Tuesday February 11

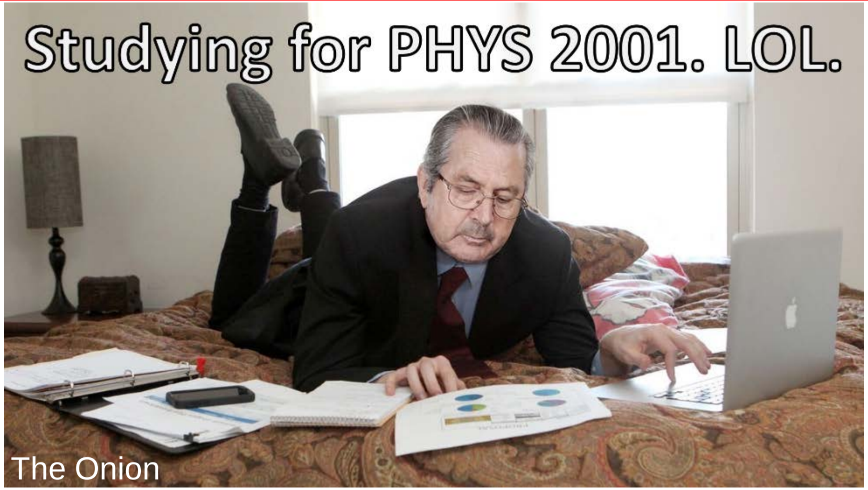
Topics for this Lecture:

Forces: Friction on Planes,
Tension With Pulleys

- *Write these equations in your notes if they're not already there.*
- *You will want them for Exam 1 & the Final.*

$$F_{\text{friction}} = \mu_{\textbf{kinetic}} F_{\text{normal}}$$

$$F_{\text{friction}} \leq \mu_{\textbf{static}} F_{\text{normal}}$$



- Assignment 5 due Friday
  Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 227, Mon & Tue 5:20-6:10pm
  & Morton 235 Thurs. 5:20-7:10pm
- Review session, Sunday Feb 16, 6-9pm this room
- Office Hours: 204 EAL, Tue 11am-Noon
  or by appointment (meisel@ohio.edu)
- **Exam Monday February 17.**
  Morton 201   7:15-9:15PM
  - Email me ASAP if you have a class conflict or need special accommodations through AS
  - ***Study!***

What is the magnitude of the upward force of the table on the box?

(A) 100N

(B) 100N + F cos(θ)

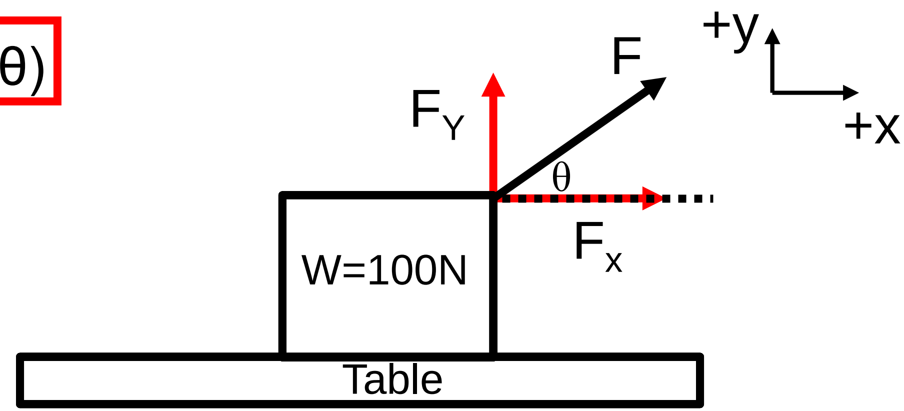
(C) 100N - F cos(θ)

(D) 100N + F sin(θ)

(E) 100N - F sin(θ)

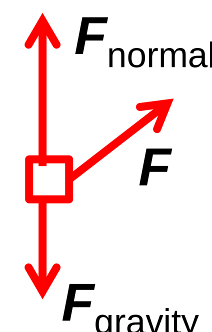
(F) F cos(θ)

(G) F sin(θ)



Solution:

1. Note the box is in equilibrium with regards to vertical motion (i.e. it is not moving vertically)
2. Break the pulling force ***F*** into components
   1. $F_x = F\cos(\theta)$; $F_y = F\sin(\theta)$
3. Use vertical equilibrium to find the table's normal force:
   1. $\sum F_y = ma_y = 0 = F_{\text{normal}} + F_y - F_{\text{gravity}} = F_{\text{normal}} + F_y - W$
   2. $0 = F_{\text{normal}} + F_y - W = F_{\text{normal}} + F\sin(\theta) - W$
   3. $F_{\text{normal}} = W - F\sin(\theta) = 100\text{N} - F\sin(\theta)$

