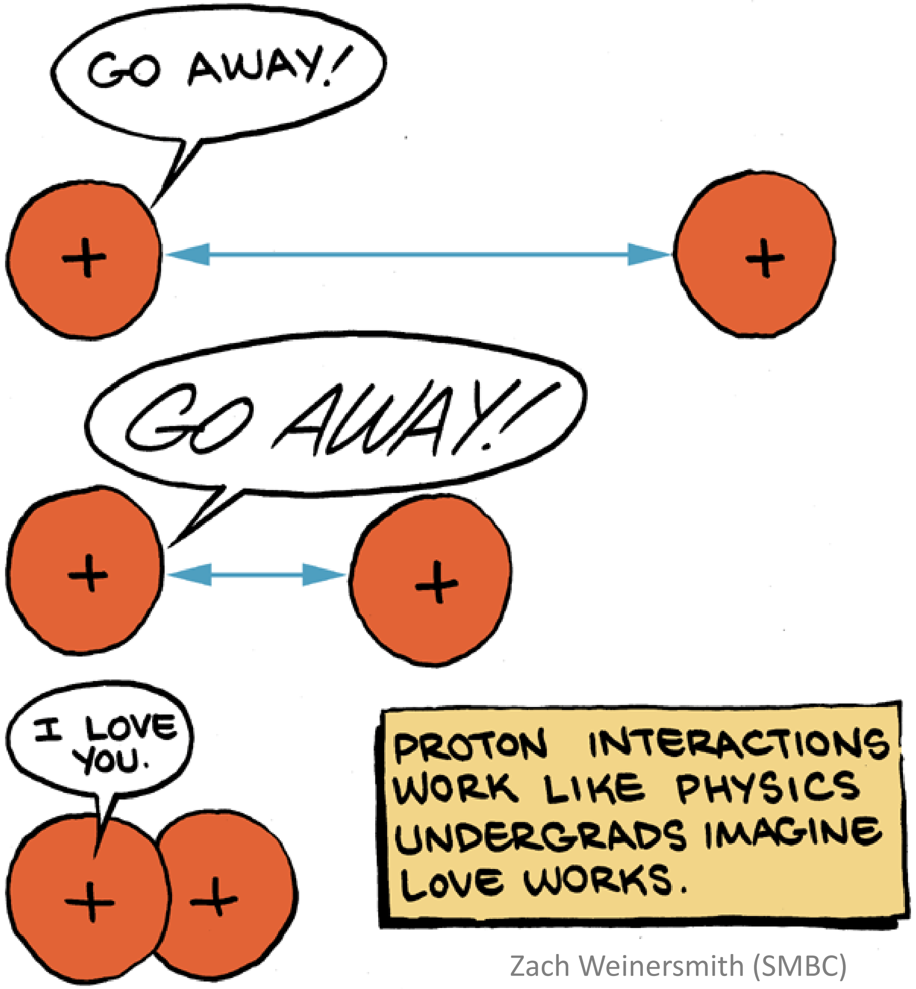
Lecture 11: Nucleon-Nucleon Interaction

- Basic properties
- The deuteron
- NN scattering
- Meson exchange model



Zach Weinersmith (SMBC)

Apparent properties of the strong force

Some basic observations provide us with general properties of the strong force:

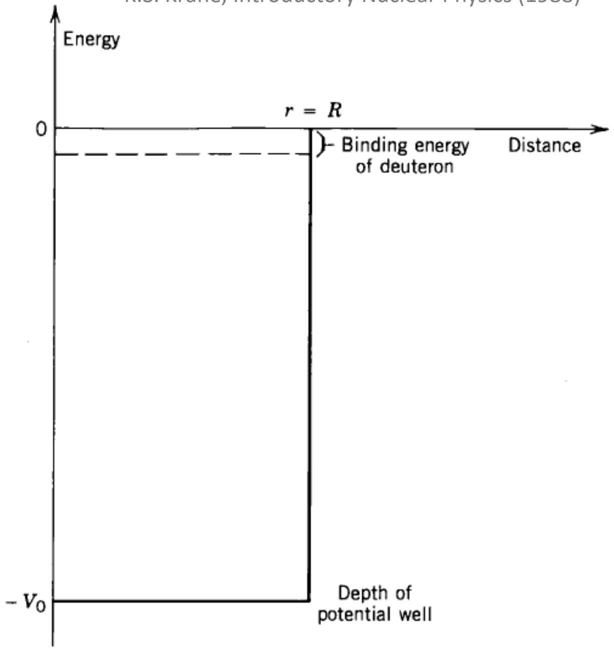
- Nuclei exist with several protons within close proximity
  - ➢ *The strong force must be stronger than the Coulomb force at close distances*
- Interactions between atoms/molecules can be understood by only considering their electrons
  - ➢ *The strong force must be weaker than the Coulomb force at "long" distances (≳atomic size)*
- Nuclei are larger than a single nucleon
  - ➢ *The strong force must be repulsive at very short distances (≲ nucleon size)*
- Chemistry is (generally) the same for all isotopes of a given element
  - ➢ *Electrons (and, by inference, perhaps other particles) are immune from the strong force*
- After correcting for Coulomb effects, protons are no more/less bound than neutrons
  - ➢ *The strong force must be mostly charge-independent*

