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ATOMIC NUCLEI

NO PUBLISHER NECESSARY



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Preface

This reference was created for use in a graduate-level introductory nuclear physics course (PHYS 7501: Paricles & Nuclei 1) at Ohio University. The intended audience is advanced undergraduate and beginning graduate students who have not yet had a formal nuclear physics course. The format follows the tufte-book LATEXclass, found at https://tufte-latex.github.io/tufte-latex/. The format features wide margins to make room for reader notes and occasional side notes.

When I first taught nuclear physics, I had trouble finding a textbook that had adequate coverage of the field at the appropriate level. Most references tend to either emphasize arduous calculations or engage in copious hand-waving. My preference is to present derivations when they are enlightening, but to otherwise emphasize back-of-theenvelope calculations. Many books also choose to emphasize nuclear structure, reactions, radioactive decay, applications, or astrophysics, but few texts provide a beginning picture of all topics. I wanted something that covered all of these at an introductory level. This is partly due to convenience, we have one semester to cover it all at Ohio University, but also to emphasize inter-connectivity. My hope is that this book achieves these goals.

Of course, no book stands alone and this text is heavily inspired by many other books. Each chapter is peppered with references to the literature and is concluded with a set of reference material which I primarily relied upon for that chapter. Such reference material includes published texts as well as open-access references, including other instructors' course notes, databases, and manuals.

NOTE: THIS IS A LIVING DOCUMENT. Corrections, improvements, and additions are all anticipated. Please mind the version number (listed on the previous page) when referring to this text.

IF YOU CHOOSE TO USE THIS BOOK IN YOUR COURSE, please let me know. It helps provide impetus to keep things going. If you find problems, inaccuracies, or glaring omissions, please contact me and I can fix them.

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Introduction

The atomic nucleus, hereafter referred to as the nucleus, plays a central role in the universe as well as every day life. Though not the most fundamental unit of matter, nuclei generally defy description from more microscopic building blocks and are thus an important subject of study in their own right. Conceptually, we can divide the study of nuclei into four themes: How ARE THEY PUT TOGETHER?, HOW DO THEY FALL APART?, HOW DO THEY INTERACT?, and HOW ARE THEY MADE AND DETECTED?. These are the four organizing themes of this book.

Part 1: How are they put together?

1 General Nuclear Properties

The nucleus is a conglomeration of *nucleons*, which is the general name for protons and neutrons. A particular grouping of protons and neutrons, known as a *nuclide*, is referred to using the following notation: ${}^{A}S_{N}$. Here Z is the number of protons, N is the number of neutrons, A = Z + N is the number of nucleons, and S is the chemical element symbol from the periodic table, e.g. H for hydrogen. Since A and S contain all the information you need, it is more common to see the notation ${}^{A}S$, particularly in nuclear physics journals¹. For instance, the most common form of carbon (Z = 6) in the universe has N = 6and is thus written as 12 C. Conversationally, we read this as "carbon twelve".

Nuclides with the same Z but different N are referred to as *isotopes*². Nuclides with the same N but different Z are *isotones*. Nuclides with the same A are *isobars*. Some light nuclides have nicknames that provide a convenient short-hand due to their regular presence in astrophysics and applications:

${}^{A}\mathcal{S}$	Nickname	Symbol
	neutron	п
$^{1}\mathrm{H}$	proton	р
$^{2}\mathrm{H}$	deuteron	d
³ H	triton	t
³ He	helion ³	h
⁴ He	alpha	α

THE SIZE OF A NUCLEUS can be estimated via phenomenology. The mass is roughly

$$M(Z,A) = Au, \tag{1.1}$$

where $u = 931.494 \text{ MeV}/c^2 \approx 1.66 \times 10^{-24} \text{ g}$ is the atomic mass unit and *c* is the speed of light in vacuum. The atomic mass unit is defined⁴

¹ An *m* following *A* indicates an isomer, the name for a long-lived excited state (sometimes called metastable state) of a nucleus. E.g. ^{26m}Al from astrophysics or ^{99m}Tc from the medical isotope realm.

² Often *isotope* is used in lieu of *nuclide*.

³ Rarely used...but let's bring it back!