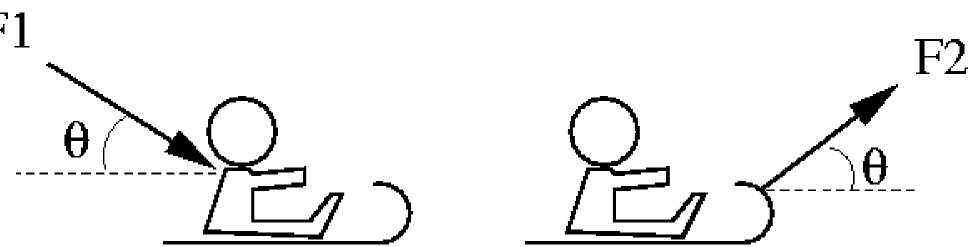


Consider moving a person on a sled,
where friction between the bottom of the sled and the ground is NOT negligible.
You can either push forward and down at an angle θ
or pull up and forward at the same angle.
If $F_1=F_2=F$ and the angles are the same, which situation has the greater acceleration?

(A) pushing

(B) pulling

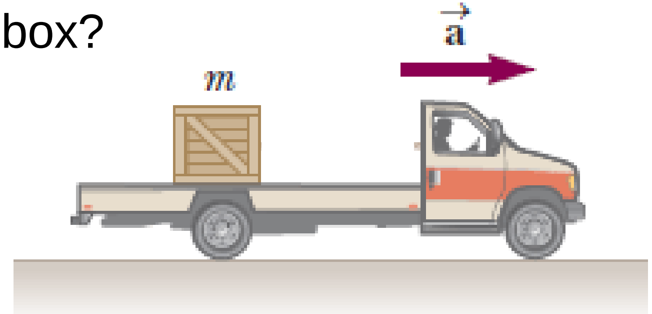
(C) both are equal



Find $\boldsymbol{F}_{net}$ in order to determine $\boldsymbol{a}$:

1. $\Sigma F_x = F_{x,net} = ma_x$
2. $a_x = F_{x,net}/m$ …so which situation has the largest $F_{x,net}$?
3. $F_{x,net} = F\cos(\theta) - F_{friction}$
4. So the situation with the smallest $F_{friction}$ will have a larger acceleration
5. $F_{friction} = \mu F_{normal}$
6. For pushing: $F_{normal} = W+F\sin(\theta)$ … For pulling: $F_{normal} = W-F\sin(\theta)$
7. F_{normal} is smaller for pulling, so acceleration will be greater

A box with a mass of 5 kg is sitting on the flat bed of a truck, but is not tied down. The truck accelerates at $2 m/s^2$, as does the box (so it's not slipping). The coefficients of friction between the box and the bed of the truck are μ_s=0.6 and μ_k=0.4.
What is the magnitude of the frictional force acting on the box?

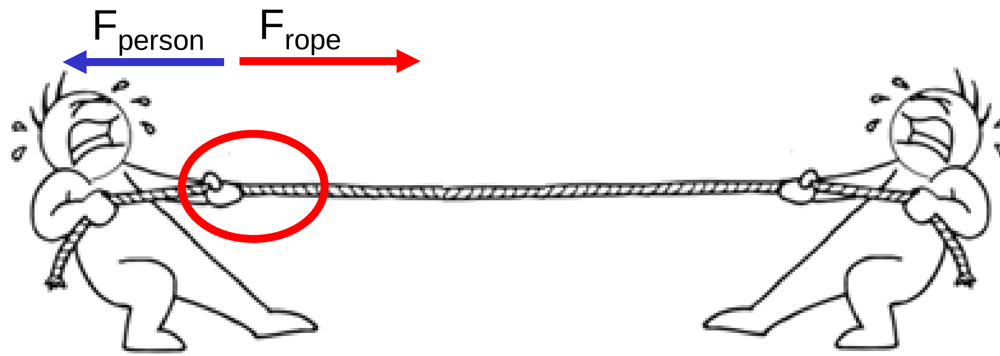


(A) 2 N
(B) 49 N
(C) 10 N
(D) 16 N
(E) 20N
(F) 29 N

<u>Does static friction apply? If so, how hard does it have to push to maintain ***a***?</u>:

1. $F_{friction} = \mu F_{normal}$
2. $F_{normal} = mg = (5kg)(9.8m/s^2) = 49N$
3. So: $F_{f,static}$ could push as hard as $F_{f,static} \leq \mu_s F_{normal} = (0.6)*(49N) = 29.4N$
4. But, $F_{f,static}$ only reacts as hard as it has to in order to maintain ***a***
5. $F_{net} = ma = (5kg)(2m/s^2) = 10N$ …which is less than the maximum $F_{f,static}$
6. So, $F_{f,static}$ will only match the applied action force (from F_{net}).
7. Meaning: $F_{f,static} = 10N$

Two people pull on opposite ends of a massless rope.
Each pulls with a force of 40N.
What is the tension in the rope?