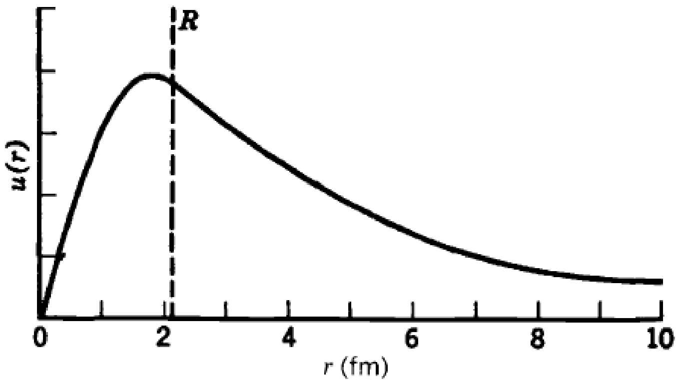
The simplest two-nucleon system: *the deuteron*

K.S. Krane, Introductory Nuclear Physics (1988)

- ^{2}H is the only bound two nucleon system, so it's a good place to start for understanding properties of the nucleon-nucleon interaction
- Using the simplest possible potential, a 3D square well, we can write down the radial form of the wavefunction, $\Psi(r) = {}^{u(r)}/_{r}$, inside & outside of the well
  - Inside: $u(r) = A\sin(kr) + B\cos(kr)$, where $k = \sqrt{{}^{2m(V_0+E)}/_{\hbar^2}}$
  - Outside: $u(r) = Ce^{-\kappa r} + De^{\kappa r}$, where $\kappa = \sqrt{{}^{-2mE}/_{\hbar^2}}$

    (note $E < 0$ for bound states)
- To keep $\frac{u(r)}{r}$ finite at $r \to 0$ and $r \to \infty$, $B = D = 0$
- Employing continuity in the wavefunction and it's derivative at $r = R$, $k\cot(kR) = -\kappa$
- Employing the measured binding energy of the ground state $E = 2.224MeV$, we have a transcendental equation relating V_0 and R
- For the measured $R \approx 2.14fm$, we find $V_0 \approx 35MeV$





It turns out, when solving the Schrödinger equation for a 3D square-well, to have a bound state: $V_0 \geq (\pi^2\hbar^2)/(8M_{red}R^2)$

The simplest two-nucleon system: *the deuteron*

- Hold it right there, you low-budget Houdini, you assumed radial symmetry of the wavefunction, i.e. $l = 0$, but how do you know that's valid!?
- From experiments, we know the deuteron has $J^{\pi} = 1^{+}$, which is created from the addition of the neutron and proton spin angular momenta and their respective orbital angular momentum, $\vec{J} = \overrightarrow{s_n} + \overrightarrow{s_p} + \vec{l}$ and $\pi = \pi_n \pi_p (-1)^l$, where protons and neutrons are $½^+$
- The positive deuteron parity therefore implies $l = even$
- Taking into account angular momentum addition, $l = 0 \; or \; 2$
- Now consider the implications of l on the magnetic moment *(See Lectures 2& 3)*
- For an odd nucleon, $g_j = \left(\frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)}\right) g_l + \left(\frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)}\right) g_s$
- If $l = 0$, the total magnetic moment of the deuteron is just $\mu = \mu_n + \mu_p \approx 0.88\mu_N$
- The experimental value is $\mu \approx 0.86\mu_N$
- Therefore, the deuteron ground state is mostly $l = 0$, but there is some admixture with $l = 2$ (To get the correct μ, $\Psi = a_0 \psi(l = 0) + a_2 \psi(l = 2)$, where $a_0^2 = 0.96, a_2^2 = 0.04$, i.e. 96% $l = 0$)

