Thursday January 26

- Assignment 3: due Friday by 11:59pm
  …like every Friday
- SI Sessions: Morton 326
  - Mon/Wed 7:15PM-8:45PM
- Help Room Wed/Thurs 6-9PM
  - Walter 245

Today:

- Projectile motion

Tips for homework:

- Break problems into components
- Link information with time

- *Write these equations in your notes if they're not already there.*
- *You will want them for Exam 1 & the Final.*

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

FunctionPlotResponse

PhET - Projectile

Proj LC HTML5

Last time, on PHYS2001 section 101…

Detailed Example: Soccer Ball (Part 2)

A soccer ball is kicked on a level patch of ground.

It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

2) How much time is the ball in the air?

Know:

- $y=y_0$, because starts & ends on ground.
- a_y=-9.8m/s^2,because projectile motion.
- $v_{y,0} = v_0$*sin(θ), from SOHCAHTOA

Want:

- *t* when ball returns to ground (*y*=0)

Best equation?:

- Really just solving free-fall
- Equation 4 looks like a good choice

Solution:

1. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
2. $y - y_0 - v_{y,0}t = \frac{1}{2}a_y t^2$
3. $\frac{2(y-y_0-v_{y,0}t)}{a_y} = t^2$
4. …since *y*=*y*$_0$=0: $\frac{-2v_{y,0}\cancel{t}}{a_y} = t^{\cancel{2}}$
5. $\boxed{t = \frac{-2\left(20\frac{m}{s}\right)sin(30)}{-9.8m/s^2} = 2.04s}$

Alternative Solution:

1. $v_y = v_{y,0} + a_y t$
2. $-v_{y,0} = v_{y,0} + a_y t$
3. $-\frac{2v_{y,0}}{a_y} = t$
4. $\boxed{t = \frac{-2\left(20\frac{m}{s}\right)sin(30)}{-9.8m/s^2} = 2.04s}$

2-ways to get to same solution

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

A firefighter is spraying water on a building.
Water leaves the hose at 35m/s and an angle of 30° above the horizontal.
The nozzle of the hose is 1.0m above the ground, and the building is 22m away.
The building is 40m tall.
In what vertical direction is the water traveling when it hits the building?

[✓ (A) up] (B) level (C) down

We need to know the vertical component of velocity at the time the water strikes the building.

We found the water strikes the building at 0.73s, so we need the velocity then:

$$v_y = v_{y,0} + a*t = v_0*\sin(\theta) - g*t = (35\mathrm{m/s})\sin(30°) - (9.8\mathrm{m/s^2})0.73\mathrm{s} = 10.4\mathrm{m/s}$$

Is this vertical velocity component greater than zero?

Yes. So the water must still be heading upward.

A firefighter is spraying water on a building.
Water leaves the hose at 35m/s and an angle of 30° above the horizontal.
The nozzle of the hose is 1.0m above the ground, and the building is 22m away.
The building is 40m tall.
What is the speed of the water just before it hits the building?

2

(A) 17.5m/s (B) 30.3m/s (C) 32.0m/s (D) 35.0m/s (E) 47.8m/s

We need the x & y velocity components.

We just found the vertical velocity component:

$$v_y = v_{y,0} + a*t = v_0*\sin(\theta) - g*t = (35\text{m/s})\sin(30°) - (9.8\text{m/s}^2)0.73\text{s} = 10.4\text{m/s}$$

And we previously found the horizontal velocity component:

$$v_x = v_{x,0} = v*\cos(\theta) = (35\text{m/s})\cos(30°) = 30.3\text{m/s}$$

So now we use the Pythagorean theorem: $v = \sqrt{v_x^2 + v_y^2} = 32.0m/s$

You're meandering. First you walk 20.0m in a direction 30.0° West of North and then 50.0m in a direction 40.0° South of West.

How far North-South did you travel? I.e. what is the North-South component of the resultant?

Tip: Draw.