(A) 0N (B) 20N

(C) 40N (D) 80N

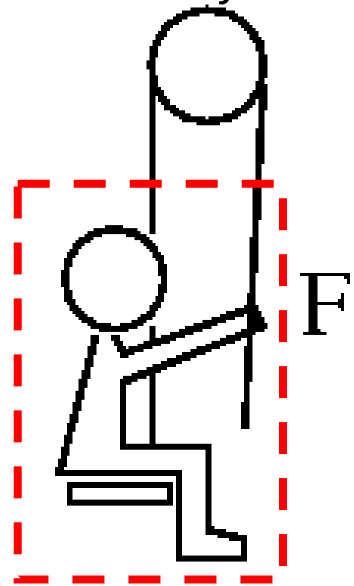
- Might be counterintuitive, so don’t rely on intuition!
  Stick to your diagrams & equations!
- Consider F=ma
  - Rope isn’t accelerating, so net forces must be balanced.
  - Force of person on rope must match force of rope on the person!
  - Tension is a reaction force.

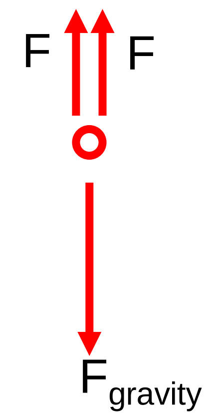
“Sneaky little hobbitses…”

“… wicked, tricksy, false!”

A person who weighs 800N is sitting on a chair that weighs10N.
The chair is supported by a rope over a pulley.
The person pulls down on the rope with a force of F to support the total weight.
What force, F, is required to hold them and the chair stationary?

(A) 400N	**(B) 405N**	(C) 800N
(D) 810N	(E) 1600N	(F) 1620N





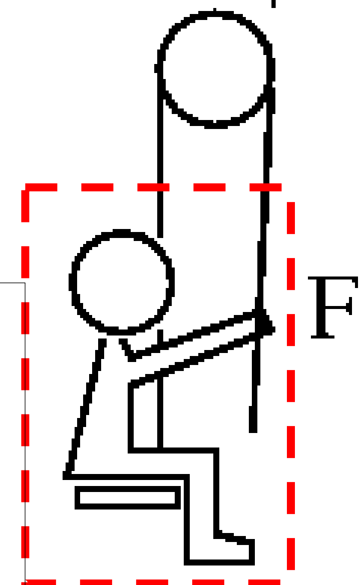
1. Pick your system & draw the forces
2. Consider that F=ma. Here a = 0.
3. Note that pulleys re-direct the tension, but maintain the tension magnitude. Therefore, the upward force is 2*F.
4. $F_{net} = \sum F = 2F - F_g = ma = 0$
5. $2F - F_g = 0$
6. 2F = W = 800N + 10N
7. F = (810N)/2 = 405N

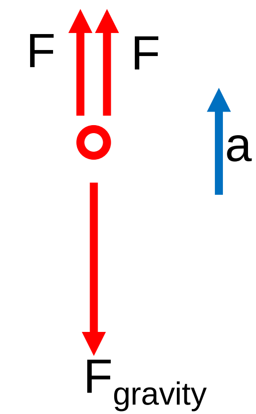
The force-multiplying power of a pulley is often referred to as its "mechanical advantage". In reality it is reduced somewhat by the pulley friction.

A person who weighs 800N is sitting on a chair that weighs10N.
The chair is supported by a rope over a pulley.
The person pulls down on the rope with a force of F.
What force, F, is required for them and the chair to accelerate upwards at 0.5 m/s^2?

(A) 26N (B) 405N (C) 362N
(D) 810N **(E) 426N** (F) 852N

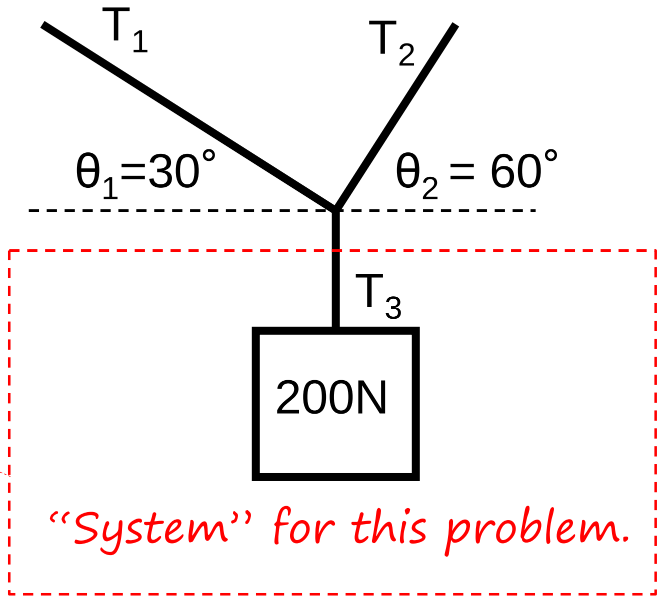
1. Pick your system & draw the forces.
2. Consider that F=ma. Here a = 0.5m/s^2.
3. Note that pulleys re-direct the tension, but maintain the tension magnitude. Therefore, the upward force is 2*F.
4. $F_{net} = \sum F = 2F - F_g = ma$
5. $2F = ma + F_g = ma + W \rightarrow \quad F = (ma + W)/2$
6. $W = mg = m(9.8 m/s^2) \rightarrow m = (810N)/(9.8m/s^2)$
7. $F = \frac{1}{2}\left((810N)\frac{0.5\frac{m}{s^2}}{9.8\frac{m}{s^2}} + 810N\right) \approx 426\text{N}$