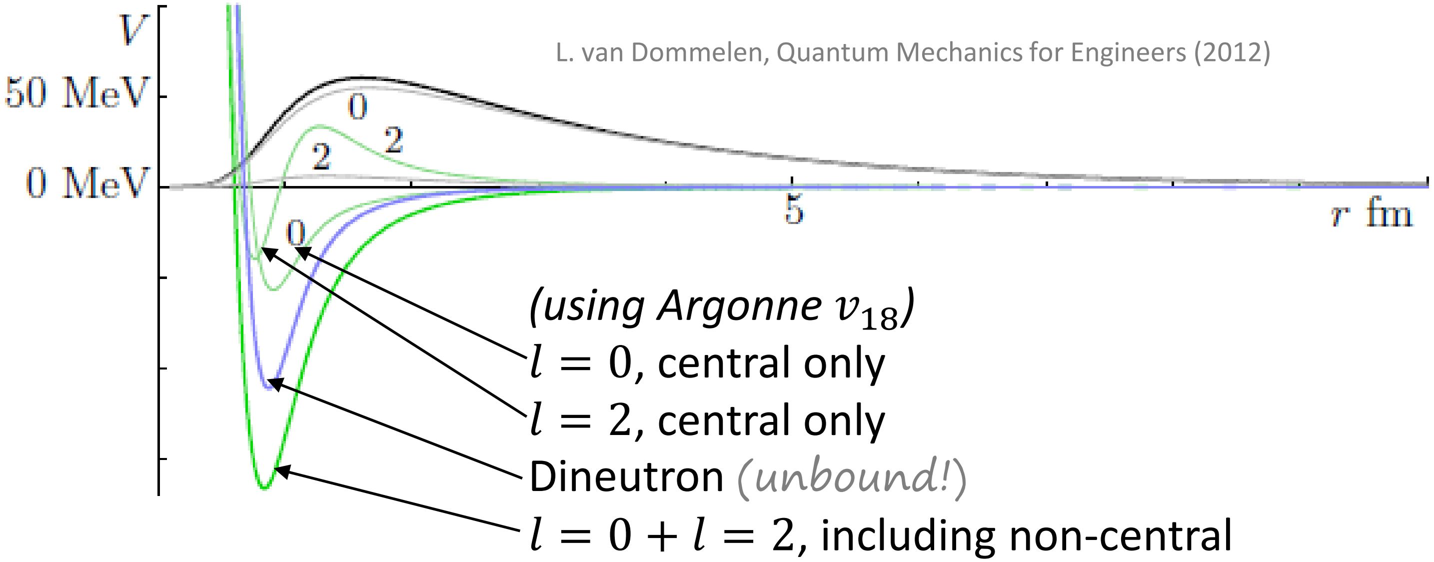
This jives with the fact that the deuteron has a non-zero electric quadrupole moment

The deuteron and the non-central (a.k.a. tensor) potential

- The non-spherical nature of the deuteron tells us something pretty interesting: the internucleon potential can't just depend on the radius r, it must depend on the direction $\vec{r}$
- The only reference axes we have are the nucleon spins $\vec{s}_n$ and $\vec{s}_p$
- In quantum mechanics, the relative angles between $\vec{r}$ and $\vec{s}_i$ are provided by the dot products:
  - $\vec{s}_n \cdot \vec{s}_p$, $\vec{s}_n \cdot \vec{r}$, and $\vec{s}_p \cdot \vec{r}$ (which with r fully describe the geometry)
- The first was taken into account to find out $l = 0$ and $l = 2$ are possible
- The second two need to be treated together, since a rotation (e.g. 180° from a parity change) will affect them in the same way
- Furthermore, it must be some 2nd order combination of the two because the strong force conserves parity and a 1st order dependence would mean this non-central contribution would change sign under a parity change
- Squaring only the 2nd or 3rd term would just result in some integer multiple of r^2, and so that won't get the job done
- Therefore, something depending on the product $(\vec{s}_n \cdot \vec{r})(\vec{s}_p \cdot \vec{r})$ is needed

