

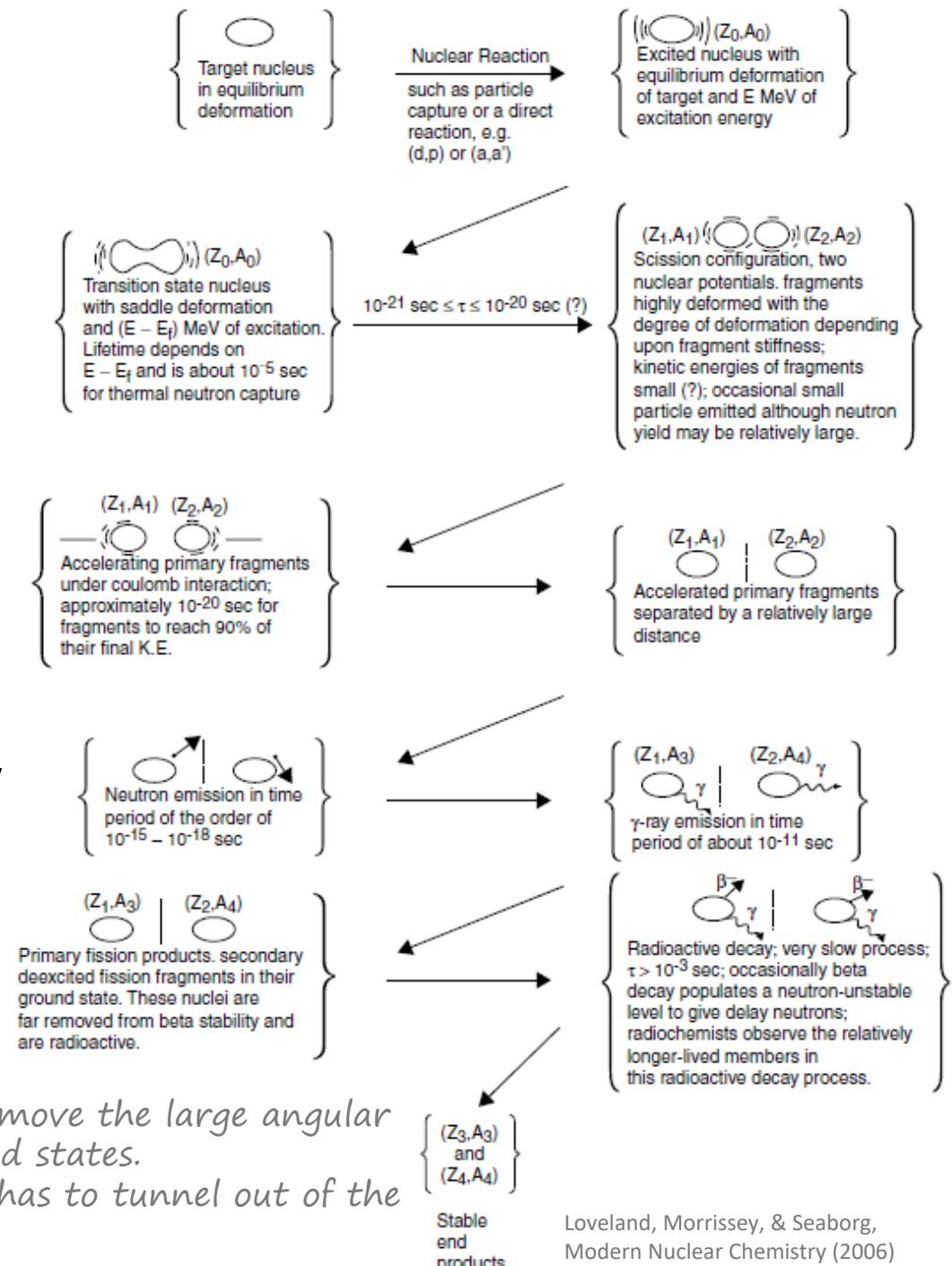
Lecture 10: Fission

- Conceptual process
- Fissionability
- Decay rate
- Decay branching
- Mass distribution
- Kinetic energy
- Neutrons



Steps of fission

1. A nucleus becomes deformed either due to an external perturbation that brings in energy or an internal cluster rattling around within the potential well
2. The energy is absorbed as a collective excitation that manifests itself as a drastic shape change, elongating the nucleus into a peanut shape
3. The separation of the two lobes of the peanut becomes great enough that the two repel each other, splitting apart at the scission point
4. The coulomb repulsion accelerates the two fragments apart
5. The two fragments are each highly excited and de-excite initially via neutron emission, followed by γ emission
(meaning prompt neutrons will be emitted along the direction of the fragments)
6. The neutron-rich fragments will then β decay back to stability, possibly emitting delayed neutrons via β -delayed neutron emission

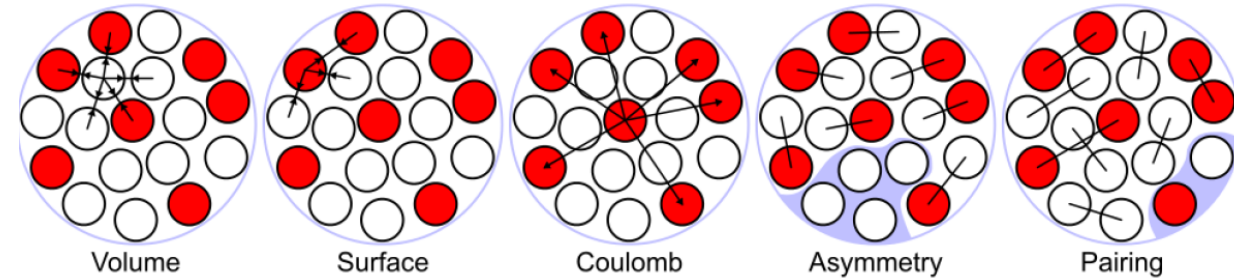


Why do you figure there is neutron emission at first and finally only γ emission?

A massive particle is better suited to remove the large angular momentum present in high-lying excited states. At lower excitation energies, a particle has to tunnel out of the nucleus, while the γ doesn't.

Energetics of shape change: *Liquid drop picture*

- Recall the contributions to the binding energy from the semi-empirical mass formula



- What happens if we deform the nucleus?

- Volume will be conserved, $A-Z$ stays the same, paired nucleons stays the same...

- But, the surface will be enlarged and the charges within the nucleus will spread apart

- In this picture, deformation will increase the binding energy penalty from the surface term, but decrease the binding energy penalty from the coulomb term

- So to understand what happens to the overall binding energy when the nucleus is deformed, we need to understand how these two terms evolve under deformation

- The change in binding energy will be $\Delta E = BE_{final} - BE_{initial} = (E'_c + E'_s) - (E_c + E_s)$, where the primed terms include the modification to coulomb and surface terms under deformation, i.e. transforming from a sphere to an ellipsoid

Energetics of shape change: *Liquid drop picture*

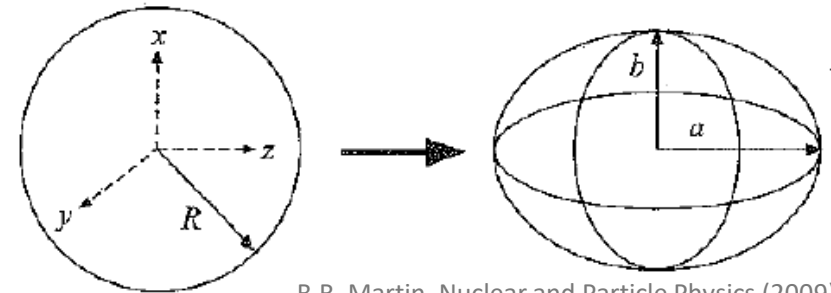
- For the deformed shape, which is an ellipsoid in this picture, the nuclear radius can be parameterized as an expansion in terms of Legendre polynomials

[which for axial symmetry will only keep the $l = 2$ term]

- $R(\theta) = R_0[1 + \alpha_2 P_2(\cos\theta)]$

- α_2 is the quadrupole distortion parameter, which is related to the quadrupole deformation by

$$\alpha_2 = \sqrt{\frac{5}{4}} \pi \beta_2, \text{ and the ellipsoid axes by: } a = R_0(1 + \alpha_2), b = R_0 \frac{1}{\sqrt{1 + \alpha_2}}$$



B.R. Martin, Nuclear and Particle Physics (2009)

- It turns out, expanding the Coulomb and surface energy terms as a power series in α_2 yields

- $E'_c \approx a_c \frac{Z^2}{A^{1/3}} \left(1 - \frac{1}{5} \alpha_2^2\right)$ and $E'_s \approx a_s A^{2/3} \left(1 + \frac{2}{5} \alpha_2^2\right)$

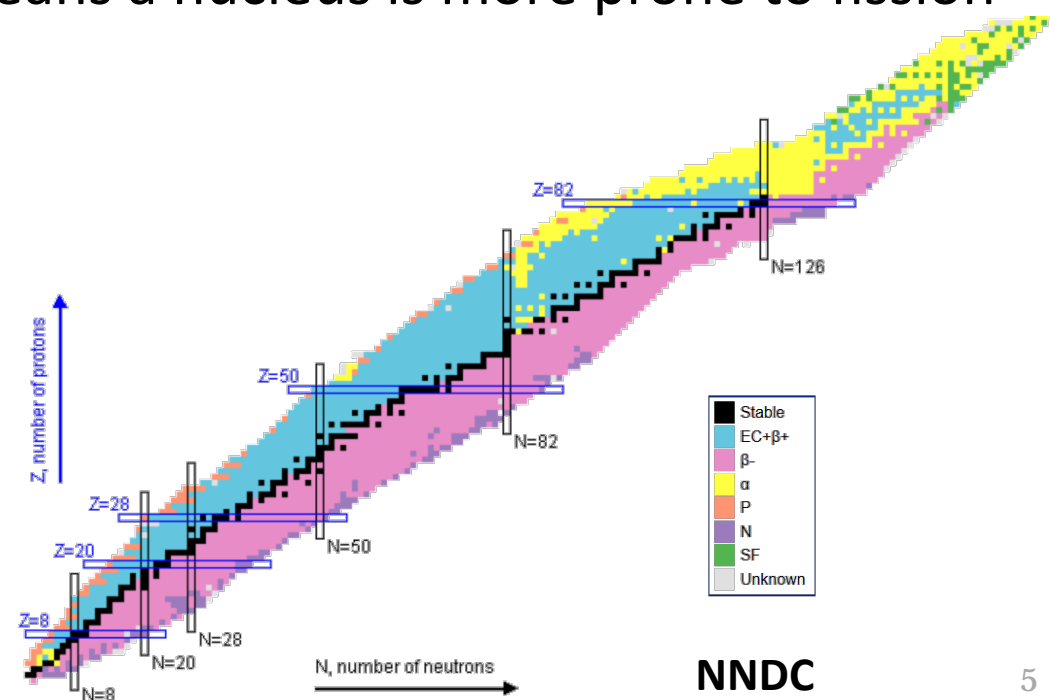
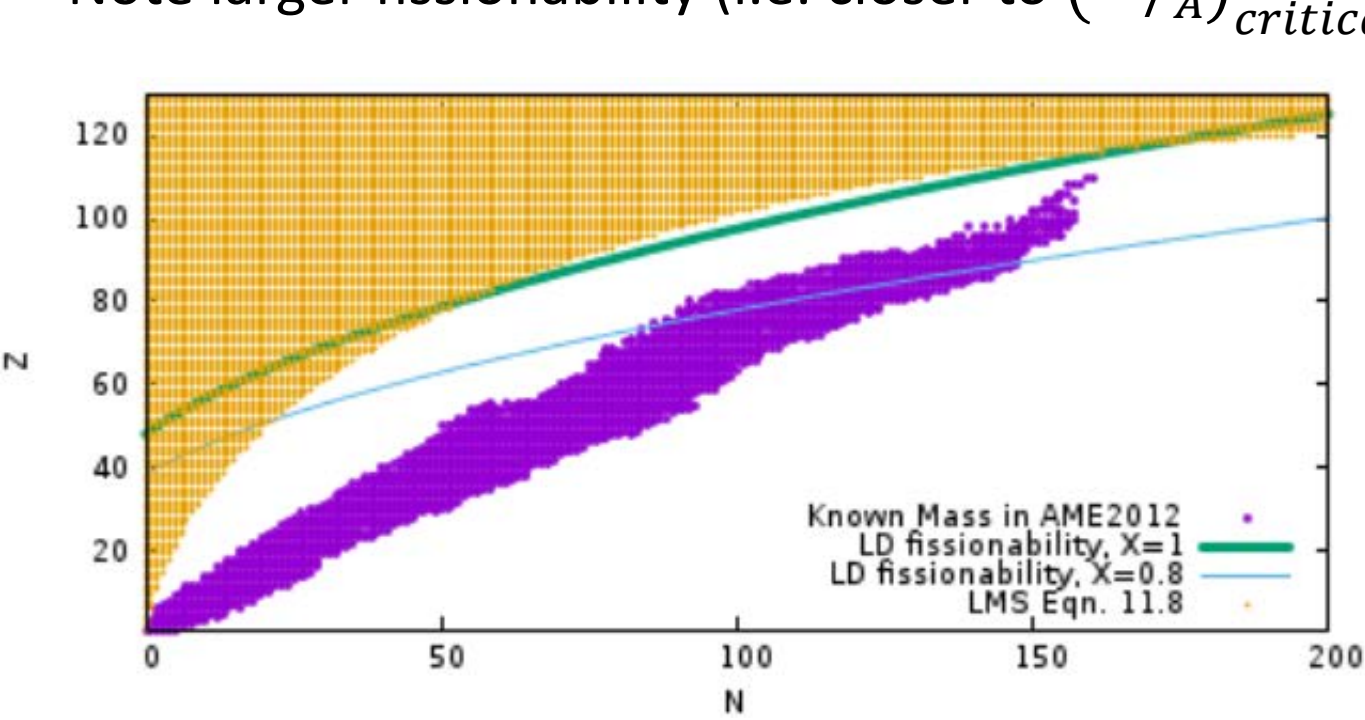
- Meaning the energy cost for deformation is $\Delta E = \frac{\alpha_2^2}{5} \left(2a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}}\right)$

- So, when the non-deformed Coulomb energy is twice the non-deformed surface energy or greater, there is zero energetic cost (or even an energetic gain) to deform (and ultimately fission!)

