

Tuesday January 24

- Assignment 3: Due Friday, 11:59pm
....like every Friday
- Pre-Class Assignment: 15min before class
...like every class
- Office Hours: Wed. 10-11am, 204 EAL
- Help Room: Wed. & Thurs. 6-9pm, here
- SI: Mon. & Wed. 7:15-8:45pm, 326 Morton

This lecture:

- 2-dimensional motion
- Components
- Projectile motion

[HTML5 Projectile](#)

[PhET - Projectile](#)

- *Write these equations in your notes if they're not already there.*
- *You will want them for Exam 1 & the Final.*

SOH-CAH-TOA

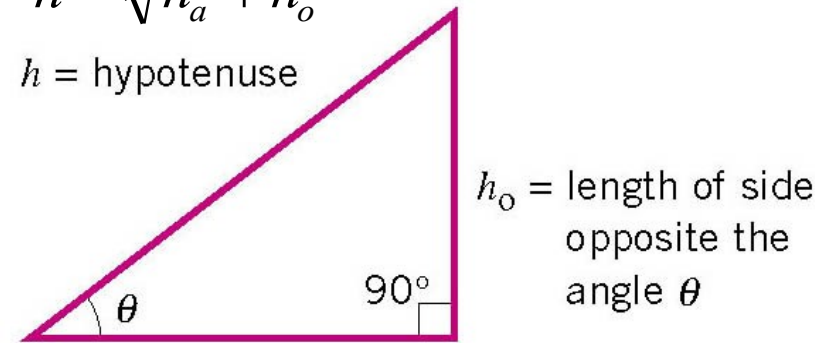
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$h = \sqrt{h_a^2 + h_o^2}$$

h = hypotenuse



h_a = length of side adjacent to the angle θ

Definitions: Velocity & Acceleration with Vectors

- These equations are 2 ways to say the same thing:

This equation: $\bar{\vec{v}} = \frac{\vec{s} - \vec{s}_0}{t - t_0} = \frac{\Delta\vec{s}}{\Delta t}$ says the same thing as this one: $\vec{s} = \vec{s}_0 + \bar{\vec{v}}\Delta t$

This equation: $\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t}$ says the same thing as this one: $\vec{v} = \vec{v}_0 + \bar{\vec{a}}\Delta t$

- The two sides of the equation are equal only if the COMPONENTS are EQUAL, so we get one formula for each dimension

Instead of: $\vec{v} = \vec{v}_0 + \bar{\vec{a}}\Delta t$ we can use:

$$v_x = v_{0x} + a_x\Delta t$$

$$v_y = v_{0y} + a_y\Delta t$$

Formulae for constant acceleration in 2-Dimensions

If acceleration is constant, can use 2 equations for each component:

$$x = x_0 + \bar{v}_x t$$

$$y = y_0 + \bar{v}_y t$$

$$\bar{v}_x = \frac{1}{2}(v_{0x} + v_x)$$

$$\bar{v}_y = \frac{1}{2}(v_{0y} + v_y)$$

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

...this is great! No need to learn new equations. We just apply our same equations for 1-Dimension to each component individually.

Projectile Motion: A special case for 2D Kinematics

- *Definition:* 2D-dimensional motion with **gravity as the only force**
- Separate components of projectile motion visualized: [HTML5](#)



Projectile Motion

This is a special, but commonly occurring, case for 2D motion. ...sort of like Free-Fall for 1D



Projectile motion is like free-fall, but with horizontal uniform motion:

- For free-fall: acceleration 9.8 m/s² downward
- IF we choose y-axis vertical (+y up typically)
- Then: $a_x = 0 \text{ m/s}^2$, $a_y = -9.8\text{m/s}^2$
- [HTML5](#)
- Our 2D equations of motion can be simplified:

$x = x_0 + \bar{v}_x t$

~~$\bar{v}_x = \frac{1}{2}(v_{0x} + v_x)$~~

~~$v_x = v_{0x} + a_x t$~~

~~$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$~~

~~$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$~~

*Uniform motion
in x:*



$v_x = v_{0x}$

$x = x_0 + v_{0x} t$

Free-fall in y:

$y = y_0 + \bar{v}_y t$

$\bar{v}_y = \frac{1}{2}(v_{0y} + v_y)$

$v_y = v_{0y} + a_y t$

$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

Steel ball A is dropped from rest.

