

Thursday January 19

- Assignment 2 – Friday, 11:59pm  
....like every Friday
- Pre-Class Assignment 15min before class ...like every class
- First Labs Next Week
- Office Hours:
  - Wed. 10-11am, 204 EAL
  - Or by appointment



This lecture:

- 2-dimensional motion
- Vectors
- Components

Given any two sides or one side and an angle can reconstruct the whole triangle

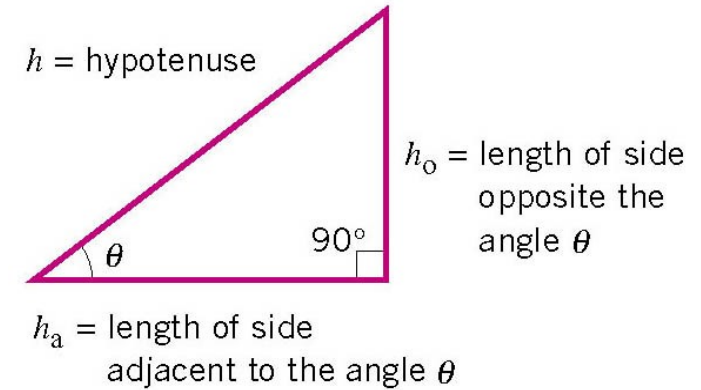
SOH-CAH-TOA

$$h = \sqrt{h_a^2 + h_o^2}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$





A person on a bridge sees a raft floating at a constant speed on the river below and decides they are going to drop a stone onto it.

The stone is released when the raft has 7.00 m more to travel before the front passes under the person on the bridge.

The stone hits the very front of the raft.

If the stone takes 3.5s to fall, what is the speed of the raft?

(A) 0.5 m/s

(B) 2 m/s

(C) 3.5 m/s

(D) 7 m/s

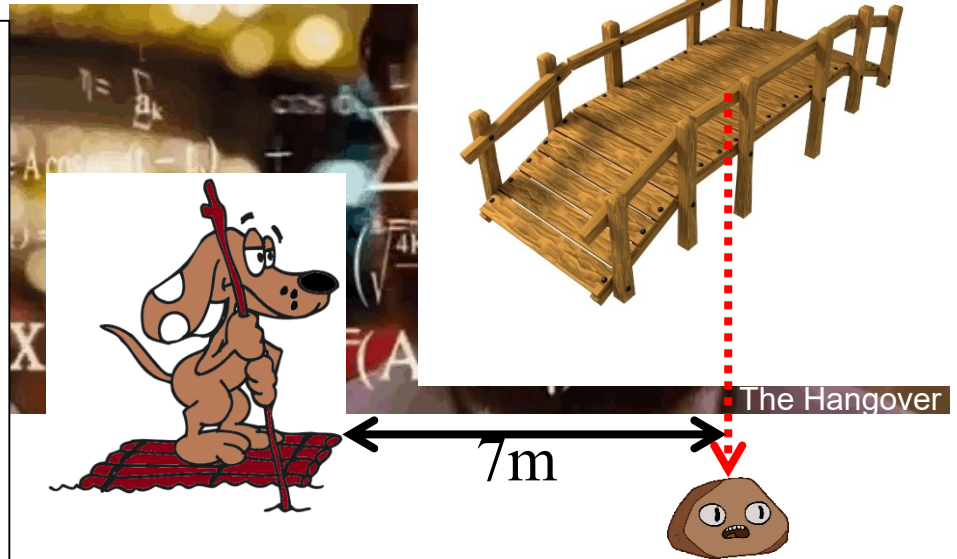
(E) 24.5 m/s

(F) need to know he

Don't make things more complicated than they have to be!

The horizontal stone position is your final position. Your initial position is 7m before that. You know you make it to the final position in 3.5s.

$$v = \text{distance}/\text{time} = 7.0\text{m}/3.5\text{s} = 2\text{m/s}$$





A person on a bridge sees a **6m-long** raft floating at a constant speed on the river below and decides they are going to drop a stone onto it. The stone is released when the raft has 7.00 m more to travel before the front passes under the person on the bridge. The stone hits the **exact middle** of the raft. If the stone takes 3.5s to fall, what is the speed of the raft?

- (A) 1.1 m/s      (B) 2 m/s      (C) 2.9 m/s  
(D) 3.5m/s      (E) 3.7 m/s

This is basically the same as the last problem, but the raft has now traveled an extra half-length of the raft, i.e. an extra 3.0m.

$$\text{So, } v = \text{distance/time} = (7.0\text{m}+3.0\text{m})/3.5\text{s} = 2.9\text{m/s}$$

A person on a bridge 75.0m-high sees a raft floating at a constant speed on the river below. This person drops a stone from rest in an attempt to hit the raft. The stone is released when the raft has 7.00 m more to travel before passing under the bridge. The stone hits the water 4.00 m in front of the raft.



What is the speed of the raft?

- (A) 1.13 m/s      (B) 2.00 m/s      (C) 2.89 m/s  
(D) 3.52m/s      (E) 3.71 m/s      (F) 0.767 m/s

1. Need to find the time for the stone to hit the water

- $y = y_0 + v_0t + (\frac{1}{2})at^2$
- $0\text{m} = 75.0\text{m} + (0\text{m/s})t + (\frac{1}{2})(-9.80\text{m/s}^2)t^2$

$$\bullet t = \sqrt{(-75.0\text{m})/(-4.90 \frac{\text{m}}{\text{s}^2})} = 3.91\text{s}$$

2. Then we can solve for our velocity

$$\bullet v = \Delta x/\Delta t = (7.00 - 4.00)/3.91 = 0.767\text{m/s}$$



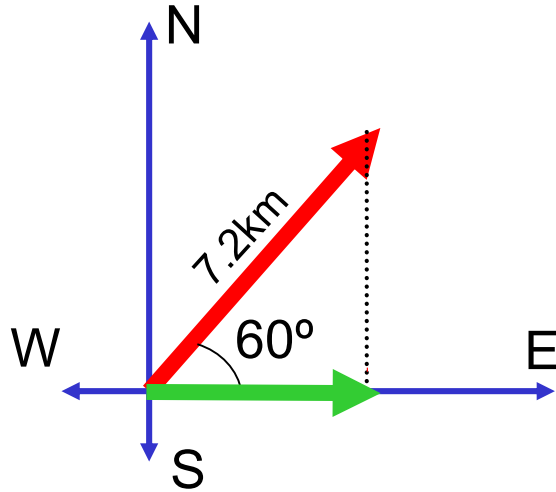
1D, a=constant Eqns:

1.  $y = y_0 + \bar{v}t$
2.  $\bar{v} = \frac{v_0+v}{2}$
3.  $v = v_0 + at$
4.  $y = y_0 + v_0t + (\frac{1}{2})at^2$
5.  $v^2 = v_0^2 + 2a(y - y_0)$

# Homework Notes:

- You don't have to work linearly.  
Consider picking-off the low-hanging fruit first.
- Solve problems like we do in class:
  - Draw the problem, what do you know?, what do you want to know?, which equation is best?, check answer for units & magnitude
- # of g's, just means the acceleration is what multiple of acceleration on earth ( $|g|=9.8\text{m/s}^2$ )
  - For example, if  $a=19.6\text{m/s}^2$ , then this is  $2g$  (because twice  $9.8\text{m/s}^2$ )
- For direction, can specify as an angle using the units "deg"
  - Like a compass direction. For example "35° north of east" means point East, then rotate your vector 35° towards the North
- Acceleration corresponds to the curvature (change in slope) on a position vs time graph

A hiker travels 7.2km in a direction  $60^\circ$  North of East.  
How far East did they travel?



- A. 2.3 km
- B. 3.6 km**
- C. 6.2 km
- D. 7.2 km
- E. 8.3 km
- F. 14.4 km

"North of East"

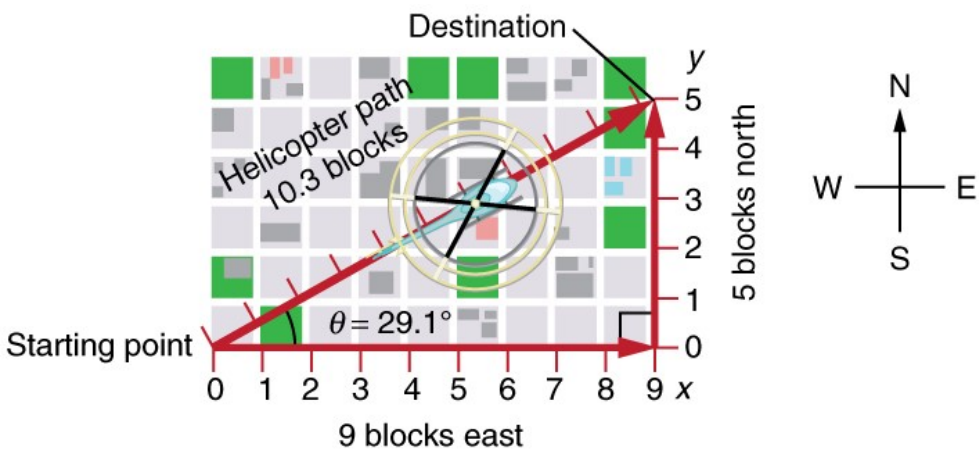
Point East, then change direction by  $60^\circ$  to the North

For angles enter units as deg on LONCAPA

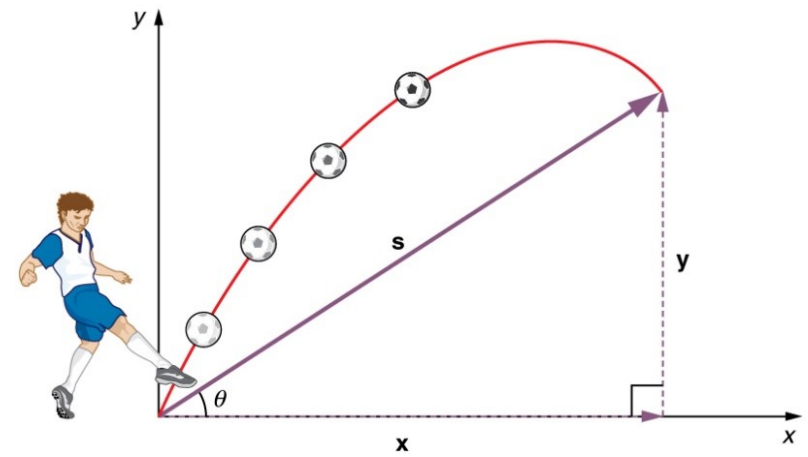
1. Looking for the length of the side that is "adjacent" to our angle
2. Recall, "SOHCAHTOA"
3. Note that we have hypotenuse "H" and want adjacent "A"
4. So cosine should do the trick (CAH):
  1.  $\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$
  2.  $\text{Adjacent} = \text{Hypotenuse} * \cos(\theta) = 7.2\text{km} * \cos(60^\circ)$
  3.  $\text{Adjacent} = 3.6 \text{ km}$

# 2D Kinematics: Describing Motion in 2 Dimensions

Use when something is not moving along one axis



Special Case: Projectile Motion  
→ gravity is the only force



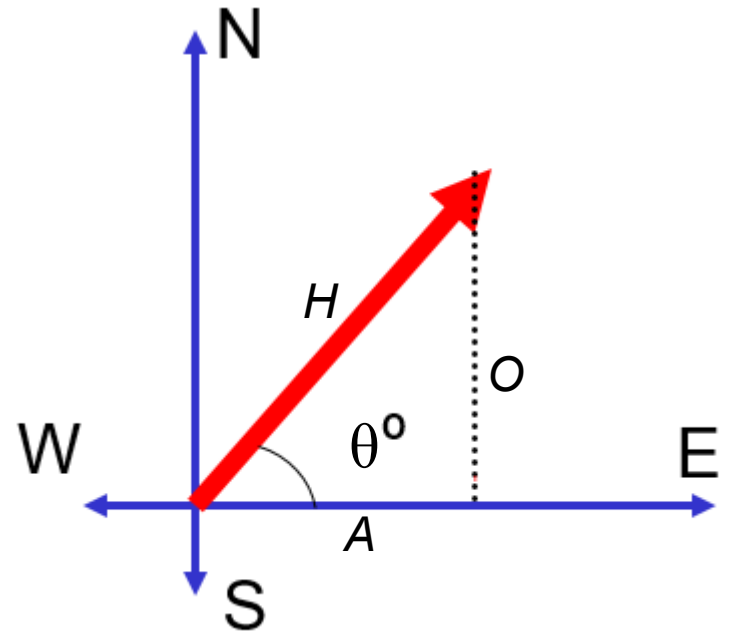
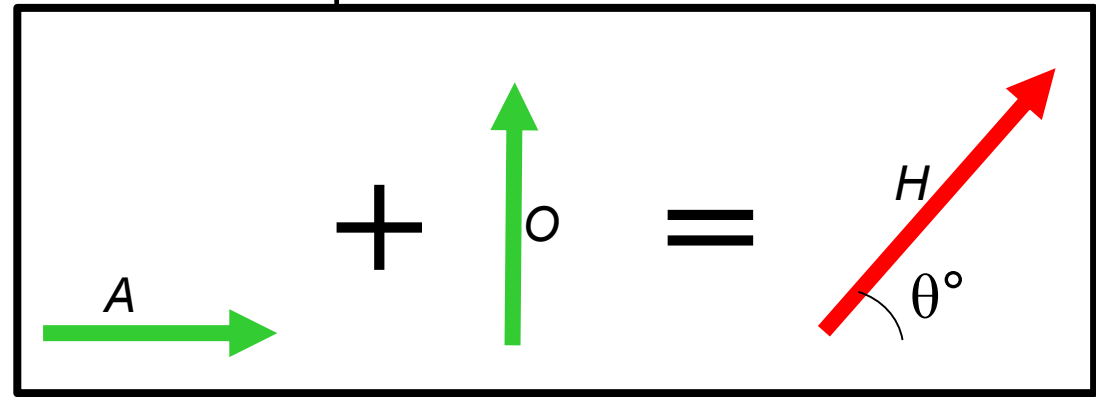
**Key:** Need to work with vectors.  
Break into components, work with each dimension individually  
and later reconstruct vectors.