(A) 14.8 m North

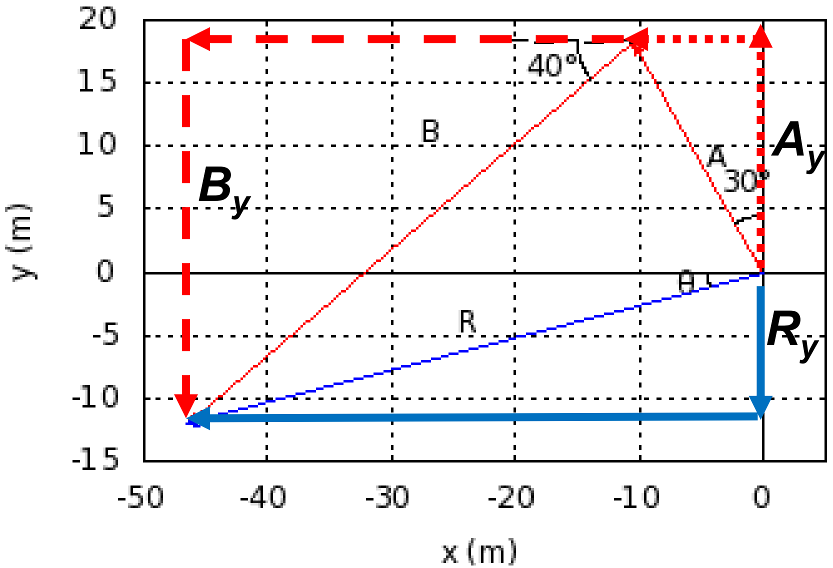
(B) 14.8 m South

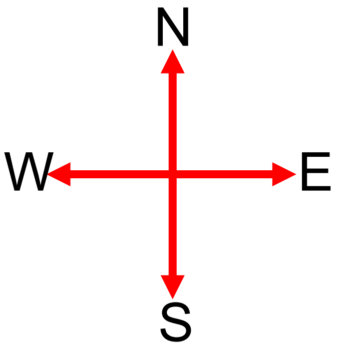
(C) 28.3 m North

(D) 28.3 m South

(E) 49.4 m North

(F) 49.4 m South





1. Find the y-component of the first part, A: A_y= (20m)cos(30) = 17.3m (North)

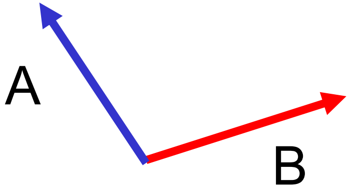
2. Find the y-component of the second part, B: B_y= (50m)sin(40) = -32.1m (South)

3. Combine the y-components of parts A and B to get the y-component for the resultant:

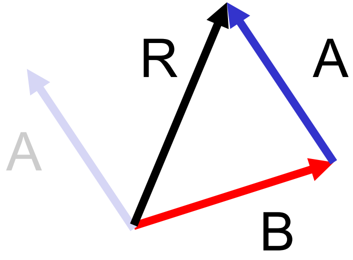
$R_y = A_y + B_y$ = 17.3m - 32.1m = -14.8m = 14.8m South

Note for Lab Next Week: Adding 2 Vectors Graphically:

Vectors we want to add:



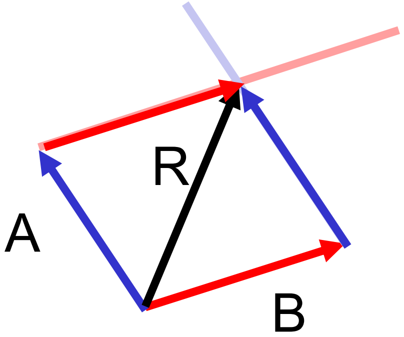
Tail-to-head Method



R is Resultant – the result of adding a group of vectors

Parallelogram Method

- Works for 2 vectors
- Create parallelogram out of two vectors
- Vector from the two tails to the opposite corner is the resultant



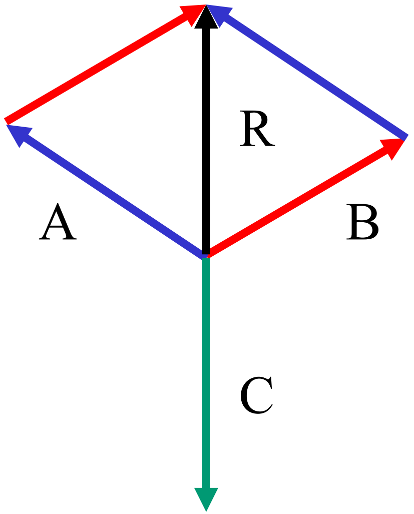
Note for Lab Next Week: Forces & Equilibrium

