Tuesday January 29

- Assignment 3: Due Friday, 11:59pm
  ….like every Friday
- Pre-Class Assignment: 15min before class
  …like every class
- Office Hours: Thurs. 3-4pm, 204 EAL
- Help Room: Wed. & Thurs. 6-9pm, here
- SI: Mon. & Tue. 6:20-7:10pm, 226 Morton

This lecture:
- 2-dimensional motion
- Components
- Projectile motion

HTML5 Projectile

PhET - Projectile

- *Write these equations in your notes if they're not already there.*
- *You will want them for Exam 1 & the Final.*

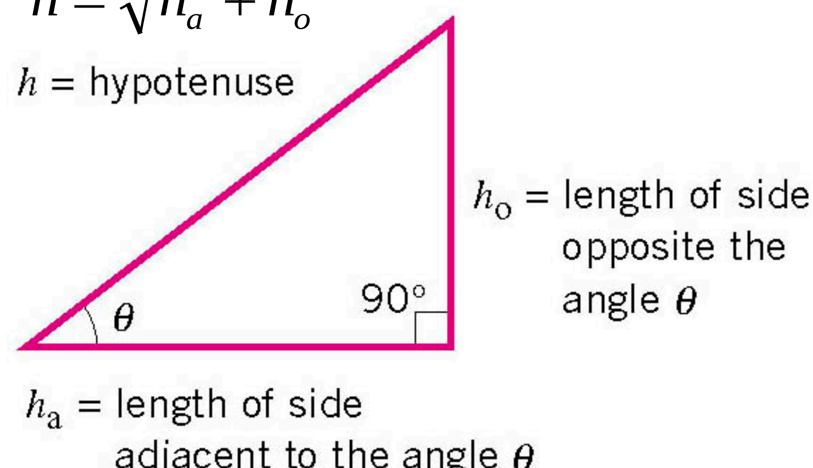
SOH-CAH-TOA

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$h = \sqrt{h_a^2 + h_o^2}$$



Definitions: Velocity & Acceleration with Vectors

- These equations are 2 ways to say the same thing:

This equation: $\overline{\vec{v}} = \dfrac{\vec{s} - \vec{s}_0}{t - t_0} = \dfrac{\Delta \vec{s}}{\Delta t}$ says the same thing as this one: $\vec{s} = \vec{s}_0 + \overline{\vec{v}} \Delta t$

This equation: $\overline{\vec{a}} = \dfrac{\Delta \vec{v}}{\Delta t}$ says the same thing as this one: $\vec{v} = \vec{v}_0 + \overline{\vec{a}} \Delta t$

- The two sides of the equation are equal only if the COMPONENTS are EQUAL, so we get one formula for each dimension

Instead of: $\boxed{\vec{v} = \vec{v}_0 + \overline{\vec{a}} \Delta t}$ we can use:

$$\boxed{\begin{aligned} v_x &= v_{0x} + a_x \Delta t \\ v_y &= v_{0y} + a_y \Delta t \end{aligned}}$$

Formulae for constant acceleration in 2-Dimensions

If acceleration is constant, can use 2 equations for each component:

$$x = x_0 + \bar{v}_x t$$

$$\bar{v}_x = \frac{1}{2}\left(v_{0x} + v_x\right)$$

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

$$y = y_0 + \bar{v}_y t$$

$$\bar{v}_y = \frac{1}{2}\left(v_{0y} + v_y\right)$$

$$v_y = v_{0y} + a_y t$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$$

…this is great! No need to learn new equations. We just apply our same equations for 1-Dimension to each component individually.

Projectile Motion: A special case for 2D Kinematics

- *Definition:* 2D-dimensional motion with **gravity as the only force**
- Separate components of projectile motion visualized: HTML5

Projectile Motion

This is a special, but commonly occurring, case for 2D motion.
...sort of like Free-Fall for 1D

Using projectile motion calculations, this person could have figured out how fast they need to be going…

https://imgur.com/gallery/m4yleux

Projectile motion is like free-fall, but with horizontal uniform motion:

