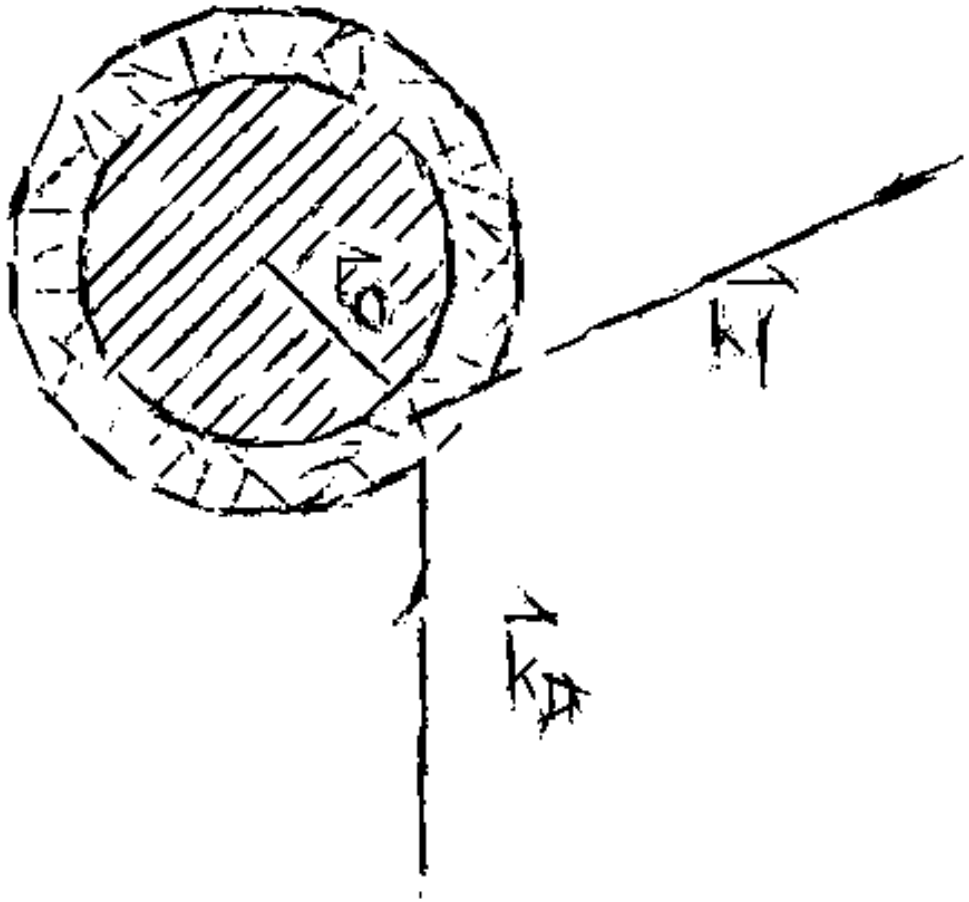
Lecture 16: Direct Reactions

- General characteristics
- Angular distribution
- Spectroscopy
- Born approximation
- Spectroscopic factor



Direct Reaction Types

- On the microscopic level, a direct reaction is one in which the incident projectile only interacts with the surface of the target
- This can happen a few ways
  - The projectile remains intact, changes angle but not energy, which is just elastic scattering
  - The projectile remains intact, changes angle and energy, this is inelastic scattering (e.g. *n,n'*)
  - Nucleons are either donated to the target from the projectile, or to the projectile from the target, as the projectile grazes the target surface. These are *transfer reactions*
    - When the projectile donates nucleon(s) to the target, this is a *stripping reaction*
    - When the target donates nucleons(s) to the projectile, this a *pickup reaction*
  - The projectile breaks apart and the target is left unscathed, this is a *break-up reaction*
  - The projectile loses a single nucleon or cluster, this is a *knockout reaction*