A 200-N box is hanging from a rope.
Two ropes attach the box to the ceiling at the angles given.
What is the tension in rope 3?

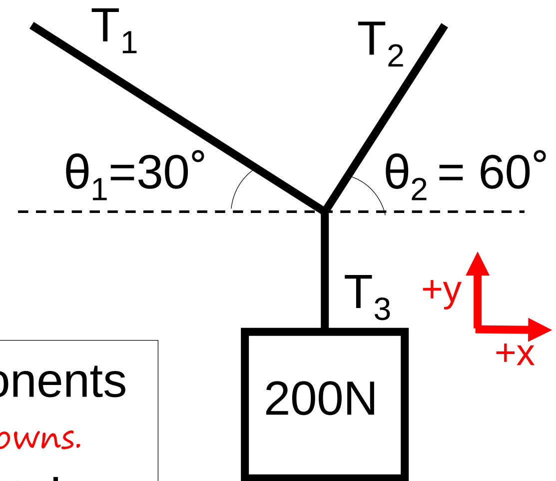
(A) 50 N
(B) 86 N
(C) 100 N
(D) 136 N
(E) 173 N
(F) 200 N



1. Pick your system: Box + rope 3.
2. Consider that F=ma. Here a = 0 m/s^2.
3. $F_{net} = \sum F = T_3 - F_g = 0$
4. $T_3 = Fg = mg = W = 200N$

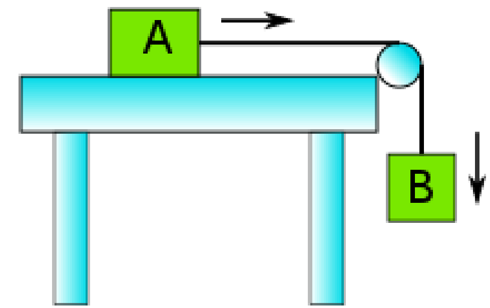
A 200-N box is hanging from a rope.
Two ropes attach the box to the ceiling at the angles given.
What are the tensions in ropes 1 and 2?

(A) 100N, 100N (B) 50N, 150N (C) 173N, 100N
(D) 150N, 50N (E) 100N, 173N (F) 200N, 200N



1. Need to break forces into components to find X & Y components two get 2 equations for T_1 & T_2. *Need 2 equations to solve 2 unknowns.*
2. Must use F=ma for X-component and Y-component separately
   1. $\sum F_x = ma_x = 0 = T_2\cos(\theta_2) - T_1\cos(\theta_1)$
   2. $\sum F_y = ma_y = 0 = T_2\sin(\theta_2) + T_1\sin(\theta_1) - T_3$
3. From (2.1): $T_2 = T_1*(\cos(\theta_1)/\cos(\theta_2))$
4. From (2.2): $T_3 = T_2\sin(\theta_2) + T_1\sin(\theta_1)$
   1. Use (3) in (4): $T3 = T_1*(\cos(\theta_1)/\cos(\theta_2))*\sin(\theta_2) + T_1\sin(\theta_1)$
   2. Therefore: $T_1 = T_3/[\sin(\theta_2)*\{\cos(\theta_1)/\cos(\theta_2)\} + \sin(\theta_1)]$
   3. $T_1 = (200N)/[0.866*\{0.866/0.5\} + 0.5] = (200N)/2 = 100N$
5. Use (4.3) in (3): $T_2 = (100N)*\{0.866/0.5\} = 173N$

Block B of mass 1.5 kg is accelerating downward at a rate of 3.0 m/s^2.
Block A is connected by a massless string.
There is <u>no friction</u> between Block A and the table.
What is the tension in the string?