# 2D Kinematics: Describing Motion in 2 Dimensions

**Key:** Need to work with vectors.  
Break into components, work with each dimension individually  
and later reconstruct vectors.

For example, *instead* of saying  
“a vector  $H$  distance  $\theta^\circ$  North of East”  
State vector components:  
“ $A$  East and  $O$  North”

Combine components to reconstruct vector:





- *Scalars*: Have magnitude (roughly speaking, a “size”)
- *Vectors*: Have magnitude ***and*** direction (relative to an origin)

Quick way to check: Does adding a geographical direction make sense?

- A force of 10 N to the East ? fine – vector
- A mass of 10kg to the West? nonsensical - scalar

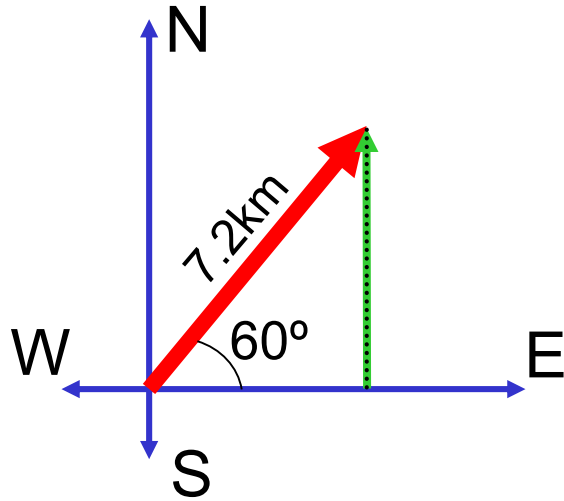
Examples of *Scalars*:

- Time
- Mass
- Energy

Examples of *Vectors*:

- Force
- Displacement
- Velocity

A hiker travels 7.2km in a direction  $60^\circ$  North of East.  
How far North did they travel?



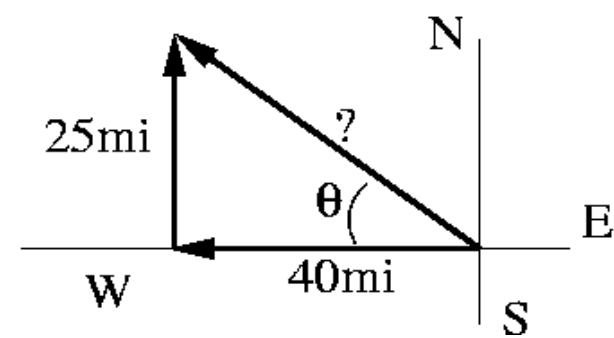
- A. 2.3 km
- B. 3.6 km
- C. 6.2 km
- D. 7.2 km
- E. 8.3 km
- F. 14.4 km

"North of East"

Point East, then change direction by  $60^\circ$  to the North

1. Looking for the length of the side that is "opposite" to our angle
2. Recall, "SOHCAHTOA"
3. Note that we have hypotenuse "H" and want opposite "O"
4. So sine should do the trick (SOH):
  1.  $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$
  2.  $\text{Opposite} = \text{Hypotenuse} * \sin(\theta) = 7.2\text{km} * \sin(60^\circ)$
  3.  $\text{Adjacent} = 6.2 \text{ km}$

You drive 40 mi due West, then switch drivers.  
Your friend then drives 25mi due North.  
What is your total displacement?



(A) 47mi 32° N of W

(B) 47mi 39° N of W

(C) 47mi 58° N of W

(D) 65mi 32° N of W

(E) 65mi 39° N of W

(F) 65mi 58° N of W

Break this into 2 parts:

(1) How far away are we?      (2) In what direction?

1) Recall the Pythagorean theorem:

•  $a^2 + b^2 = c^2$  ....or: (adjacent)<sup>2</sup> + (opposite)<sup>2</sup> = (hypotenuse)<sup>2</sup>

•  $h = \sqrt{a^2 + o^2} = \sqrt{(25mi)^2 + (40mi)^2} = 47mi$

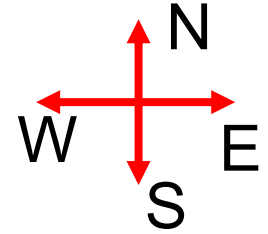
2) Recall SOHCAHTOA:

• We know opposite (O) and adjacent (A) ... so use tangent (T)

•  $\tan(\theta) = \text{opposite/adjacent} \rightarrow \tan^{-1}(\text{opposite/adjacent}) = \tan^{-1}(25mi/40mi) = 32^\circ$

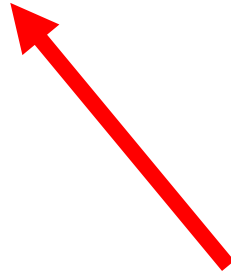
# Magnitude: “how big” ...a.k.a “size”

- Magnitude of a **scalar** is just the absolute value.
  - Drop any negative sign
- Magnitude of a **vector** is the size of that vector
  - Never negative