Surface interaction implications

- Since direct reactions only involve one or a few nucleons at the surface of the target, we can estimate typical projectile energies that lead to this case
  - By "seeing" the surface nucleon(s) and not the nucleus, this implies the de Broglie wavelength of the projectile is more nucleon-sized than nucleus-sized
    - $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$
    - So in general, higher projectile energies are going to be more prone to direct reactions
- Since the interaction is essentially only a quick grazing, we can estimate the reaction timescale as the nucleus crossing time
  - For example, ^{56}Fe(d,p) for a 15MeV incident deuteron
    - $v_d = \sqrt{\frac{2E}{m_d}} = \sqrt{\frac{2 \cdot 15MeV}{2 \cdot 931.5MeV}} c \approx 3 \times 10^7 fm/fs$
    - $R \approx 1.2 A_T^{1/3} fm = 4.6 fm$
    - $t_{crossing} = R/v = 4.6 fm/(3 \times 10^7 fm/fs) \approx 10^{-22} s$
  - Practically speaking, this means there isn't time for momentum from the collision to be shared amongst the target nucleons and there are few opportunities for multiple scatters in the nucleus

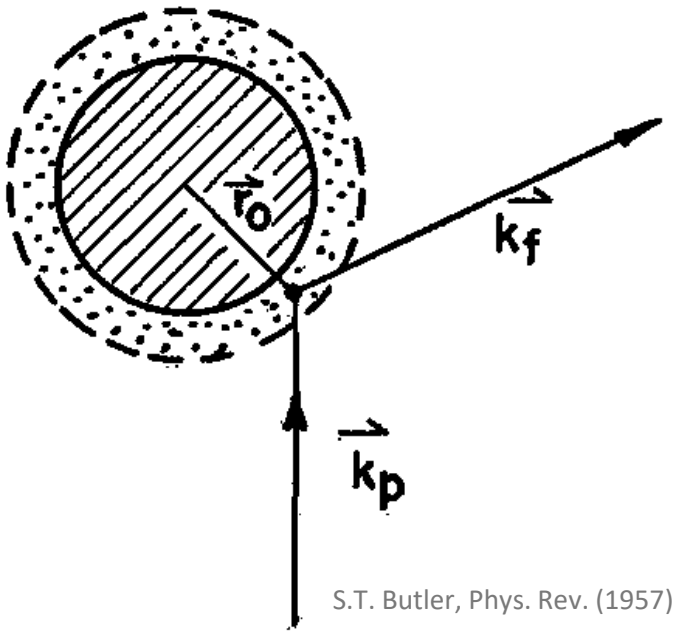
Note that the projectile need not be the lighter of the nuclides. Direct reactions happen just as well for heavy beams on light targets…which is actually how they're more commonly used in experiments today.

Other factors influencing the direct process

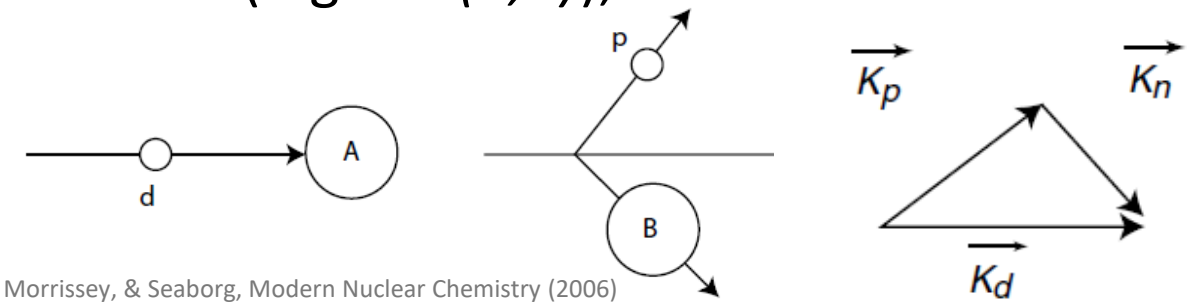
- The general characteristics of a particular reaction type allows one to estimate whether the direct reaction mechanism is important or not
- Consider a deuteron stripping reaction, *(d,p)*
  - For this case, (by definition) a charged particle needs to leave the nucleus
  - It is unlikely the charged particle is going to be able to “evaporate” out of a nucleus that has absorbed energy from a projectile and shared it among the nucleons (in a compound process), since the proton has to tunnel out of the Coulomb barrier
  - For the direct reaction, the emitted proton carries a larger portion of the reaction energy, and so tunneling out is less problematic.
  - Thus, the direct mechanism is favored for this case
- Consider America’s favorite reaction channel, *(α,n)*
  - For this case, a direct reaction mechanism would imply three nucleons are simultaneously transferred to the target
  - That process is as unlikely as it sounds, and so here the neutron evaporation via a compound process will be the primary reaction mechanism