⁴ Using a relative definition for the atomic mass unit is a valuable tool for high-precision nuclear mass measurements [Scheidenberger et al., 2002].

such that $M(^{12}C) \equiv 12$ u. The charge radius of a nucleus is roughly

$$R(Z,A) = r_0 A^{1/3}, (1.2)$$

where $r_0 \approx 1.2$ fm. For the root-mean-square (RMS) radius, an additional factor of $\sqrt{3/5}$ is needed.

As such, an estimate of the density of a nucleus is

$$n = \frac{\text{#nucleons}}{V} = \frac{A}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi (1.2\text{fm})^3 A} \approx 0.14 \text{ nucleons/fm}^3.$$
(1.3)

Note that *this estimate is independent of A*! Therefore, the density of a nucleus is independent of its size, much the way that a liquid's density is independent of the size of a liquid drop. Partly inspired by this property, some basic (and surprisingly accurate) nuclear calculations are built on the liquid drop analogy [Gamow, 1930, Weizsäcker, 1935, Bohr, 1936, Bohr and Wheeler, 1939]. Inserting the nucleon mass and expressing Equation 1.3 in more familiar units, we find the nuclear density corresponds to 2×10^{14} g/cm³. To emphasize how staggering this is, note that such a density is comparable to stuffing 100 copies of the Great Pyramid of Giza into the volume of a grape. Why is terrestrial matter then not extremely dense? Atoms are mostly empty space, with atomic electrons residing out to ångström-scale distances relative to the few-femtometer nuclear radius.

NUCLEAR TRANSMUTATION, converting one or more nuclides into one or more other nuclides, obeys some basic conservation laws. We can determine if a few of these are satisfied by tallying up *A* and the electric charge *q* before and after the transmutation. *Charge conservation* demands that $\sum q_{after} = \sum q_{before}$, where *q* results from combining the electric charges of any protons, positrons, and electrons. *Baryon⁵ conservation* requires $\sum A_{before} = \sum A_{after}$. *Lepton⁶ number conservation* stipulates that $\sum L_{before} = \sum L_{after}$, where $L = n_{\ell} - n_{\overline{\ell}}$, with n_{ℓ} and $n_{\overline{\ell}}$ as the number of leptons and anti-leptons, respectively. Further rules that determine the possibility and likelihood of a transmutation involve energetic considerations and spin/parity selection rules that we will get to in due time. Nonetheless, the aforementioned conservation rules provide a handy way to know whether a proposed transmutation is sensible at first glance.

There are two types of nuclear transmutations: *decays* and *reactions* 7. A *decay* is when a nucleus ejects radiation, such as an α , β , or γ , or splits apart as in fission. The capture of an electron is also classified with decays, since that quantum mechanical formalism is more applicable. *Reactions* are when multiple reactants interact to create one or more products. For instance, heavy nucleus *X* might combine with light nucleus *a* to produce heavy nucleus *Y* and light nucleus *b*:

⁵ A composite particle made of an oddnumber of quarks; in our case, this is always 3.

⁶ A spin-½ elementary particle (i.e. no sub-components) that does not interact via the strong force. Here this will almost always be an electron e^- , positron e^+ , electron neutrino v_e , or electron antineutrino \overline{v}_e .

⁷ The first wittingly measured radioactive decay was *α*-decay from uranium in 1896 [Becquerel, 1896]. The first nuclear reaction intentionally made in the laboratory was ¹⁴N(*α*, *p*) in 1919 [Rutherford, 1935]. $X + a \rightarrow Y + b$. A more compact notation is typically used for convenience: X(a, b)Y, where the lighter reactant and product nuclides are placed within the parentheses. Since we can employ our conservation laws to determine what nuclide *Y* is if we know *X*, *a*, and *b*, it is common to see the notation X(a, b). For example, when ¹²C fuses with an α and emits a photon, we write ¹²C(α , γ)¹⁶O, or even ¹²C(α , γ). The latter is read as "carbon-twelve alpha gamma". If instead a proton were the light product, then we write ¹²C(α , p)¹⁵N, or ¹²C(α , p), read as "carbon-twelve alpha p".