The deuteron and the non-central (a.k.a. tensor) potential

- We've established that a non-central force relying on $(\vec{s}_n \cdot \vec{r})(\vec{s}_p \cdot \vec{r})$ is needed
- The form adopted is $S_{12}V_T(r)$, where $V_T(r)$ takes care of all of the radial dependence
- $S_{12} \equiv \frac{4}{\hbar^2}\left(\frac{3}{r^2}(\vec{s}_n \cdot \vec{r})(\vec{s}_p \cdot \vec{r}) - \vec{s}_n \cdot \vec{s}_p\right),$
- $\propto r^{-2}$ is to remove the radial dependence
- $-\vec{s}_n \cdot \vec{s}_p$ is to make $S_{12} = 0$ when averaged over $\vec{r}$
- The end result is that the tensor component of the potential deepens the overall interaction potential



L. van Dommelen, Quantum Mechanics for Engineers (2012)

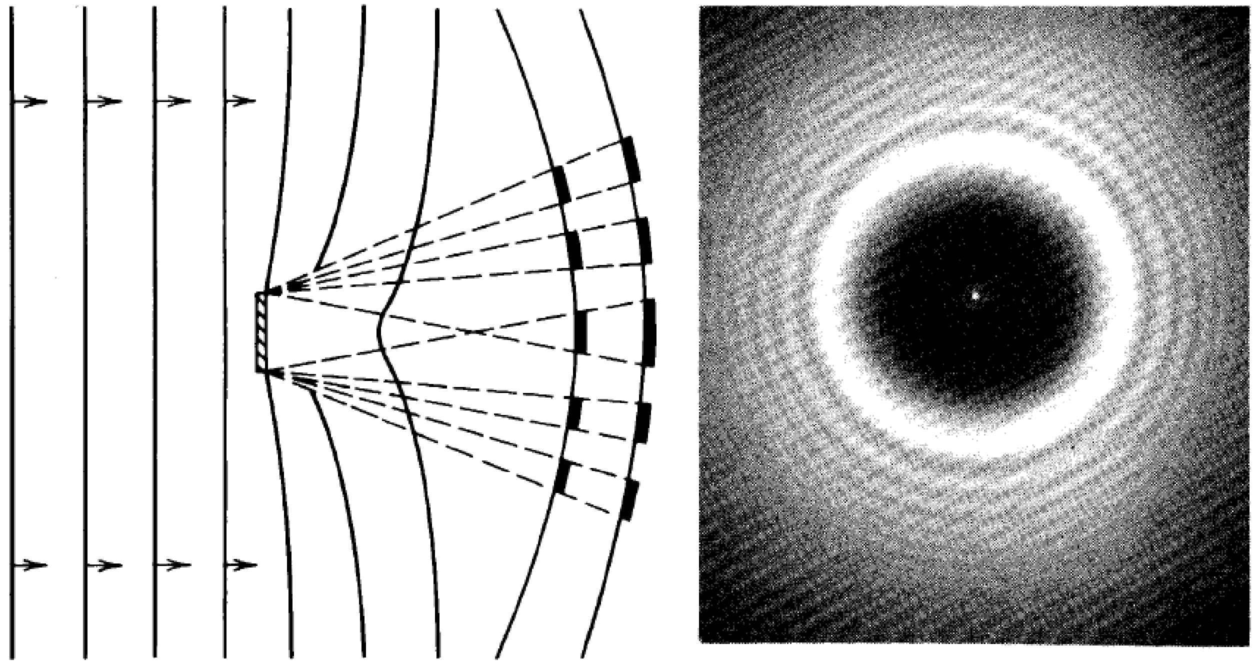
Nucleon-nucleon scattering

- To move beyond the single naturally-occurring scenario the deuteron provides us, we need to move to nucleon-nucleon (NN) scattering to learn more about the NN interaction
- To get the cleanest observables, the optimum target is a single nucleon, namely hydrogen

Why not a neutron target? Because they decay and so it would be really hard to make!

- Like with the deuteron, we'll stick to a square well, but now we're considering an incoming plane wave and an outgoing spherical wave
- To keep life simple, we'll consider an $l = 0$ (a.k.a. s-wave, if you're apart of the secret society) plane wave, which practically means a low energy ($\ll 20MeV$) beam to keep $\mu_{red} vR \ll \hbar$

K.S. Krane, Introductory Nuclear Physics (1988)



