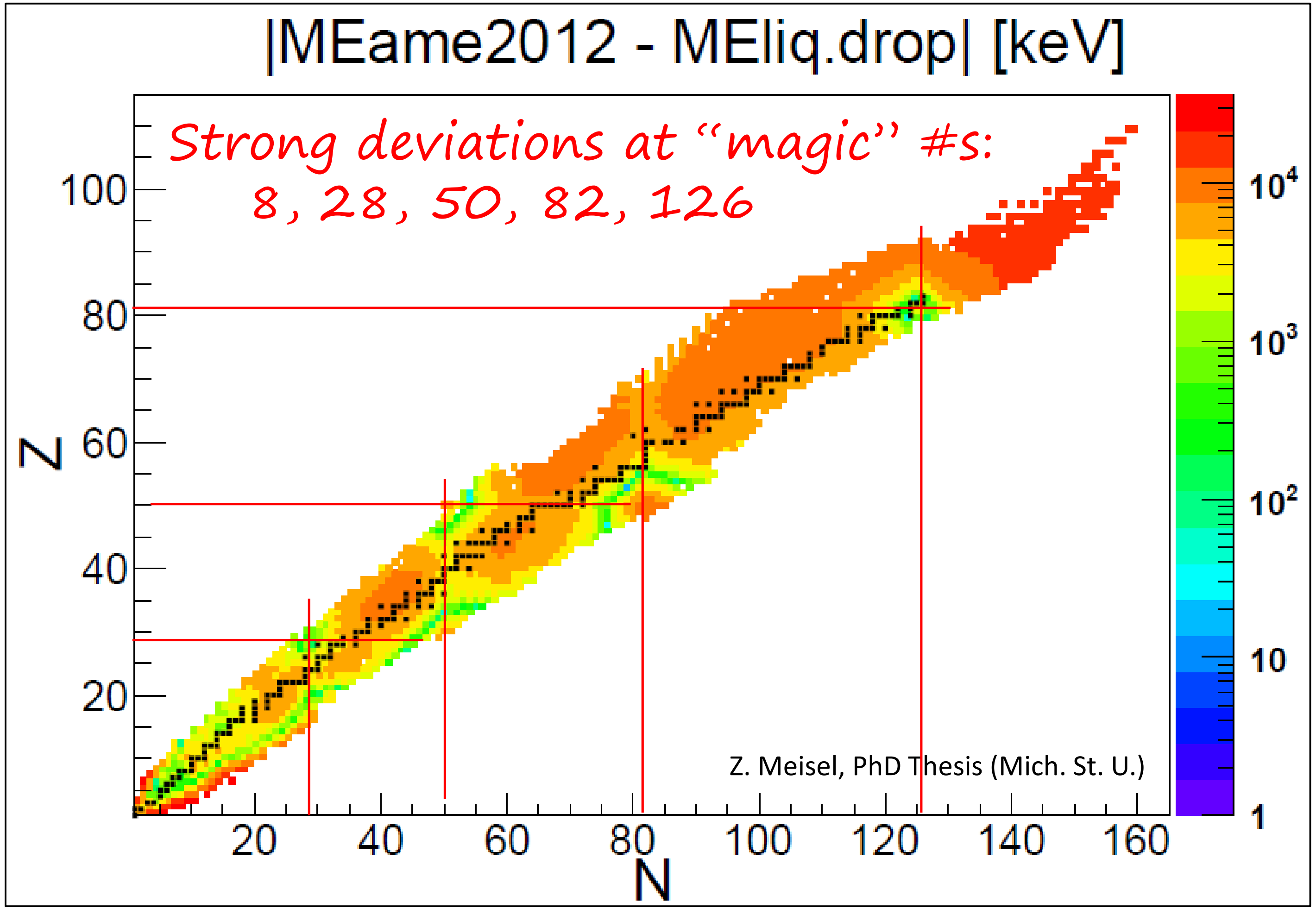
  m_n = 939.56533 MeV/c^2 is the neutron mass

  m_u = 931.494013 MeV/c^2 is the atomic mass unit
- Periodic evaluations are released for all known atomic masses, evaluated from a variety of experimental data by a group of noble souls.
  This is referred to as the “Atomic Mass Evaluation” or “AME”.
  The latest is from 2016 (M. Wang et al. Ch. Phys. C 2017)