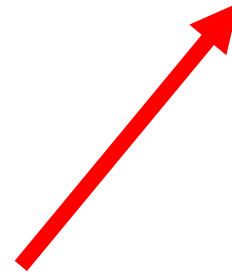


Example 1:

6km 60° N of W



6km 60° N of E



Magnitude of both vectors is 6km

Example 2:

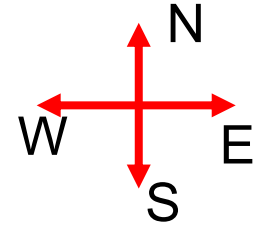
15N to West



Magnitude is 15N

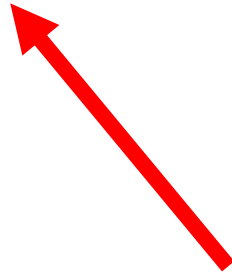
# Equality of vectors

- Two vectors are equal if magnitude **and** direction are the same
  - Does not matter where they are drawn

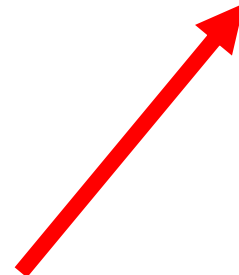


Example 1:

6km 60° N of W



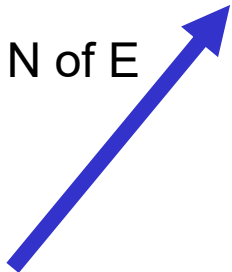
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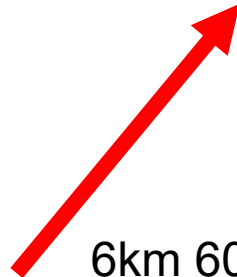
6km 60° N of E

Example 2:

6km 60° N of E



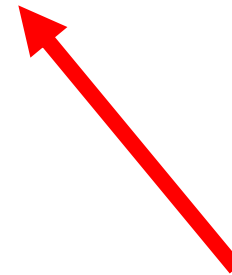
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6km 60° N of E

Example 3:

6km 60° N of W



≠

3km 60° N of W



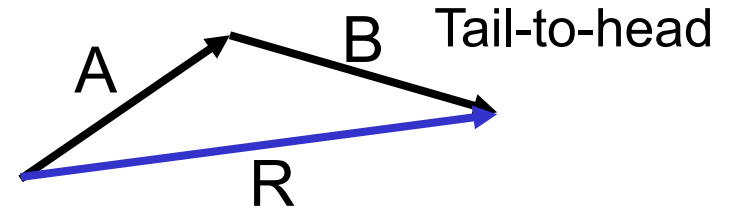
# Vector Operations

- Add: (vector 1) + (vector 2) = (vector 3)

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

- Adding Tail to Head:

- Can move vectors around to help visualize
- Works for any number of vectors (i.e. can combine more than 2)
- $\mathbf{R}$  is “Resultant” (the result of the addition)



## Example: Paths in a city

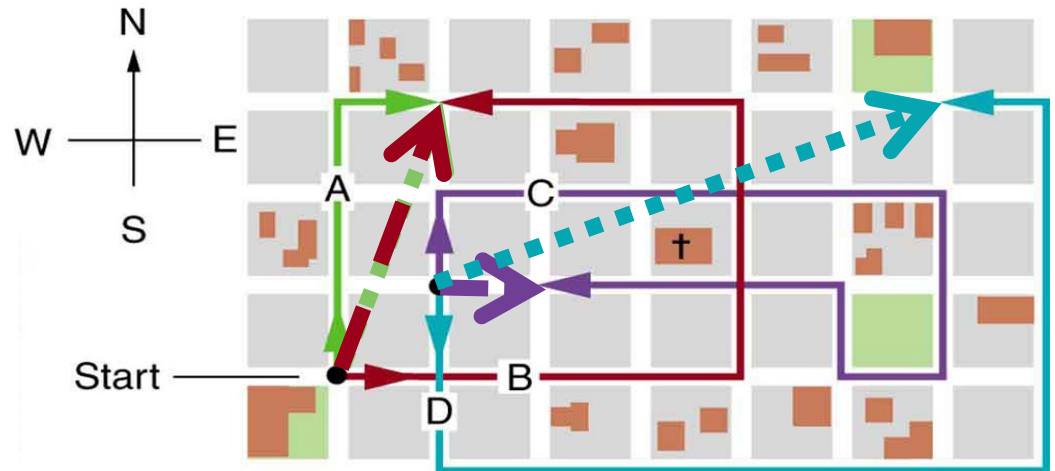
- **Vector A**

- **Vector D**

- **Vector C**

- **Vector B**

...equals vector A



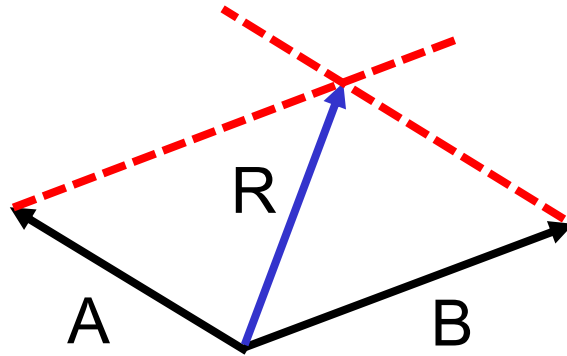
# Vector Operations

- Add: (vector 1) + (vector 2) = (vector 3)

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

Parallelogram Method (not in book, but will use in lab)

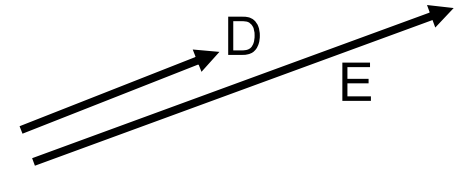
- only works with 2 vectors
- Works well with forces
- Draw parallelogram
- Tail corner to opposite corner is resultant