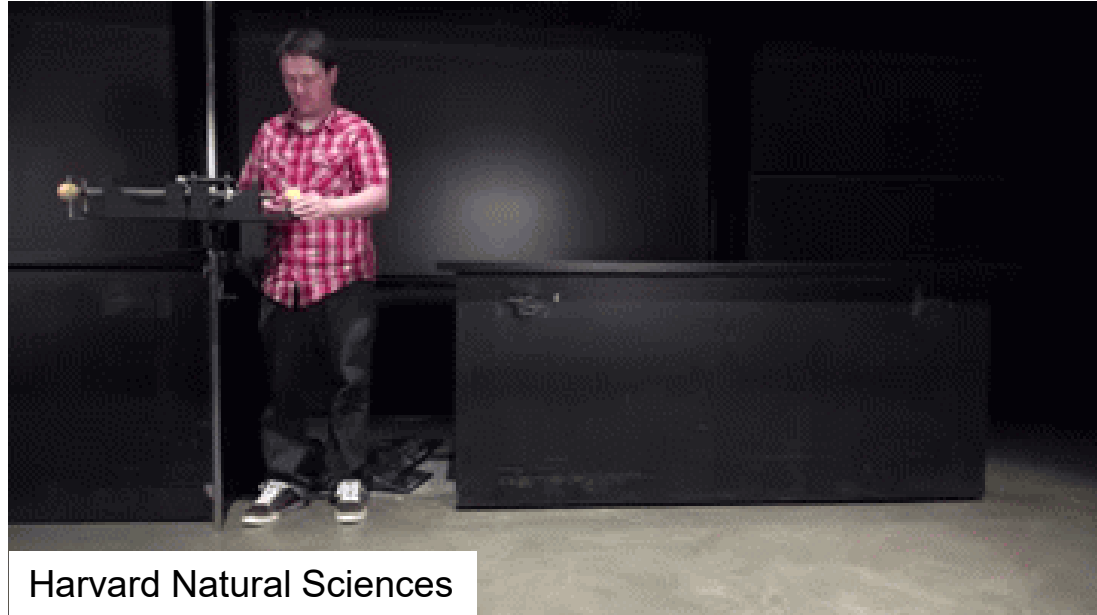
Steel ball B is shot horizontally from the same height.

Which ball hits the floor first?

A. Ball A: dropped

B. Ball B: shot

C. Both hit at the same time



<https://www.youtube.com/watch?v=zMF4CD7i3hg>

Vertical motion is independent of horizontal motion.

Therefore, the vertical behavior is the same.



Steel ball A is dropped from rest.

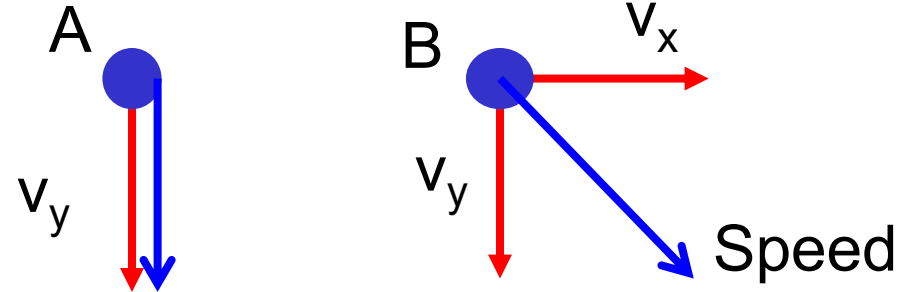
Steel ball B is shot horizontally from the same height.

Which ball is traveling the fastest before it hits the floor?

A. Ball A: dropped

B. Ball B: shot

C. They have the same speed



Ball A and B have the same v_y

Ball A: $v_x = 0$

Ball B: $v_x > 0$

Way to check for reasonableness of answer:

- Speed should be greater than or equal to magnitude of largest velocity component.*
- Speed must be larger than the magnitude of the largest component if there is more than one non-zero component.*



A cart travels at constant speed along a level track.

At a given point, a ball pops straight up from the cart.

Which of the following is the best option for the behavior of the ball.

- A. The ball will land in front of the cart
- B. The ball will land back in the cart
- C. The ball will land behind the cart
- D. The ball will land at the point on the track where the ball was launched

Horizontal motion is independent of vertical motion.