- The fissionability parameter $x = \frac{E_c}{2E_s} = \frac{a_c}{2a_s} \frac{Z^2}{A} \equiv \frac{Z^2/A}{(Z^2/A)_{critical}}$ is a measure of fission favorability

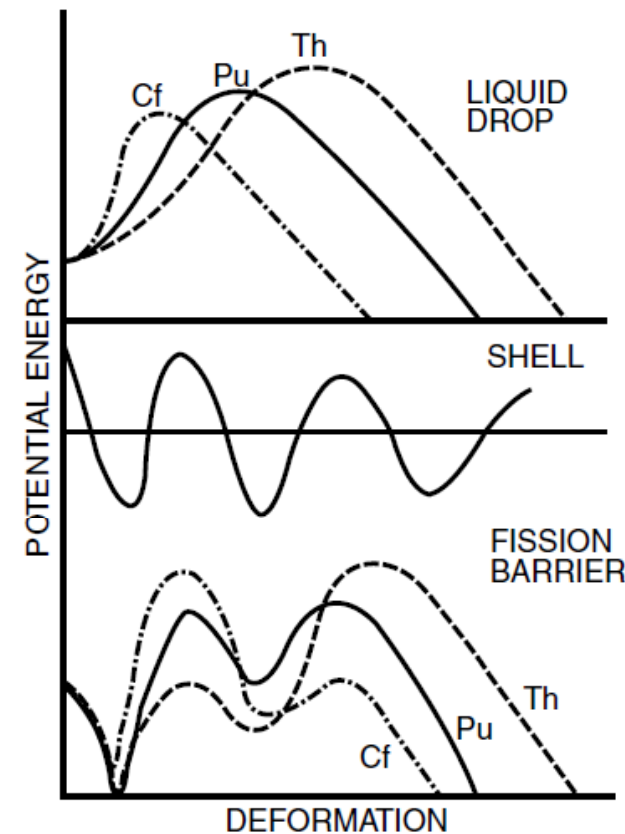
Energetics of shape change: *Liquid drop picture*

- What is the Z,A combination prone to fission? i.e. what is $(Z^2/A)_{critical}$?
 - For the liquid drop, we just defined it as $\frac{2a_s}{a_c}$...which, from our fit to masses is ≈ 48
 [that's $Z > 116, A \geq 270$]
 - Taking Loveland, Morrissey, & Seaborg's word for it "a more sophisticated treatment" yields $(Z^2/A)_{critical} = 50.8333 \left[1 - 1.7826 \left(\frac{N-Z}{A} \right)^2 \right]$
- Note larger fissionability (i.e. closer to $(Z^2/A)_{critical}$) means a nucleus is more prone to fission



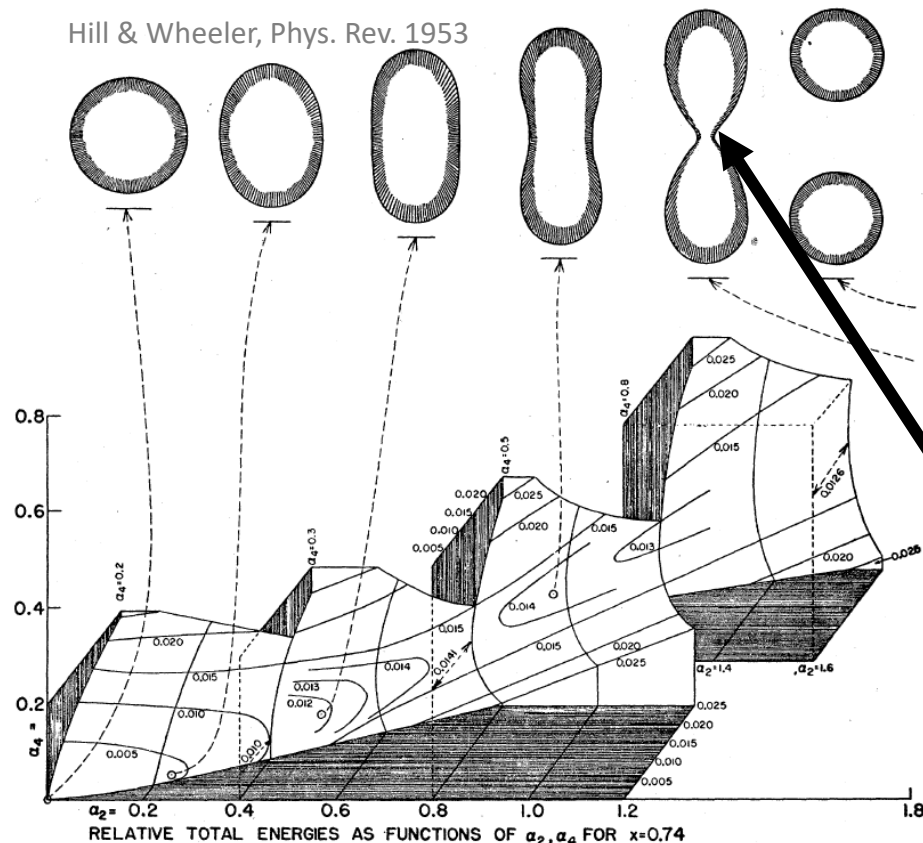
Fission barrier

- To quantify the probability of fission occurring, we can consider the potential energy associated with a particular shape of the nucleus
- This potential energy is dominated by liquid drop energetics, ...but with a major correction for enhanced binding near closed shells ...the magnitude of which is altered by the nuclear deformation
- So, the peaks and valleys in potential energy space are associated by the competition between liquid-drop and shell-model modified by deformation
- The likelihood of transitioning from one shape to another depends on the energetic cost associated with overcoming (or tunneling through) the barrier between them



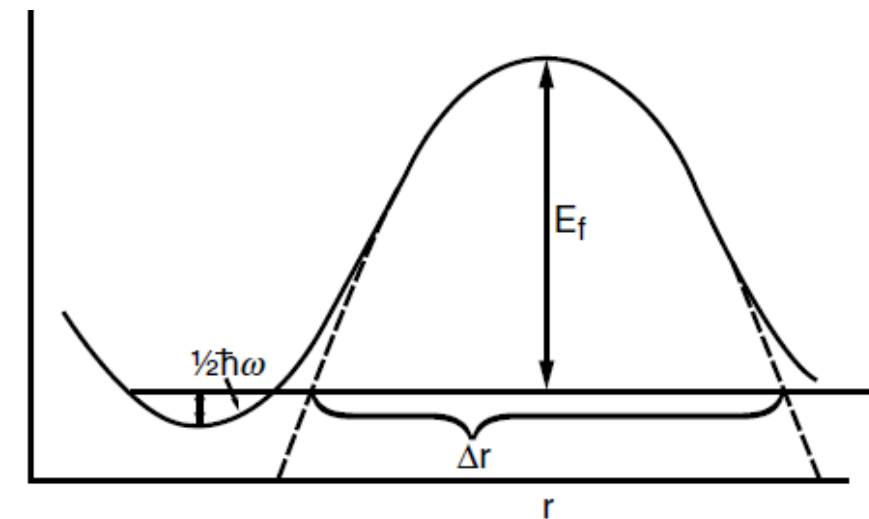
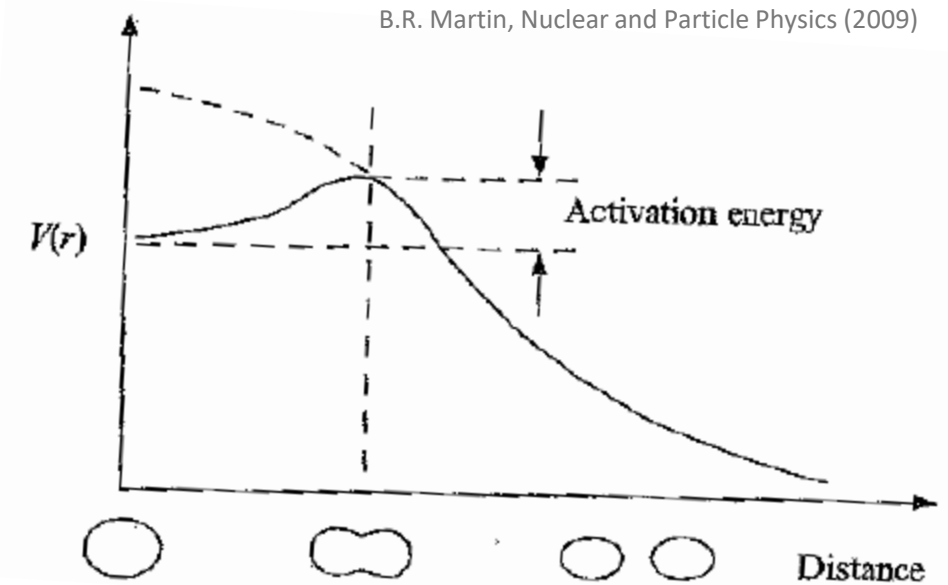
Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

scission occurs at the saddle point in the potential



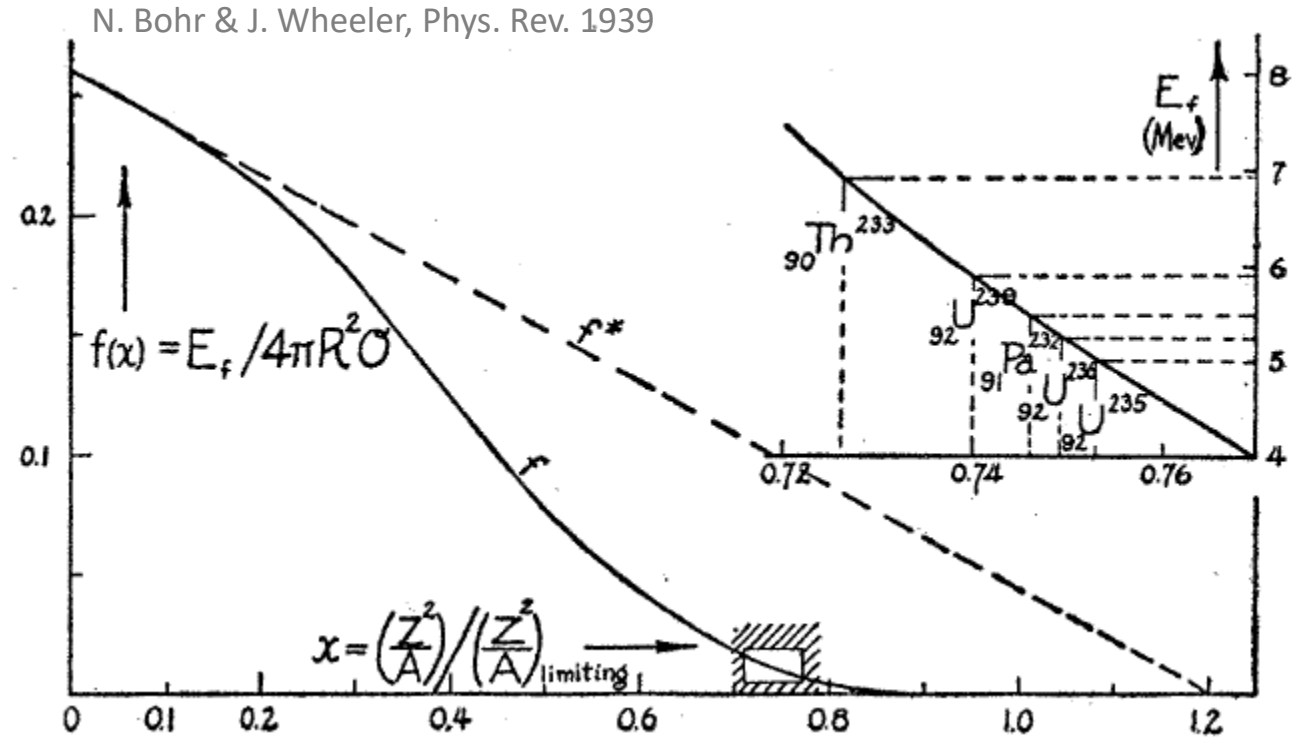
Fission barrier height / Activation energy