- "Equilibrium" means the external forces are balanced
  …therefore there is zero acceleration (***F***=*m**a***)
- If acceleration is equal to zero, then the external forces are balanced: $\sum \vec{F} = 0$
- "Weight" is the force due to gravity: $\boldsymbol{F_g}=m\boldsymbol{g}$
- "Balanced forces" means in all directions, i.e. both x and y in 2D:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Example: Forces A, B, and C acting on an object are in equilibrium

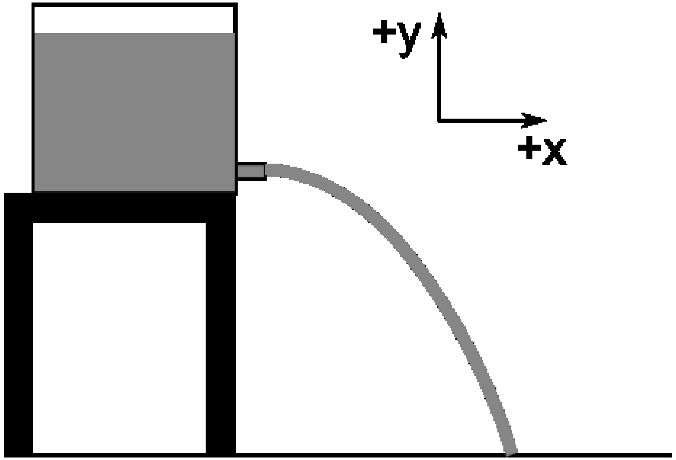


If $\vec{R} = \vec{A} + \vec{B}$

Then $\vec{R} + \vec{C}$ should be zero

Consider a Water Tank

Water leaves the tank traveling horizontally at a speed of 4 m/s. The spigot is 1 m above the ground. What is $v_{y,0}$?

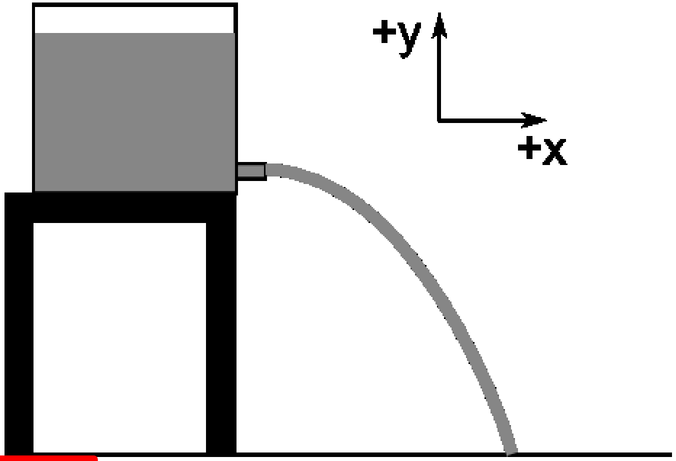


[✓ (A) 0 m/s] (B) 4 m/s (C) 9.8 m/s^2 (D) -4m/s (E) -9.8m/s^2

Since the water is initially traveling horizontally, there is no velocity in the vertical direction.

Consider a Water Tank

Water leaves the tank traveling horizontally at a speed of 4 m/s. The spigot is 1 m above the ground. What are the values of the following quantities?: $v_{x,0}$, $v_{y,0}$, a_x, a_y



[✓ (A) 4m/s, 0m/s, 0m/s^2, -9.8m/s^2]

(B) 4m/s, 0m/s, 0m/s^2, 0m/s^2

(C) -4m/s, 0m/s, 0m/s^2, -9.8m/s^2

(D) 4m/s, 0m/s, 0m/s^2, 9.8m/s^2

Pay attention to the axis convention!

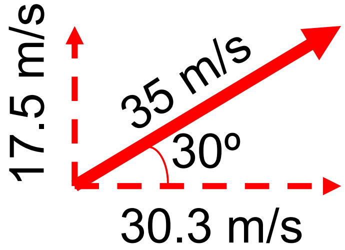
How much time does it take for the water to hit the ground?

