Tuesday February 12

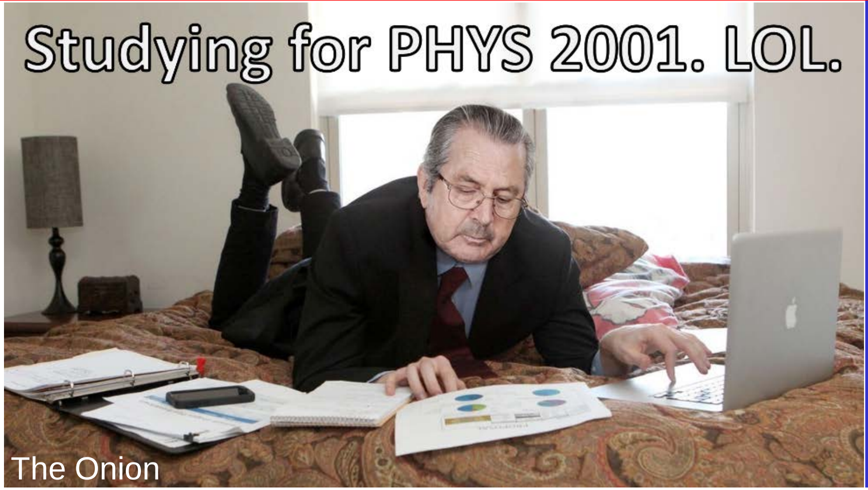
Topics for this Lecture:

Forces: Friction on Planes,
Tension With Pulleys

- *Write these equations in your notes if they're not already there.*
- *You will want them for Exam 1 & the Final.*

$$F_{\text{friction}} = \mu_{\textbf{kinetic}} F_{\text{normal}}$$

$$F_{\text{friction}} \leq \mu_{\textbf{static}} F_{\text{normal}}$$



- Assignment 5 due Friday
  …like almost every Friday
- Pre-class due 15min before class
  …like every class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 226, Tu&Th 6:20-6:10pm
  & Morton 102 Wed 6:20-8:10pm
- Office Hours: 204 EAL, 3-4pm Thurs
  or by appointment (meisel@ohio.edu)
- **Exam Monday February 18.**
  Morton 201 7:15-9:15PM
  - Email me ASAP if you have a class conflict or need special accommodations through AS
  - ***Study!***

What is the magnitude of the upward force of the table on the box?

(A) 100N

(B) 100N + F cos(θ)

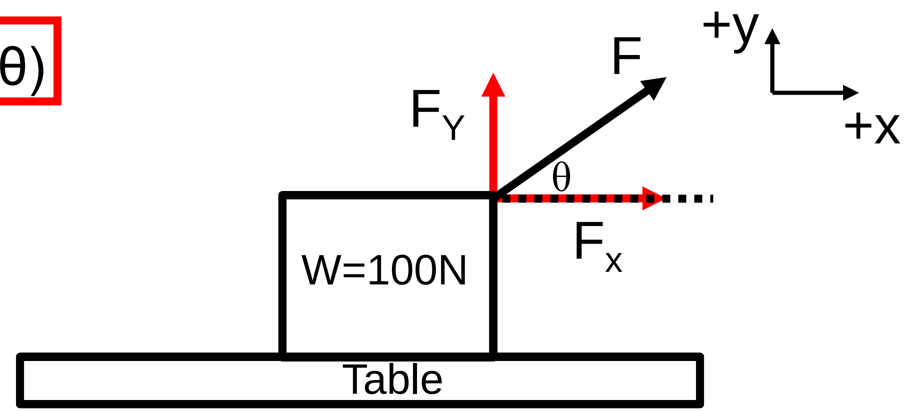
(C) 100N - F cos(θ)

(D) 100N + F sin(θ)

(E) 100N - F sin(θ)

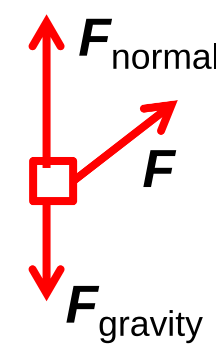
(F) F cos(θ)

(G) F sin(θ)



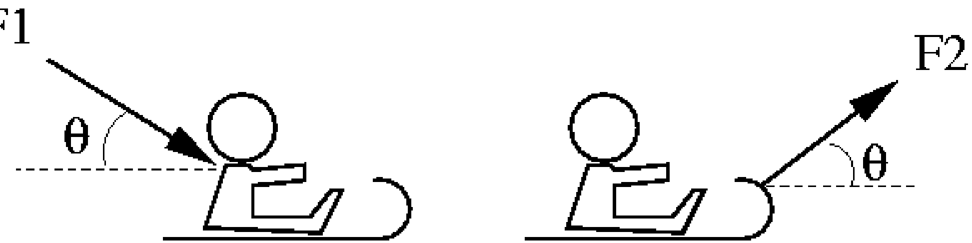
Solution:

1. Note the box is in equilibrium with regards to vertical motion (i.e. it is not moving vertically)
2. Break the pulling force ***F*** into components
   1. $F_x = F\cos(\theta)$; $F_y = F\sin(\theta)$
3. Use vertical equilibrium to find the table's normal force:
   1. $\sum F_y = ma_y = 0 = F_{\text{normal}} + F_y - F_{\text{gravity}} = F_{\text{normal}} + F_y - W$
   2. $0 = F_{\text{normal}} + F_y - W = F_{\text{normal}} + F\sin(\theta) - W$
   3. $F_{\text{normal}} = W - F\sin(\theta) = 100\text{N} - F\sin(\theta)$



Consider moving a person on a sled,
where friction between the bottom of the sled and the ground is NOT negligible.
You can either push forward and down at an angle θ
or pull up and forward at the same angle.
If $F_1 = F_2 = F$ and the angles are the same, which situation has the greater acceleration?

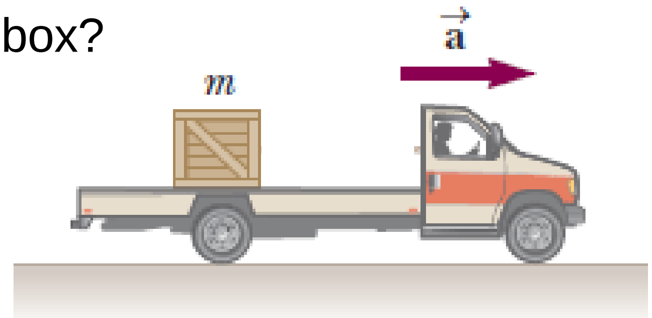
(A) pushing
(B) pulling
(C) both are equal



Find $\boldsymbol{F}_{net}$ in order to determine ***a***:

1. $\Sigma F_x = F_{x,net} = ma_x$
2. $a_x = F_{x,net}/m$ …so which situation has the largest $F_{x,net}$?
3. $F_{x,net} = F\cos(\theta) - F_{friction}$
4. So the situation with the smallest $F_{friction}$ will have a larger acceleration
5. $F_{friction} = \mu F_{normal}$
6. For pushing: $F_{normal} = W + F\sin(\theta)$ … For pulling: $F_{normal} = W - F\sin(\theta)$
7. F_{normal} is smaller for pulling, so acceleration will be greater

A box with a mass of 5 kg is sitting on the flat bed of a truck, but is not tied down.
The truck accelerates at $2 m/s^2$, as does the box (so it's not slipping).
The coefficients of friction between the box and the bed of the truck are μ_s=0.6 and μ_k=0.4.
What is the magnitude of the frictional force acting on the box?



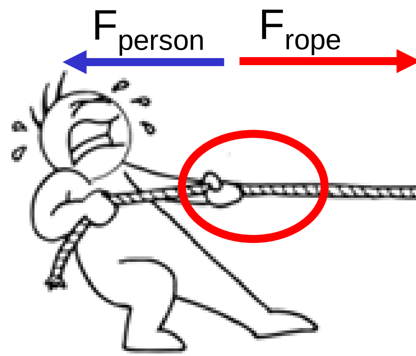
(A) 2 N
(B) 49 N
(C) 10 N
(D) 16 N
(E) 20N
(F) 29 N

Does static friction apply? If so, how hard does it have to push to maintain ***a***?:

1. $F_{friction} = \mu F_{normal}$
2. $F_{normal} = mg = (5kg)(9.8 m/s^2) = 49N$
3. So: $F_{f,static}$ could push as hard as $F_{f,static} \leq \mu_s F_{normal} = (0.6)*(49N) = 29.4N$
4. But, $F_{f,static}$ only reacts as hard as it has to in order to maintain ***a***
5. $F_{net} = ma = (5kg)(2 m/s^2) = 10N$ …which is less than the maximum $F_{f,static}$
6. So, $F_{f,static}$ will only match the applied action force (from F_{net}).
7. Meaning: $F_{f,static} = 10N$

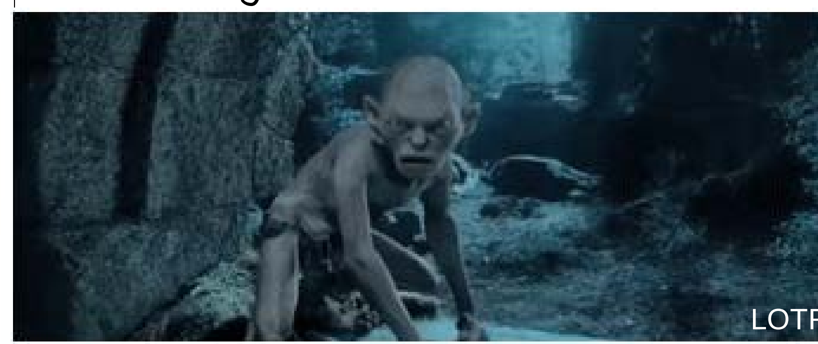
Two people pull on opposite ends of a massless rope.
Each pulls with a force of 40N.
What is the tension in the rope?