- For free-fall: acceleration 9.8 m/s^2 downward
- IF we choose y-axis vertical (+y up typically)
- Then: $a_x = 0$ m/s^2, $a_y = -9.8$m/s^2
- HTML5
- Our 2D equations of motion can be simplified:

$$x = x_0 + \bar{v}_x t$$

$$\cancel{\bar{v}_x = \frac{1}{2}(v_{0x} + v_x)}$$

$$v_x = v_{0x} + \cancel{a_x t}$$

$$x = x_0 + v_{0x} t + \cancel{\frac{1}{2} a_x t^2}$$

$$\cancel{v_x^2 = v_{0x}^2 + 2a_x(x - x_0)}$$

Uniform motion in x:

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x} t$$

Free-fall in y:

$$y = y_0 + \bar{v}_y t$$

$$\bar{v}_y = \frac{1}{2}(v_{0y} + v_y)$$

$$v_y = v_{0y} + a_y t$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

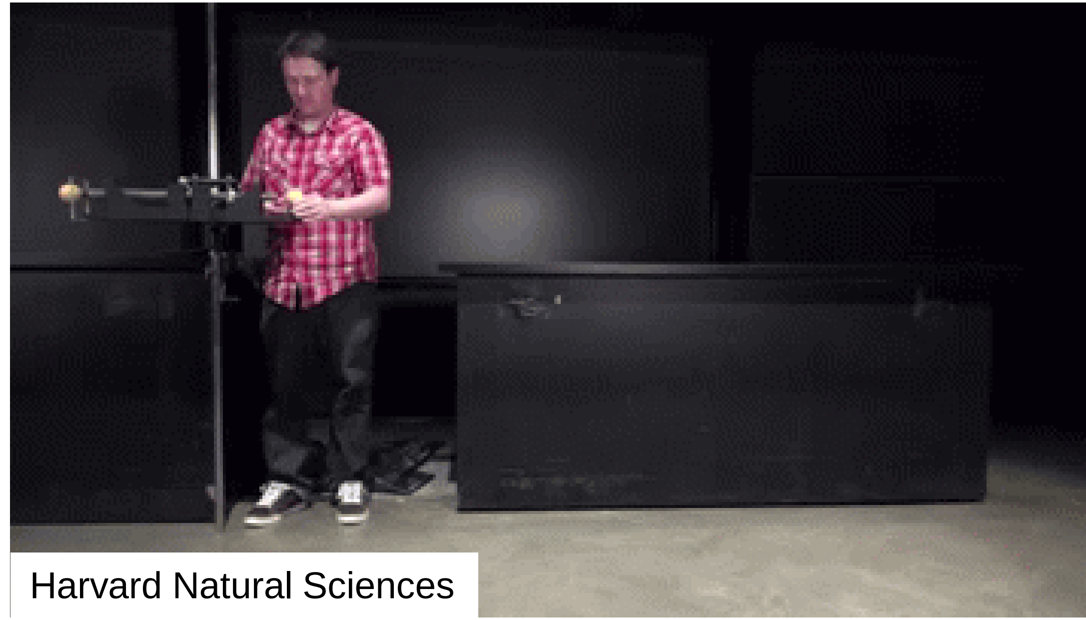
$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

Steel ball A is dropped from rest.
Steel ball B is shot horizontally from the same height.
Which ball hits the floor first?

A. Ball A: dropped

B. Ball B: shot

C. Both hit at the same time



https://www.youtube.com/watch?v=zMF4CD7i3hg

Vertical motion is independent of horizontal motion.

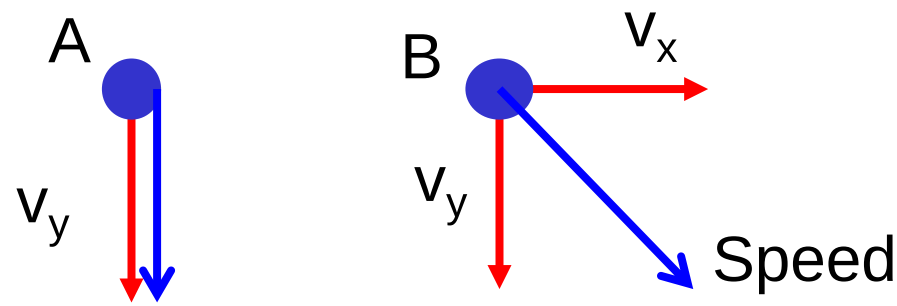
Therefore, the vertical behavior is the same.