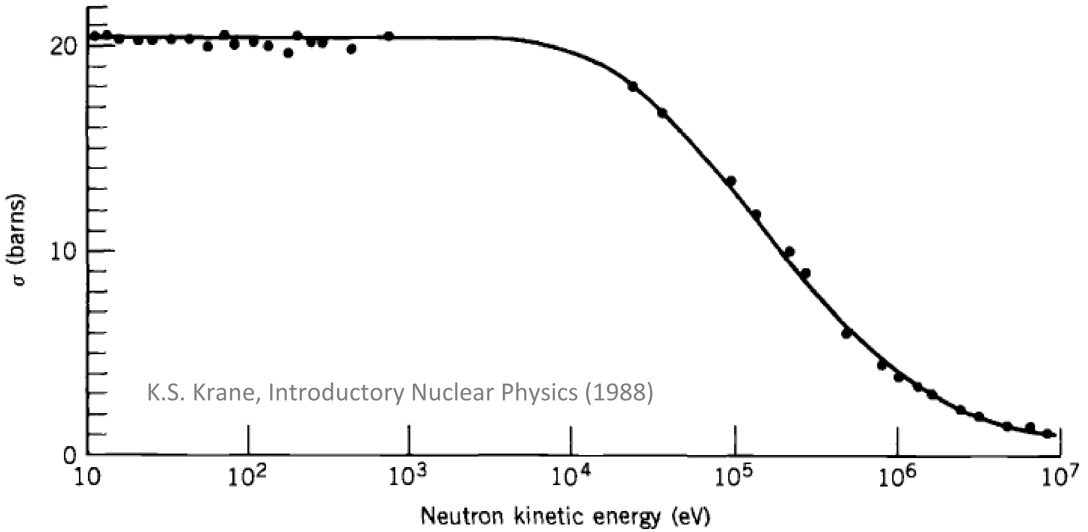
Nucleon-nucleon scattering

- To solve for properties of our system, we'll again do the wavefunction matching gymnastics we did for the deuteron
- The wavefunction inside the barrier is like for the deuteron, $u_i(r) = A\sin(k_1 r)$, where $k_1 = \sqrt{2m(V_0+E)/\hbar^2}$, but here E is set by the center of mass energy for the scattering
- Outside the barrier, $u_o(r) = \mathrm{C}'\sin(k_2 r) + \mathrm{D}'\cos(k_2 r)$, where $k_2 = \sqrt{2mE/\hbar^2}$
- This is generally rewritten as $u_o(r) = C\sin(k_2 r + \delta)$, so $C' = C\cos(\delta)$ and $D' = D\sin(\delta)$, where δ is the "phase shift" we'll care about later in the semester when we discuss scattering
- Matching $u(r)$ and it's derivative at R and doing some algebra results in the transcendental equation $k_2 \cot(k_2 R + \delta) = k_1 \cot(k_1 R)$, relating V_0 and R for a given E
- Defining $\alpha \equiv -k_1\cot(k_1 R)$ and flexing your trigonometry skills, $\sin^2(\delta) = \frac{\cos(k_2 R)+\left(\frac{\alpha}{k_2}\right)\sin(k_2 R)}{1+\alpha^2/k_2^2}$
- By inspection, the amplitude of the D' term of the outgoing wavefunction is going to tell us what the total scattering was, since the C' term describes a plane wave like the incident one, so $\sigma_{scatt} \propto \sin^2(\delta)$ …so we can predict the overall scattering cross section

Nucleon-nucleon scattering

- For $V_0 = 35MeV$ (from the deuteron) and $E \ll V_0$ (e.g. 10's of keV), $k_1 \approx 0.92fm^{-1}, k_2 \lesssim 0.016fm^{-1}$
- For reasons we'll get into in the scattering lecture, $\sigma_{scatt} = \frac{4\pi sin^2(\delta)}{k^2}$
- Then, using $R \approx 2fm$ from the deuteron (which means, for our k_1, $\alpha \approx 0.2fm^{-1}$), $\sigma \approx \frac{4\pi}{\alpha^2}(1+\alpha R) = 4.6\ barns$
- Comparing to data, at low energy, $\sigma \approx 20.4b$
- Where did we go wrong!?
- We didn't consider the fact that the nucleon-nucleon interaction could be spin-dependent!

Maybe our 1st grade teacher was right and finger-painting was our true calling.
Anyhow, it's too late for that.