(A) 0N (B) 20N
(C) 40N (D) 80N



- Might be counterintuitive, so don't rely on intuition!
  Stick to your diagrams & equations!
- Consider F=ma
  - Rope isn't accelerating, so net forces must be balanced.
  - Force of person on rope must match force of rope on the person!
  - Tension is a reaction force.

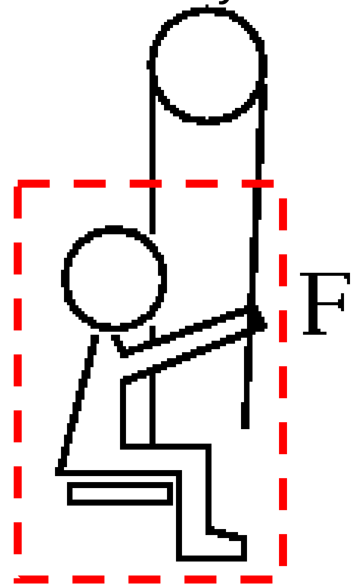
"Sneaky little hobbitses..."

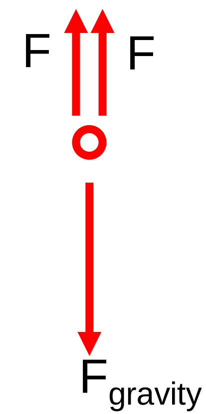


"... wicked, tricksy, false!"

A person who weighs 800N is sitting on a chair that weighs10N.
The chair is supported by a rope over a pulley.
The person pulls down on the rope with a force of F to support the total weight.
What force, F, is required to hold them and the chair stationary?

(A) 400N | **(B) 405N** | (C) 800N
(D) 810N | (E) 1600N | (F) 1620N



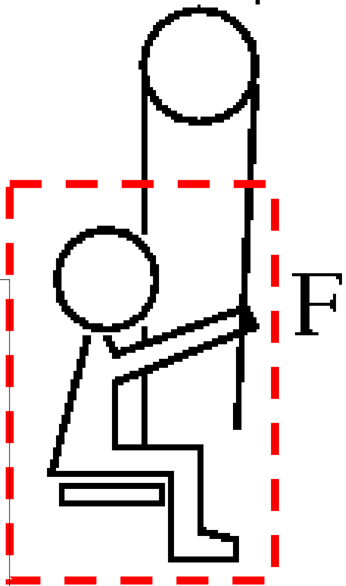


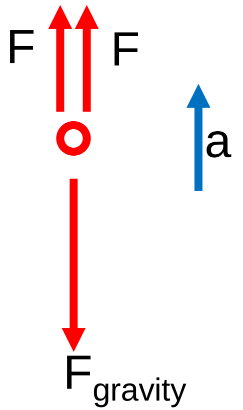
1. Pick your system & draw the forces
2. Consider that F=ma. Here a = 0.
3. Note that pulleys re-direct the tension, but maintain the tension magnitude. Therefore, the upward force is 2*F.
4. $F_{net} = \sum F = 2F - F_g = ma = 0$
5. $2F - F_g = 0$
6. $2F = W = 800N + 10N$
7. $F = (810N)/2 = 405N$

The force-multiplying power of a pulley is often referred to as its "mechanical advantage". In reality it is reduced somewhat by the pulley friction.

A person who weighs 800N is sitting on a chair that weighs10N.
The chair is supported by a rope over a pulley.
The person pulls down on the rope with a force of F.
What force, F, is required for them and the chair to accelerate upwards at 0.5 m/s^2?