Angular distribution

- Due to the quick crossing time, there is little chance for many scattering-type events to happen for the projectile within the target
- As such, it is expected that the direct reaction products should be forward-peaked [i.e. along the beam direction], as we've seen for elastic scattering
- Consider the case where an incident projectile interacts with only the outer layer of a nucleus
  [where all deeper interactions correspond to a different reaction mechanism]
  without worrying about what the ejectile is
  [i.e. it could be the same particle as the projectile, or it could be something else]
- For a surface interaction, it's difficult to impart much momentum to the target, so generally low-lying excitations(including no excitation) will occur
- Considering a momentum triangle for the reaction (e.g. for *(d,n)*), it's clear that low-lying excitations imply forward-peaked reaction products



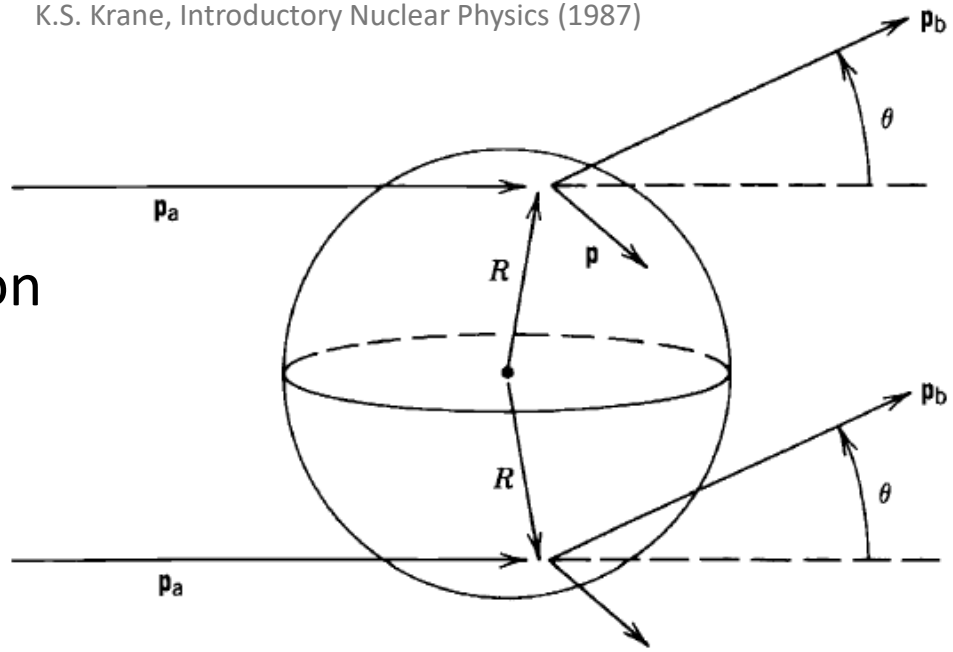
S.T. Butler, Phys. Rev. (1957)