Binding Energy & Magic Numbers



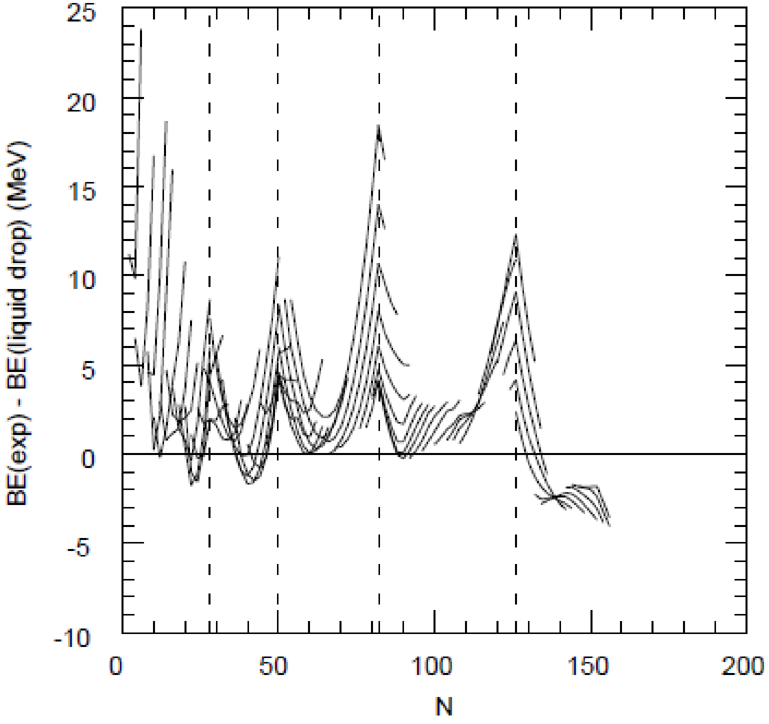
- Binding energy per nucleon looks like a pretty smooth surface, what happens if we fit a smooth function to it & plot the residual? (we'll worry about what this smooth function is next time)



Z. Meisel, PhD Thesis (Mich. St. U.)

Even easier to see in 1D:

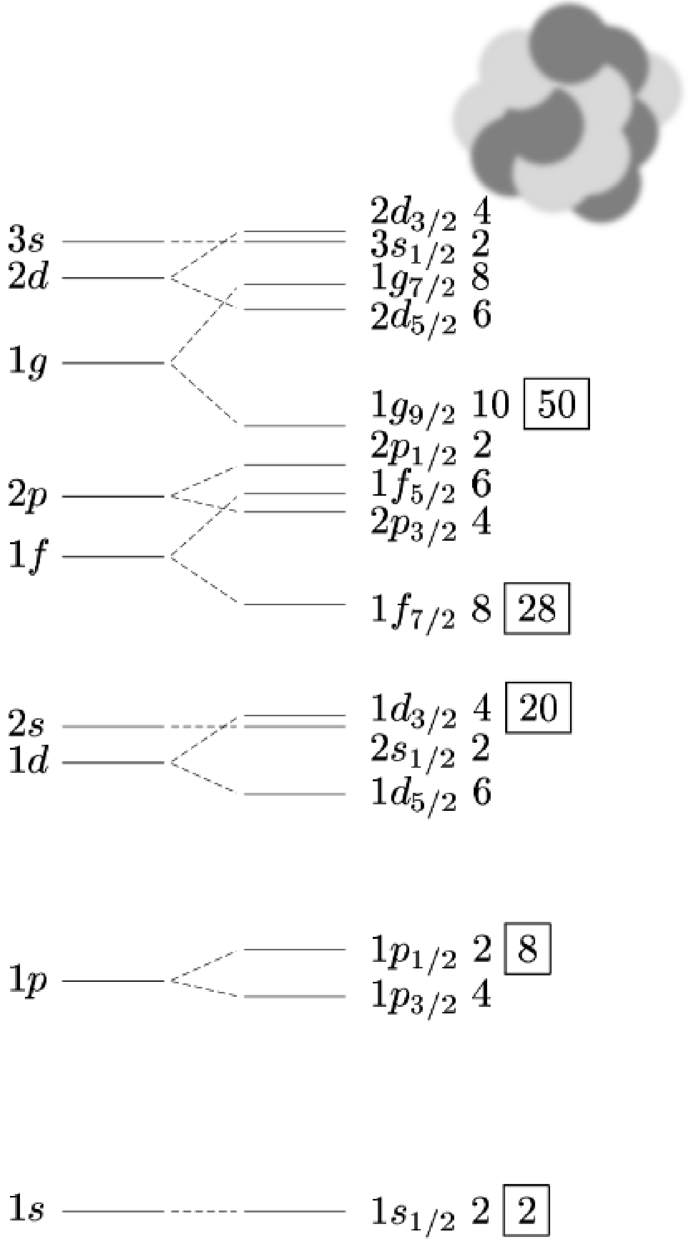
(here even-even nuclei only)