(A) 26N (B) 405N (C) 362N
(D) 810N (E) 426N (F) 852N

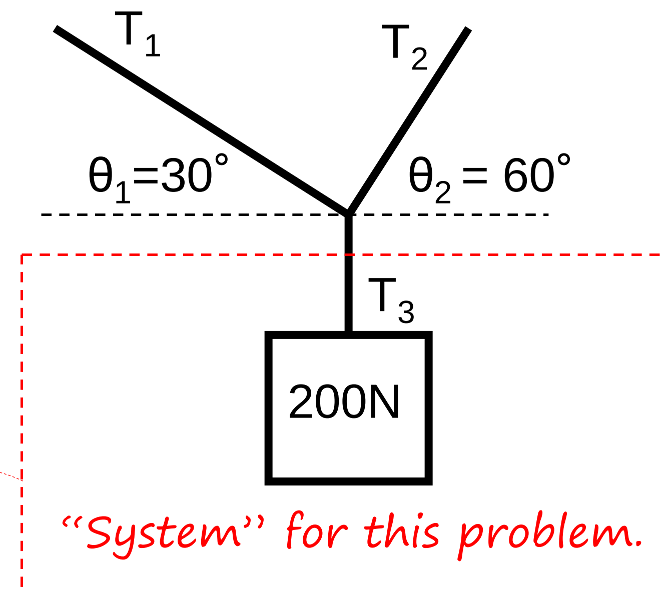




1. Pick your system & draw the forces.
2. Consider that F=ma. Here a = 0.5m/s^2.
3. Note that pulleys re-direct the tension, but maintain the tension magnitude. Therefore, the upward force is 2*F.
4. $F_{net} = \sum F = 2F - F_g = ma$
5. $2F = ma + F_g = ma + W \rightarrow F = (ma + W)/2$
6. $W = mg = m(9.8 m/s^2) \rightarrow m = (810N)/(9.8 m/s^2)$
7. $F = \frac{1}{2}\left((810N)\frac{0.5\frac{m}{s^2}}{9.8\frac{m}{s^2}} + 810N\right) \approx 426\text{N}$

A 200-N box is hanging from a rope.
Two ropes attach the box to the ceiling at the angles given.
What is the tension in rope 3?

(A) 50 N (B) 86 N (C) 100 N
(D) 136 N (E) 173 N (F) 200 N



1. Pick your system: Box + rope 3.
2. Consider that F=ma. Here $a = 0\ m/s^2$.
3. $F_{net} = \sum F = T_3 - F_g = 0$
4. $T_3 = Fg = mg = W = 200N$

A 200-N box is hanging from a rope.
Two ropes attach the box to the ceiling at the angles given.
What are the tensions in ropes 1 and 2?

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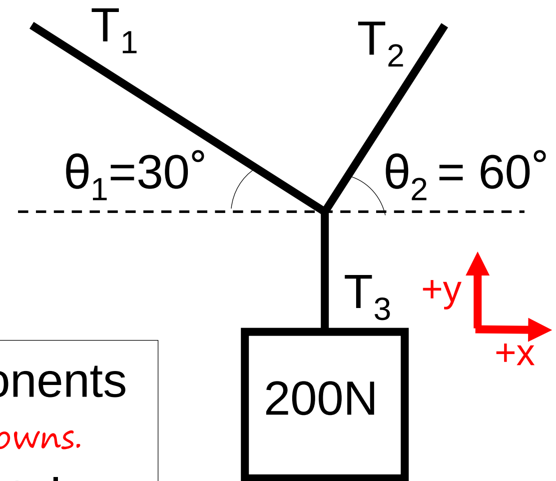

(A) 100N, 100N (B) 50N, 150N (C) 173N, 100N
(D) 150N, 50N **(E) 100N, 173N** (F) 200N, 200N

1. Need to break forces into components to find X & Y components two get 2 equations for T_1 & T_2. *Need 2 equations to solve 2 unknowns.*
2. Must use F=ma for X-component and Y-component separately
   1. $\sum F_x = ma_x = 0 = T_2\cos(\theta_2) - T_1\cos(\theta_1)$
   2. $\sum F_y = ma_y = 0 = T_2\sin(\theta_2) + T_1\sin(\theta_1) - T_3$
3. From (2.1): $T_2 = T_1*(\cos(\theta_1)/\cos(\theta_2))$
4. From (2.2): $T_3 = T_2\sin(\theta_2) + T_1\sin(\theta_1)$
   1. Use (3) in (4): $T3 = T_1*(\cos(\theta_1)/\cos(\theta_2))*\sin(\theta_2) + T_1\sin(\theta_1)$
   2. Therefore: $T_1 = T_3/[\sin(\theta_2)*\{\cos(\theta_1)/\cos(\theta_2)\} + \sin(\theta_1)]$
   3. $T_1 = (200N)/[0.866*\{0.866/0.5\} + 0.5] = (200N)/2 = 100N$
5. Use (4.3) in (3): $T_2 = (100N)*\{0.866/0.5\} = 173N$

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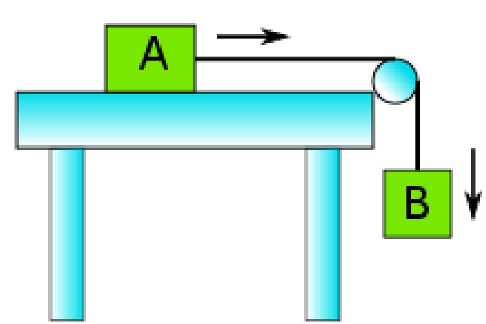
Block B of mass 1.5 kg is accelerating downward at a rate of 3.0 m/s^2.
Block A is connected by a massless string.
There is <u>no friction</u> between Block A and the table.
What is the tension in the string?