Steel ball A is dropped from rest.
Steel ball B is shot horizontally from the same height.
Which ball is traveling the fastest before it hits the floor?

A. Ball A: dropped

B. Ball B: shot

C. They have the same speed



Ball A and B have the same v_y

Ball A: $v_x = 0$

Ball B: $v_x > 0$

Way to check for reasonableness of answer:

- *Speed should be greater than or equal to magnitude of largest velocity component.*
- *Speed must be larger than the magnitude of the largest component if there is more than one non-zero component.*

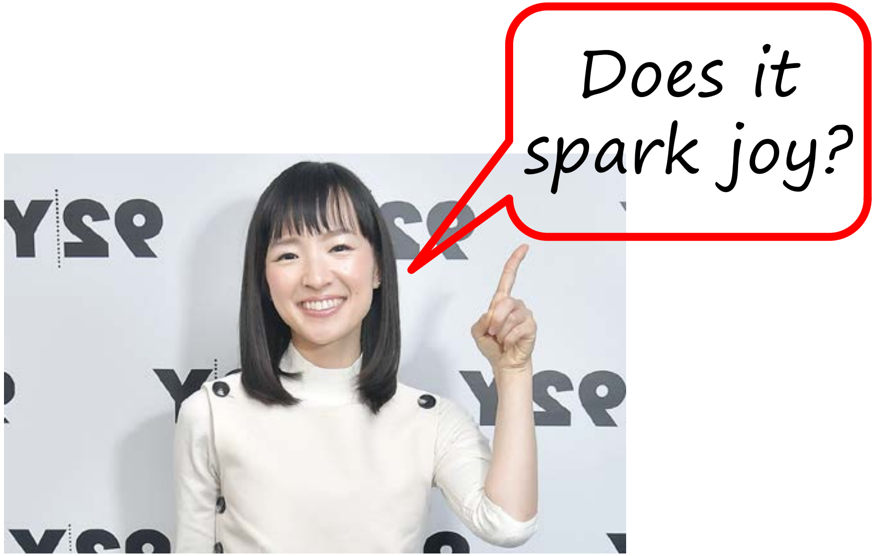
A cart travels at constant speed along a level track.
At a given point, a ball pops straight up from the cart.
Which of the following is the best option for the behavior of the ball.

A. The ball will land in front of the cart
B. The ball will land back in the cart
C. The ball will land behind the cart
D. The ball will land at the point on the track where the ball was launched

***Horizontal motion is independent of vertical motion*.**
In this case, the cart & ball have the same (constant) horizontal velocity.

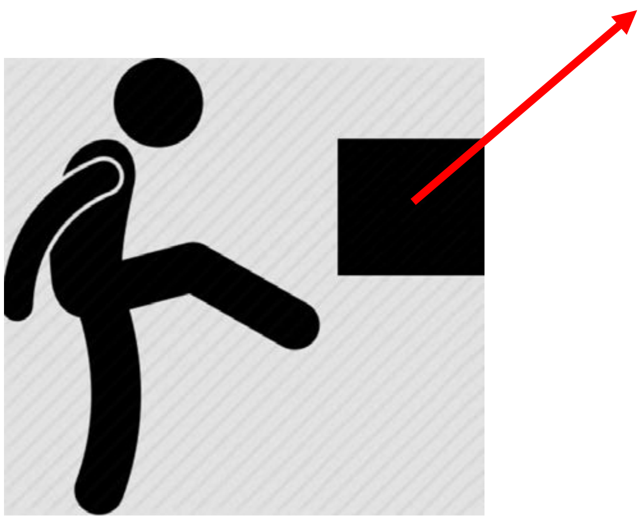
https://www.youtube.com/watch?v=XawCNYpeKD4

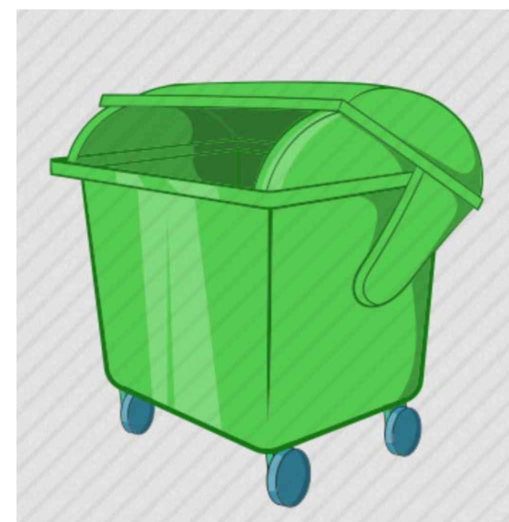
Doug Smith



You have been binge watching Marie Kondo lately and have decided this box of junk does not spark joy.
So, you decide to kick the box into the dumpster.