In this case, the cart & ball have the same (constant) horizontal velocity.

<https://www.youtube.com/watch?v=XawCNYpeKD4>



Doug Smith



A soccer ball is kicked on a level playing field. Its initial velocity is 20m/s at an angle of 30° above the horizontal. What is the most accurate statement about the speed of the ball?

- A. It is a minimum at maximum height
- B. It is zero at maximum height
- C. It is a minimum just before it lands
- D. It is a minimum just after it is kicked
- E. It has the same speed throughout the trajectory

v_x is constant. Speed depends on both v_x and v_y . So, smallest speed when $v_y=0$.

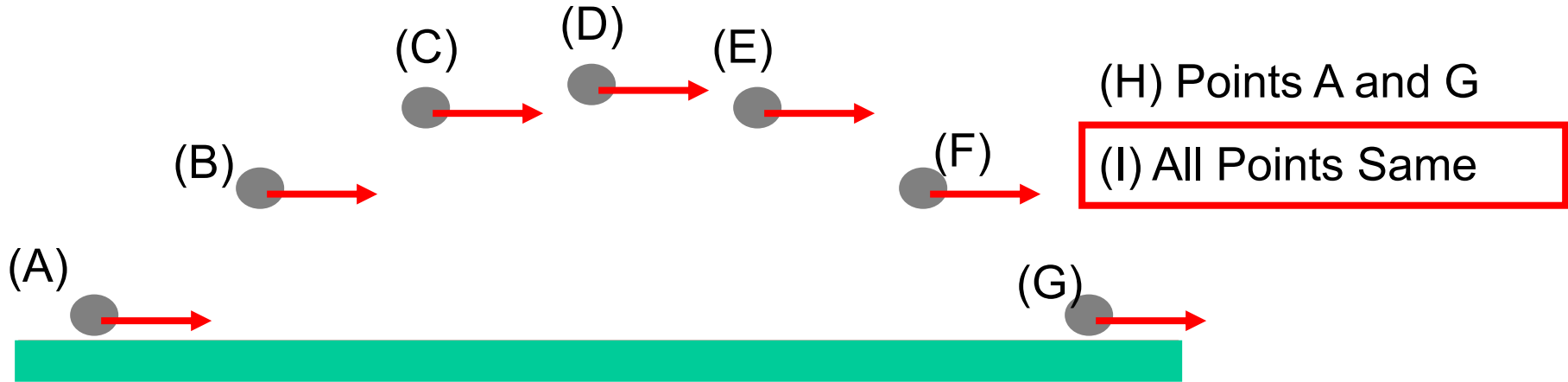


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The motion diagram of a kicked soccer ball is pictured here.

The snapshots (gray circles) are for 1 second intervals.

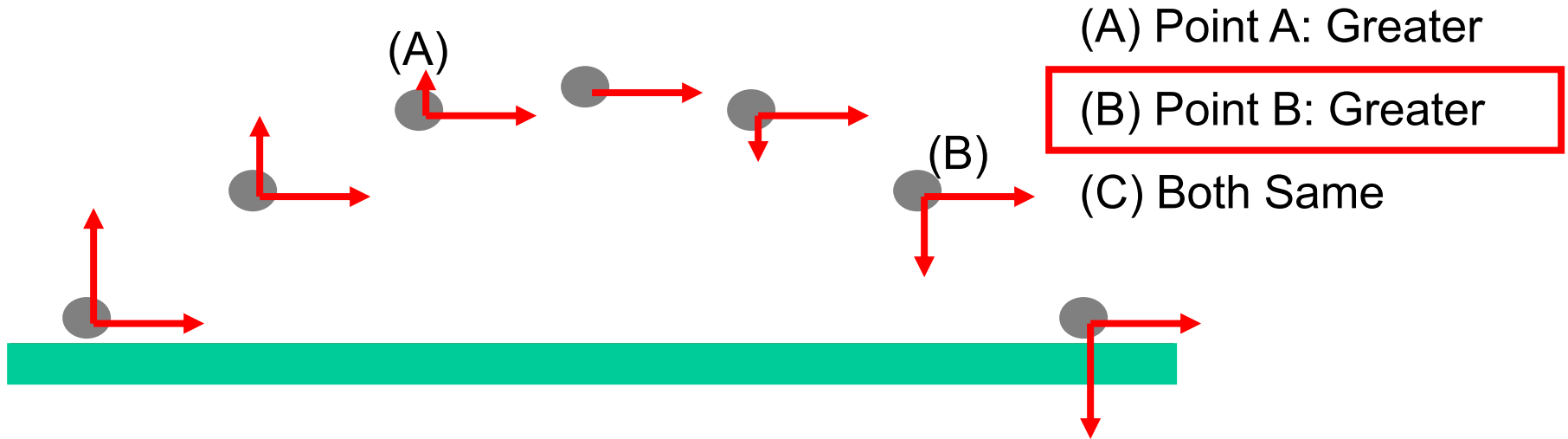
At which point is the magnitude of the horizontal component of the velocity the greatest? (Ignore air resistance.)