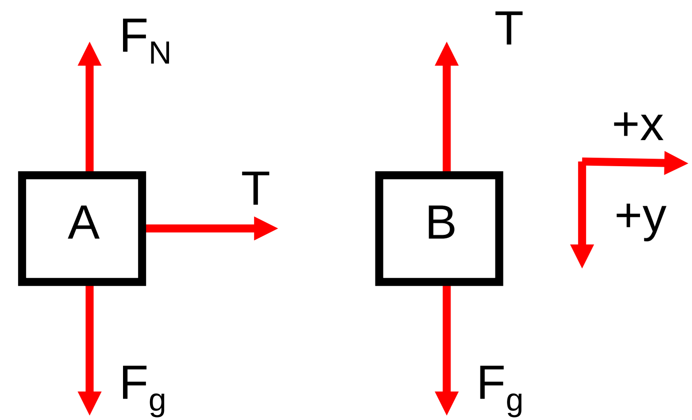


(A) 0 N (B) 1.5 N (C) 3.0 N (D) 4.5 N
(E) 8.4 N **(F) 10.2 N** (G) 14.7 N (H) 19.2 N

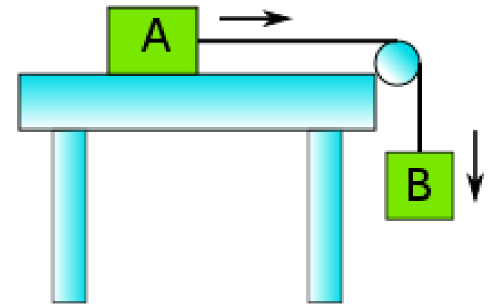
1. The string is massless, so the pulley just re-directs tension.
   Also A has no friction, so is not resisting the vertical acceleration.
   Therefore, just focus on Block B.

2. $\sum F_y = ma_y$
   1. $F_g - T = ma$
   2. $T = F_g - ma = mg + ma = m(a-g)$
   3. $T = (1.5\text{kg})(9.8\text{m/s}^2 - 3.0\text{m/s}^2)$
   4. $T = 10.2\text{N}$

Here we subtract tension from gravity because we defined downward to be +y!

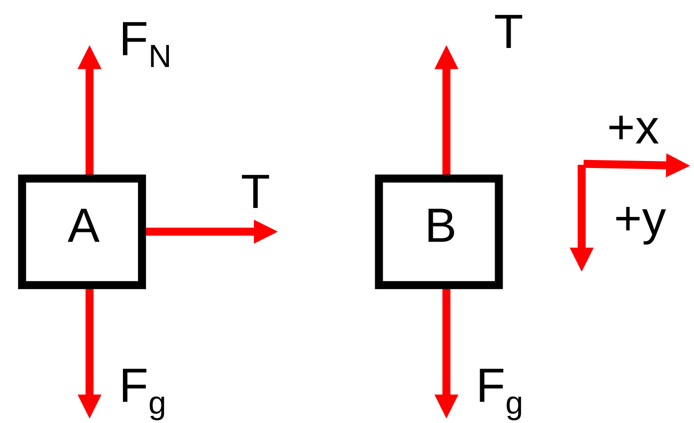


Block B of mass 1.5 kg is accelerating downward at a rate of 3.0 m/s^2.
Block A is connected by a massless string.
There is no friction between Block A and the table.
What is the mass of Block A?



(A) 3.4 kg (B) 1.5 kg (C) 6.4 kg (D) 4.5 kg

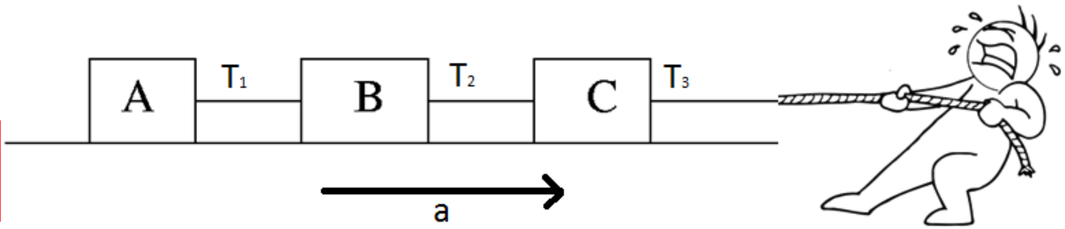
1. The system is just block A and the tension of the string on it.
2. The pulley just redirects the tension, so we can use the tension we just found for F.
3. F=ma
4. m = F/a = T/a = (10.2N)/(3.0m/s^2) = 3.4kg



Three boxes are accelerating to the right at a rate of $2.0 m/s^2$. The mass of block A is 2kg. The mass of block B is 1kg. The mass of block C is 2kg. The friction between the blocks and the ground is described by coefficients $\mu_{Static} = 0.45$ and $\mu_{Kinetic} = 0.35$. What is the tension in rope 1 (T_1)?