(A) 1s (B) 4.9s (C) 0.2s [✓ (D) 0.5s]

We can ignore the horizontal motion, and just use $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$.

$$\ldots -1\text{m} = 0m + (0m/s)t + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)t^2 \quad \rightarrow\ t = \sqrt{(-1m)/(-4.9\frac{m}{s^2})} \approx 0.5s$$

Reminder: Velocity and Speed

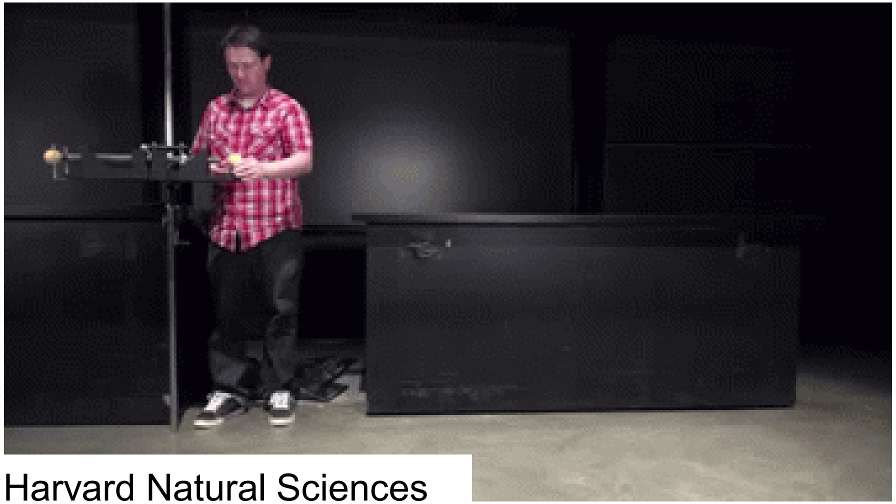


- Speed is magnitude (size) of velocity vector. Here it is 35m/s.
- Velocity would be 35 m/s 30º above the horizontal.
- Vertical (y) component of velocity would be +17.5m/s.
- Horizontal (x) component of velocity would be +30.3m/s.
- If you need to find the speed, then you probably need to find the two velocity components and reconstruct the speed from there.
- At the beginning of the problem, you'll likely have to break the velocity into components. This will make the problem much easier to solve (or make it possible to solve in the first place).

Student A is hanging on to a tree limb. Student B is on a nearby hillside aiming a water balloon directly horizontally at Student A. Student B lets out a loud scream and releases the water balloon. Student A lets go of the branch at the same time the water balloon is released.

The water balloon will:

1. Pass above Student A
2. [✓ Hit Student A]
3. Pass below Student A



Harvard Natural Sciences

https://www.youtube.com/watch?v=zMF4CD7i3hg

Only the vertical velocity components matter!

The initial vertical velocity component is zero for both the student and the water balloon. Therefore, they will both fall at the same rate.

Three projectiles are launched as described.
Which one reaches the highest altitude?

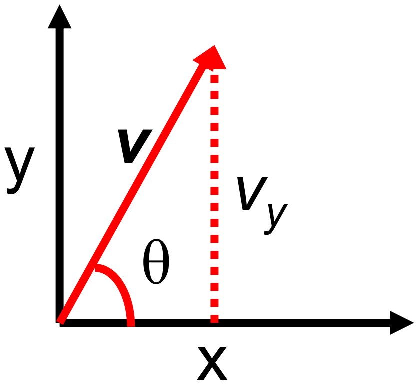
1. A 5kg ball launched straight up at 20m/s
2. A 4kg ball launched at 24m/s at an angle 70° above the horizontal
3. A 2kg ball launched at 27m/s at an angle 50° above the horizontal
4. All three reach the same maximum altitude

Compare vertical components of velocity:

1) $v_y = \mathbf{v}*\sin(\theta) = (20\text{m/s})*\sin(90^\circ) = 20\text{m/s}$

2) $v_y = \mathbf{v}*\sin(\theta) = (24\text{m/s})*\sin(70^\circ) = 23\text{m/s}$

3) $v_y = \mathbf{v}*\sin(\theta) = (27\text{m/s})*\sin(50^\circ) = 21\text{m/s}$



Largest v_y will go the highest.