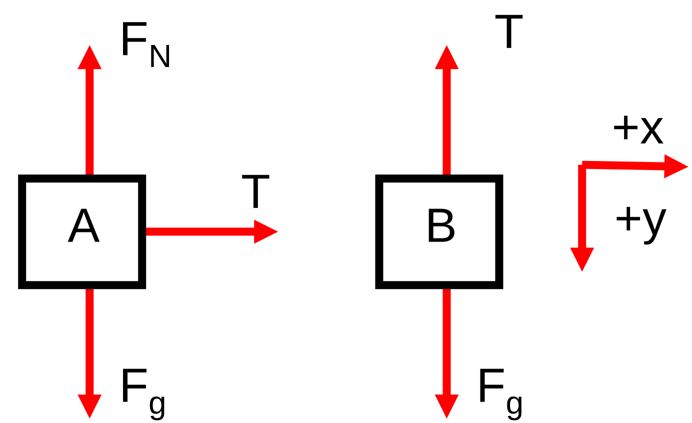


(A) 0 N (B) 1.5 N (C) 3.0 N (D) 4.5 N
(E) 8.4 N **(F) 10.2 N** (G) 14.7 N (H) 19.2 N

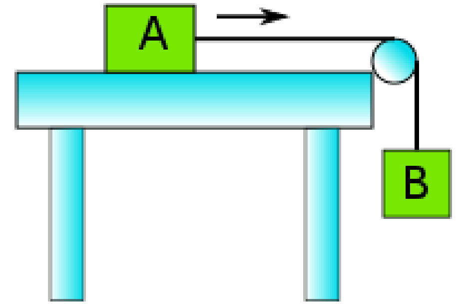
1. The string is massless, so the pulley just re-directs tension.
   Also A has no friction, so is not resisting the vertical acceleration.
   Therefore, just focus on Block B.

2. $\sum F_y = ma_y$
   1. $F_g - T = ma$
   2. $T = F_g - ma = mg + ma = m(a-g)$
   3. $T = (1.5\text{kg})(9.8\text{m/s}^2 - 3.0\text{m/s}^2)$
   4. $T = 10.2\text{N}$

Here we subtract tension from gravity because we defined downward to be +y!

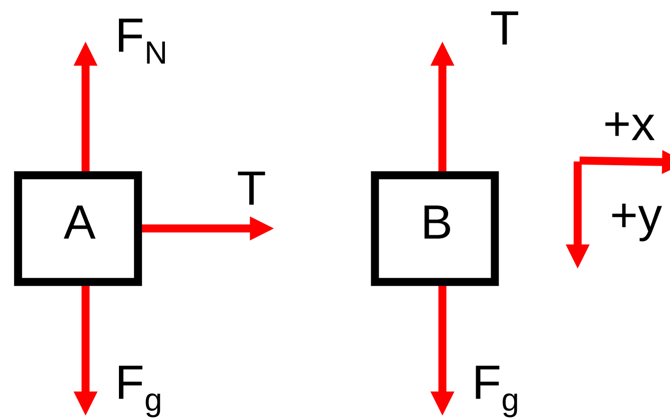


Block B of mass 1.5 kg is accelerating downward at a rate of 3.0 m/s^2.
Block A is connected by a massless string.
There is <u>no friction</u> between Block A and the table.
What is the mass of Block A?



(A) 3.4 kg (B) 1.5 kg (C) 6.4 kg (D) 4.5 kg

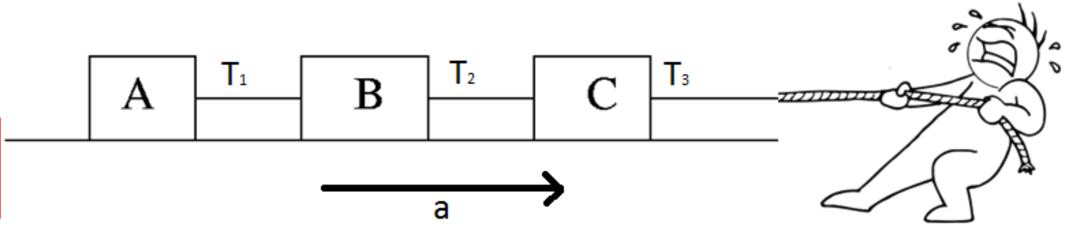
1. The system is just block A and the tension of the string on it.
2. The pulley just redirects the tension, so we can use the tension we just found for F.
3. F=ma
4. m = F/a = T/a = (10.2N)/(3.0m/s^2) = 3.4kg



Three boxes are accelerating to the right at a rate of $2.0 m/s^2$.
The mass of block A is 2kg. The mass of block B is 1kg.
The mass of block C is 2kg. The friction between the blocks and the ground is described by coefficients $\mu_{Static} = 0.45$ and $\mu_{Kinetic} = 0.35$.
What is the tension in rope 1 (T_1)?