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

Angular distribution

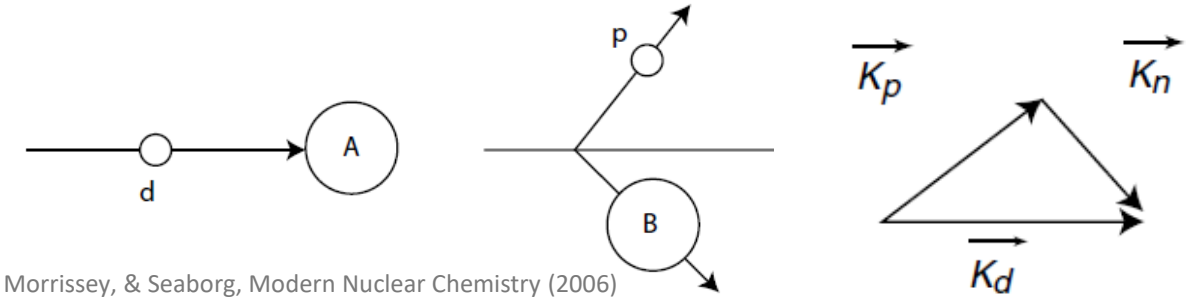
K.S. Krane, Introductory Nuclear Physics (1987)



- To get slightly more detailed predictions, consider the angular momentum transfer in a direct reaction
- If a projectile with momentum $\vec{p}_a$ interacts with the target near the surface (i.e. $b \approx R_T$) [recall b is the impact parameter], resulting in an ejectile with momentum $\vec{p}_b$, then the recoil nucleus has a momentum $\vec{p} = \vec{p}_a - \vec{p}_b$
- From momentum conservation, $p^2 = p_a^2 + p_b^2 - 2p_a p_b \cos(\theta)$
- Supposing this is a stripping reaction (e.g. *(d,p)*), the transferred nucleon will begin orbiting the recoil nucleus with angular momentum $l\hbar = Rp$,
- So, $p = l\hbar/R = \sqrt{p_a^2 + p_b^2 - 2p_a p_b \cos(\theta)}$
- As you can see, the orbital angular momentum l can be determined by choosing a particular beam energy and measuring the ejectile energy and angle

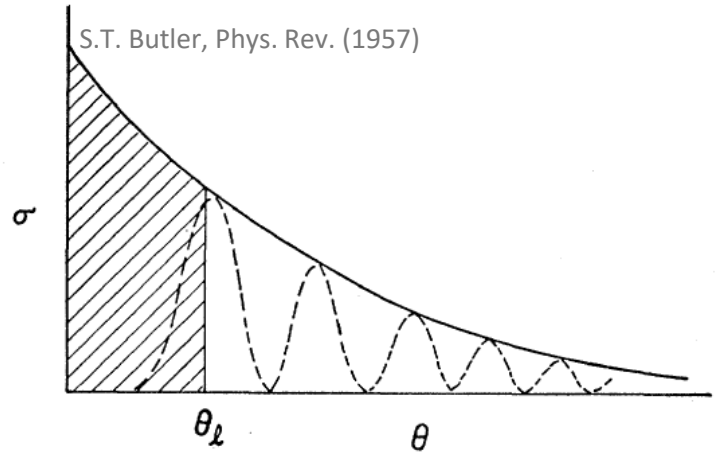
Who cares?
This means we can do spectroscopy!