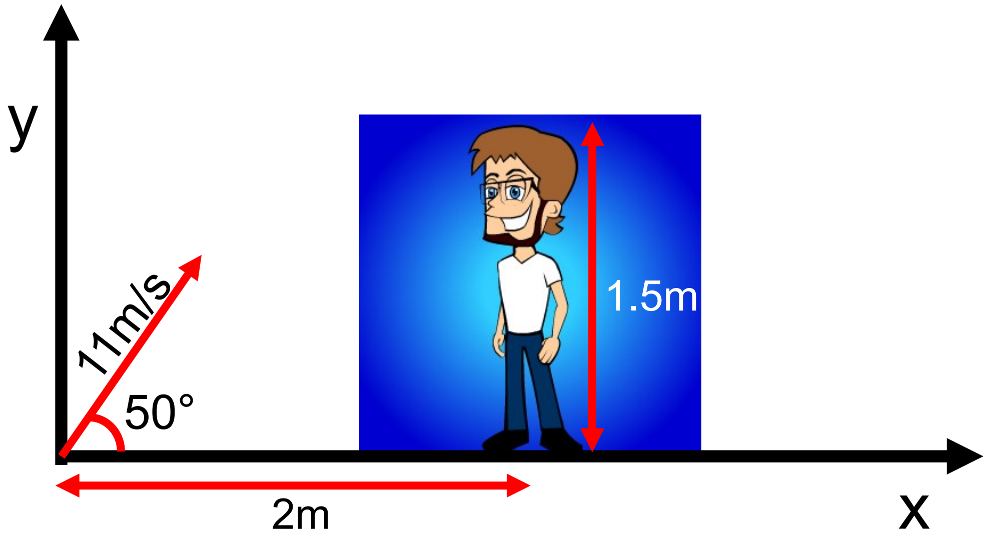
Object mass doesn't matter
for projectile motion!

A water balloon is launched at a speed of 11m/s from ground level at an angle of 50 degrees above the horizontal.
The water balloon is launched toward an unsuspecting bystander who is 1.5m tall and standing 2m away.

By how much does the water balloon clear (i.e. go over) the person's head?

(A) 2m (B) 2.4m

[✓ (C) 0.5m] (D) 0.4m



Trying to answer:
What is ***y*** *at the time* ***x****=2m?*

1. What is t at x=2m?
   - $x = x_0 + v_{x,0}t = (11\text{m/s})\cos(50°)t$
   - $\rightarrow t = (2\text{m})/(7.07\text{m/s}) = 0.28\text{s}$
2. What is $y(t=0.28\text{s})$?
   - $y = y_0 + v_{y,0}t + (1/2)a_y t^2 = (0\text{m}) + (11\text{m/s})\sin(50°)(0.28\text{s}) + (0.5)(-9.8\text{m/s}^2)(0.28\text{s})^2$
   - $y = 2.36\text{m} - 0.38\text{m} = 1.98\text{m}$
3. So we cleared their head by $(1.98\text{m} - 1.50\text{m}) \approx 0.5\text{m}$

A water balloon is launched at a speed of 11 m/s from ground level at an angle of 50 degrees above the horizontal.

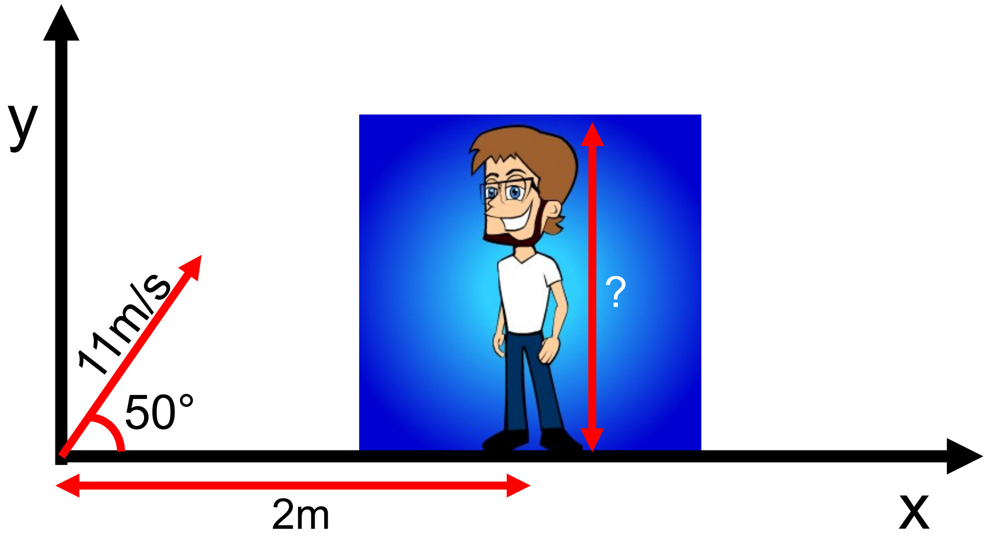
The water balloon is launched toward an unsuspecting bystander who is **some height** and standing 2m away.