The motion diagram of a kicked soccer ball is pictured here.

The snapshots (gray circles) are for 1 second intervals.

How does the magnitude (size) of the vertical component of the velocity compare between Point A and Point B?



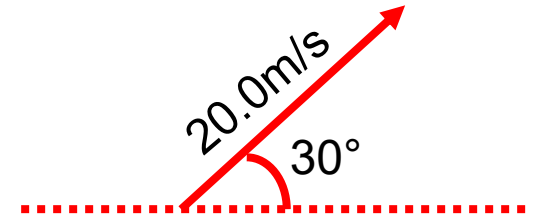
Note the symmetry. If there is no air resistance (always the case for this class), at a fixed height magnitude of velocity as it comes down is the same as when it was going up (Though the directions are different.)

At fixed height $|v_y|$ is the same.

A soccer ball is kicked on a level patch of ground. Its initial velocity is 20.0 m/s at an angle of 30° above the horizontal. What is the speed of the ball at maximum height?

- A. 0 m/s
- B. 4.5 m/s
- C. 10.0 m/s
- D. 12.4 m/s
- E. 17.3 m/s**
- F. 20.0 m/s

1. For “maximum height”, we know: $v_y = 0$.
2. Projectile motion, so we know $v_x = \text{constant} = v_{x0}$.
3. Recall **SOHCAHTOA**,



4. $v_{x0} = (20.0\text{m/s}) \cos(30^\circ) = 17.3 \text{ m/s}$

A soccer ball is kicked on a level patch of ground.
It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

- 1) What is the maximum height the ball reaches?
- 2) How much time is the ball in the air?
- 3) How far away does it hit the ground? (what is the range)
- 4) What is the final speed of the ball just before it hits the ground?

Equations for Projectile Motion in 2D

$$v_x = v_{0x}$$
$$x = x_0 + v_{0x}t$$

*Uniform motion
for horizontal
motion*

*Free-fall
for
vertical
motion*

$$v_y = v_{0y} + a_y t$$
$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$
$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$
$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

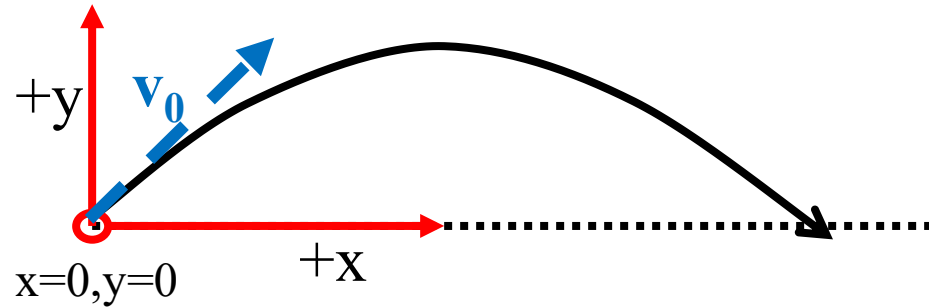
Detailed Example: Soccer Ball (Part 1)

A soccer ball is kicked on a level patch of ground.

It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

1) What is the maximum height the ball reaches?

Draw, (choose axes & origin)



Know? Want to know?

- $v_0 = 20.0\text{m/s}$ at 30° angle
- “Max height”: $v_y = 0$
- Projectile motion: $a_y = -9.8\text{m/s}^2$

Which equation to use?

- Know: $v_y, v_{y,0}, a_y$. Want $y - y_0$so use Eqn. 6.