# Vector Operations

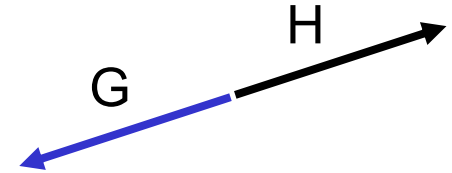
- (scalar) \* (vector) = (vector)

$$\mathbf{E} = 2\mathbf{D}$$



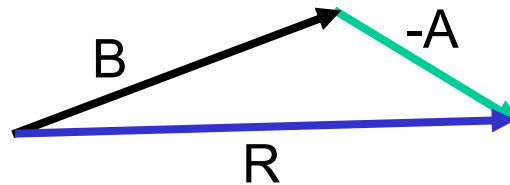
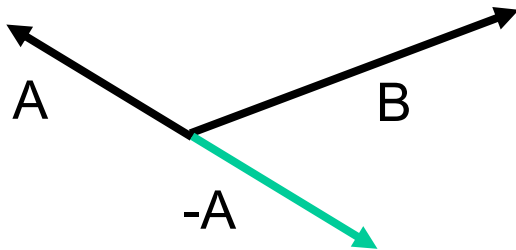
- $(-1) * (\text{vector}) = \text{vector in opposite direction}$ 
  - *remember: magnitude is never negative*

$$\mathbf{G} = -\mathbf{H}$$

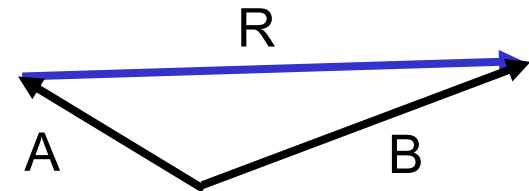


- Subtraction: (vector) +  $(-1)(\text{vector}) = (\text{vector})$

$$\mathbf{R} = \mathbf{B} - \mathbf{A}$$



$$(\mathbf{B} = \mathbf{A} + \mathbf{R})$$





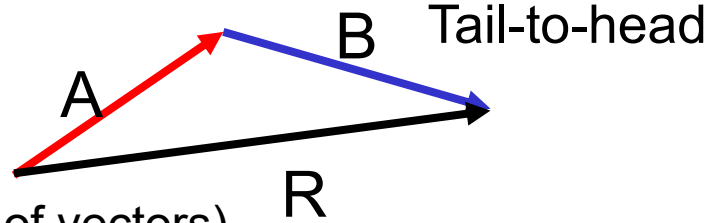
# Adding Vectors: The Math

- Add: (vector) + (vector) = (vector)

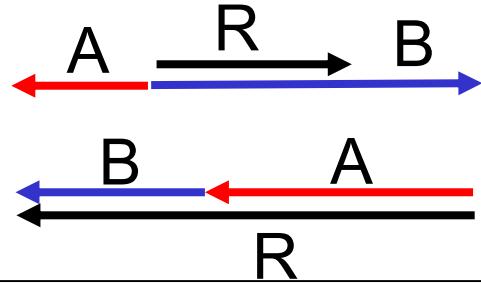
$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$\vec{R} = \vec{A} + \vec{B}$$

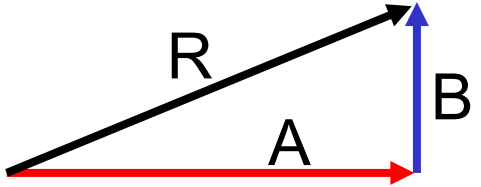
(R for Resultant – the result of adding a group of vectors)



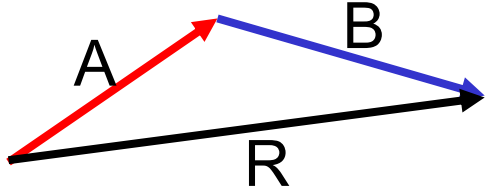
If co-linear (parallel or anti-parallel), add arithmetically.  
Pretty straightforward.



If perpendicular, treat as triangle.  
Messier, but just need a bit of trigonometry.



If neither, find way to apply easier cases.  
Break down into "components".



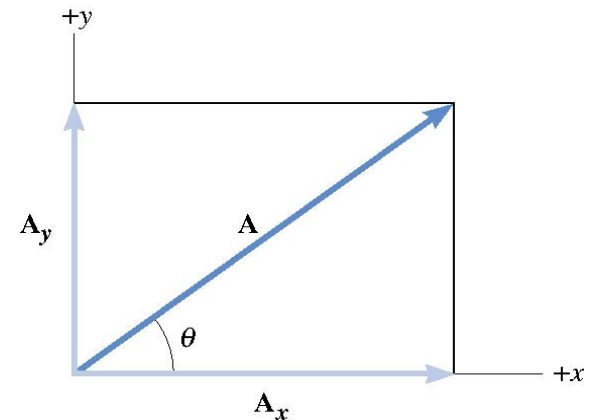
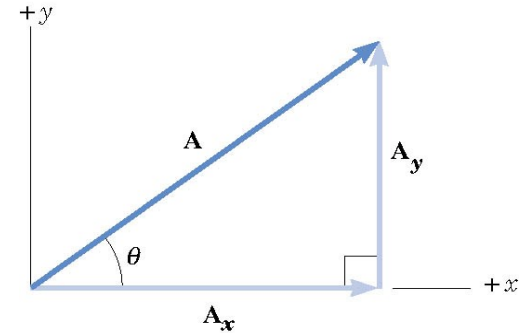
*\*see next few slides*

# Components

- Break a vector down into parts parallel to each axis.
  - "How far North, how far East?"
  - "How far up, how far right?"

If you know the magnitude and direction, you can find the components.

If you know the components, you can find the magnitude and direction.

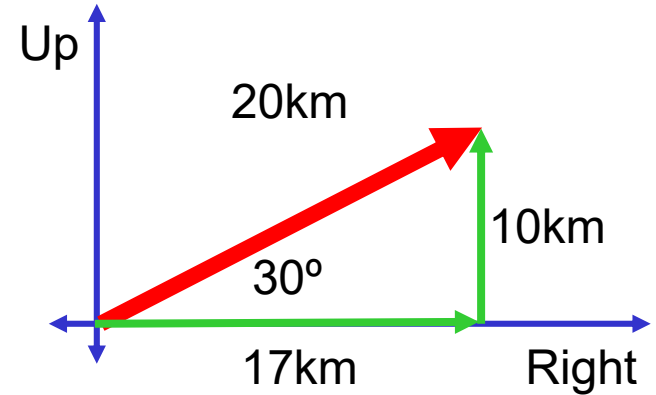


Can the same vector in different ways.  
Components will be the same.

From the upper to lower picture,  
all we did was move  $A_y$  to the left.