- Quantitatively, the energy that must be paid (or at least approached [tunneling]) is an activation energy
- The more common parlance is to refer to the fission barrier height (E_f in the picture), which is measured relative to the ground-state of the original system, as quantified as a harmonic oscillator $\frac{1}{2}\hbar\omega$
- Rather than overcome the barrier, it is also possible to tunnel through it, in which case the relevant quantity is the barrier width
- This width will be related to the barrier curvature $\hbar\omega$, where a large curvature means a thin barrier (easier to tunnel through) and a small curvature means a thick barrier (harder to tunnel through)
- Overcoming the barrier (or overcoming part of it and tunneling through the top) corresponds to *induced* fission, while tunneling through the barrier corresponds to *spontaneous* fission



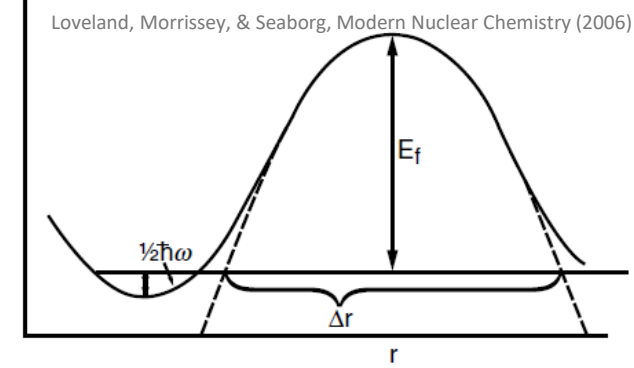
Fission barrier height

- In a liquid drop estimate, the fission barrier height is the difference between the ground-state binding energy of a system and the energy of two spherical nuclei made by splitting that system
- Near the longest-lived isotopes of Uranium and Thorium, $E_f \sim 5 - 7 \text{ MeV}$
- Note this simple method yields one useful prediction already:
 - ^{235}U has a much shorter fission barrier height than ^{238}U and so the $A = 235$ isotope is more useful when a lot of fission is what you're looking for



Spontaneous fission rate

- Spontaneous fission is akin to α decay (or proton or cluster decay, for that matter), where a barrier is “assaulted” at some rate and there is probability for tunneling through the barrier
- Here, the difference is that the potential is not from the nucleus, but rather the potential energy surface for the landscape of possible shapes. The nucleus itself is tunneling through the barrier.
- $t_{1/2} = \frac{\ln(2)}{fP}$, where f is the assault frequency and P is the tunneling probability
- f corresponds to the rate at which the nuclear shape is changing, i.e. the frequency of surface oscillations, which is $\sim 10^{20} \text{ s}^{-1}$ (Hill & Wheeler, Phys. Rev. 1953)

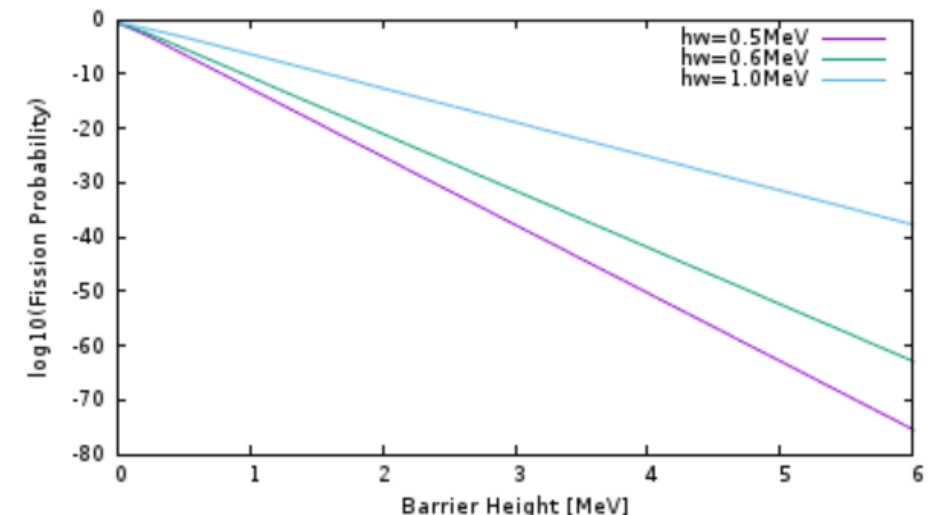


- For the simplest case of a one-humped barrier, approximated as an inverted parabola, (due to the liquid-drop + single particle potential for deformation, See Lec. 4), the tunneling probability is (Hill & Wheeler, Phys. Rev. 1953)

$$P = \frac{1}{1 + \exp(2\pi E_f / \hbar\omega)}$$

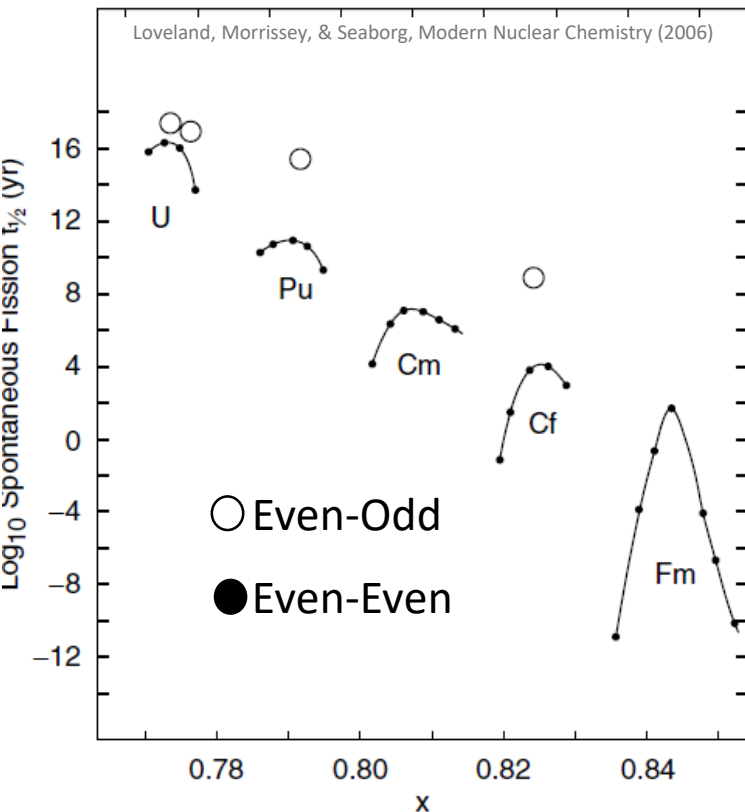
Extremely sensitive to predictions of E_f

- So, $t_{1/2} \approx 10^{-21} \exp(2\pi E_f / \hbar\omega)$

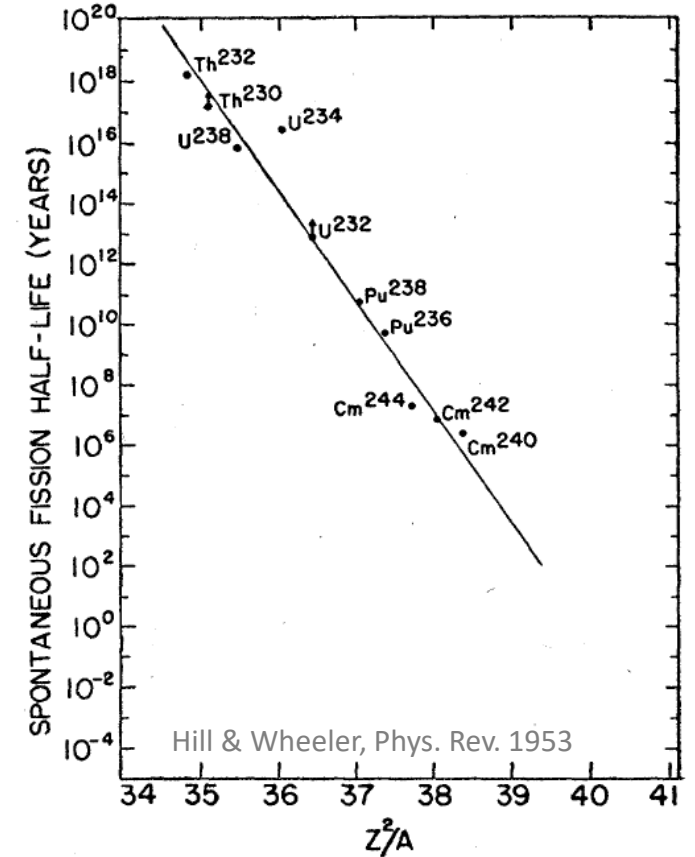
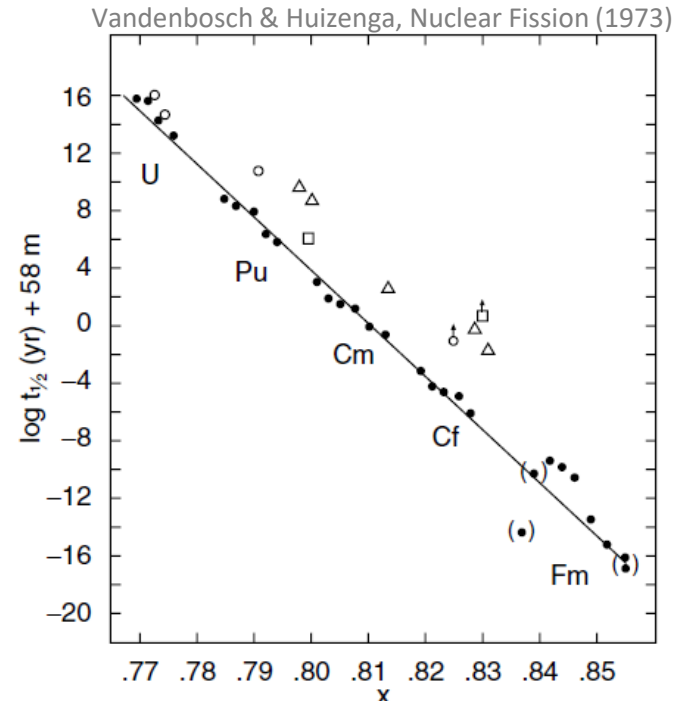


Spontaneous fission rate

- The height of the fission barrier is related to the fissionability parameter x (recall $x \sim 1$, i.e. $Z^2/A \sim 48$, means fission immediately)
- So without resorting to fancy calculations of E_f , the half-life for spontaneous fission can be ball-parked
- But this method is extremely rough



....using shell-corrected masses gives some improvement:



Fission decay branching ratio

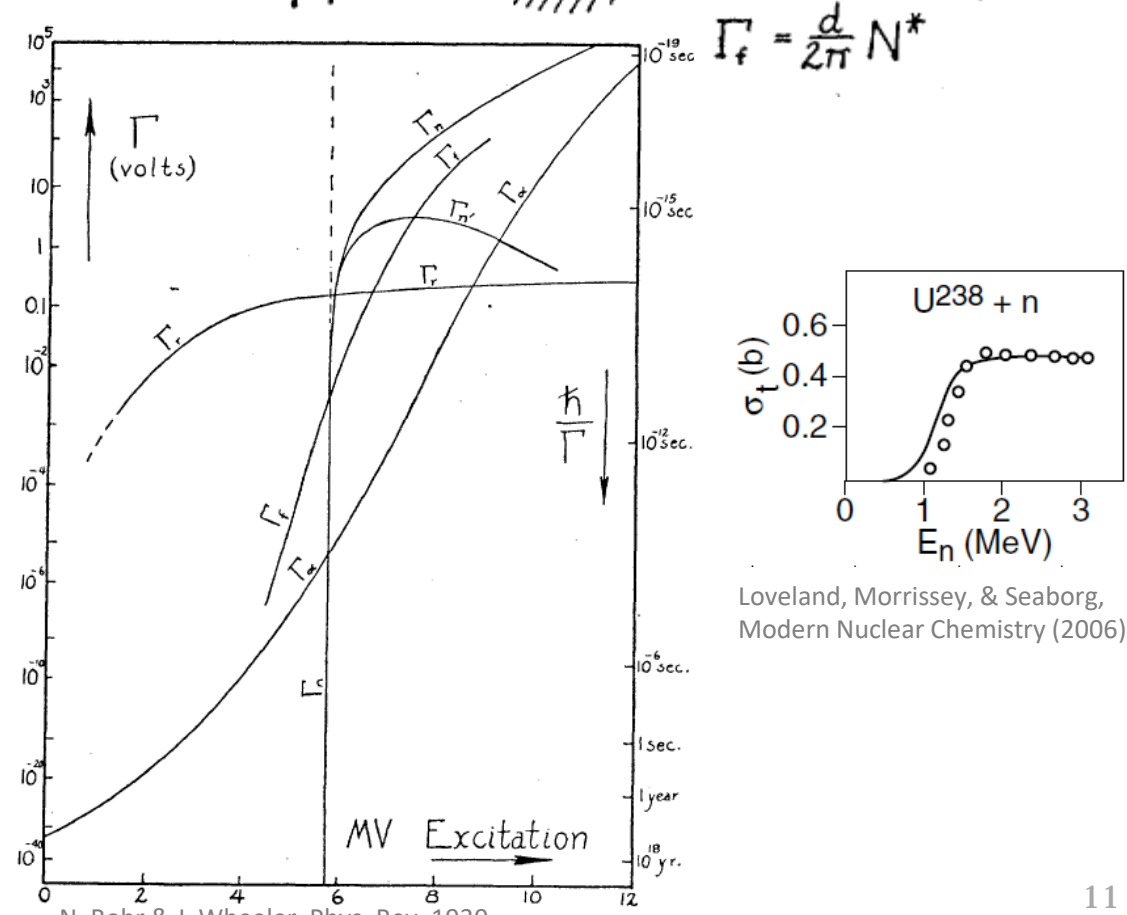
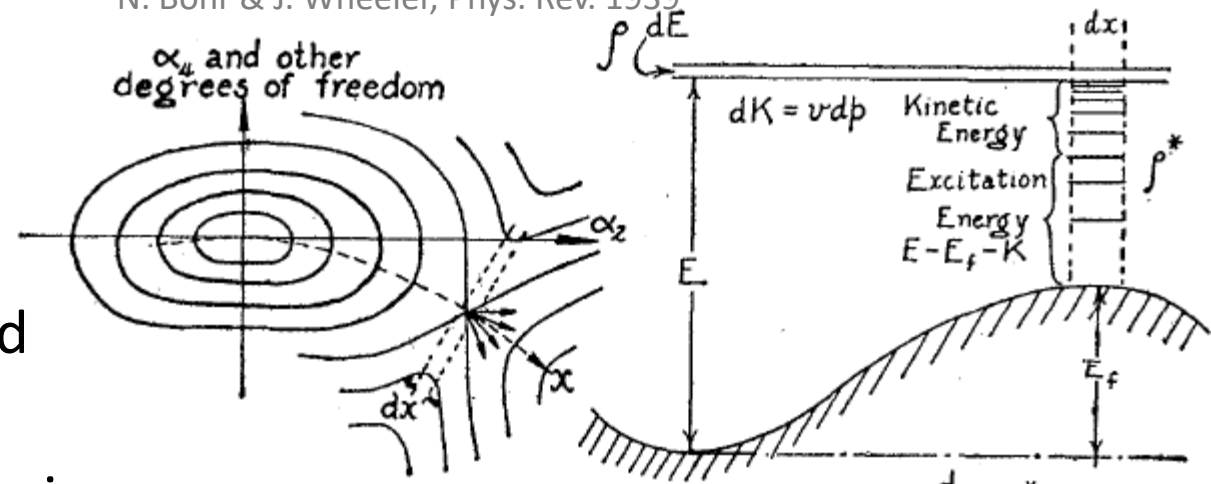
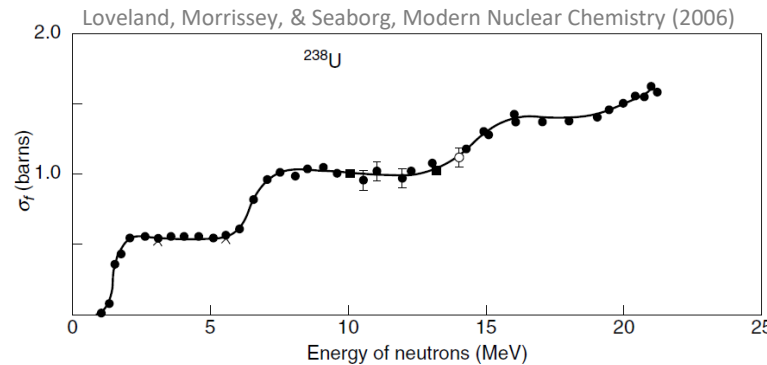
- Fission is a means of de-excitation and, as such, competes with other decay modes
- Recall, the lifetime of an excited state can instead be specified in terms of a width $\Gamma = \hbar/\tau = \hbar\lambda$
- The fraction of decays proceeding through fission is

$$BR_f = \frac{\Gamma_f}{\sum_{modes} \Gamma_{mode}} = \frac{\Gamma_f}{\Gamma_f + \Gamma_n + \Gamma_\gamma + \Gamma_\alpha + \Gamma_{other\ c.p.}}$$

where the Γ_i are related to the density of final states available for that decay mode and the probability for transitioning from the initial to final state (e.g. tunneling probability for charged particles. See Lectures 5,7,9.)

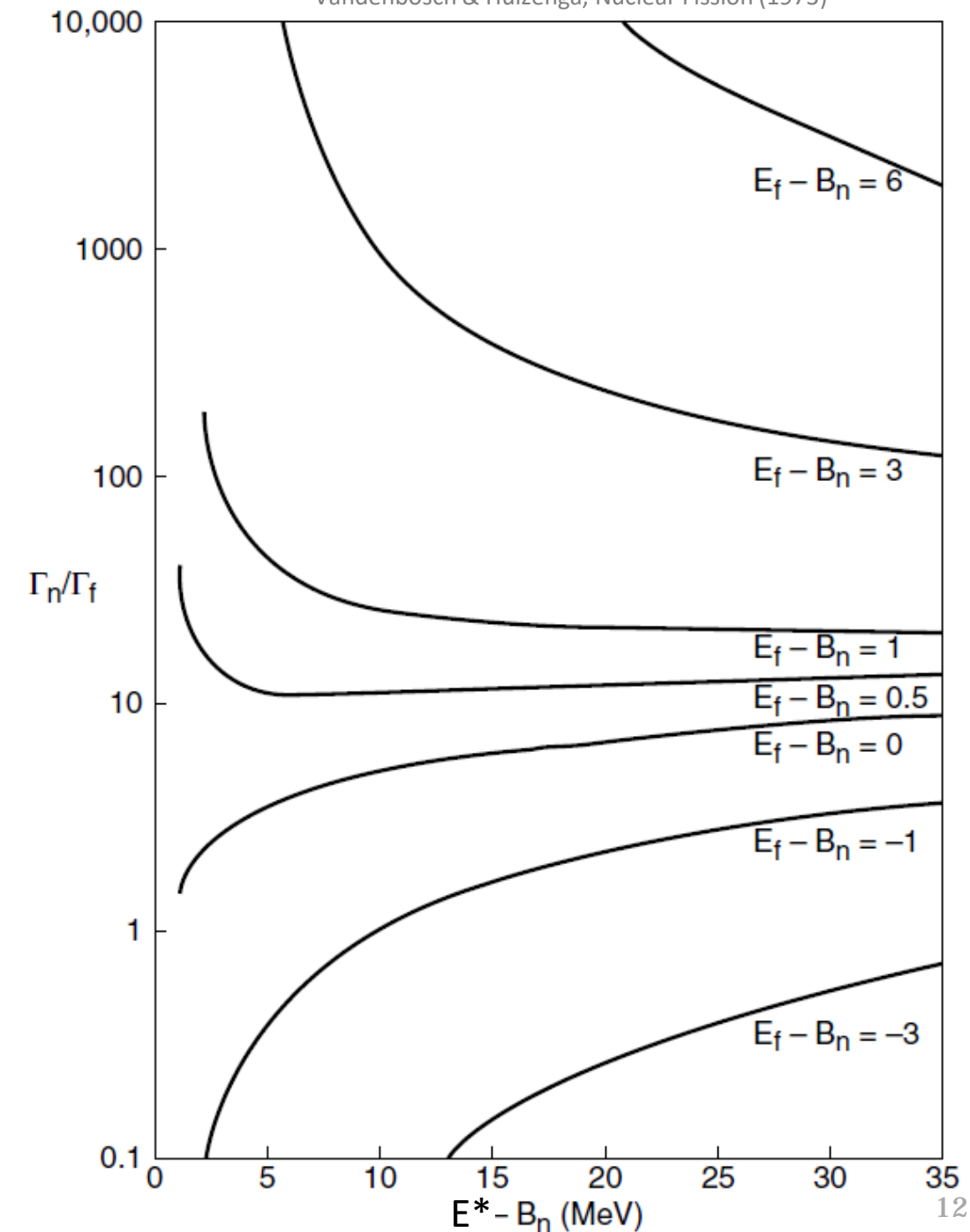
- When multi-neutron emission channels open, the fission cross section actually goes up!

[2nd, 3rd,chance fission]



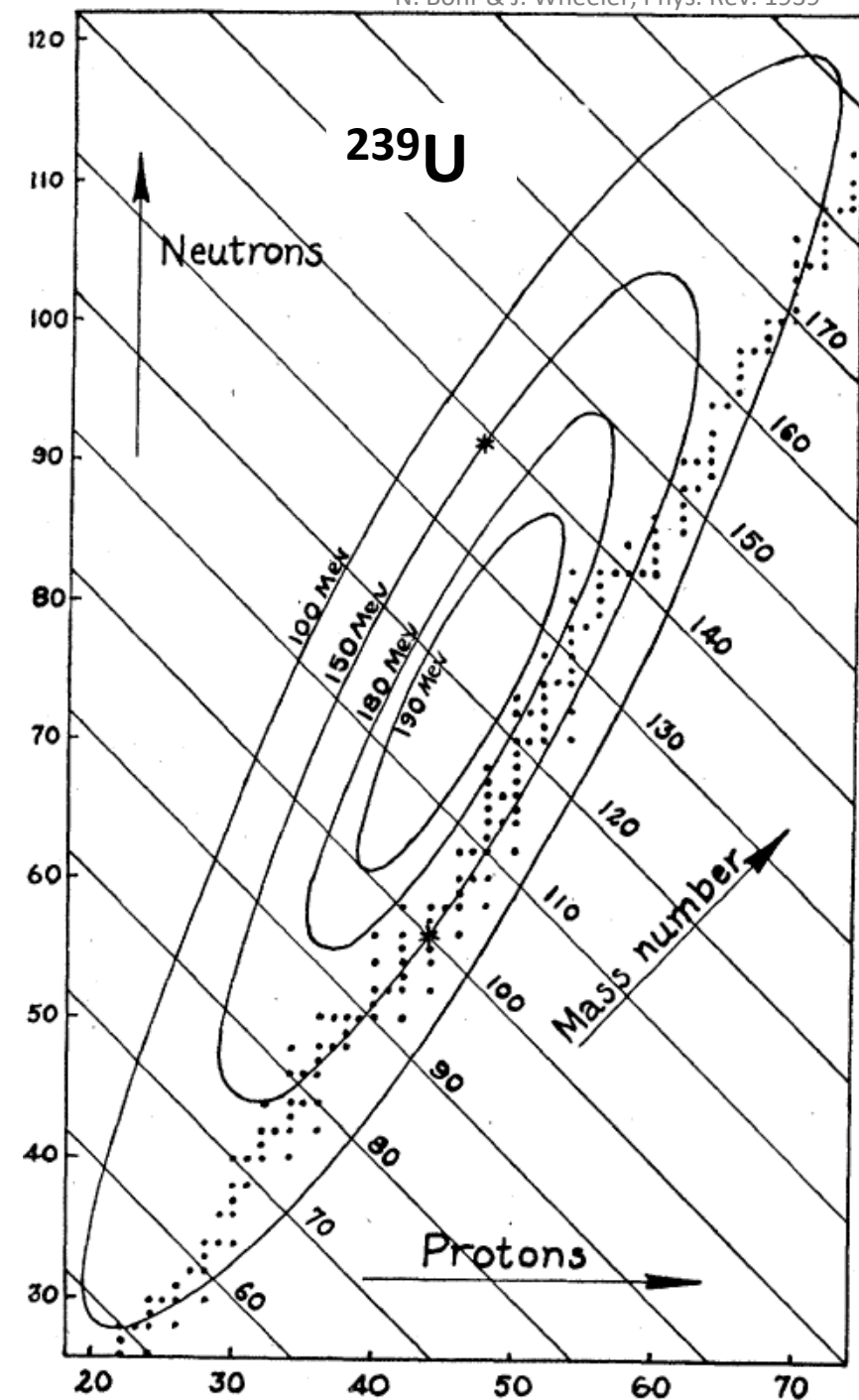
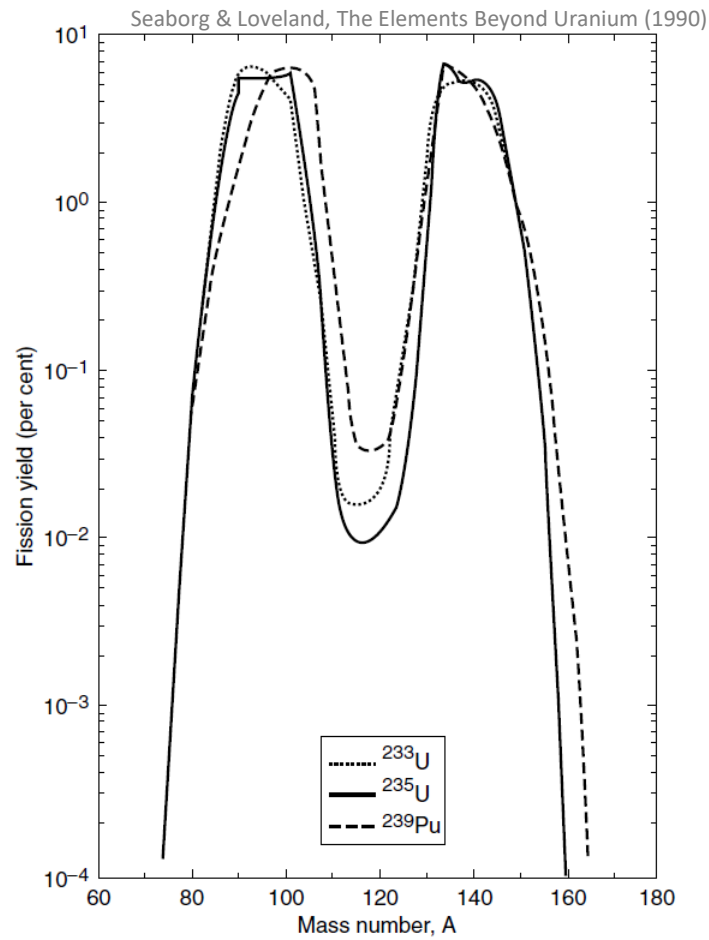
Fission decay branching ratio