Solve:

1. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$
2. $(0\text{m/s})^2 = ((20\text{m/s}) \cdot \sin(30^\circ))^2 + 2(-9.8\text{m/s}^2)(y)$
3. $y = (-100\text{m}^2/\text{s}^2) / (-19.6\text{m/s}^2) = 5.1\text{m}$

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_yt$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_yt^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

Detailed Example: Soccer Ball (Part 2)

A soccer ball is kicked on a level patch of ground.

It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

2) How much time is the ball in the air?

Know:

- $y=y_0$, because starts & ends on ground.
- $a_y=-9.8\text{m/s}^2$, because projectile motion.
- $v_{y,0} = v_0 \sin(\theta)$, from SOHCAHTOA

Solution:

$$1. \quad y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$$

$$2. \quad y - y_0 - v_{y,0}t = \frac{1}{2}a_y t^2$$

$$3. \quad \frac{2(y - y_0 - v_{y,0}t)}{a_y} = t^2$$

$$4. \quad \dots \text{since } y=y_0=0: \frac{-2v_{y,0}t}{a_y} = t^2$$

$$5. \quad t = \frac{-2\left(20\frac{\text{m}}{\text{s}}\right)\sin(30)}{-9.8\text{m/s}^2} = 2.04\text{s}$$

Want:

- t when ball returns to ground ($y=0$)

Best equation?:

- Really just solving free-fall
- Equation 4 looks like a good choice

2D Projectile motion eqns.

$$1. \quad v_x = v_{x,0}$$

$$2. \quad x = x_0 + v_{x,0}t$$

$$3. \quad v_y = v_{y,0} + a_y t$$

$$4. \quad y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$$

$$5. \quad y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$$

$$6. \quad v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$$

Detailed Example: Soccer Ball (Part 3)

A soccer ball is kicked on a level patch of ground.

It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

3) How far away does it hit the ground? (what is the range)

Know:

- $y=y_0$, because starts & ends on ground.
- $a_y=-9.8\text{m/s}^2$, because projectile motion.
- $v_{y,0} = v_0 \sin(\theta)$, from SOHCAHTOA

Solution:

1. $x = x_0 + v_{x,0}t$
2. $x = 0\text{m} + \left(20 \frac{\text{m}}{\text{s}}\right) \cos(30^\circ)t$
3. ...just solved for $t = 2.04\text{s}$
4. $x = \left(17.32 \frac{\text{m}}{\text{s}}\right) (2.04\text{s}) = 35.3\text{m}$

Want:

- Final horizontal position, x

Best Equation?

- There is just uniform motion in x
- Equation 2 is best

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

Detailed Example: Soccer Ball (Part 4)

A soccer ball is kicked on a level patch of ground.

It's initial velocity is 20.0m/s at an angle of 30° above the horizontal.

4) What is the final speed of the ball just before it hits the ground?

Know:

- $a_y = -9.8 \text{ m/s}^2$, because projectile motion.
- $v_x = v_{x,0}$, because uniform horiz. Motion.
- $v_{y,0} = v_0 \sin(\theta)$, from SOHCAHTOA.

Solution:

1. $v_x = v_{x,0} = v_0 \cos(\theta) = \left(20 \frac{\text{m}}{\text{s}}\right) \cos(30^\circ) = 17.3 \text{ m/s}$

2. $v_y = v_{y,0} + a_y t = \left(20 \frac{\text{m}}{\text{s}}\right) \sin(30^\circ) + \left(-9.8 \frac{\text{m}}{\text{s}^2}\right) t$

3. ...solved for t already, so

$$v_y = 10 \frac{\text{m}}{\text{s}} - \left(9.8 \frac{\text{m}}{\text{s}^2}\right) (2.04 \text{ s}) = 9.99 \text{ m/s}$$

4. Use the Pythagorean theorem:

$$v_x^2 + v_y^2 = v^2 \quad \dots \text{ so, } v = \sqrt{(17.3 \text{ m/s})^2 + (10 \frac{\text{m}}{\text{s}})^2} = 20 \text{ m/s}$$

Want:

- Final speed, $v = \sqrt{v_x^2 + v_y^2}$

Best Equation?