Let’s investigate the trajectory.





A box, which does not spark joy, is kicked towards a dumpster. Its initial velocity is 20m/s at an angle of 30° above the horizontal. What is the most accurate statement about the speed of the box?

A. It is a minimum at maximum height
B. It is zero at maximum height
C. It is a minimum just before it lands
D. It is a minimum just after it is kicked
E. It has the same speed throughout the trajectory

v_x is constant. Speed depends on both v_x and v_y. So,smallest speed when v_y=0.

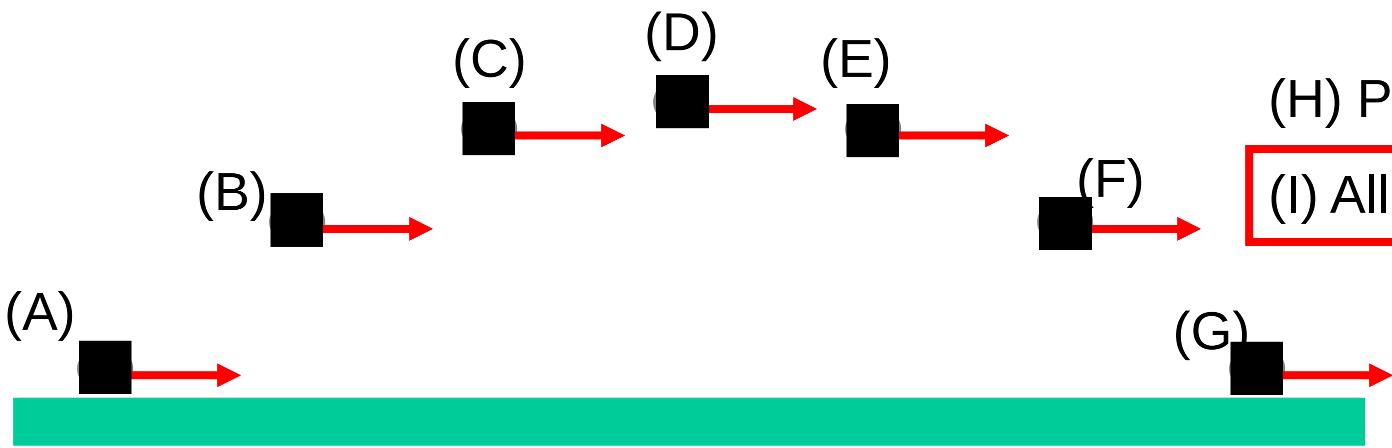
As with free-fall, "maximum height" corresponds to zero **vertical** velocity.

HTML5

The motion diagram of a kicked box, which does not spark joy, is pictured here.

The snapshots (black boxes) are for 1 second intervals.