Angular distribution example



Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

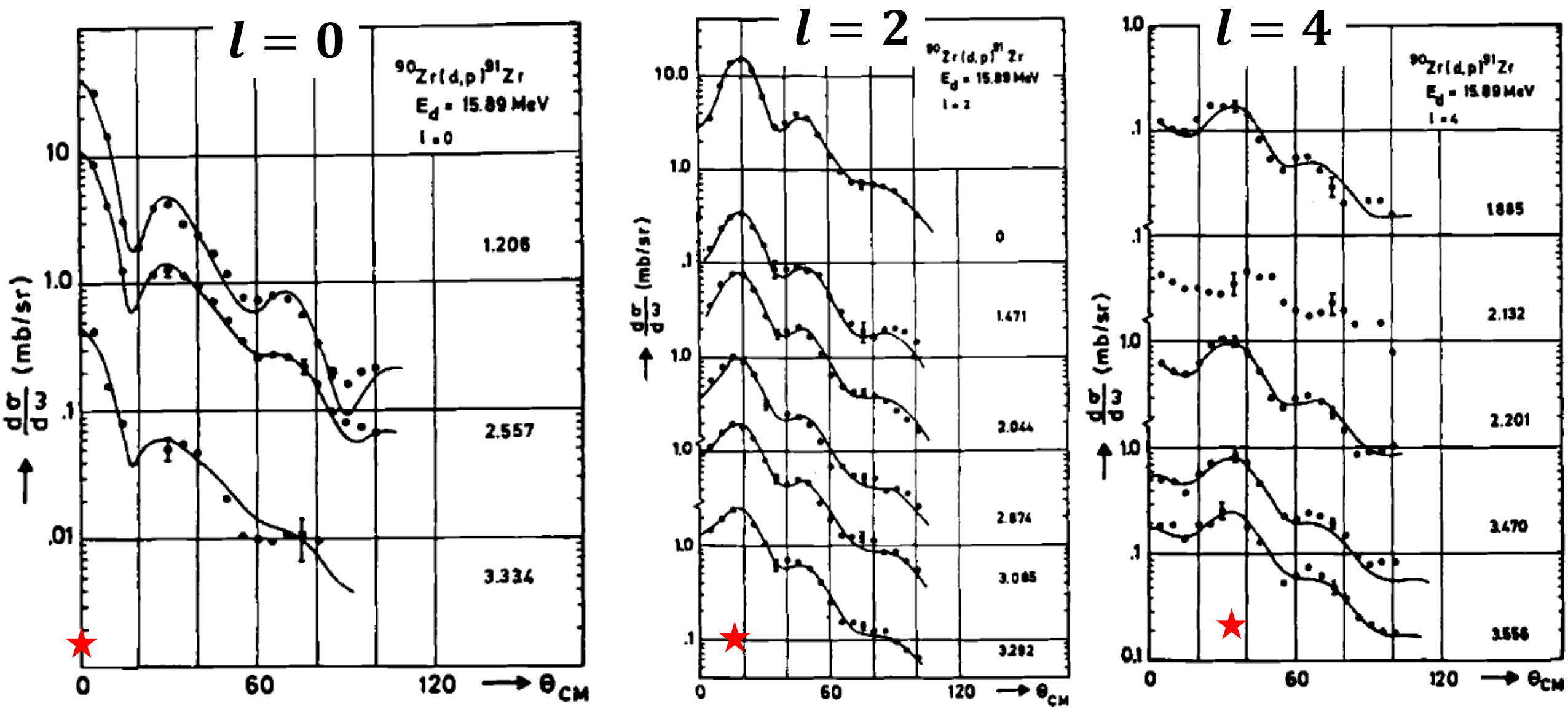
- Consider the deuteron stripping reaction ^{90}Zr(d,p) for a 5MeV deuteron
  - $p_d = \sqrt{2m_d E_d} \approx 140MeV$
  - The reaction Q-value is 4.97MeV, so $p_p \approx \sqrt{2m_p\left(E_d + Q - E_{xs,Zr91}\right)} \approx 140MeV$
  - Note that $p^2 = p_a^2 + p_b^2 - 2p_a p_b \cos(\theta) = (p_a - p_b)^2 + 2p_a p_b(1 - \cos(\theta))$
  - So, $p \approx \sqrt{2p_a p_b(1 - \cos(\theta))}$ and it's still true that $p = l\hbar/R$
  - Meaning, $l \approx \frac{c}{\hbar c} R\sqrt{2p_a p_b(1 - \cos(\theta))}$
  - For this case $l = \frac{c}{197MeVfm} r_0 90^{1/3}\sqrt{2(140MeV/c)(140MeV/c)(1 - \cos(\theta))} \approx 8\sin\left(\frac{\theta}{2}\right)$
  - I.e. $l = 0$ at $0°$, $l = 1$ at $14°$, etc.
- This of course is a classical estimate, what it really tells us is the angle θ_l at which the angular distribution for a given l transfer will peak



S.T. Butler, Phys. Rev. (1957)

Angular distribution example

★ $= \theta_l$



K.S. Krane, Introductory Nuclear Physics (1987)