- SOHCAHTOA for v_x
- Equation 3 for v_y

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0} t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0} t + \frac{1}{2} a_y t^2$
5. $y = y_0 + \frac{1}{2} (v_{y,0} + v_y) t$
6. $v_y^2 = v_{y,0}^2 + 2 a_y (y - y_0)$

How to interpret a problem:

Often you will have to glean extra information from the problem to find out what you know, what you don't know, and what is asked. (This is almost always the case in real life)

- “Speed”: magnitude of velocity
- “Max height”: vertical velocity is zero
- “Just after ___ leaves ___”: initial information ($t = 0$)
- “Just after ___ hits ___”: final information ($t = t_{final}$)
- Look for clues about coordinates: “horizontal”, “vertical”
- Note if there are any other forces other than gravity
 - If other forces are there, acceleration will not be 9.8m/s^2

A firefighter is spraying water on a building.

Water leaves the hose at 35m/s and an angle of 30° above the horizontal.

The nozzle of the hose is 1.0m above the ground, and the building is 22m away.

The building is 40m tall.

What are the horizontal (x) and vertical (y) components of the initial velocity?

(i.e. just after the water leaves the hose)



(A) $v_{x0} = 30.3 \text{ m/s}$; $v_{y0} = 30.3 \text{ m/s}$

(B) $v_{x0} = 17.5 \text{ m/s}$; $v_{y0} = 17.5 \text{ m/s}$

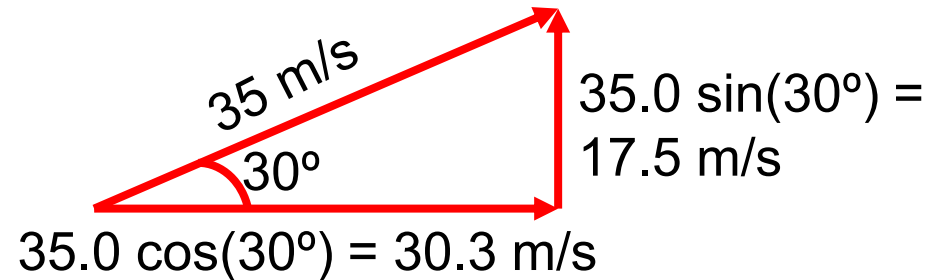
(C) $v_{x0} = 17.5 \text{ m/s}$; $v_{y0} = 30.3 \text{ m/s}$

(D) $v_{x0} = 30.3 \text{ m/s}$; $v_{y0} = 17.5 \text{ m/s}$

(E) $v_{x0} = 35.0 \text{ m/s}$; $v_{y0} = 17.5 \text{ m/s}$

(F) $v_{x0} = 35.0 \text{ m/s}$; $v_{y0} = 35.0 \text{ m/s}$

SOHCAHTOA





A firefighter is spraying water on a building.

Water leaves the hose at 35m/s and an angle of 30° above the horizontal.

The nozzle of the hose is 1.0m above the ground, and the building is 22m away.

The building is 40m tall.

How much time does it take for the water to reach the building?

(A) 0.63s

(B) 0.73s

(C) 1.00s

(D) 1.32s

(E) 1.78s

We just need to know how much time it takes for the water to cover the horizontal distance between the firefighter & building.

The horizontal motion is described by uniform motion, so we only need to determine the initial horizontal velocity:

$$v_x = v_{x,0} = v \cdot \cos(\theta) = (35\text{m/s})\cos(30^\circ) = 30.3\text{m/s}$$

Then we can determine how long it takes to cover the distance at that speed:

$$v_x = \Delta x / \Delta t \rightarrow t = \Delta x / v_x = (22\text{m}) / (30.3\text{m/s}) = 0.73\text{s}$$



A firefighter is spraying water on a building.

Water leaves the hose at 35m/s and an angle of 30° above the horizontal.

The nozzle of the hose is 1.0m above the ground, and the building is 22m away.

The building is 40m tall.

In what vertical direction is the water traveling when it hits the building?

(A) up

(B) level

(C) down

We need to know the vertical component of velocity at the time the water strikes the building.

We found the water strikes the building at 0.73s, so we need the velocity then:

$$v_y = v_{y,0} + a*t = v_0*\sin(\theta) - g*t = (35\text{m/s})\sin(30^\circ) - (9.8\text{m/s}^2)0.73\text{s} = 10.4\text{m/s}$$

Is this vertical velocity component greater than zero?

Yes. So the water must still be heading upward.