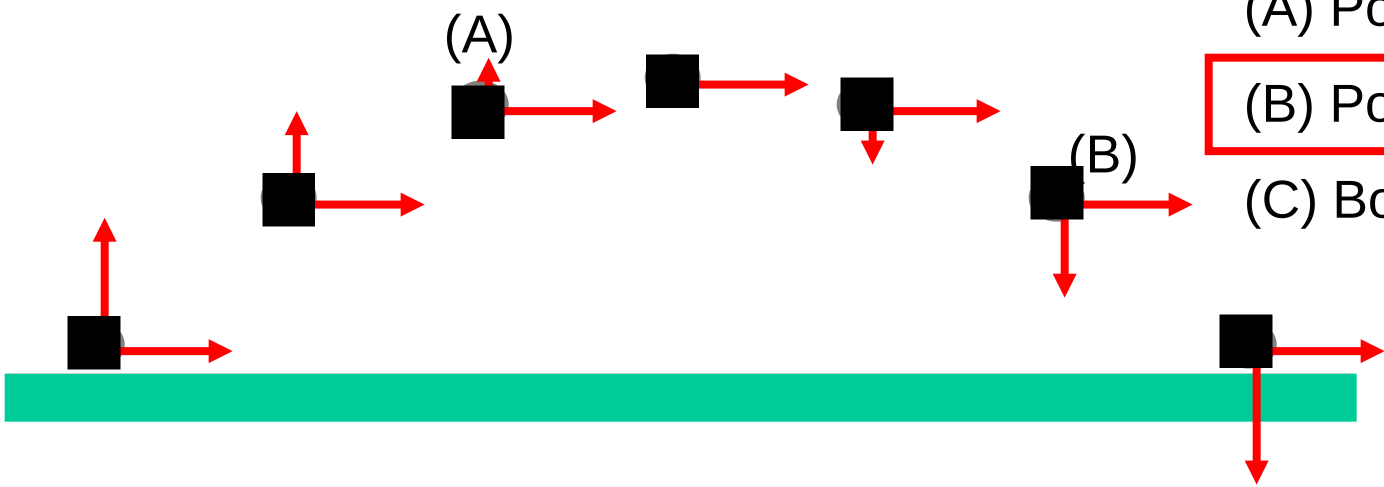
At which point is the magnitude of the horizontal component of the velocity the greatest? (Ignore air resistance.)



(H) Points A and G

(I) All Points Same

The motion diagram of a kicked box, which does not spark joy, is pictured here.
The snapshots (black boxes) are for 1 second intervals.
How does the magnitude (size) of the vertical component of the velocity compare between Point A and Point B?



(A) Point A: Greater

(B) Point B: Greater

(C) Both Same

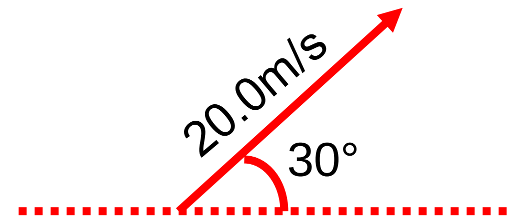
Note the symmetry. If there is no air resistance (always the case for this class), at a fixed height magnitude of velocity as it comes down is the same as when it was going up (Though the directions are different.)

At fixed height $|v_y|$ is the same.

A box, which does not spark joy, is kicked.
Its initial velocity is 20.0 m/s at an angle of 30° above the horizontal.
What is the speed of the box at maximum height?

A. 0 m/s
B. 4.5 m/s
C. 10.0 m/s
D. 12.4 m/s
E. 17.3 m/s
F. 20.0 m/s

1. For “maximum height”, we know: $v_y = 0$.
2. Projectile motion, so we know v_x = constant = v_{x0}.
3. Recall SOH**CAH**TOA,



4. $v_{x0} = (20.0\text{m/s})\cos(30^{o}) = 17.3$ m/s

HTML5

Detailed Example: Soccer Ball

Sect 3.3

A soccer ball is kicked on a level patch of ground.
It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

1) What is the maximum height the ball reaches?
2) How much time is the ball in the air?
3) How far away does it hit the ground? (what is the range)
4) What is the final speed of the ball just before it hits the ground?

Equations for Projectile Motion in 2D

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x} t$$

Uniform motion for horizontal motion

Free-fall for vertical motion

$$v_y = v_{0y} + a_y t$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$y = y_0 + \frac{1}{2}\left(v_{0y} + v_y\right) t$$

$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$$

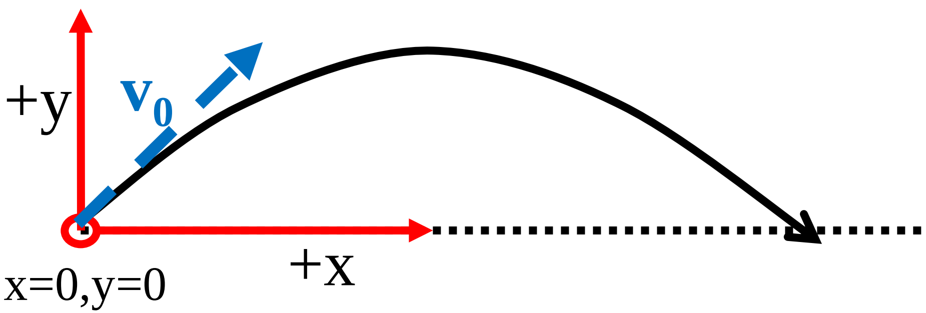
HTML5

Detailed Example: Soccer Ball (Part 1)

A soccer ball is kicked on a level patch of ground.
It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

1) What is the maximum height the ball reaches?