- Since neutrons have no Coulomb barrier to tunnel through and can generally carry away more angular momentum than γ rays, the neutron decay branch is the primary competition for fission
- The relative branching ratio depends on the difference between the fission barrier and neutron binding energy $E_f - B_n$ and the difference between the compound nucleus (captured particle + original nucleus) excitation energy and the neutron binding energy $E^* - B_n$
- The excitation energy will be the sum of the center-of-mass energy for the capture reaction and the Q-value for the capture reaction $E^* = E_{CM} + Q$
- E.g. ^{238}U bombarded by a 42MeV α
 - $E^* = 42 \frac{^{238}}{242} + (ME(U238) + ME(\alpha) - ME(Pu242))c^2 = 36.3\text{MeV}$
 - $B_n(Pu242) = (ME(Pu241) + ME(n) - ME(Pu242))c^2 = 6.3\text{MeV}$
 - $E_f(Pu242) = 5.3\text{MeV}$ [*..could estimate from Z^2/A*]
 - $E^* - B_n = 30\text{MeV}$; $E_f - B_n = -1 \rightarrow \frac{\Gamma_n}{\Gamma_f} \approx 3$
 - ... so fission occurs 25% of the time for this case



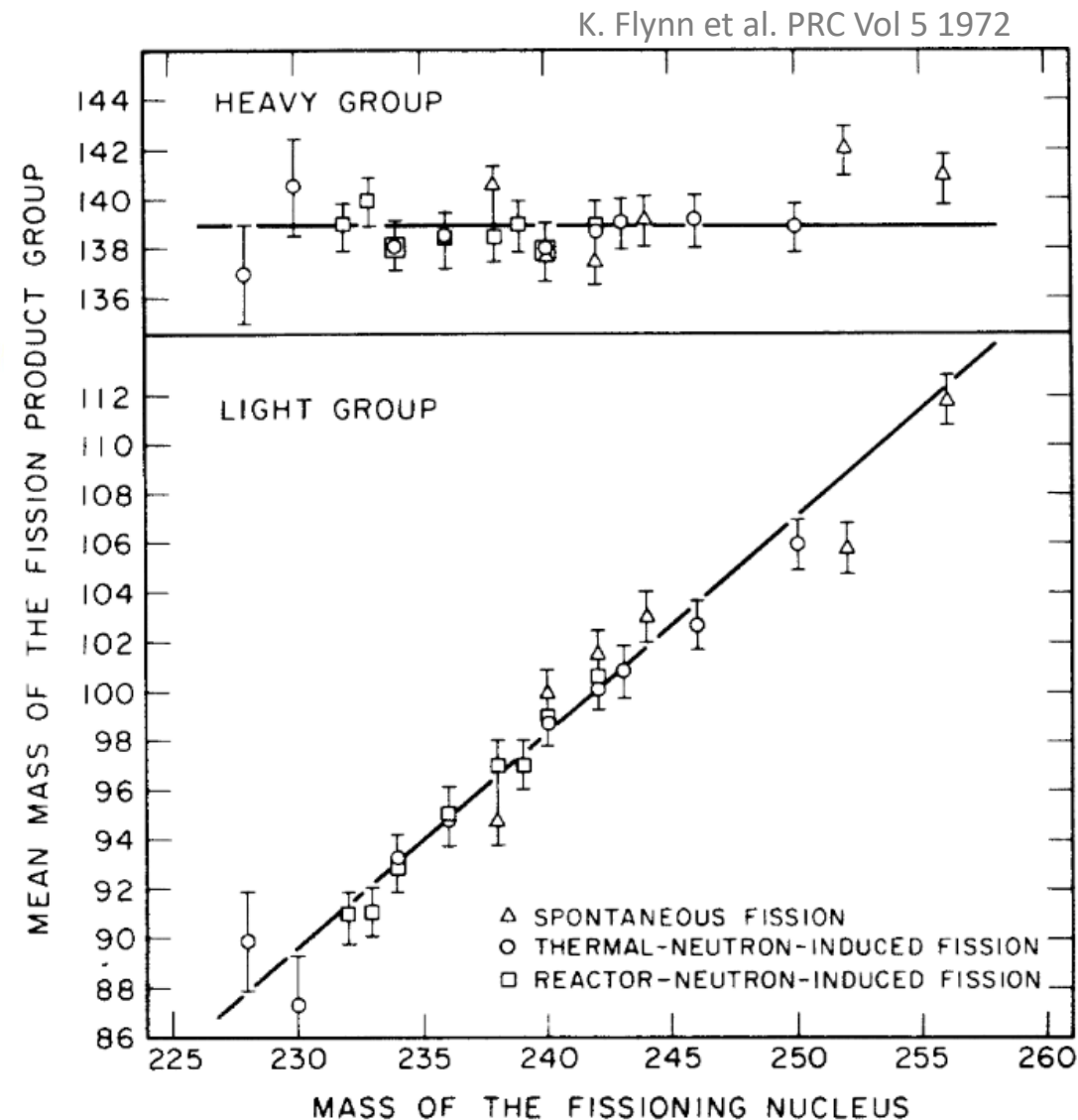
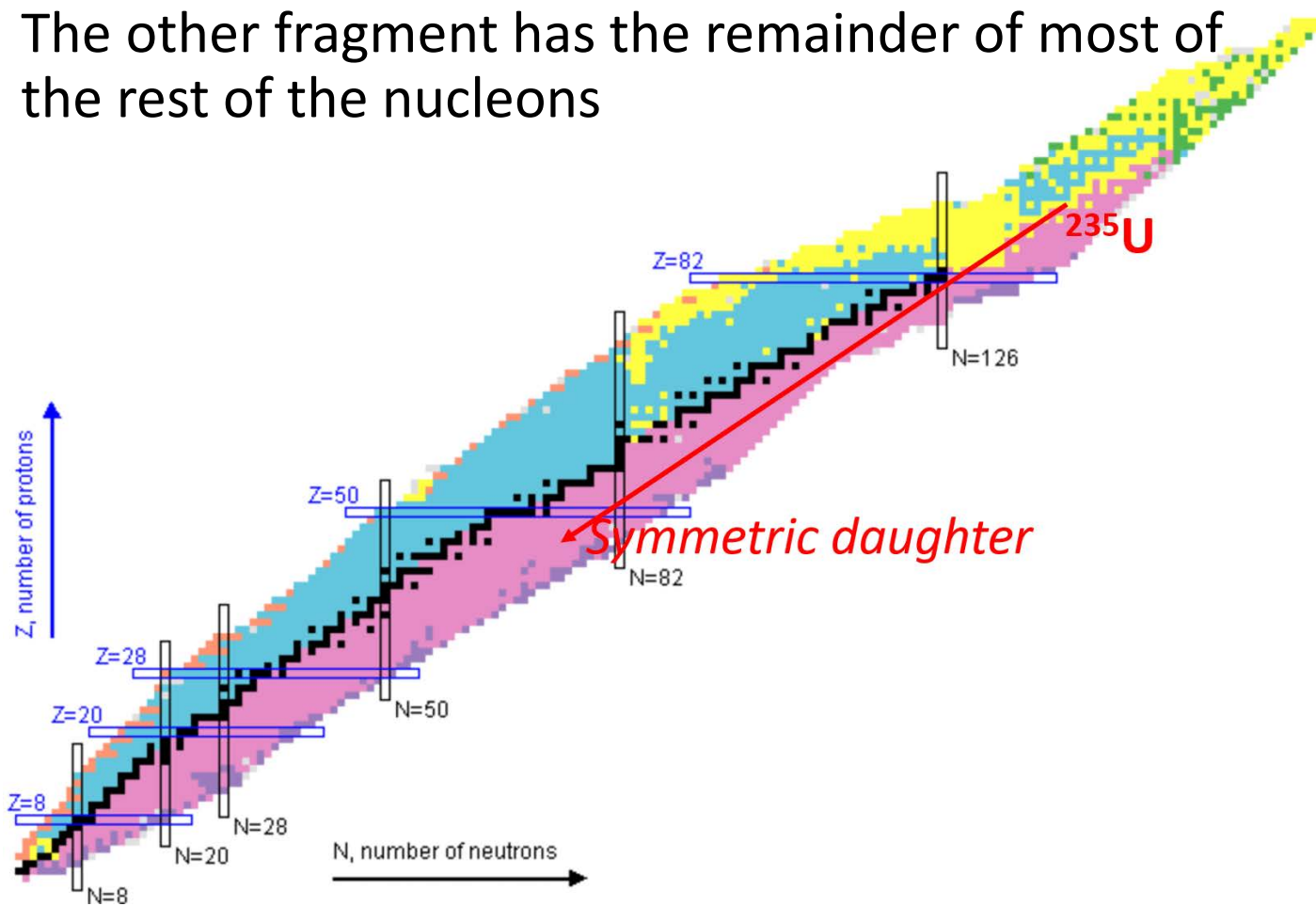
Fission fragment mass distribution

- To first order, the fission fragments will be those that maximize the total energy release
[which can be estimated from the liquid drop model]
- This would always result in a case of a symmetric distribution of the matter
- However, often the observed mass distributions are instead quite asymmetric:
- The individual fragments approximately maintain the original A/Z (because neutrons and protons separate in the same way), but often high- A and low- A peaks exist