NUCLEAR FORCES are the mechanisms for binding and transmutation of nuclei. Of the four fundamental forces in nature, gravitational,electromagnetic, weak, and strong, we are primarily concerned with the last three. The effective range and relative strength of the four forces span many orders of magnitude:

Force	Range (m)	Relative Strength	Force Carrier
Gravitational	∞	10^{-38}	Graviton
Weak	10^{-18}	10^{-5}	W^{\pm},Z^{0}
Electromagnetic	∞	$\alpha_{\mathrm{fs}} \approx 1/137$	Photon
Strong	10^{-15}	1	Gluon, Pion

The force range is a result of the Heisenberg uncertainty principle. The force carrier must momentarily be created to mediate the force, which briefly requires violating energy conservation by an energy increment $\Delta E \ge M_{\rm fc}c^2$, where $M_{\rm fc}$ is the mass of the force carrier. This violation is only allowed for the time increment Δt specified by the uncertainty principle, $\Delta E \Delta t \ge \hbar/2$, where \hbar is the reduced Planck constant. As such, the force carrier must be reabsorbed before $\Delta t \le \hbar/(2\Delta E)$ has elapsed. Since the force carrier cannot move faster than c, the furthest range the force carrier could travel is $R \le \Delta tc = \hbar c/(2\Delta E) = \hbar c/(2M_{\rm fc}c^2) = \hbar/(2M_{\rm fc}c)$. Since this is all a bit hand-wavy, we usually drop the factor of two and employ the definition $R \equiv \hbar/(M_{\rm fc}c)$.

We are now equipped to do order of magnitude estimates⁸. The graviton and photon are massless, so $R = \infty$ for their respective forces. For the weak force, $M_{\rm fc} \sim 100 \,{\rm GeV}/c^2$, so $R_{\rm weak} \sim 10^{-3}$ fm, suggesting this is something that takes place within a nucleon. The pion mass is roughly 100 MeV/ c^2 and so $R_{\rm strong} \sim 1$ fm, which is comparable to the nucleon size.

⁸ The key to these calculations is that $\hbar c \approx 197$ MeVfm. This is convenient because nuclear masses are generally available in units MeV/ c^2 .

THE NUCLEAR LANDSCAPE is typically depicted using a Segrè plot, more commonly known as the nuclear chart, or the chart of the nuclides, or sometimes the table of isotopes. Figure 1.1 is a typical example, with each box representing an individual nuclide, where the horizontal coordinate indicates N and the vertical coordinate Z. Orientation is often provided by indicating which rows and columns correspond to "magic" numbers of protons and neutrons, respectively. Magicity is an important quantity that will be discussed in detail later on. For now, suffice it to say that the magic nucleon numbers are N or Z = 8, 20, 28, 50, 82, 126. Figure 1.1 uses the box color to indicate the primary mode of radioactive decay for that nuclide. Black boxes do not undergo decay and are thus known as stable. For reasons that will be discussed shortly, the band of stable nuclides is known as the valley of stability. Isotopes with a relative neutron deficiency are known as proton-rich, whereas those with a relative neutron surplus are known as neutron-rich.



Figure 1.1: The chart of nuclides with magic numbers for *Z* and *N* indicated on the right and left-hand side of the colorful boxes, respectively. The color map is courtesy of Edward Simpson's The Colourful Nuclide Chart. A similar chart that is conveniently linked to a large number of nuclide properties is available at the National Nuclear Data Center (NNDC).

The nuclear chart is a powerful tool for comparing the evolution of properties across the nuclear landscape. For instance, Figure 1.2 contrasts radioactive decay half-lives, emphasizing the orders-of-magnitude range as well as the relative constancy across wide swaths of N and Z. Figure 1.3 highlights the gap between nuclides accessed by present nuclear physics facilities, near-future facilities, and the predicted extent of the nuclear landscape.

Predicting the extent of the nuclear landscape will be discussed in the following chapter. Nonetheless, it is interesting to consider how little we know about the full landscape at present. As of late 2019, \sim 3000 nuclides have been observed in the laboratory. However, depending on



Figure 1.2: Decay half-lives from Audi et al. [2017], displayed using The Colourful Nuclide Chart.