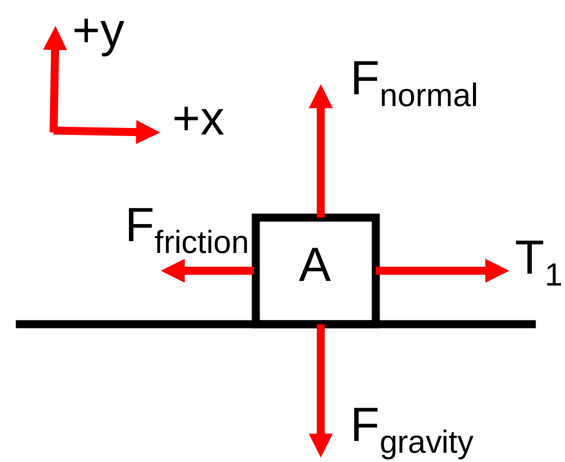
(A) 5.43N (B) 4.41N (C) 12.8N

(D) 4.00N (E)10.0N **(F) 10.9N**



1. Focus! Your system is just Block A and Rope 1.
2. $\sum F = T_1 - F_{friction} = ma$
3. $T_1 = ma + F_{friction} = ma + \mu F_{normal}$
4. Here, only vertical downward force is from the Weight. So your normal force (which is a reaction force) will be equal in magnitude & opposite in direction to this.
   $F_{normal} = F_{gravity} = mg$
5. $T_1 = ma + \mu F_{normal} = ma + \mu mg = m(a + \mu g)$
   $= 2kg*(2.0 m/s^2 + 0.35*(9.8 m/s^2)) \approx 10.9\ N$

Use kinetic friction because the block is moving.

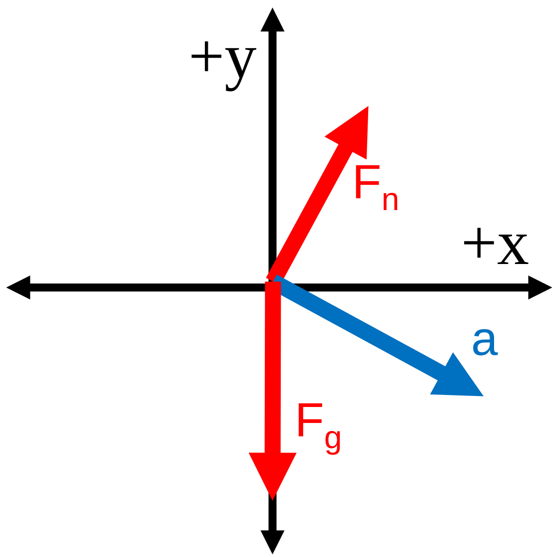


A note on coordinate axes:

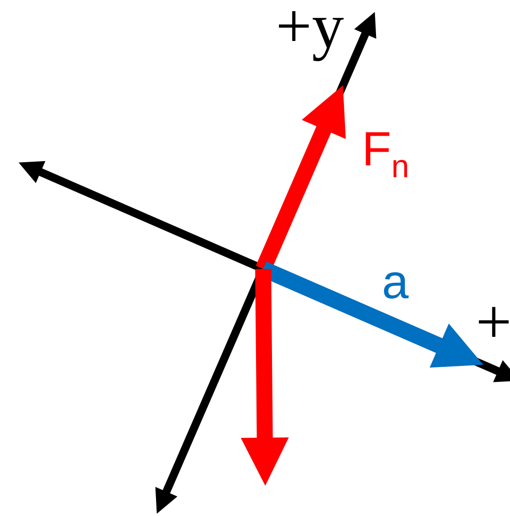
- Coordinate axes are your friend!
- Axes are an artificial constraint you place on the world to make the math used to describe your situation as easy as possible.
- You can orient axes however you want.
- The only rule is that they must be perpendicular to each other.

Example:

Consider a block sliding down a frictionless ramp.

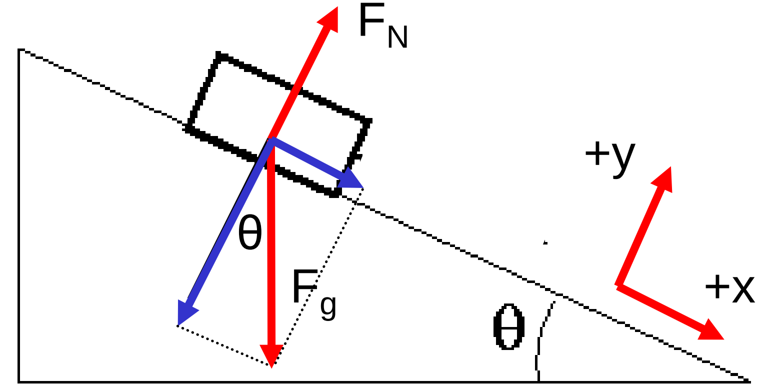


OK choice.



More convenient choice.

A block on an incline has a weight of 2.0N. The incline is at an angle θ of 30º.
What is the component of the force due to gravity in the x direction, with x as defined here?