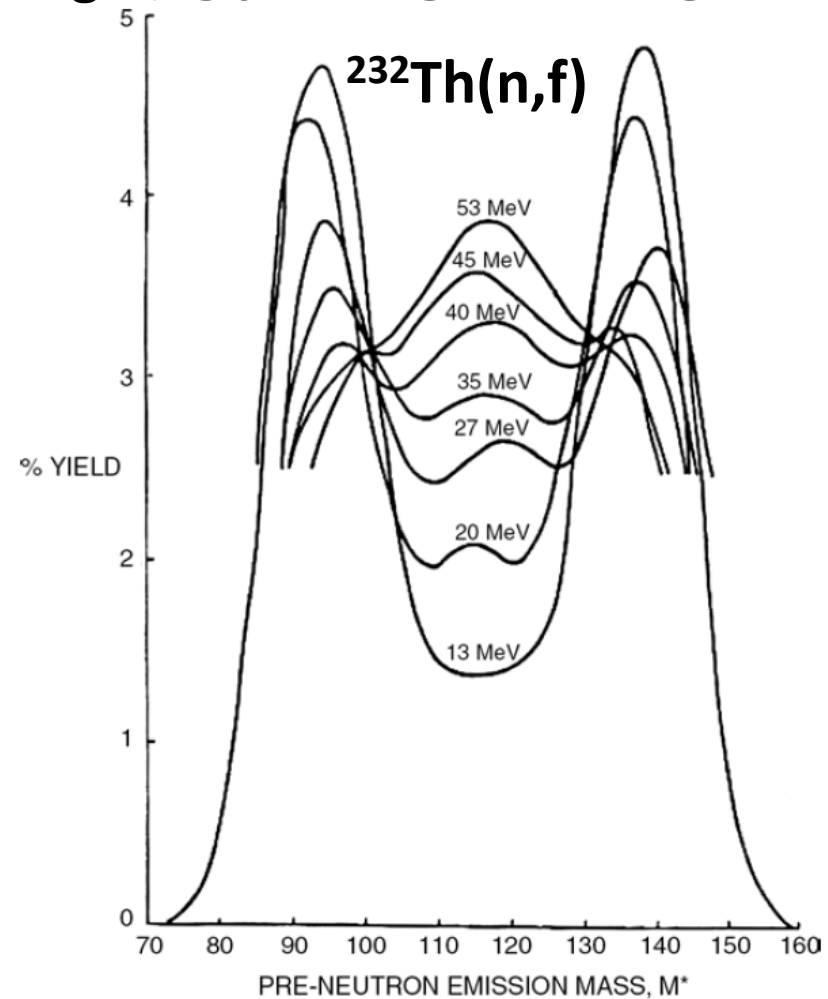
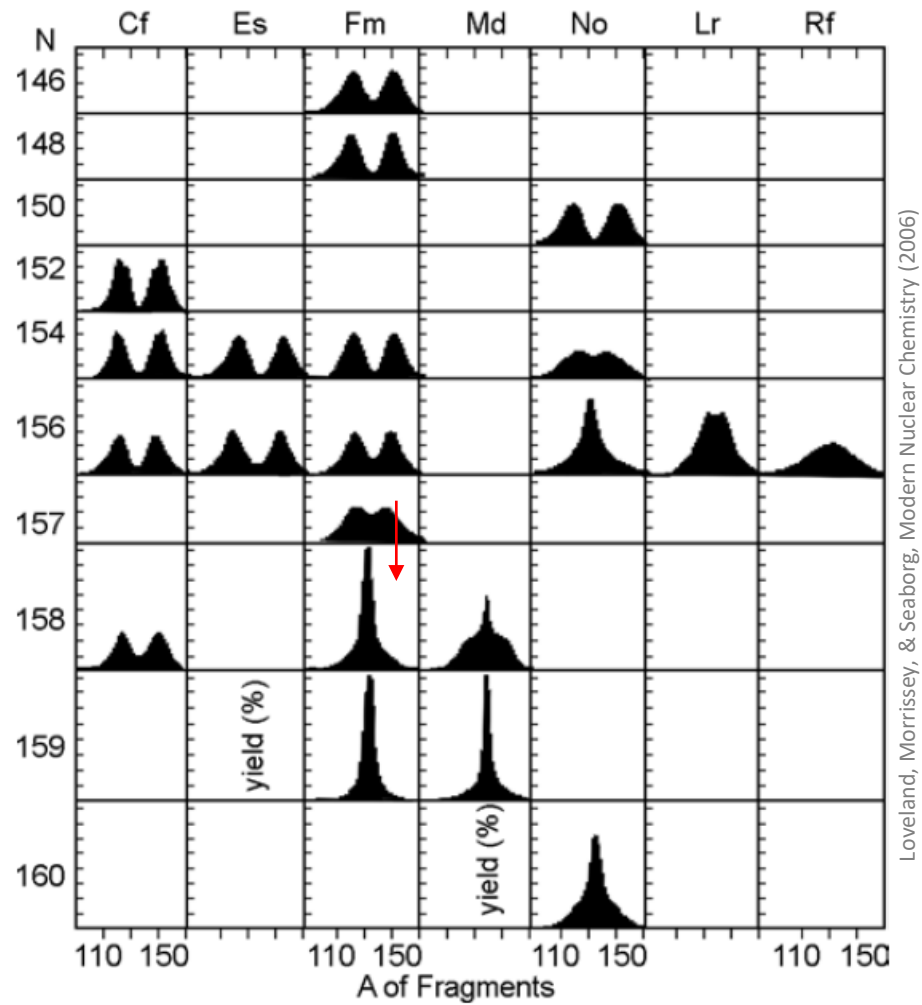
Fission fragment mass distribution

- The culprit for the asymmetric mass distribution are the $Z = 50, N = 82$ shell closures, which favors nuclei in this range for one of the fission fragments.
- The other fragment has the remainder of most of the rest of the nucleons

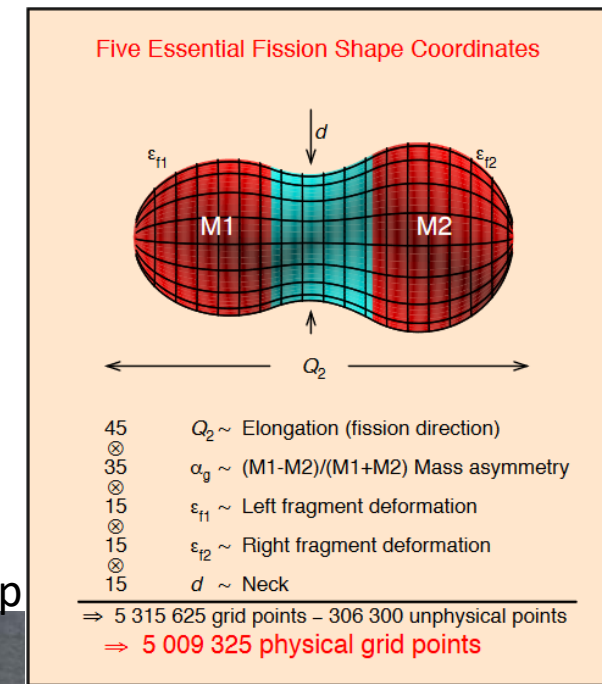
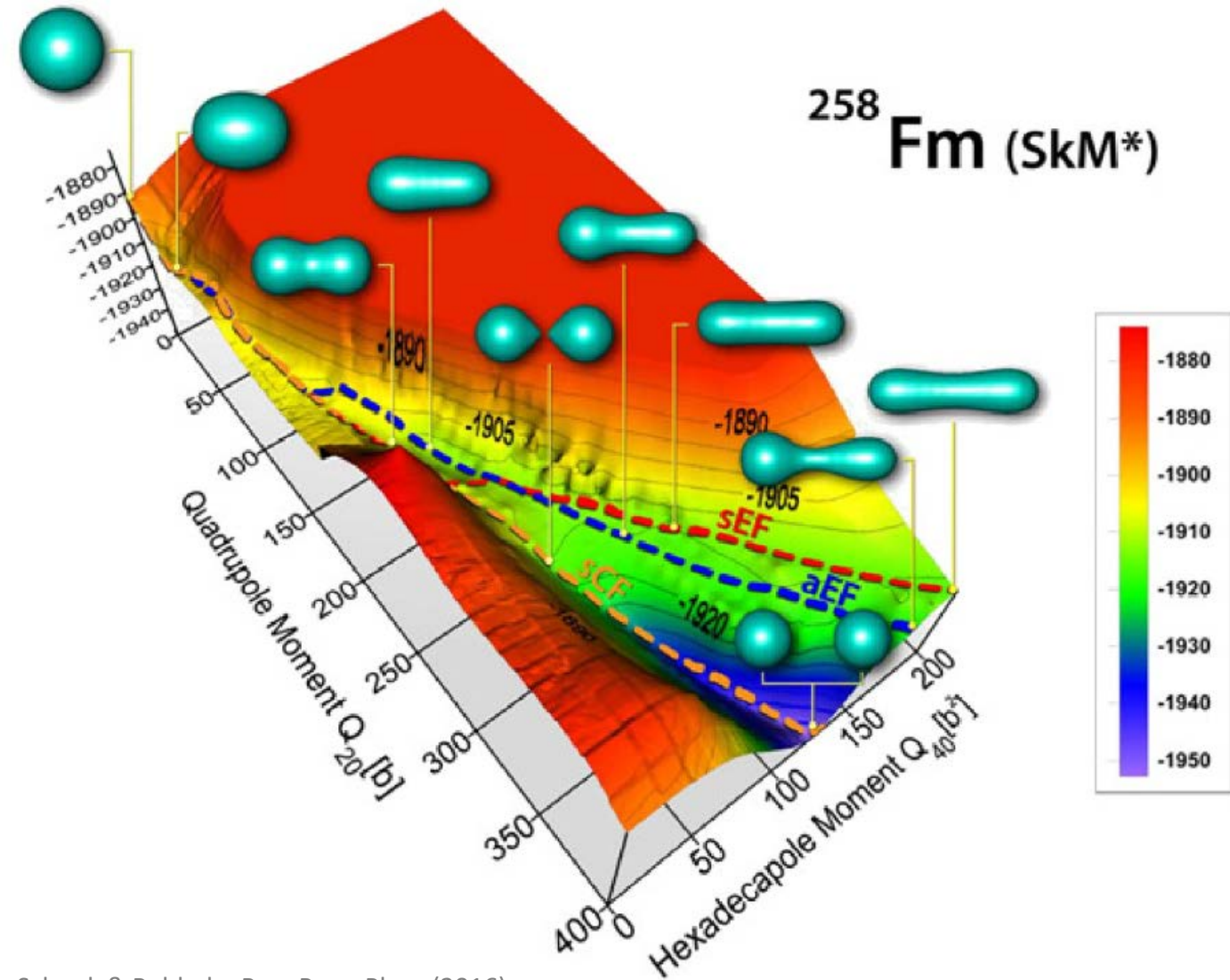


Fission fragment mass distribution

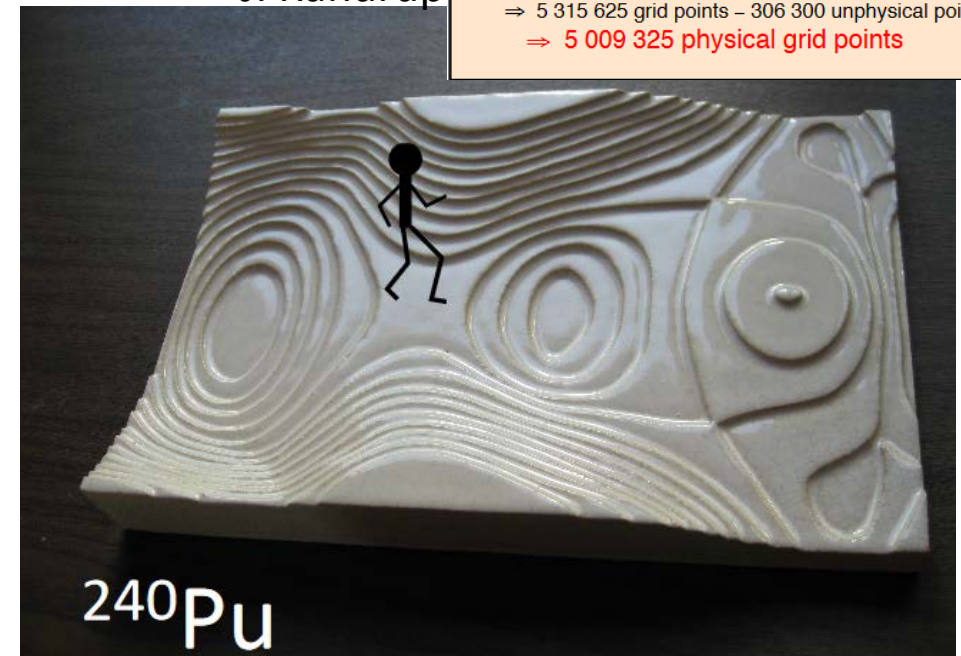
- However, the signature of the closed shells is erased if the symmetric fragment distribution is too far from the shell closure...
- ...or if the excitation energy of the fissile nucleus is high (*e.g. from a high- E incoming neutron*)