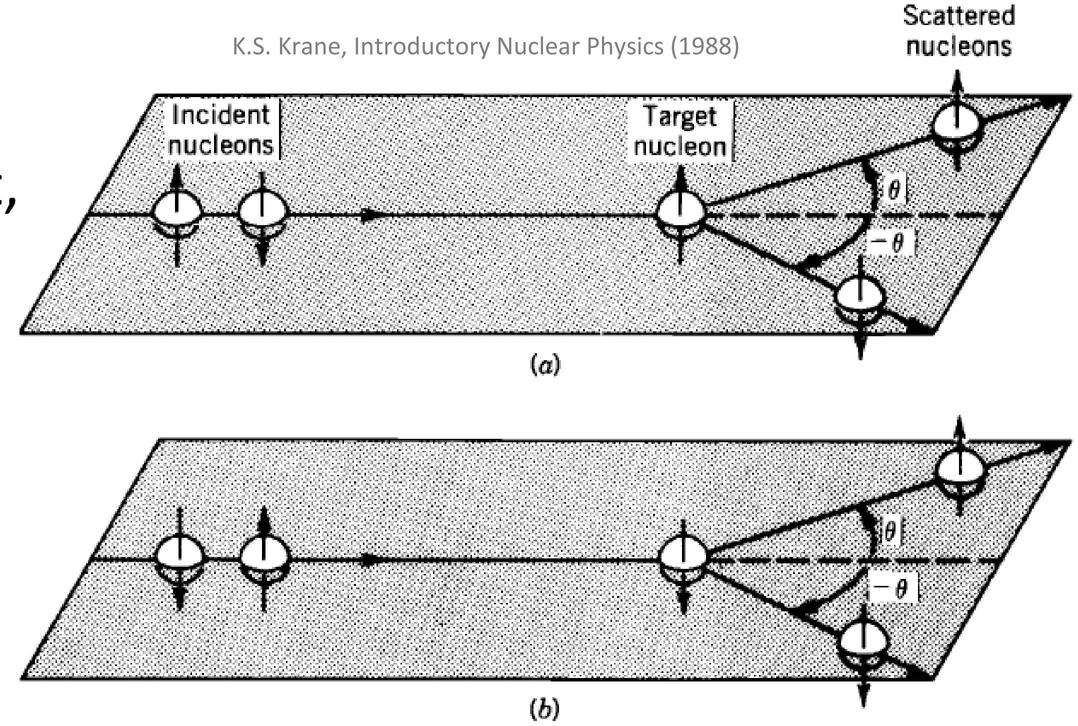
- The deuteron, with $J^{\pi} = 1^+$ and mostly $l = 0$, is in the $S = 1$ triplet state (because it has 3 spin projections), but nucleons can also be anti-aligned in the $S = 0$ singlet state (one spin projection)
- Good old degeneracy dictates that $S = 1$ will contribute 3× as much as the $S = 0$, so $\sigma = \frac{3}{4}\sigma_{S=1} + \frac{1}{4}\sigma_{S=0}$
- Our calculation + data then imply, $\sigma_{S=1} = 4.6b$, $\sigma_{S=0} = 67.8b$

The upshot is that the central potential is spin-dependent

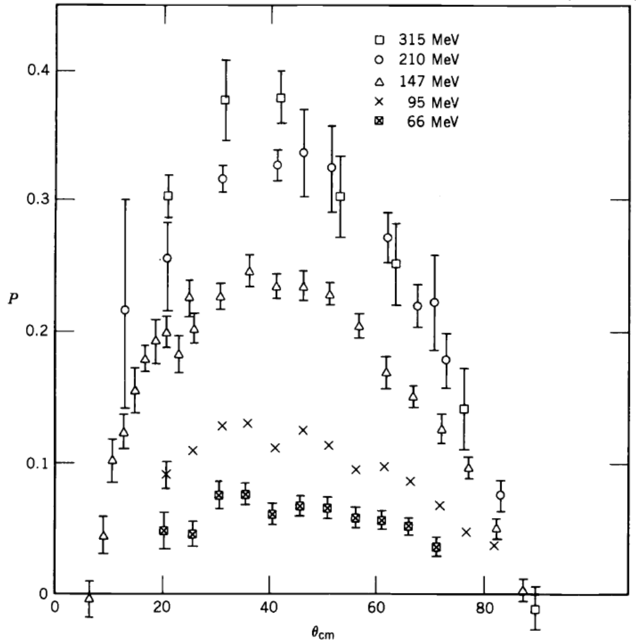
More nucleon-nucleon scattering

- Since the nucleon-nucleon potential is spin-dependent, let's keep an open mind and assume spin-orbit interactions can play a role too
- We can consider a spin orbit potential $V_{so}(r)\vec{l}\cdot\vec{S}$
- Then, for a given l, whether an incident nucleon is spin-up or spin-down will change the sign of the interaction
- A spin-up nucleon would be scattered one way, while a spin-down would be scattered another
- Indeed, such an effect is seen, where the polarization $P = \frac{N(\uparrow)-N(\downarrow)}{N(\uparrow)+N(\downarrow)}$ is non-zero, in particular for larger energies, where higher l play a role ($l = 0$ would not yield polarization)
- This means polarized beams can be formed just using scattering