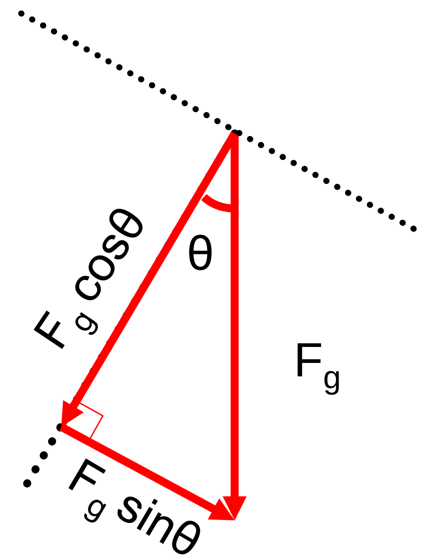


(A) 0.5 N
(B) 1.0 N
(C) 1.7 N
(D) 2.0 N
(E) 2.3 N
(F) 4 N

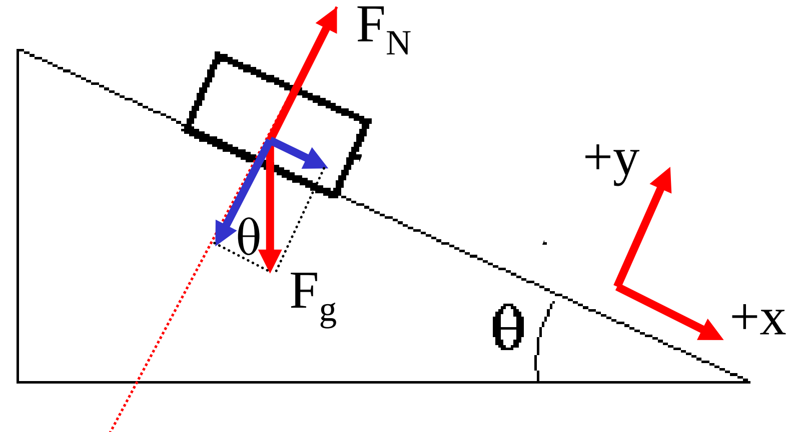
For a Weight of 2.0N, using **SOH**CAHTOA, the component parallel to the incline would be:

$$F_g \sin\theta = (2.0\text{N})^*\sin(30^o)$$

$$= 1.0\ \text{N}$$



Comments on problems with inclines:



Why tilt the coordinate axes?

- Gravity points down, but can get parallel and perpendicular components.
- The normal force is perpendicular to the surface.
- Friction is parallel to the surface.
- So, *tilting axes makes 2/3 forces along an axis*.

How does mass affect motion on incline?

- ***IF*** gravity & friction are the only forces,
- Then all forces involved are scaled by the same mass:
  - $F_{gravity,x} = m*g*\sin(\theta)$
  - $F_{friction} = \mu F_{normal} = \mu F_{gravity,y} = \mu*m*g*\cos(\theta)$
- So acceleration: $F = \cancel{m}a = F_{gravity,x} - F_{friction} = \cancel{m}*g*\sin(\theta) - \mu*\cancel{m}*g*\cos(\theta)$
- Therefore, for this situation: $a = g*[\sin(\theta) - \mu*\cos(\theta)]$

<u>If</u> only gravity & friction are involved, then motion on a plane is independent of an object's mass.

Suppose you increase the angle θ.

What happens to the x component of F_g and the normal force F_N?

(A) $F_{g,x}$ increases; F_N increases

(B) $F_{g,x}$ increases; F_N same

(C) $F_{g,x}$ increases; F_N decreases

(D) $F_{g,x}$ decreases; F_N increases

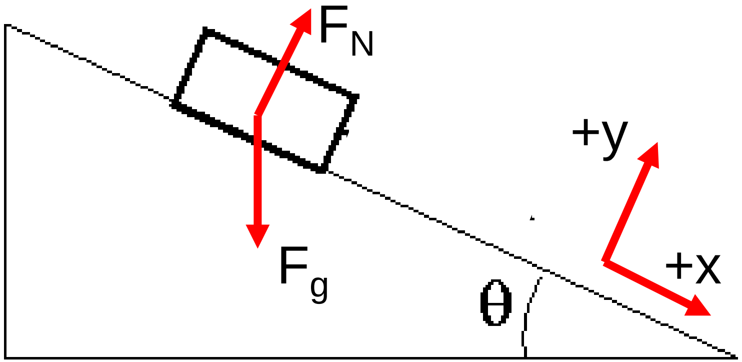
(E) $F_{g,x}$ decreases; F_N same

(F) $F_{g,x}$ decreases; F_N decreases

(G) $F_{g,x}$ same; F_N increases

(H) $F_{g,x}$ same; F_N same

(I) $F_{g,x}$ same; F_N decreases



Math Solution:

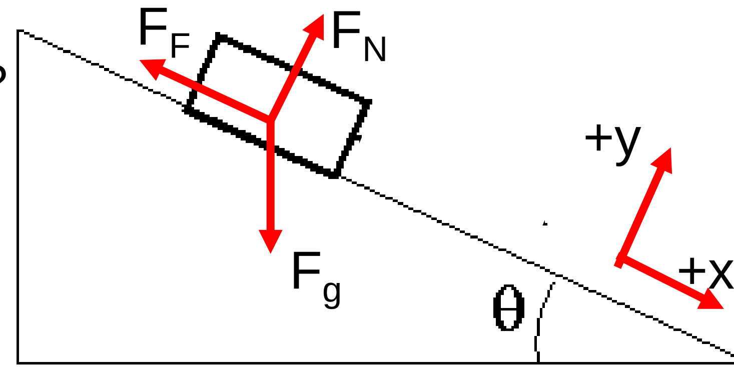
- The normal force is a reaction force to the opposing perpendicular force.
  - $|F_N| = |F_{g,y}| = mg*\cos(\theta)$
  - As $\theta \to 90°$, $\cos(\theta) \to 0$.
- The x-component of gravity, from SOHCAHTOA, is:
  - $|F_{g,x}| = mg*\sin(\theta)$
  - As $\theta \to 90°$, $\sin(\theta) \to 1$.

Logic Solution:

- For θ=90°, just free-fall:
  - $F_{gravity,x}$ pointing straight down & so maximized
  - Not pressing against a surface and so no normal force
- For θ=0°, just sitting on a plane:
  - No horizontal gravity
  - Gravity fully vertical, so F_{normal} maximized

A 30kg (294N) crate is sliding down an incline at an angle 30° below the horizontal. The kinetic coefficient of friction is 0.3 between the crate and the ramp.
What is the acceleration of the crate along the ramp?

(A) $9.80 m/s^2$ (B) $8.49 m/s^2$
(C) $4.90 m/s^2$ **(D) $2.35\ m/s^2$**
(E) $2.54 m/s^2$ (F) $0.00 m/s^2$



Solution:

1. Draw & choose +x to be along the ramp. +y to be perpendicular to ramp.
2. Want a_x, so need $F_{net,x}$ in order to use $\sum F_x = ma_x$.
   1. $\sum F_x = F_{gravity,x} - F_{friction}$
   2. From SOHCAHTOA: $F_{gravity,x} = m*g*\sin(\theta)$
   3. But $F_{friction} = \mu F_{normal}$ …so we need the normal force (which is in +y)
3. The normal force is a reaction force which opposes that are perpendicular to & toward the surface (here just the y-component of the weight):
   1. $|F_{normal}| = |F_{gravity,y}| = m*g*\cos(\theta)$
   2. Therefore, $F_{friction} = \mu*m*g*\cos(\theta)$
4. Going back to F=ma …then a = F/m.
   1. $a = (Fg,x - Ff)/m = (m*g*\sin(\theta) - \mu*m*g*\cos(\theta))/m$
   2. $a = g*\{\sin(\theta) - \mu*\cos(\theta)\} = (9.8 m/s^2)\{0.5 - (0.3)*0.866\} = 2.35 m/s^2$

*As promised earlier.
Mass doesn't matter here.*

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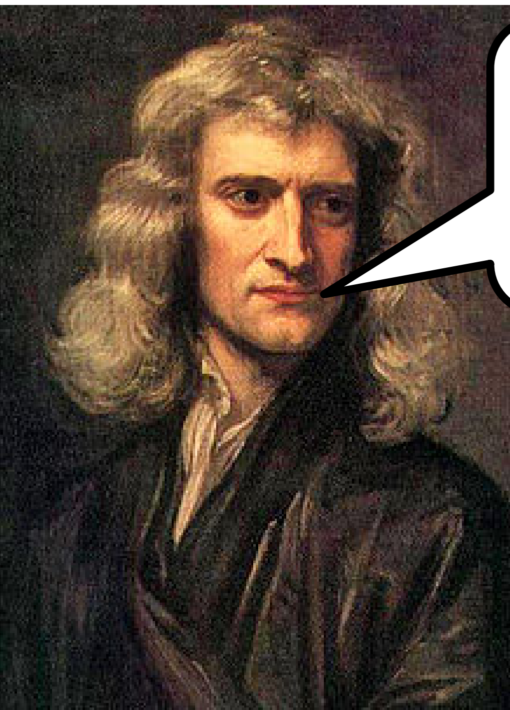
A block with a weight of 10N is sitting at rest on an incline which is tilted at an angle of 30º. The force of friction is 5.0N. What is the net force acting on the block?

(1) 0 N
(2) 5N down the incline
(3) 5N up the incline
(4) 10N straight down
(5) 5N straight up
(6) 15N straight down



A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity, ***unless*** acted upon by a net external force.

“At rest” = no acceleration = no net force.