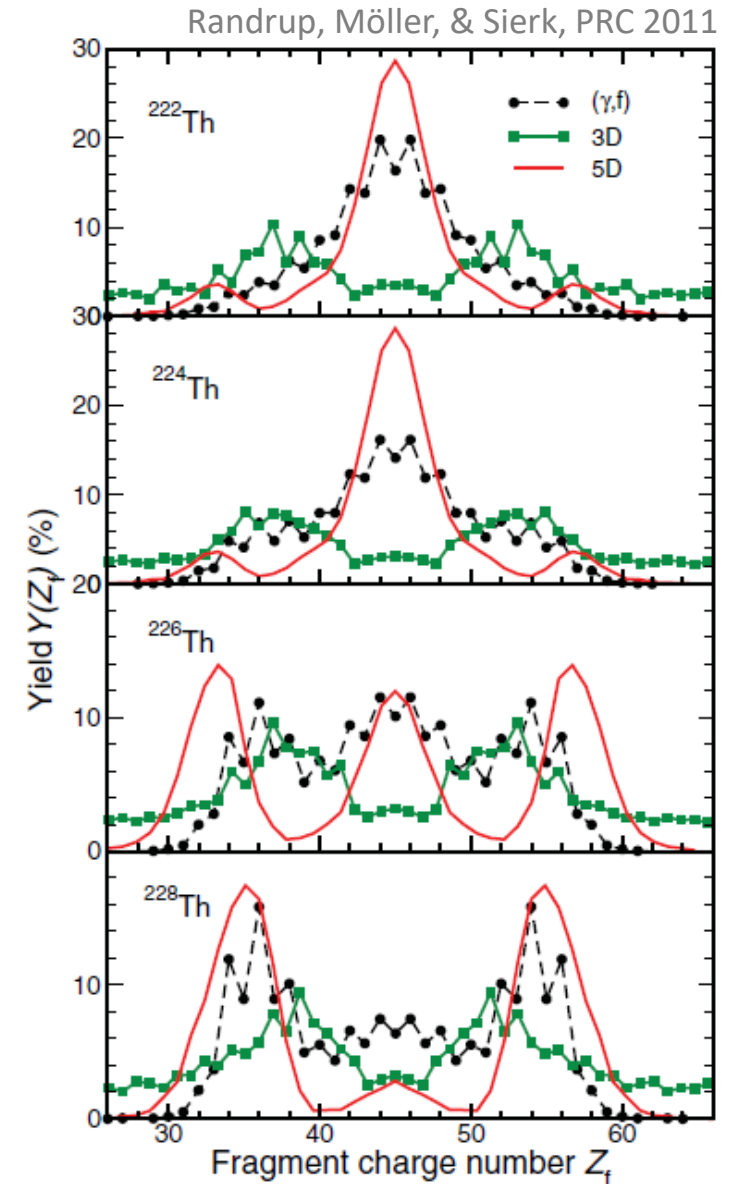
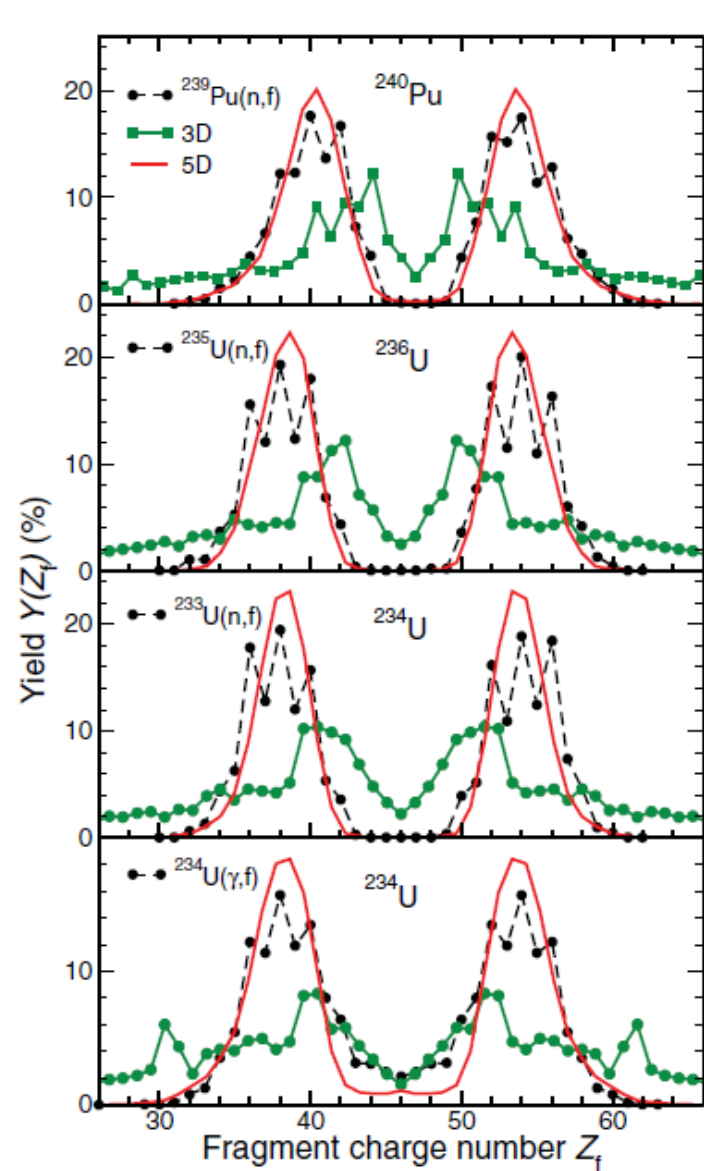
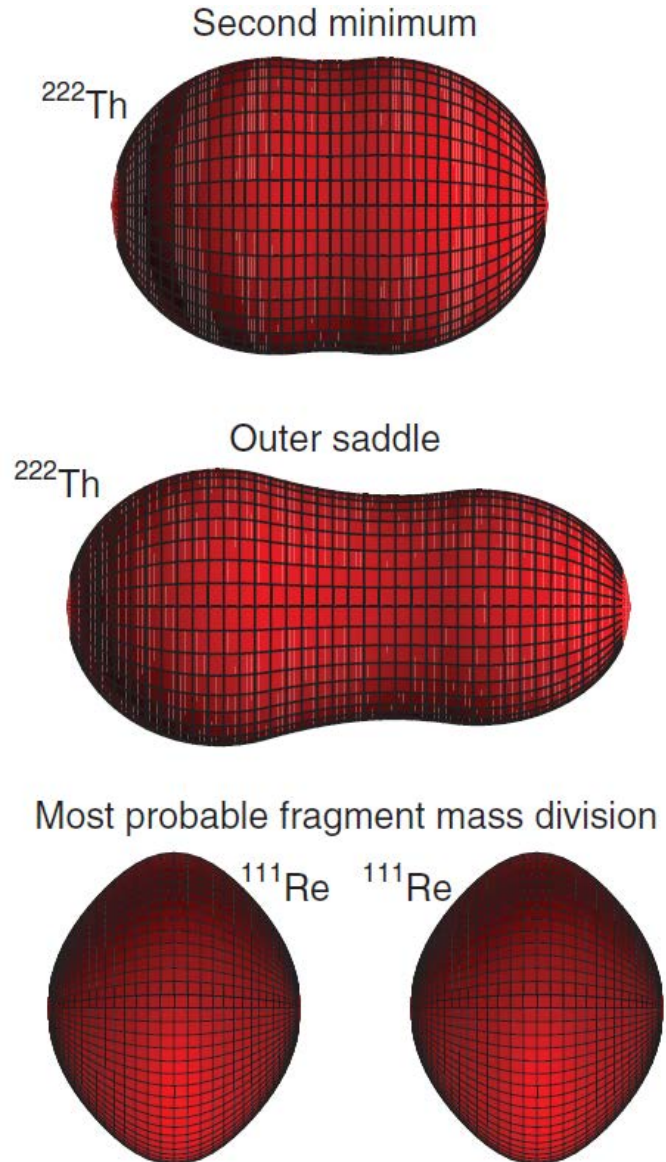
Modern fission fragment calculations: *Random walk in a shape-based energy space*



J. Randrup

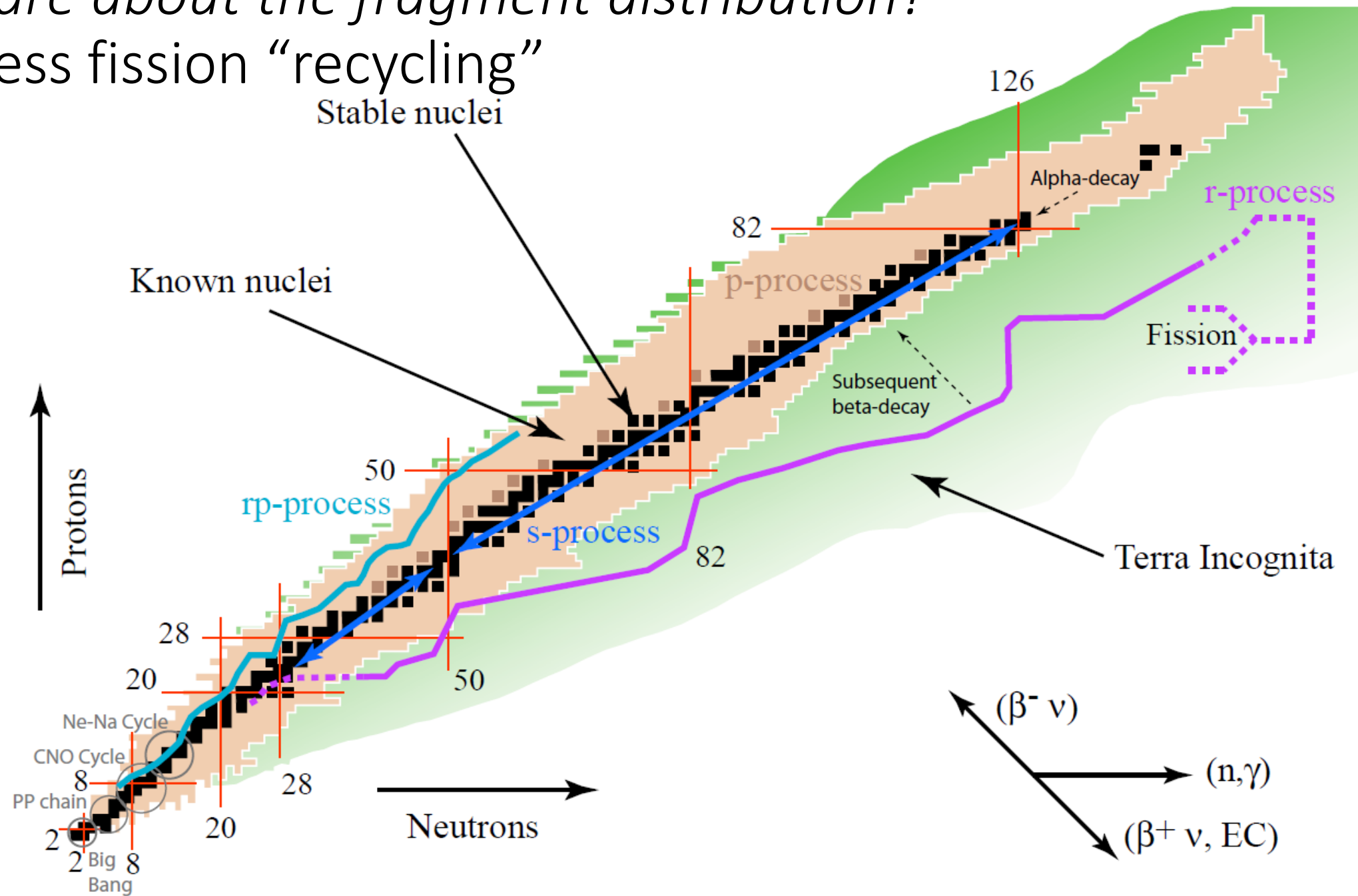


Modern fission fragment calculations: *Random walk in a shape-based energy space*



Why care about the fragment distribution?

r-process fission “recycling”



Fission fragments: *kinetic energy*

- The total kinetic energy of fission products will roughly be the coulomb repulsion energy of the two main fission fragments,

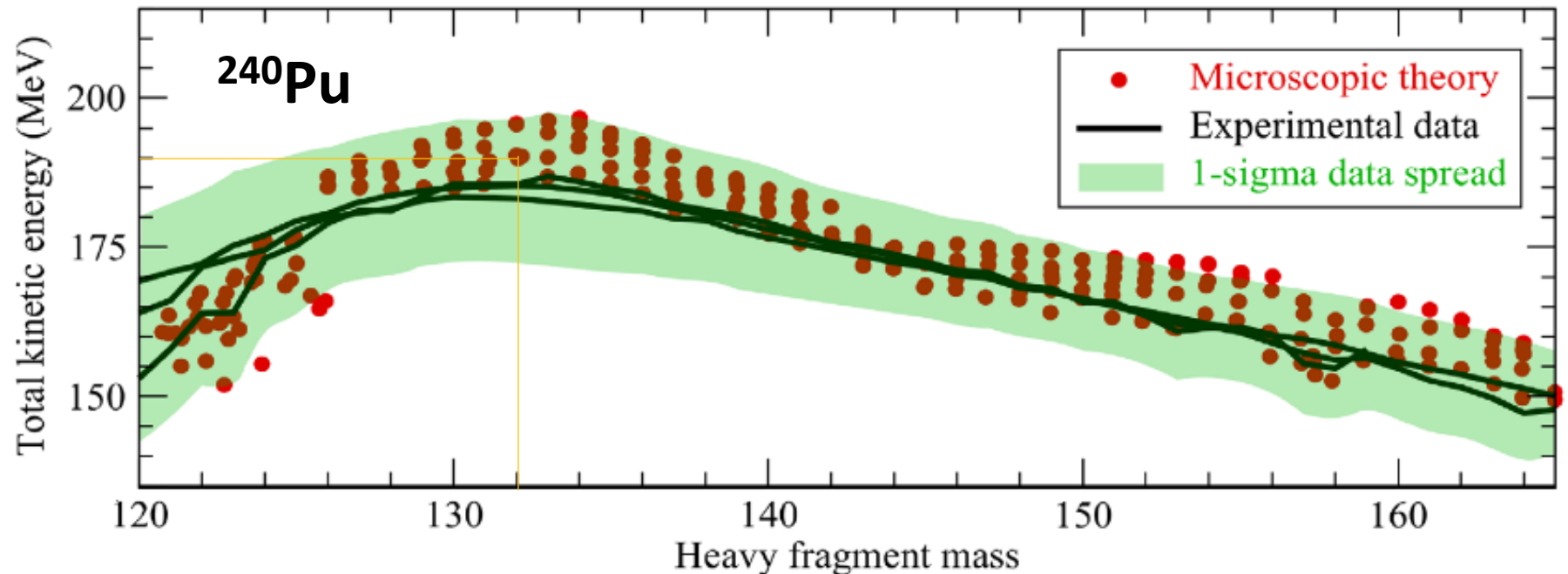
- $TKE = \frac{Z_1 Z_2 \alpha \hbar c}{1.8(A_1^{1/3} + A_2^{1/3})}$...where $r_0=1.8\text{fm}$ is used instead of 1.2 because of the strong deformation at scission