K.S. Krane, Introductory Nuclear Physics (1988)



R. Wilson, The Nucleon-Nucleon Interaction (1963)



The nucleon-nucleon potential

- Bringing it all together, the contributions to the nucleon-nucleon potential V_{NN} are,
  - Central component of strong force [which is spin-dependent]
  - Coulomb potential
  - Non-central (tensor), spin-spin potential
  - Spin-orbit potential
- $V_{NN}(\vec{r}) = V_{nn}(r) + V_{Coul}(r) + V_{ss}(\vec{r}) + V_{so}(\vec{r})$

- Coming up with an adequate form that describes observables from nuclei is an outstanding problem in nuclear physics
- A further complication which the data seems to require is an inclusion of higher-order interactions, i.e. so-called 3-body interactions
- Practically speaking, this means we're a far way off from describing observables for a large range of nuclei starting purely from a theoretical description of the NN interaction

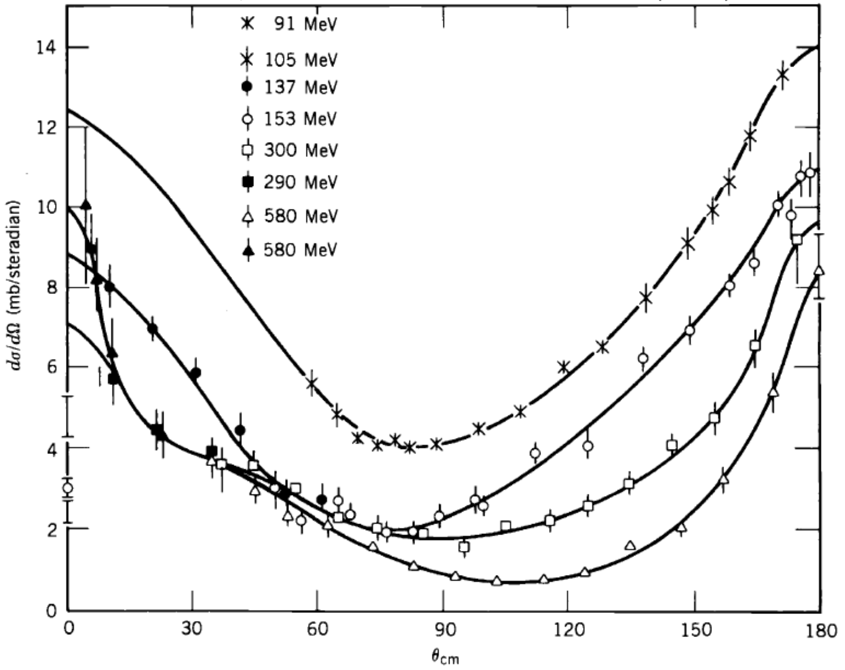
Hurray for job security!

The NN interaction as a derivative of meson exchange

- Like all forces, the nucleon-nucleon interaction is a result (a.k.a. derivative) of some underlying particle exchange, which itself is described by some interaction
- Evidence for this fact is provided by the angular distribution of np scattering

A bit of secret-handshake trivia: The backward peak is referred to as the "saturation of nuclear forces"

R. Wilson, The Nucleon-Nucleon Interaction (1963)



- The average deflection angle should be,

$$\theta \approx \sin(\theta) = \frac{\Delta p}{p} = \frac{F\Delta t}{p} = \frac{\left(\frac{dV}{dr}\right)\Delta t}{p} \approx \frac{\left(\frac{V_0}{R}\right)\left(\frac{R}{v}\right)}{p} = \frac{V_0}{2KE}$$

- For $KE \gtrsim 100 MeV,\ \theta \lesssim 10°$ *…so a backward peak is unexpected*
- The interpretation is that some force swaps the neutron & proton positions, meaning a forward-moving neutron is converted to a forward moving proton
- At the most fundamental level, gluon exchange between quarks within the nucleons is responsible for the nuclear force. This is described by quantum chromodynamics (QCD), which is complex enough as to not be all that useful at the moment.
- Since quarks and gluons aren't found in isolation anyways, a net force description is going to be used, where groups of quarks mediate interactions between other groups of quarks

Why *meson* exchange?