Figure 1.3: Nuclear mass measurement uncertainty anticipated for the Facility for Rare Isotope Beams (FRIB) relative to known nuclear masses and the twoneutron dripline predicted by Moller et al. [1995]. Based on Meisel [2016].

the choice of theoretical mass model, on the order of \sim 7000 nuclides are thought to be possible [Erler et al., 2012].

THE TERM "VALLEY OF STABILITY" is used due to the trend in nuclear binding energies across the landscape. Early mass spectrometry showed that nuclear masses are nearly integer multiples of the hydrogen-atom mass, which was known as the "whole number rule" [Aston, 1919]. However, high-precision work showed that deviations existed on the sub-percent level [Aston, 1924]. The deviation, shown in Figure 1.4, is known as the *mass defect*, which means that the nuclear mass is a little bit less than what you would obtain from summing the constituent nucleon masses.

The mass defect is a consequence of Einstein's postulate $E = mc^2$. The nucleons within a nucleus are bound together, which requires some binding energy. The nuclear binding energy *BE* is paid for via the reduced mass of the nucleus. That the conglomeration of bound nucleons is a lower-energy state relative to the same set of nucleons un-bound is the only reason nucleons cluster together to form nuclei.

The binding energy is defined as

$$BE(Z, N) \equiv Z(m_p + m_e) + Nm_n - M(Z, N),$$
 (1.4)

where $m_p = 938.272 \text{ MeV}/c^2$ is the proton mass, $m_e = 0.511 \text{ MeV}/c^2$ is the electron mass, is $m_n = 939.565 \text{ MeV}/c^2$ is the neutron mass, and M(Z, N) is the nuclear mass. It is just as common to instead see the <u>atomic</u> mass excess⁹ used:

$$ME(Z,N) \equiv M(Z,N) - (Z+N)u, \qquad (1.5)$$

where *u* is the previously encountered atomic mass unit. Regular evaluations of *ME* and *BE* are available from the Atomic Mass Data Center. Each evaluation, based on essentially all pertinent literature up to that point, is known as the Atomic Mass Evaluation (AME), e.g. [Wang et al., 2012, 2017].

The first trend we notice from Equation 1.4 is that $BE \propto A$, since each nucleon has its own binding bonus and more nucleons means a bigger bonus. To see more details, BE/A is used. Since the lowestenergy state is generally favored, all else being equal, it is informative to look at -BE/A. Figure 1.4 shows -BE/A near nickel, where there the "valley" is clearly visible. It is also clear that here $BE/A \sim 8$ MeV, which is also the case for most other nuclides with $A \gtrsim 30$. As we will see later, the slope of the valley walls directly impacts β -decay rates, and hence the longer name "valley of β -stability" is often used. Additionally, the binding energy features of the valley, sometimes called the *nuclear mass surface*, can provide a useful mental tool to estimate



Figure 1.4: Deviation of experimental masses in Wang et al. [2012] from the whole-number rule. To calculate: $\Delta M = [M(Z,N) - [ZM(^1H) + Nm_n]]/[ZM(^1H) + Nm_n].$

⁹ *ME* is used here, though arguably Δ is more common in the literature. The way I see it, we're already using *BE* for binding energy and Δ is overused as it is. Plus, we need a way to feel like rebels.



Figure 1.5: -BE/A in the nickel region, revealing the "valley" of β -stability. Data are from Wang et al. [2012].

which reaction types will be favored over others, e.g. (α, p) vs. (α, n) , for a particular nuclide.

The binding energy trend looks smooth to the point of being boring. What happens if we fit the surface with a smooth function¹⁰ and plot the residual? As shown in Figure 1.6, striking trends emerge near specific isotonic and isotopic chains. Namely, we see the emergence of the magic numbers, a first indication of what magicity implies.



¹⁰ Exactly which smooth function is discussed in the following chapter.

Figure 1.6: Absolute value of the difference between *ME* from Wang et al. [2012] and the liquid drop model *ME* in MeV. Based on Meisel [2015].

A NUCLIDE CAN EXIST IN ONE OF SEVERAL ENERGY STATES, where the lowest is the *ground state* and all others are *excited states*. The states represent possible configurations of the nucleons within the nucleus. The discrete structure is a feature of the quantum mechanical nature of the nucleus, i.e. a continuum of internal energies is not possible. This is a familiar feature from our knowledge of electron orbitals and the resultant chemical similarity of elements in a column of the periodic table.