Spectroscopy

- Since the angular distribution of the ejectile is directly related to the l transfer in the reaction, we can use direct reactions to do spectroscopy
- If J^{π} is known for the target (which is presumably in the ground state) and l is the angular momentum brought into the nucleus by the particle stripped from the projectile, these can combine to form a state of some spin in the recoil nucleus
- For, e.g. *X(d,p)Y*, the allowed spin for the excited state populated is in the range:

$$\left|\left(|J_X - l_n| - \tfrac{1}{2}\right)\right| \leq J_{Y^*} \leq J_X + l_n + \tfrac{1}{2}$$

with parity constrained by $\pi_X \pi_{Y^*} = (-1)^l$

- Note there that the transferred angular momentum is $l \pm s$, where l and s correspond to the transferred nucleon

The Born Approximation

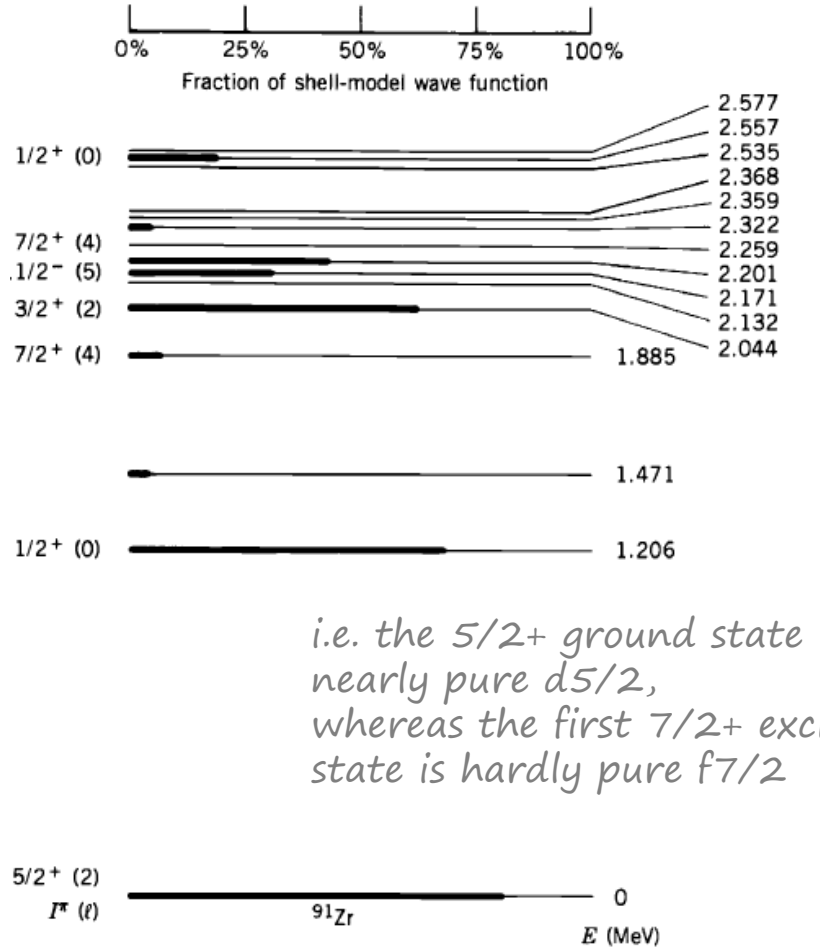
Perhaps this is the title for the straight-to-DVD film in the Jason Bourne series.

- Supposing we want to actually calculate the angular distribution, we need to get a bit fancier
- The matrix element describing the amplitude for the transition for the initial state of our reaction, *a+X*, to the final state, *b+Y*, is defined in the usual way $M = \int \psi_Y^* \psi_b^* V \psi_X \psi_a dv$, where V describes the interaction potential
- The Born Approximation is to treat the projectile and ejectile as plane-waves, i.e. $\psi \propto e^{i\vec{p}\cdot\vec{r}/\hbar}$ for *a* and *b*, expanded as $\propto \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos(\theta))$
- Additionally, the assumption is made that this is a surface-only interaction, and so the matrix element integral is evaluated for $r = R$, meaning M will scale as the spherical Bessel function $j_l(kR)$ and the cross section will depend on $(j_l(kR))^2$
- This will result in the wiggly curves plotted two slides ago