Draw, (choose axes & origin)



Know? Want to know?

- $\boldsymbol{v_0}$ = 20.0m/s at 30° angle
- "Max height": $v_y = 0$
- Projectile motion: a_y=-9.8m/s²

Which equation to use?

- Know: v_y, $v_{y,0}$, a_y. Want $y-y_0$. …so use Eqn. 6.

Solve:

1. $v_y^2 = v_{y,0}^2 + 2a_y(\boldsymbol{y} - y_0)$
2. $(0\,\text{m/s})^2 = ((20\,\text{m/s})*\sin(30°))^2 + 2(-9.8\,\text{m/s}^2)(\boldsymbol{y})$
3. $y = (-100\,\text{m}^2/\text{s}^2)/(-19.6\,\text{m/s}^2) = 5.1\,\text{m}$

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

Detailed Example: Soccer Ball (Part 2)

Sect 3.3

A soccer ball is kicked on a level patch of ground.
It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

2) How much time is the ball in the air?

Know:

- $y=y_0$, because starts & ends on ground.
- a_y=-9.8m/s^2,because projectile motion.
- $v_{y,0} = v_0$*sin(θ), from SOHCAHTOA

Want:

- t when ball returns to ground (y=0)

Best equation?:

- Really just solving free-fall
- Equation 4 looks like a good choice

Solution:

1. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
2. $y - y_0 - v_{y,0}t = \frac{1}{2}a_y t^2$
3. $\frac{2(y - y_0 - v_{y,0}t)}{a_y} = t^2$
4. …since $y=y_0=0$: $\frac{-2v_{y,0}\cancel{t}}{a_y} = t^{\cancel{2}}$
5. $t = \frac{-2\left(20\frac{m}{s}\right)sin(30)}{-9.8m/s^2} = 2.04s$

<u>2D Projectile motion eqns.</u>

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

Detailed Example: Soccer Ball (Part 3)

A soccer ball is kicked on a level patch of ground.
It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

3) How far away does it hit the ground? (what is the range)

Know:
- $y=y_0$, because starts & ends on ground.
- a_y=-9.8m/s^2,because projectile motion.
- $v_{y,0} = v_0$*sin(θ), from SOHCAHTOA

Want:
- Final horizontal position, x

Best Equation?
- There is just uniform motion in x
- Equation 2 is best

Solution:
1. $x = x_0 + v_{x,0}t$
2. $x = 0m + \left(20\frac{m}{s}\right)\cos(30°)t$
3. …just solved for t = 2.04s
4. $x = \left(17.32\frac{\mathrm{m}}{\mathrm{s}}\right)(2.04\mathrm{s}) = 35.3\mathrm{m}$

<u>2D Projectile motion eqns.</u>
1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}\left(v_{y,0} + v_y\right)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

Detailed Example: Soccer Ball (Part 4)

A soccer ball is kicked on a level patch of ground.
It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

4) What is the final speed of the ball just before it hits the ground?

Know:

- a_y=-9.8m/s^2, because projectile motion.
- v_x=$v_{x,0}$, because uniform horiz. Motion.
- $v_{y,0}$ = v_0*sin(θ), from SOHCAHTOA.

Want:

- Final speed, $v = \sqrt{v_x^2 + v_y^2}$

Best Equation?