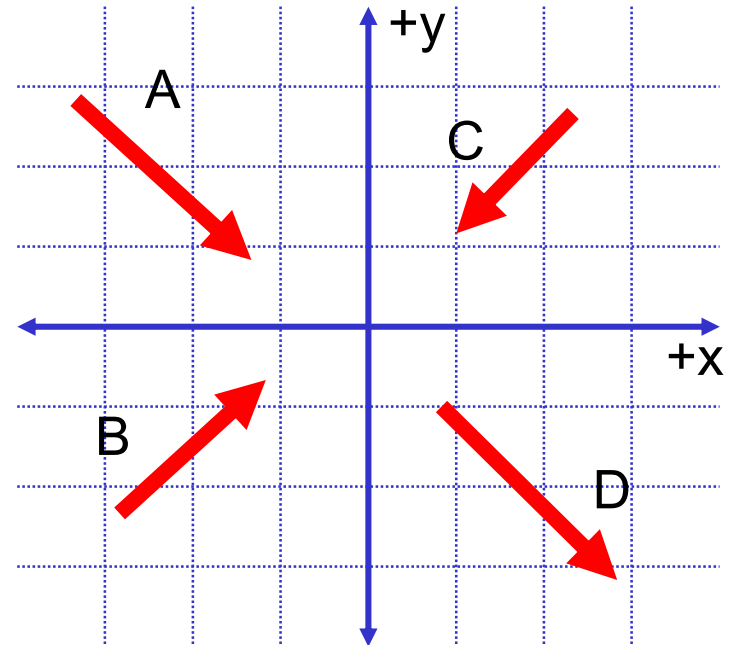
# Vector and Scalar Components

- Can refer to vector components
  - “x-component”: 17km to the right
  - “y-component”: 10km up
- Since we know the direction of a component (because it is along a single dimension) we can treat the **magnitude of a component as a scalar** and use the **sign (+ or -) to represent direction** (up/down or left/right)
  - x-component: +17km
  - y-component: +10km
- With these two components, our vector is fully defined



Which of the following vectors have a positive x-component and a negative y-component?

- (A) A    (B) B    (C) C    (D) D  
(E) A and C    (F) A and B  
(G) B and D    (H) A and D  
(I) C and D    (J) A, B, D



Since we only care about the components, the location of the vector in our graph doesn't matter.

Only its direction matters.

Which formulae give you the X and Y components of the vector **A**?

(A)  $A_x = A \sin(\theta)$ ;  $A_y = A \cos(\theta)$ ;

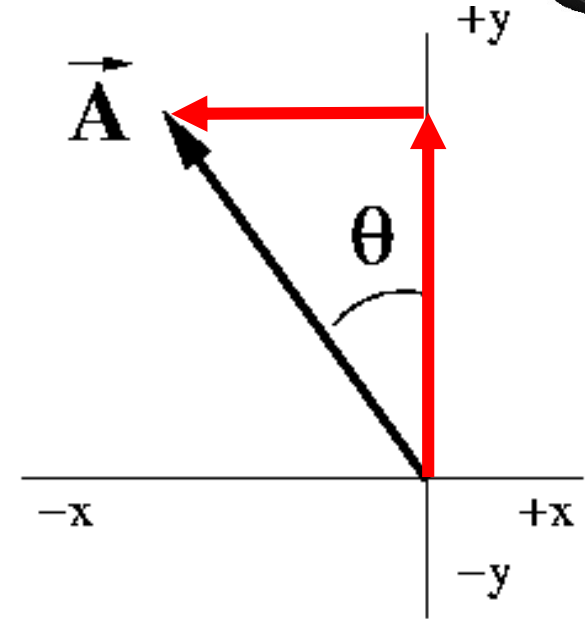
(B)  $A_x = -A \sin(\theta)$ ;  $A_y = A \cos(\theta)$ ;

(C)  $A_x = A \cos(\theta)$ ;  $A_y = A \sin(\theta)$ ;

(D)  $A_x = A \cos(\theta)$ ;  $A_y = A \sin(\theta)$ ;

(E)  $A_x = A \cos(\theta)$ ;  $A_y = -A \sin(\theta)$ ;

(F)  $A_x = A \sin(\theta)$ ;  $A_y = -A \cos(\theta)$ ;



Note that the angle doesn't increase from the +x-axis, like you may be used to.

Always draw the triangle to be sure.

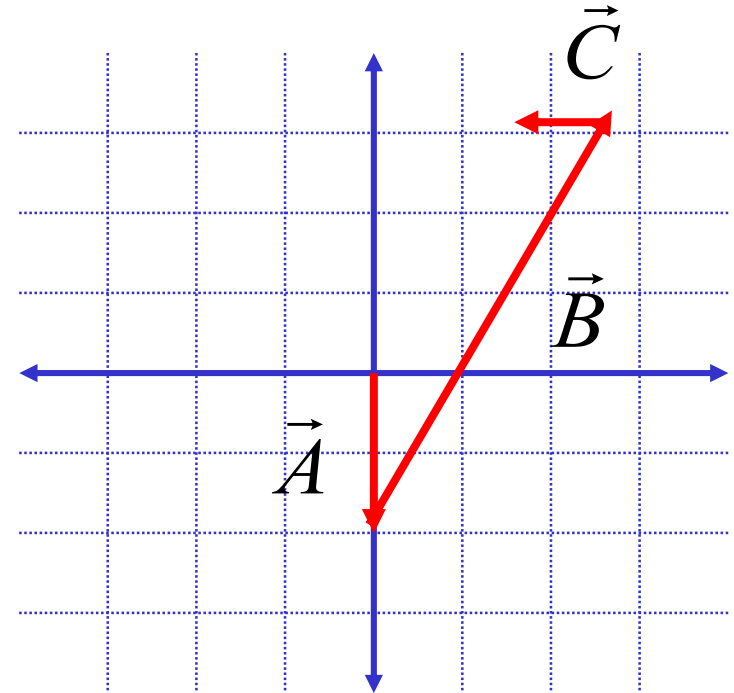
# Adding vectors by components:

1. Break into components
2. Add components
3. Reconstruct vector

For example,

You travel 10km South,  
then 30km  $60^\circ$  N of E,  
and then 5km West.

How far and in what direction are you  
away from where you started?



You travel 10km South, then 30km 60° N of E, and then 5km West. How far and in what direction are you away from where you started?