Much like electrons in the atom, nucleons have distinct orbitals which they can occupy, known as *single particle states*. Filling these in the most orderly way possible results in the ground state. Exciting one or more nucleons to higher orbitals results in an excited state. The lifetime of a ground state depends on the properties of a nucleus relative to neighboring nuclei in the nuclear landscape. The lifetime of an excited state additionally depends on the properties of other states within the same nucleus. Each of these topics are covered in the second part of this book.

From Figure 1.7 it is apparent that the plot of energy states in a nucleus, known as a level scheme, can get pretty complicated. A few



Figure 1.7: Level scheme and known γ decays for ²⁸Si, generated using David Radford's RadWare with data from the NNDC Evaluated Nuclear Structure Data File (ENSDF).

general features are worth noting here. Each state has an important set of properties: *excitation energy*, the energy surplus over the ground state; *width*, which is related to the time it takes for that state to decay; *spin*, which is the intrinsic angular momentum; and *parity*, which describes the symmetry of the wave function representing that state. Note that the number of states increases dramatically with excitation energy. This *nuclear level density* increases exponentially with excitation energy, which will be explained later. The only reason the density of levels begins to thin-out at higher excitation energies in Figure 1.7 is that the experimental techniques used to identify levels tend to struggle when many neighboring levels exist.

FOR HISTORICAL PERSPECTIVE, the following is a selected list of major developments from roughly the first half-century of nuclear physics¹¹:

- 1896: Radioactivity discovered¹² [Becquerel, 1896]
- 1905: Mass-energy equivalence proposed [Einstein, 1905]
- 1911: Nuclear model of atom proposed [Rutherford, 1911]
- 1913: Mass spectrometry invented¹³ [Thomson, 1913]
- 1919: Isotope existence discovered [Aston, 1919]
- 1920: Nuclear transmutation proposed to power the sun [Edding-ton, 1920]
- 1928: *α*-decay theory¹⁴ [Gamow, 1928]
- 1929: Liquid drop model of nucleus proposed [Gamow, 1930]
- 1932: Neutron discovered [Chadwick, 1932]
- 1932: Nuclei proposed to be interacting nucleons¹⁵ [Heisenberg, 1932]
- 1932: Nuclear transmutation using a particle accelerator¹⁶ [Cock-croft et al., 1932]
- 1934: Theory of β -decay [Fermi, 1934]¹⁷
- 1935: Semi-empirical mass formula developed¹⁸ [Weizsäcker, 1935]
- 1936: Fusion theory¹⁹ [Bohr, 1936]
- 1939: Fission measured [Hahn and Strassmann, 1939, Abelson, 1939]
- 1939: Fission theory²⁰ [Bohr and Wheeler, 1939, Meitner and Frisch, 1939]
- 1949: Magic number explanation [Mayer, 1949, Haxel et al., 1949]
- 1951/1952: Statistical nuclear reaction formalism [Wolfenstein, 1951, Hauser and Feshbach, 1952]
- 1957: Comprehensive theory of nucleosynthesis [Burbidge et al., 1957, Cameron, 1957]
- 1957: Parity violation discovered [Wu et al., 1957]

General References for This Chapter:

- Chapters 1 & 2: Loveland et al. [2006]
- Chapters 1 & 2: Martin [2006]
- Chapter 1: Brown [2005]

¹¹ Of course many exciting things have happened since and they will be covered throughout the book. Also, there's no need to insult the living whose accomplishments would inadvertently (or deliberately!) be left out.

¹² Due to an accident. Uranium salts were placed next to photographic plates.
¹³ Discovery of the proton.

¹⁴ This is the first application of quantum mechanics to a real world problem.

¹⁵ Based on the discovery of the neutron.

^{16 7}Li $(p, \alpha)\alpha$.

¹⁷ This paper was rejected from Nature.

- ¹⁸ Based on the liquid drop model.
- ¹⁹ Based on the liquid drop model.

²⁰ Based on the liquid drop model.

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