How tall can a person be and still have the balloon go over their head?

[✓ (A) 1.98m] (B) 2.4m

(C) 0.50m (D) 0.40m

…just worked this out on the previous slide



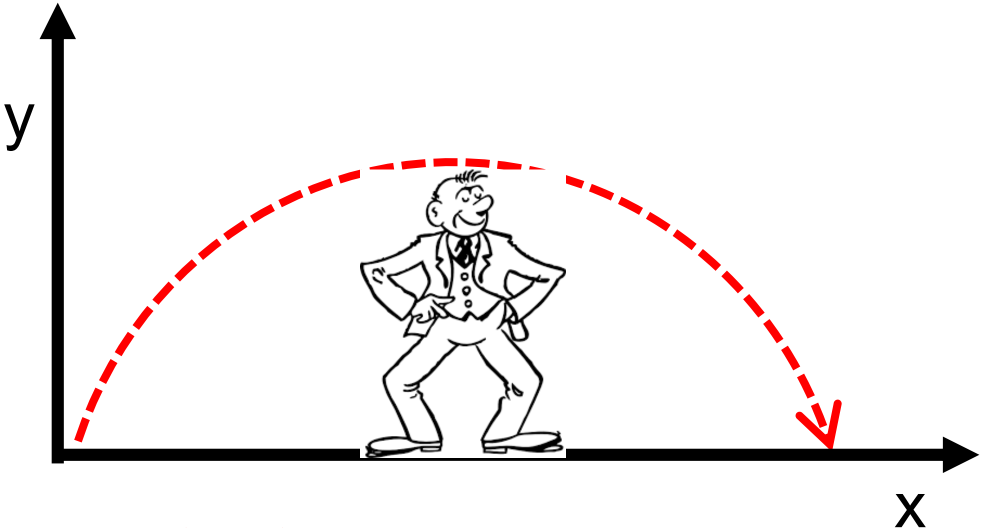
A water balloon is launched at a speed of 11 m/s from ground level at an angle of 50 degrees above the horizontal.
The water balloon is launched toward an unsuspecting bystander who is **some height** and standing **some distance** away.

What is the tallest a person can be and still have the balloon go over their head if they can stand wherever they want?

(A) 1.98m [✓ (B) 3.62m]

(C) 0.50m (D) 8.42m

This is really just asking for the maximum height of the balloon!



1. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$ Want max-height, so v_y=0.
2. $(0\,\frac{\mathrm{m}}{\mathrm{s}})^2 = ((11\,m/s)\ \sin(50°))^2 + 2(-9.8\,\frac{m}{s^2})(y - 0m)$
3. $y = \frac{-71.0m^2/s^2}{-19.6m/s^2} = 3.62m$

A water balloon is launched at a speed of 11 m/s from ground level at an angle of 50 degrees above the horizontal.
The water balloon is launched toward an unsuspecting bystander who is **1.5m** tall and standing **2m** away.

How fast is the balloon traveling when it is directly over the bystander's head?