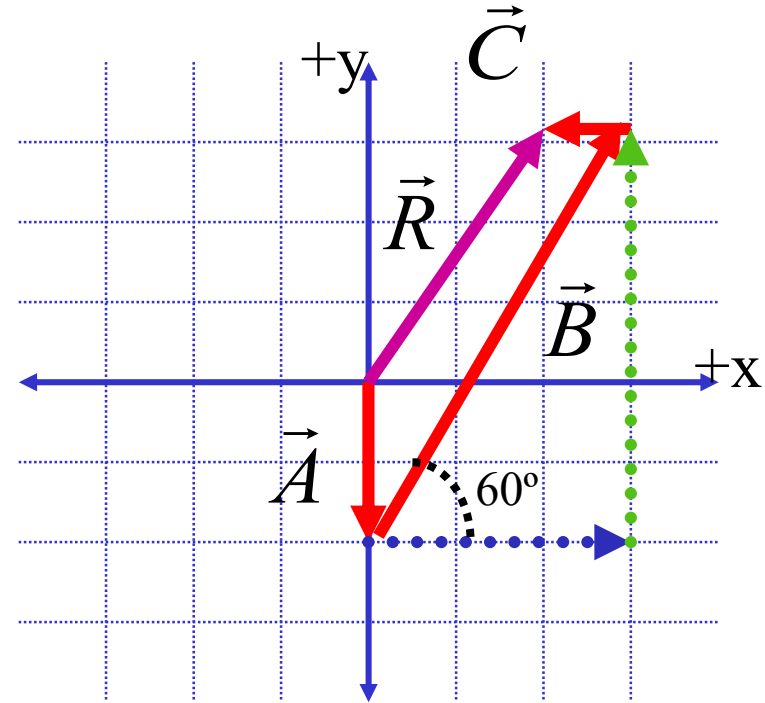
- Break into components & Add them

	x-component	y-component
A	0.0 km	-10.0 km
B	+30 km cos(60°) = <b>+15.0 km</b>	+30km sin(60°) = <b>+26.0 km</b>
C	-5.0 km	0.0 km
<b>R</b>	<b>+10.0 km</b>	<b>+16.0 km</b>

- Reconstruct the vector

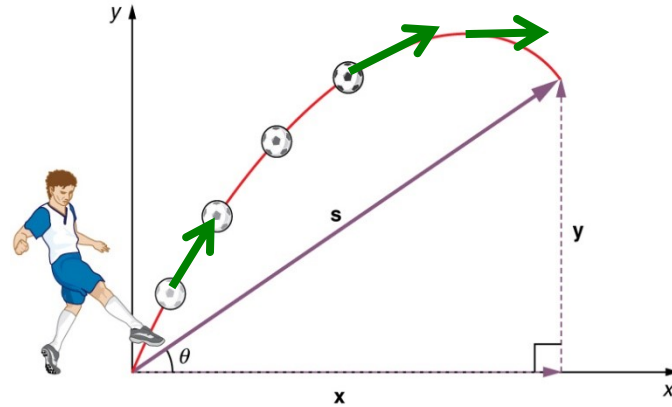
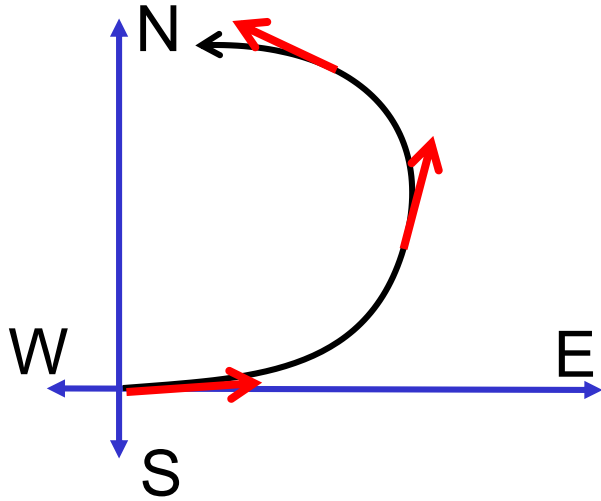
$$|\vec{R}| = \sqrt{(+10.0\text{km})^2 + (+16.0\text{km})^2} = 18.9\text{km}$$

$$\theta = \tan^{-1}\left(\frac{+16.0\text{km}}{+10.0\text{km}}\right) = 58^\circ \text{ N of E}$$



# 2D Kinematics and Velocity Vectors

- Velocity at a point is given by the **tangent** to path (or trajectory)



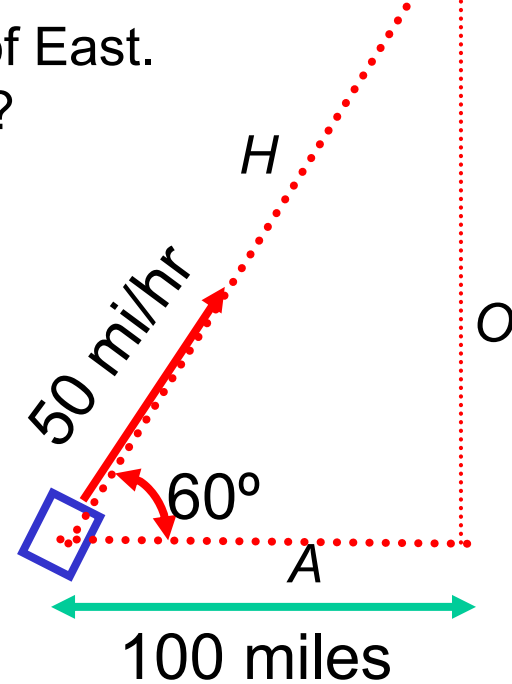
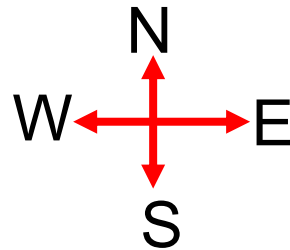
Just like displacement, can break velocity vectors into components:

- How fast traveling East? North?
- How fast traveling vertically? horizontally?



A car is traveling 50 mi/hr in a direction  $60^\circ$  North of East.  
How much time will it take to travel 100 miles East?