- SOHCAHTOA for v_x
- Equation 3 for v_y

Solution:

1. $v_x = v_{x,0} = v_0 \cos(\theta) = \left(20\frac{m}{s}\right)\cos(30°) = 17.3m/s$
2. $v_y = v_{y,0} + a_y t = \left(20\frac{m}{s}\right)\sin(30°) + \left(-9.8\frac{m}{s^2}\right)t$
3. …solved for t already, so
$$v_y = 10\frac{m}{s} - \left(9.8\frac{m}{s^2}\right)(2.04s) = 9.99m/s$$
4. Use the Pythagorean theorem:
$v_x^2 + v_y^2 = v^2$ … so, $v = \sqrt{(17.3m/s)^2 + (10\frac{m}{s})^2} = 20m/s$

<u>2D Projectile motion eqns.</u>

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

How to interpret a problem:

Often you will have to glean extra information from the problem to find out what you know, what you don't know, and what is asked.(This is almost always the case in real life)

- "Speed": magnitude of velocity
- "Max height": vertical velocity is zero
- "Just after __ leaves __": initial information ($t = 0$)
- "Just after __ hits __": final information ($t = t_{final}$)
- Look for clues about coordinates: "horizontal", "vertical"
- Note if there are any other forces other than gravity
  - If other forces are there, acceleration will not be $9.8 m/s^2$

A firefighter is spraying water on a building.
Water leaves the hose at 35m/s and an angle of 30° above the horizontal.
The nozzle of the hose is 1.0m above the ground, and the building is 22m away.
The building is 40m tall.
What are the horizontal (x) and vertical (y) components of the initial velocity?
(i.e. just after the water leaves the hose)

(A) v_{x0} = 30.3 m/s; v_{y0}= 30.3 m/s

(B) v_{x0} = 17.5 m/s; v_{y0}= 17.5 m/s

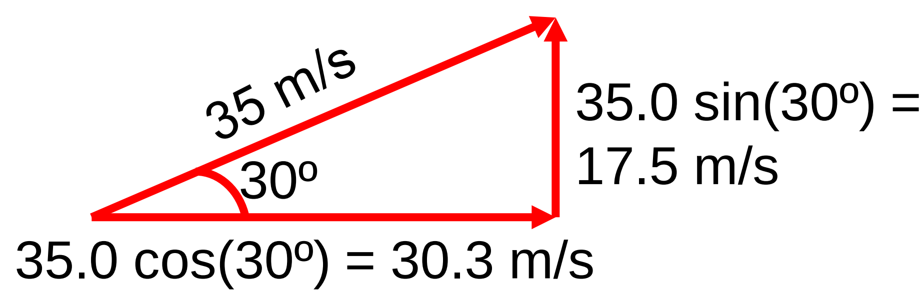
(C) v_{x0} = 17.5 m/s; v_{y0}= 30.3 m/s

(D) v_{x0} = 30.3 m/s; v_{y0}= 17.5 m/s

(E) v_{x0} = 35.0 m/s; v_{y0}= 17.5 m/s

(F) v_{x0} = 35.0 m/s; v_{y0}= 35.0 m/s

SOHCAHTOA



A firefighter is spraying water on a building.
Water leaves the hose at 35m/s and an angle of 30° above the horizontal.
The nozzle of the hose is 1.0m above the ground, and the building is 22m away.
The building is 40m tall.
How much time does it take for the water to reach the building?

(A) 0.63s (B) 0.73s (C) 1.00s (D) 1.32s (E) 1.78s

We just need to know how much time it takes for the water to cover the horizontal distance between the firefighter & building.

The horizontal motion is described by uniform motion, so we only need to determine the initial horizontal velocity:

$$v_x = v_{x,0} = v^*\cos(\theta) = (35\text{m/s})\cos(30°) = 30.3\text{m/s}$$

Then we can determine how long it takes to cover the distance at that speed:

$$v_x = \Delta x/\Delta t \quad \rightarrow \quad t = \Delta x/v_x = (22\text{m})/(30.3\text{m/s}) = 0.73\text{s}$$

A firefighter is spraying water on a building.
Water leaves the hose at 35m/s and an angle of 30° above the horizontal.
The nozzle of the hose is 1.0m above the ground, and the building is 22m away.
The building is 40m tall.
In what vertical direction is the water traveling when it hits the building?

$\boxed{\text{(A) up}}$ (B) level (C) down

We need to know the vertical component of velocity at the time the water strikes the building.

We found the water strikes the building at 0.73s, so we need the velocity then:

$$v_y = v_{y,0} + a*t = v_0*\sin(\theta) - g*t = (35\text{m/s})\sin(30°) - (9.8\text{m/s}^2)0.73\text{s} = 10.4\text{m/s}$$

Is this vertical velocity component greater than zero?

Yes. So the water must still be heading upward.