- E.g. ^{240}Pu

- $A/Z \approx 2.55$. If $Z_{high} = 50, N_{high} = 82$, then $Z_{low} = 94 - 50 = 44, N_{low} = 240 - 132 - 44 = 64$

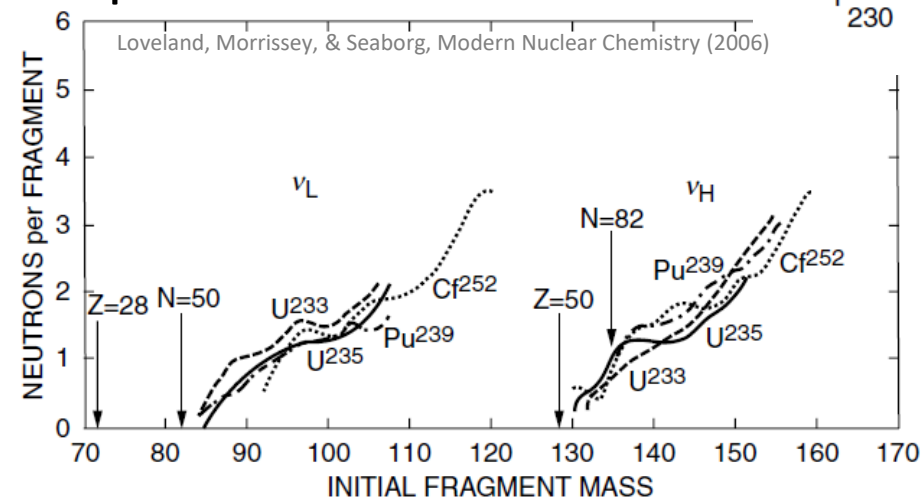
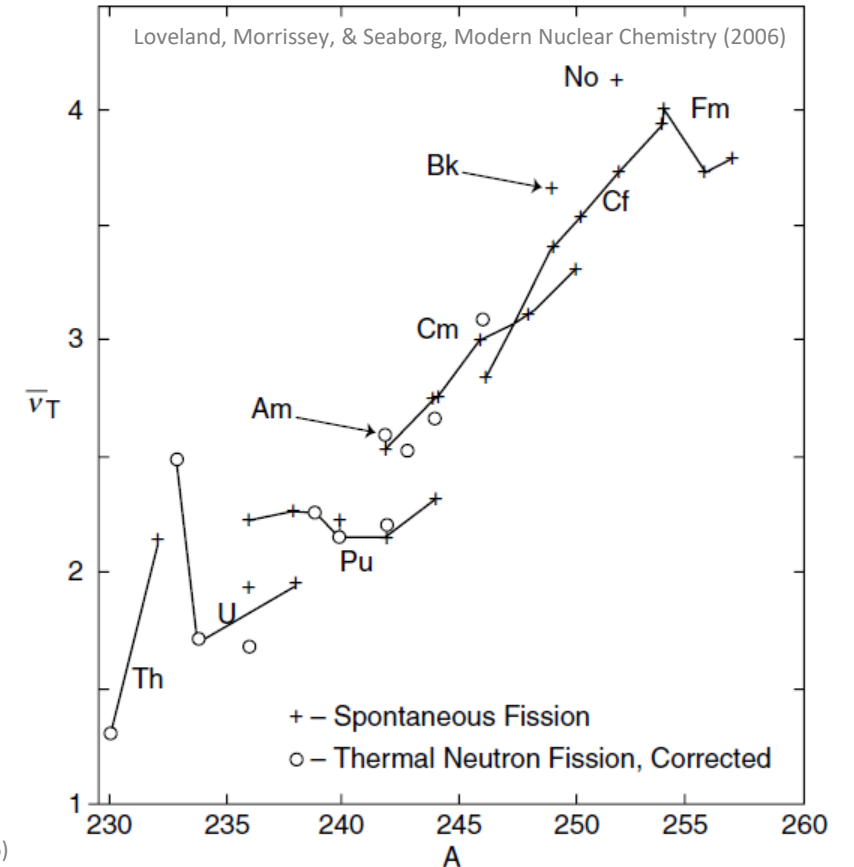
- $TKE = \frac{(50)(44)\alpha\hbar c}{1.8(132^{1/3} + 108^{1/3})} \approx 178\text{MeV}$

Schunk & Robledo, Rep. Prog. Phys. (2016)



Fragment de-excitation & neutron emission

- The difference between the parent and fragment binding energies ΔBE needs to be spent
- Typically the difference between ΔBE and TKE leaves several tens of MeV left over
- A rough estimate is that $\sim 10 MeV$ of excitation energy is removed from γ emission and β -decay and the associated ν emission
- The bulk of the difference is carried away by creating neutrons ($\sim 5.5 MeV$ to create 1 neutron), where a few neutrons are emitted per fission
- *Though the number depends on the fragment mass (due to shell closure effects)*



Fission neutrons kinetic energy

- Neutrons are emitted from the nearly fully accelerated fission fragments and so are emitted along the direction of the fragments
- The neutron kinetic energy (typically $\sim 2\text{MeV}$) is just described by a Maxwell-Boltzmann distribution,

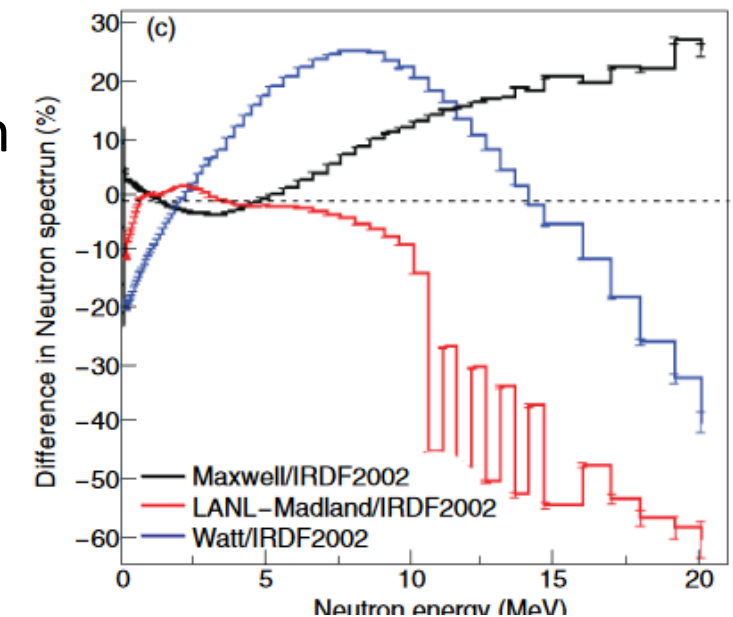
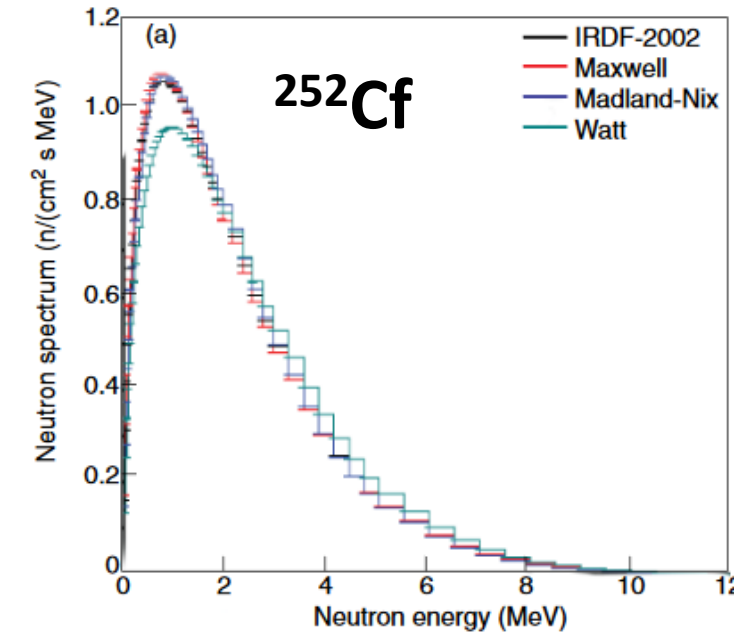
$$P(KE_n) = KE_n \exp(-E_n/k_B T)$$
- Transforming into the lab frame, the result is the Watt fission spectrum,

$$P(KE_n) = \exp(-KE_n/k_B T) \sinh\left(\sqrt{4E_n E_f / (k_B T)^2}\right), \text{ where } E_n$$

and E_f are the laboratory neutron and fission fragment energies in

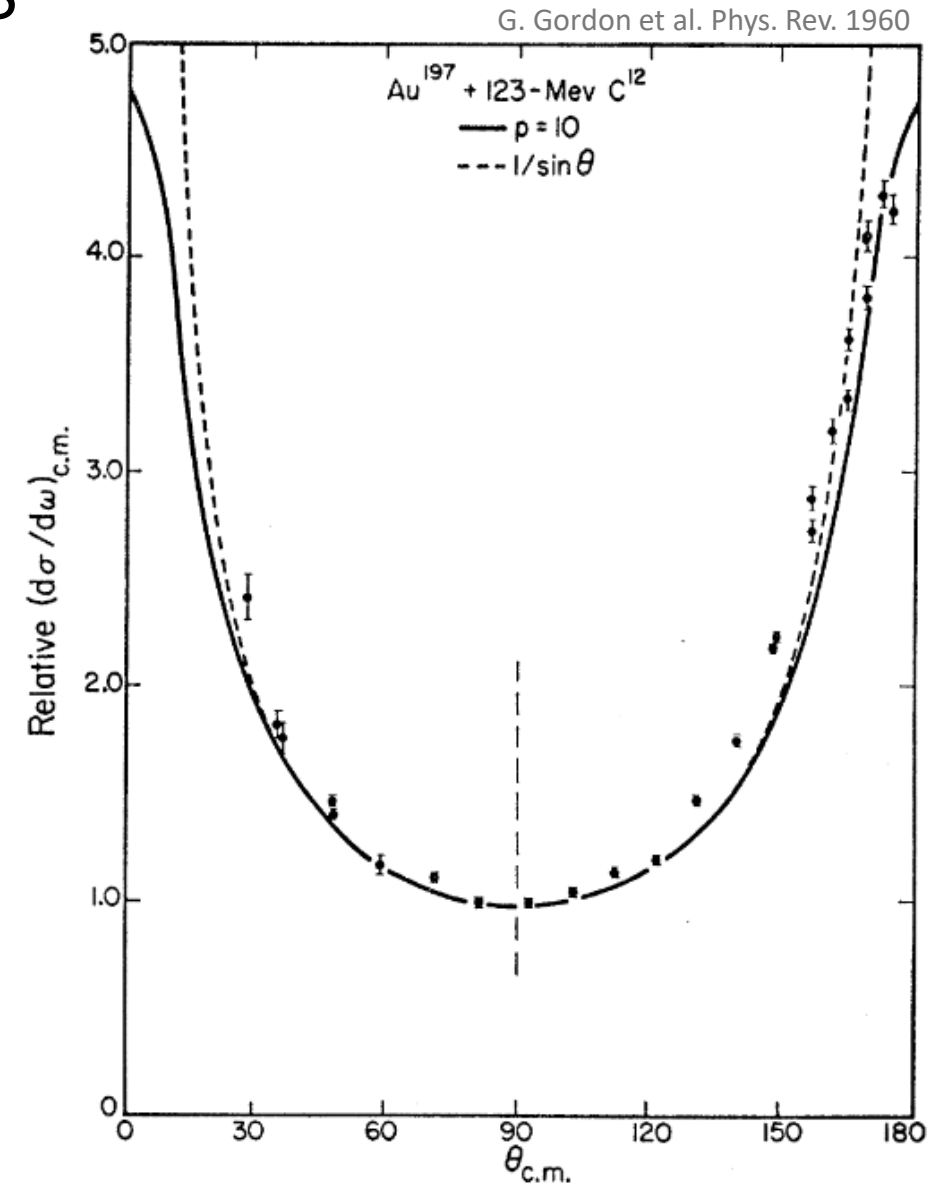
MeV/u and $T = \sqrt{\frac{8E^*}{A}}$ is the nuclear temperature

- Parameterized forms of the Watt fission spectrum, with the associated parameter values for a given fission fragment are available. E.g. in the appendix of the MCNP user manual



Fission fragment angular distributions

- Fission is slow enough that the nucleus reaches statistical equilibrium
- This results in an angular distribution of fission fragments that is symmetric about 90° in the center of mass, where 90° marks the plane perpendicular to the direction of motion of the fissioning system
- The formula for the angular distribution is complicated and related to the deformation of the fissioning nucleus
[See Loveland, Morrissey, & Seaborg]
- From momentum conservation, we know that the fragments themselves are emitted 180° apart (in the CM), which is handy for gating on fission events in reaction-induced fission studies



Further Reading

- Chapter 11: Modern Nuclear Chemistry (Loveland, Morrissey, Seaborg)
- Chapters 2 & 9 : Nuclear & Particle Physics (B.R. Martin)