(A) 5.43N (B) 4.41N (C) 12.8N

(D) 4.00N (E)10.0N (F) 10.9N



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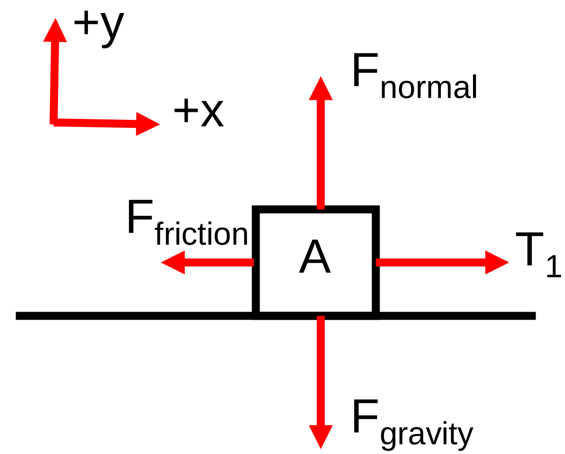
1. Focus! Your system is just Block A and Rope 1.
2. $\sum F = T_1 - F_{friction} = ma$
3. $T_1 = ma + F_{friction} = ma + \mu F_{normal}$
4. Here, only vertical downward force is from the Weight. So your normal force (which is a reaction force) will be equal in magnitude & opposite in direction to this.
   $F_{normal} = F_{gravity} = mg$
5. $T_1 = ma + \mu F_{normal} = ma + \mu mg = m(a + \mu g)$
   $= 2kg*(2.0 m/s^2 + 0.35*(9.8 m/s^2)) \approx 10.9\ N$

Use kinetic friction because the block is moving.

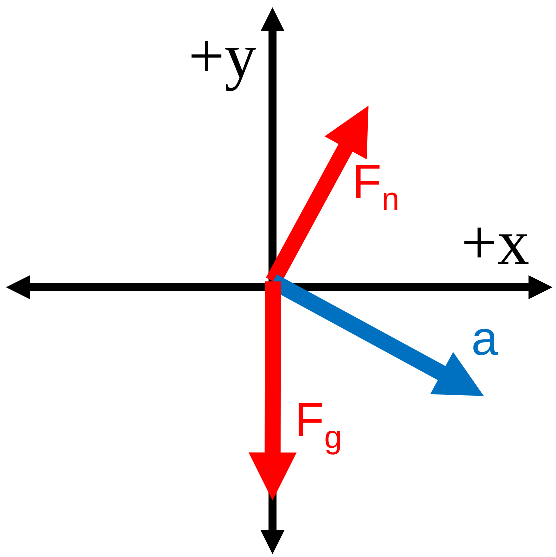


A note on coordinate axes:

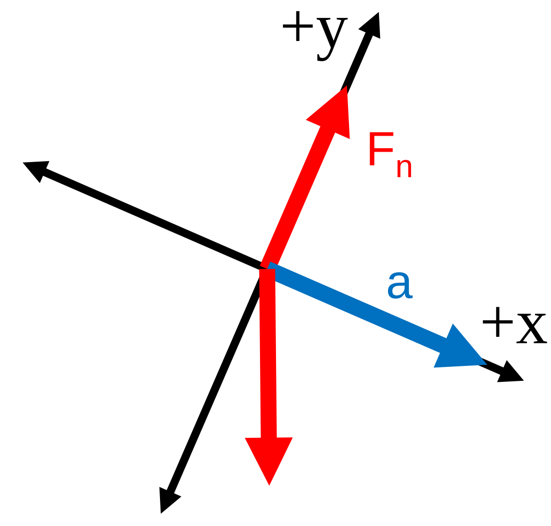
- Coordinate axes are your friend!
- Axes are an artificial constraint you place on the world to make the math used to describe your situation as easy as possible.
- You can orient axes however you want.
- The only rule is that they must be perpendicular to each other.

<u>Example:</u>

Consider a block sliding down a frictionless ramp.

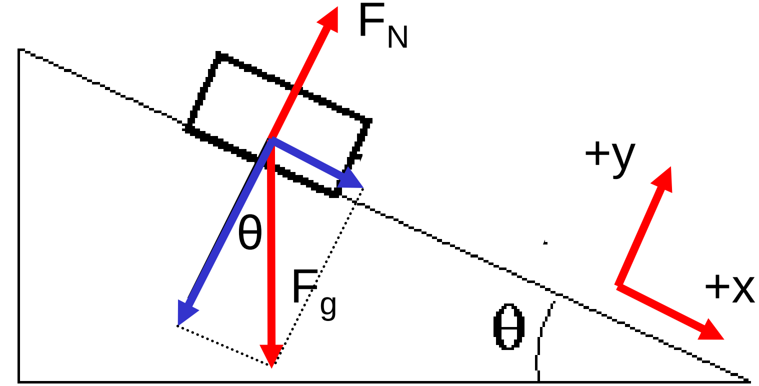


OK choice.



More convenient choice.

A block on an incline has a weight of 2.0N. The incline is at an angle θ of 30º.
What is the component of the force due to gravity in the x direction, with x as defined here?