B.A. Brown, Lecture Notes in Nuclear Structure Physics, 2005.

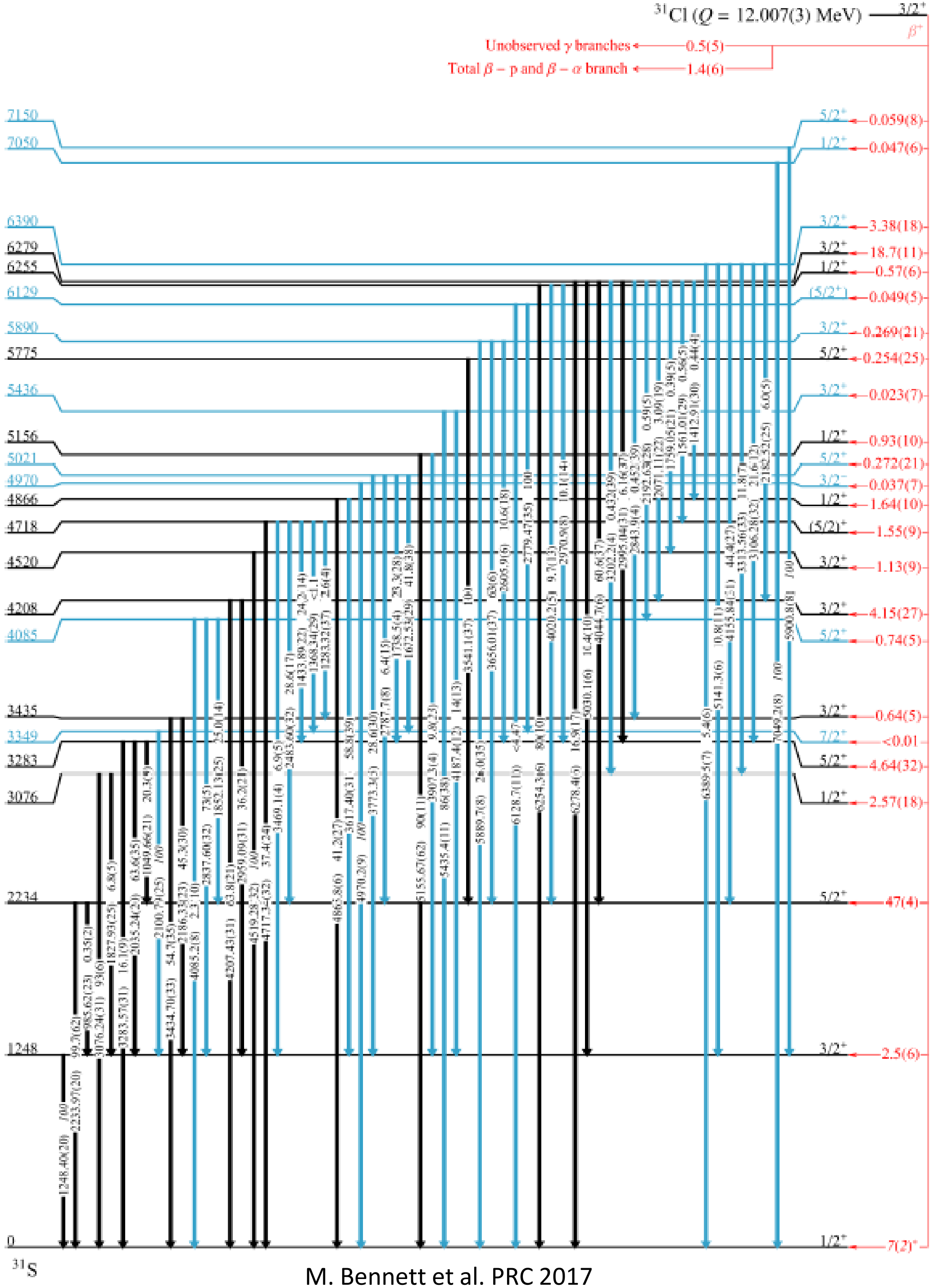
Magic Numbers and Nuclear Structure

- Nucleons can only be combined in discrete configurations, signified as a “state”
- The lowest-energy state is the “ground state”.
- A nucleus populated in a higher-lying state, an “excited state” , will often γ-decay to de-excite to the ground state (though it may decay to a state of another nucleus).
- In the shell model (which we’ll get to in later weeks), the properties of a state are calculated by considering neutrons and protons filling orbitals, like electrons for an atom.
- These are called single-particle levels:
- Note the gaps at the magic numbers.
  (We’ll talk about how those come to be later)
- Exciting nucleons (or pairs) into other levels gives rise to excited states.



Nuclear Excited States

- The example on the right is for ^{31}S, where the levels shown were populated by β-decay from ^{31}Cl
- Note that the density of levels increases with excitation energy pretty dramatically.
- The only reason it doesn't keep increasing with excitation energy here is because, as we'll learn, this particular mechanism isn't suited for populating the higher lying states.
- Properties of the states include not only the excitation energy, but also
  - Width: related to time to decay via γ or particle emission
  - Spin: intrinsic angular momentum
  - Parity: related to symmetry of wave function



M. Bennett et al. PRC 2017

Timeline of Selected Major Early Developments in Nuclear Physics

- 1896: Radioactivity discovered (uranium) (H. Becquerel, Comptes Rendus)
- 1905: Mass-energy equivalence proposed (A. Einstien, Annalen der Physik)
- 1911: Nuclear model of atom proposed (E. Rutherford, Phil. Mag.)