The Born Distortion

$$M = \int \psi_Y^* \psi_b^* V \psi_X \psi_a dv$$

- Are we goldfish!? Not but one lecture ago did we note that reactions (other than elastic) modify the outgoing wavefunction!
- Taking this into account is done by the Distorted-Wave Born Approximation (DWBA)
- The scattered wavefunction is calculated using the optical model, as we discussed last lecture
- An added level of realism is to use wavefunctions calculated via the shell model for ψ_Y^*
- In general we don't expect final states to directly correspond to those calculated by the shell-model, but rather to be a mixture of many shell-model states
- As such, the differential cross section will be a combination of the differential cross sections calculated for pure shell model states, where the weighting factor corresponds to the fraction of a "pure" shell-model state contributing to the actual final state's wavefunction

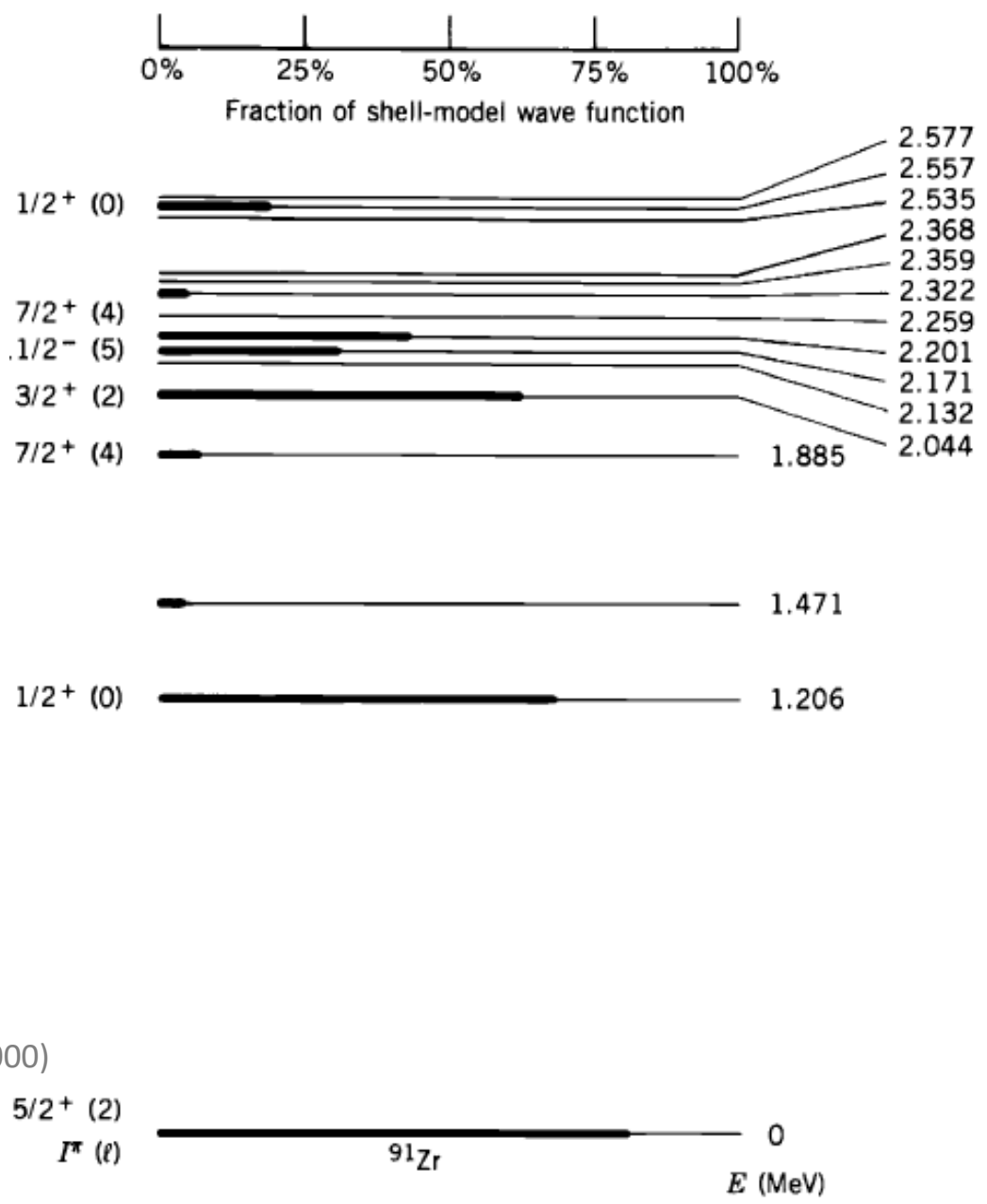


i.e. the 5/2+ ground state is nearly pure d5/2, whereas the first 7/2+ excited state is hardly pure f7/2

K.S. Krane, Introductory Nuclear Physics (1987)

Spectroscopic Factor, S

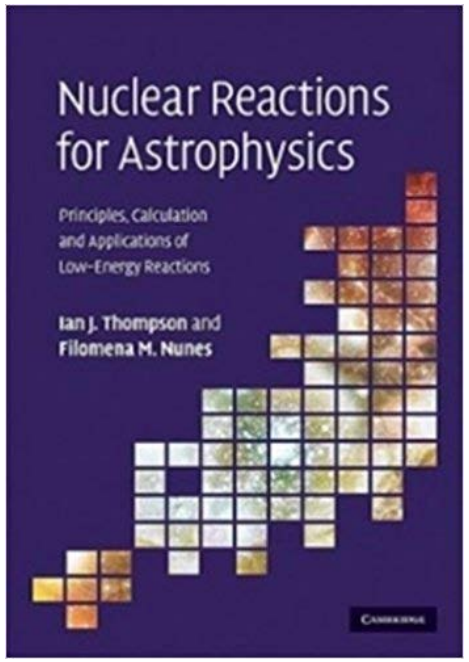
- To quantify the fraction of a pure shell model state that contributes to the differential cross section for a reaction involving a particular final nuclear state, the Spectroscopic Factor is introduced
- $\left(\frac{d\sigma}{d\Omega}(\theta)\right)_{measured} = S\left(\frac{d\sigma}{d\Omega}(\theta)\right)_{calculated}$
- For a pure shell-model state, $S = 1$
- This factor is of course model dependent!
- But, it allows you to use the shell-model calculation results to calculate other quantities you may be interested where what you need is a wavefunction