(A) 0.5 N

(B) 1.0 N

(C) 1.7 N

(D) 2.0 N

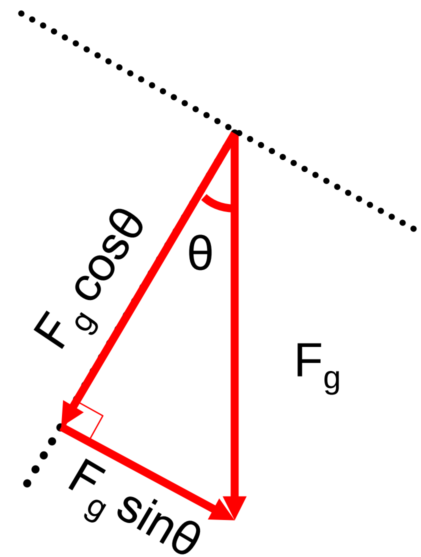
(E) 2.3 N

(F) 4 N

For a Weight of 2.0N, using **SOH**CAHTOA, the component parallel to the incline would be:

$$F_g \sin\theta = (2.0\text{N})*\sin(30^{o})$$

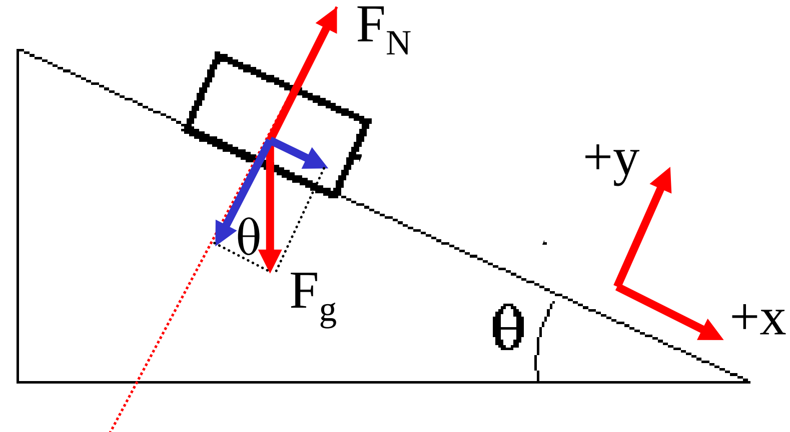
$$= 1.0 \text{ N}$$



Comments on problems with inclines:

Why tilt the coordinate axes?

- Gravity points down, but can get parallel and perpendicular components.
- The normal force is perpendicular to the surface.
- Friction is parallel to the surface.
- So, *tilting axes makes 2/3 forces along an axis*.



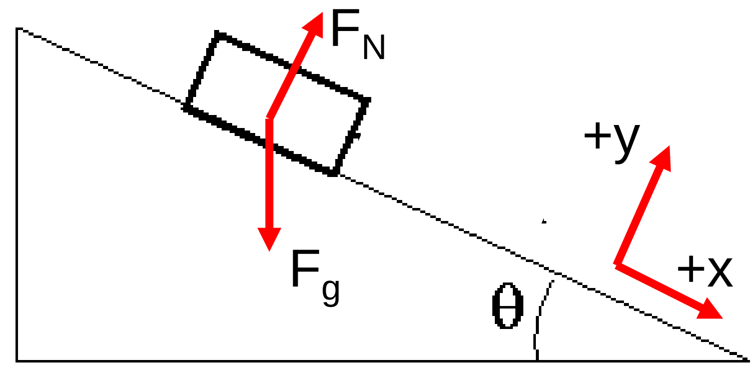
How does mass affect motion on incline?

- ***IF*** gravity & friction are the only forces,
- Then all forces involved are scaled by the same mass:
  - $F_{gravity,x} = m*g*\sin(\theta)$
  - $F_{friction} = \mu F_{normal} = \mu F_{gravity,y} = \mu*m*g*\cos(\theta)$
- So acceleration: $F = \cancel{m}a = F_{gravity,x} - F_{friction} = \cancel{m}*g*\sin(\theta) - \mu*\cancel{m}*g*\cos(\theta)$
- Therefore, for this situation: $a = g*[\sin(\theta) - \mu*\cos(\theta)]$

If only gravity & friction are involved, then motion on a plane is independent of an object's mass.

Suppose you increase the angle θ.
What happens to the x component of F_g and the normal force F_N?