- 1913: Mass spectrometry invented (proton discovered) (J. Thomson, Proc. Phys. Soc. London)
- 1919: Isotope existence discovered (F. Aston, Nature)
- 1920: Nuclear transmutation proposed to power the sun (A. Eddington, The Observer)
- 1928: α-decay theory & barrier penetration (1st application of QM to real world problem) (G. Gamow, Z. Phys.)
- 1929: Liquid drop model of nucleus proposed (G. Gamow, Proc. Roy. Soc. A)
- 1932: Neutron discovered (J. Chadwick, Nature)
- 1932: Nuclei proposed to be interacting nucleons (based on neutron discovery) (W. Heisenberg, Z. Phys.)
- 1932: Nuclear transmutation using a particle accelerator (^{7}Li(p,α)α) (J. Cockroft & E. Walton, Proc. Phys. Soc. London)
- 1934: Theory of β decay (rejected from Nature & caused Fermi to switch to nuclear experiment) (E. Fermi, Z. Phys.)
- 1935: Semi-empirical mass formula developed (based on liquid drop) (C. Wiezsäcker, Z. Phys)
- 1936: Fusion theory (based on liquid drop) (N. Bohr, Nature)
- 1937: Fission proposed (based on liquid drop) (N. Bohr & J. Wheeler, Science)
- 1939: Fission measured (O. Hahn & F. Strassmann, Die Naturwissenschaften)
- 1939: Fission theory (L. Meitner & O. Frisch, Nature)
- 1949: Magic number explanation (M. Goeppert-Mayer, Phys. Rev.)
- 1952: Compound nuclear reaction formalism (W. Hauser & H. Feshbach, Phys. Rev.)
- 1957: Comprehensive theory of nucleosynthesis (M. Burbidge, G. Burbidge, W. Fowler, F. Hoyle, Rev Mod Phys.)
- 1957: Parity violation discovered (C. Wu et al., Phys. Rev)

Further Reading

- Chapters 1 & 2: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapters 1 & 2: Nuclear & Particle Physics (B.R. Martin)
- Chapter 1: Lecture Notes in Nuclear Structure Physics (B.A. Brown)