- Considering the fact that particles are waves, waves transmit fields (e.g. E&M), and quantum theory require small-scale phenomena to be quantized, quantum field theory postulates that nucleons (or at least the quarks within them) interact by exchanging quanta of the nuclear field
- The mass of the quanta can be surmised by considering the fact that quanta will be "*virtual particles*", which are only allowed to exist for a finite time Δt, subject to the uncertainty principle $\Delta E \Delta t \geq \hbar$
- The furthest range a force can be mediated by a particle is, since it must have $v \leq c$, and minimum energy $E = mc^2$, is $R \leq tc = \frac{\hbar c}{Mc^2} \approx \frac{197MeV}{Mc^2} fm$
- So, to mediate a force across the nucleon ($R \sim 1fm$), $Mc^2 \sim 200MeV$
- Since this is between the electron and nucleon masses, it was coined *meson*, for the Greek "meso" for "middle"

You expect originality from the people who only ever wear checkered-patterned shirts!?

- These turn-out to be quark pairs, the lightest (and most important) of which is the pi-meson (pion), which is a quark-antiquark pair in the ground state
- There are three types of pions, with charge +1, -1, or 0: $\pi^{\pm}(\sim 140MeV)$ and $\pi^{0}(\sim 135MeV)$

The Yukawa potential

- The preceding considerations lead us to the conclusion that a nucleon generates a pion field, where the associated interaction is described by a pion potential
- A nearby nucleon will have its own pion field, which the first nucleon will interact with, and which will interact with the pion field of the first nucleon
- Like the Coulomb potential, we have an interaction generated by some central force, and so we expect $V_{meson} \propto \frac{1}{r}$
- However, the finite mass of the force-mediating particle means the finite range must be taken into account with a penalty for interaction distances beyond the particle range, thus
  - $V_{meson} = V_{Yukawa} = -C\frac{e^{-r/R}}{r}$, where r is the internucleon distance, C is fit to data, and $R \approx 1.4fm$ is the pion range
- So, even though protons in the nucleus are all capable of attracting each other by the strong force, the strong force range is very limited, while the Coulomb force is not.
  Hence, Coulomb wins and too many protons splits a nucleus apart.

One-Pion Exchange Potential (OPEP)

- Killjoys with an enthusiasm for rigor will notice a spin and charge dependent parts are required to explain pion-nucleon interactions
- The pion has $\pi = -$ and the nucleon has $\pi = +$, so, since the nucleon field is generating the pion, a pion of any old l can't be generated
- Also, charged pions exist and charge is conserved, so if, e.g. a proton generates a π^+, it must change into a neutron
- A lot of hard (but logical) work (See L van Dommelen Appendix 41) leads to the OPEP:

$$V_{\mathrm{OPEP}} \sim \frac{g_\pi^2}{12}\left(\frac{m_\pi}{m_\mathrm{p}}\right)^2 m_\pi c^2 \;\; \vec{\tau}_1 \cdot \vec{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12} V_\mathrm{T}\right] \frac{e^{-r/R}}{r/R}$$

σ and τ are spin and isospin (here, charge) flipping operators, respectively

$$S_{12} \equiv \frac{3}{r^2}(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \qquad V_\mathrm{T} \equiv 1 + 3\frac{R}{r} + 3\frac{R^2}{r^2}$$

$$r \equiv |\vec{r}| \equiv |\vec{r}_2 - \vec{r}_1| \quad R \equiv \frac{\hbar c}{m_\pi c^2} \approx 1.4 \text{ fm} \quad m_\pi c^2 \approx 138 \text{ MeV} \quad m_\mathrm{p} c^2 \approx 938 \text{ MeV}$$

$g_\pi^2 \approx 15$ is an empirical constant

Further Reading

- Chapter 5: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Appendices 40 & 41: Quantum Mechanics for Engineers (L. van Dommelen)
- Chapter 11: Lecture Notes in Nuclear Structure Physics (B.A. Brown)
- Chapter 10: The Atomic Nucleus (R.D. Evans)
- Chapter 4: Introductory Nuclear Physics (K.S. Krane)
- Chapter 11: Introduction to Special Relativity, Quantum Mechanics, and Nuclear Physics for Nuclear Engineers (A. Bielajew)