(A) $F_{g,x}$ increases; F_N increases
(B) $F_{g,x}$ increases; F_N same
(C) $F_{g,x}$ increases; F_N decreases
(D) $F_{g,x}$ decreases; F_N increases
(E) $F_{g,x}$ decreases; F_N same
(F) $F_{g,x}$ decreases; F_N decreases
(G) $F_{g,x}$ same; F_N increases
(H) $F_{g,x}$ same; F_N same
(I) $F_{g,x}$ same; F_N decreases



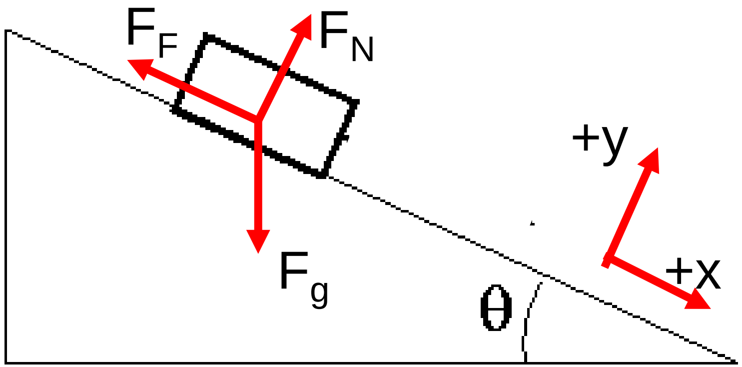
<u>Math Solution:</u>

- The normal force is a reaction force to the opposing perpendicular force.
  - $|F_N| = |F_{g,y}| = mg*\cos(\theta)$
  - As $\theta \rightarrow 90°$, $\cos(\theta) \rightarrow 0$.
- The x-component of gravity, from SOHCAHTOA, is:
  - $|F_{g,x}| = mg*\sin(\theta)$
  - As $\theta \rightarrow 90°$, $\sin(\theta) \rightarrow 1$.

<u>Logic Solution:</u>

- For $\theta=90°$, just free-fall:
  - $F_{gravity,x}$ pointing straight down & so maximized
  - Not pressing against a surface and so no normal force
- For $\theta=0°$, just sitting on a plane:
  - No horizontal gravity
  - Gravity fully vertical, so F_{normal} maximized

A 30kg (294N) crate is sliding down an incline at an angle 30° below the horizontal. The kinetic coefficient of friction is 0.3 between the crate and the ramp.
What is the acceleration of the crate along the ramp?



(A) $9.80 m/s^2$ (B) $8.49 m/s^2$
(C) $4.90 m/s^2$ **(D) $2.35\ m/s^2$**
(E) $2.54 m/s^2$ (F) $0.00 m/s^2$

Solution:

1. Draw & choose +x to be along the ramp. +y to be perpendicular to ramp.
2. Want a_x, so need $F_{net,x}$ in order to use $\sum F_x = ma_x$.
   1. $\sum F_x = F_{gravity,x} - F_{friction}$
   2. From SOHCAHTOA: $F_{gravity,x} = m*g*\sin(\theta)$
   3. But $F_{friction} = \mu F_{normal}$ …so we need the normal force (which is in +y)
3. The normal force is a reaction force which opposes that are perpendicular to & toward the surface (here just the y-component of the weight):
   1. $|F_{normal}| = |F_{gravity,y}| = m*g*\cos(\theta)$
   2. Therefore, $F_{friction} = \mu*m*g*\cos(\theta)$
4. Going back to F=ma …then a = F/m.
   1. $a = (Fg,x - Ff)/m = (m*g*\sin(\theta) - \mu*m*g*\cos(\theta))/m$
   2. $a = g*\{\sin(\theta) - \mu*\cos(\theta)\} = (9.8 m/s^2)\{0.5 - (0.3)*0.866\} = 2.35 m/s^2$

*As promised earlier.
Mass doesn't matter here.*

A block with a weight of 10N is sitting at rest on an incline which is tilted at an angle of 30°. The force of friction is 5.0N. What is the net force acting on the block?

(1) 0 N

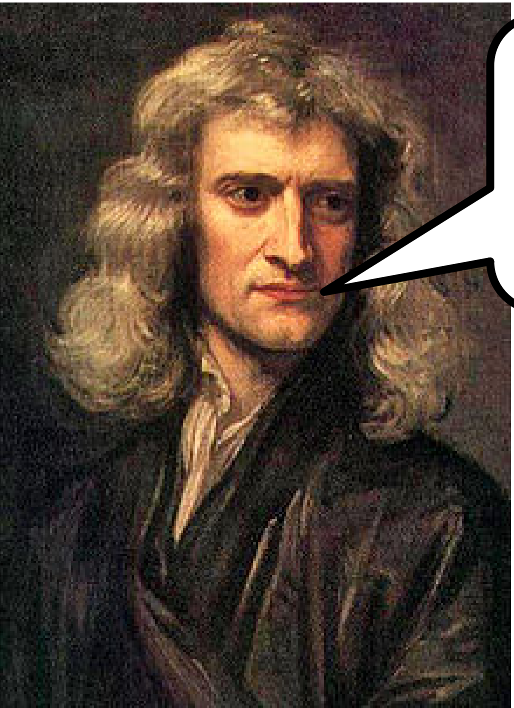
(2) 5N down the incline

(3) 5N up the incline

(4) 10N straight down

(5) 5N straight up

(6) 15N straight down



A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity, ***unless*** acted upon by a net external force.

"At rest" = no acceleration = no net force.