(A) 5.68m/s (B) 7.07m/s (C) 11.0m/s (D) 9.06m/s

Trying to answer:
*What is the magnitude of **v** at the time **x**=2m?*

1. What is t at x = 2m?
   - $x = x_0 + v_{x,0}t$ =(11m/s)cos(50°)t
   - → t = (2m)/(7.07m/s) = 0.28s
2. What is $v(t$ = 0.28s)? (Really, what are the velocity components?)
   - $v_y = v_{y,0} + a_y t$ = (11m/s)sin(50°)+(-9.8m/s^2)(0.28s) = 8.42m/s - 2.74m/s = 5.68m/s
   - $v_x = v_{x,0}$ = (11m/s)cos(50°) = 7.07m/s
   - $v = \sqrt{v_x^2 + v_y^2} = \sqrt{49.99 + 32.26} = 9.06m/s$

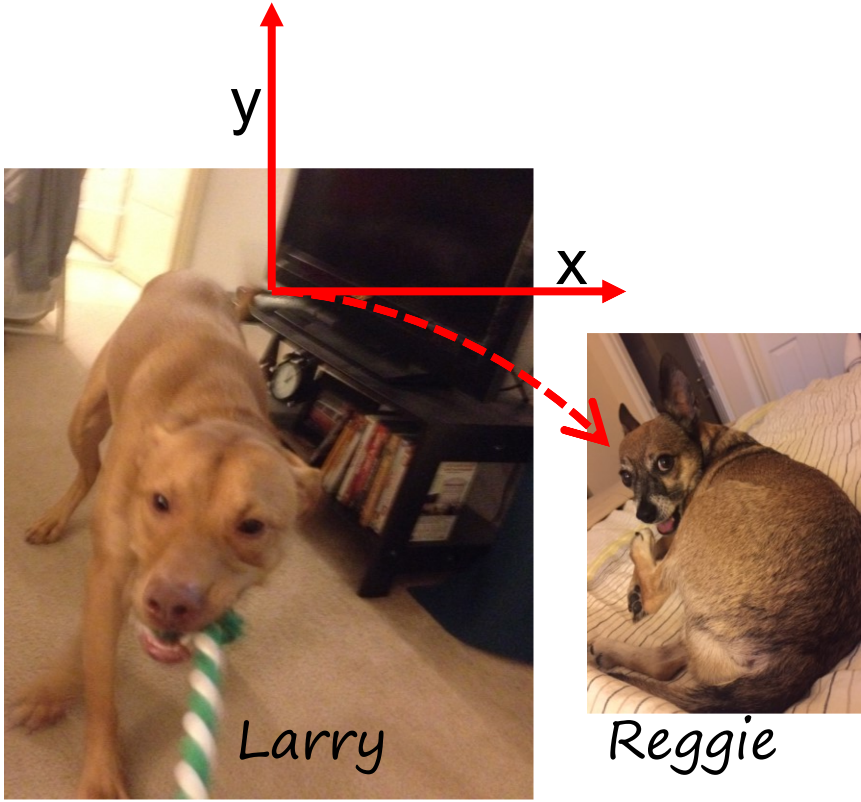
Larry (the dog) is wildly wagging his tail. Larry's tail launches an object off of a table top directly horizontally, right toward poor Reggie (the dog) and hits him in the head. Reggie's head is 1.0m horizontally from where the object is launched and 0.5m below Larry's tail height.

How much time is the object in free fall?

(A) 0.1s
(B) 0.3s
(C) 1s
(D) 3s

Vertical motion is independent of the horizontal motion.
So this is just a free-fall question.

1. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
2. $\frac{2(y-y_0-v_{y,0}t)}{a_y} = t^2$
3. $t = \sqrt{\frac{2(-0.5m)}{-9.8m/s^2}} = 0.3s$



Larry (the dog) is wildly wagging his tail. Larry's tail launches an object off of a table top directly horizontally, right toward poor Reggie (the dog) and hits him in the head. Reggie's head is 1.0m horizontally from where the object is launched and 0.5m below Larry's tail height.

How fast must the tip of Larry's tail been swinging?

(A) 9.8m/s (B) 1.0m/s

(C) 0.3m/s (D) 3.3m/s

The horizontal motion is uniform motion, so solving for the horizontal velocity at any time gives us the initial velocity.

We know the object covered the horizontal distance in the free-fall time we just solved for (0.3s). So:

- $x = x_0 + v_{x,0}t \rightarrow (x - x_0)/t = v_{x,0} = v_x = 3.3\text{m/s}$