  - E.g. for resonant reaction rates (P. Descouvemont, Astrophys. J. 2000)



K.S. Krane, Introductory Nuclear Physics (1987)

Ok, nice hand-waving, how do I do useful calculations?

- This book:



has all the gory details
and, tells you how to use the code FRESCO, which is mainly for coupled-channels calculations, but can do DWBA as well

Fresco

Coupled Reaction Channels Calculations

www.fresco.org.uk

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Fresco Input Examples

If a simple click on these files does not download them, *then* right-click and select 'download' or 'save as' or 'save link as'.

From Appendix B of the book:

		Input Files	Expected Output Files
B1	Elastic Scattering	B1-example-el.in	B1-example-el.out
B2	Inelastic Scattering	B2-example-inel2.in	B2-example-inel2.out
B3	Breakup (long form)	B3-example-br-long.in	B3-example-br-long.out
B4	Breakup (short form)	B4-example-br-short.in	B4-example-br-short.out
B5	Transfer	B5-example-tr.in	B5-example-tr.out
B6	Capture	B6-example-capture.in	B6-example-capture.out
B7 B8 B9	Parameter search	B7-p-cd.frin B8-p-cd.search B9-p-cd.min	B9-p-cd.out B9-p-cd-init.plot B9-p-cd-fit.plot

Note that there are some misprints in listing the inputs in the book. The above input files are those which do work: they give the output files.

Computer Program FRESCO

Further Reading

- Chapter 10: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapter 11: Introductory Nuclear Physics (K.S. Krane)
- Chapter 17: Introduction to Special Relativity, Quantum Mechanics, and Nuclear Physics for Nuclear Engineers (A. Bielajew)
- S.T. Butler, Phys. Rev., 106, 272 (1957)