- (A) 1 hour
- (B) 2 hours
- (C) 3 hours
- (D) 4 hours



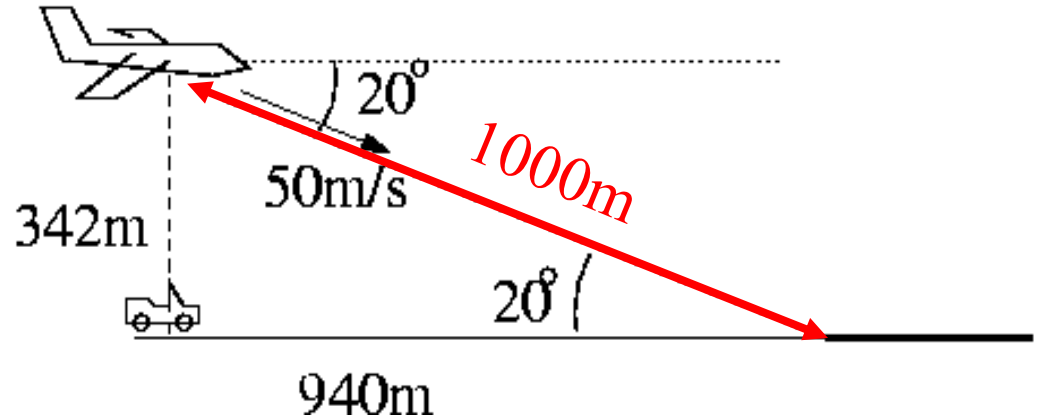
- Think components & recall SOHCAHTOA to get x-component of velocity:
  - East-West Component of Velocity:  $(50 \text{ mi/hr}) \cdot (\cos 60^\circ) = 25 \text{ mi/hr}$
  - North-South Component of Velocity:  $(50 \text{ mi/hr}) \cdot (\sin 60^\circ) = 43.3 \text{ mi/hr}$
- Now solve for time given the x-velocity:
  - If traveling 25 miles East every hour, takes 4 hours to travel 100 miles East.

A plane is landing at a speed of 50m/s. The plane is 1000m from the runway. The plane is descending at an angle of 20 deg. As part of a stunt for a movie, a Jeep wants to travel directly underneath the plane the entire time that the plane approaches the runway.



What speed will the jeep need to travel?

- (A) 50m/s
- (B)  $(50\text{m/s})\sin 20^\circ$
- (C)  $(50\text{m/s})\cos 20^\circ$
- (D)  $(50\text{m/s})/\sin 20^\circ$
- (E)  $(50\text{m/s})/\cos 20^\circ$



Find horizontal component of velocity of plane.

The Jeep will match that.

(from SOHCAHTOA:  $x = \text{adjacent} = \text{hypotenuse} * \cos(\theta)$ )

Note that **speed involves both components** and is greater than or equal to largest component.

*Way to check for reasonableness of answer: Speed should be greater than or equal to magnitude of largest velocity component.*

A soccer ball is kicked at an initial speed of 20.0 m/s at an angle  $30^\circ$  above horizontal (+x direction). What are the horizontal (x) and vertical (y) components of the initial velocity (just after the ball leaves the foot)?

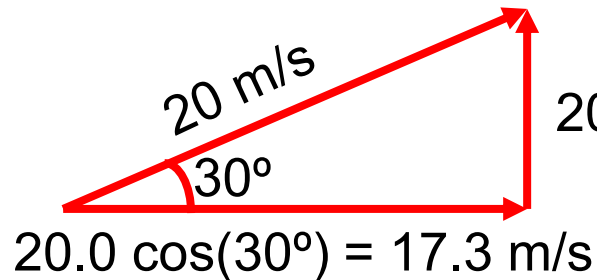
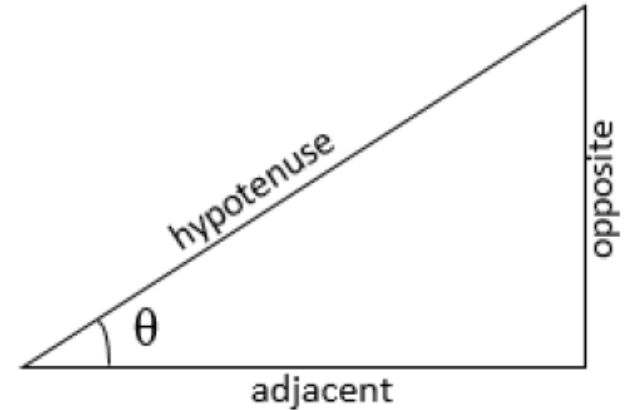
- (A)  $v_{x0} = 10.0$  m/s;  $v_{y0} = 10.0$  m/s
- (B)  $v_{x0} = 17.3$  m/s;  $v_{y0} = 17.3$  m/s
- (C)  $v_{x0} = 10.0$  m/s;  $v_{y0} = 17.3$  m/s
- (D)  $v_{x0} = 17.3$  m/s;  $v_{y0} = 10.0$  m/s**
- (E)  $v_{x0} = 20.0$  m/s;  $v_{y0} = 10.0$  m/s
- (F)  $v_{x0} = 20.0$  m/s;  $v_{y0} = 20.0$  m/s

“SOHCAHTOA”:

Sine = Opposite/Hypotenuse = “ $\sin(\theta)$ ”

Cosine = Adjacent/Hypotenuse = “ $\cos(\theta)$ ”

Tangent = Opposite/Adjacent = “ $\tan(\theta)$ ”



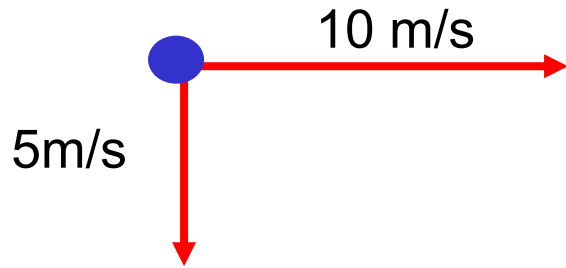
$$20.0 \sin(30^\circ) = 10.0 \text{ m/s}$$

Just before it hits the ground, a rock is traveling with a horizontal component of the velocity equal to 10 m/s to the East and a vertical component of the velocity equal to 5 m/s downward.

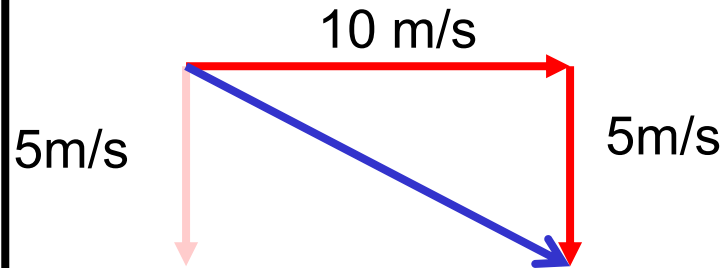
How fast is the rock traveling? (i.e. what is the magnitude of the velocity?)

- (A) 8.7 m/s    (B) 11.2 m/s    (C) 13.5 m/s    (D) 14.3 m/s    (E) 15.0 m/s

1. Draw & find components



2. Recombine components (head to tail)



3. Calculate property for reconstructed vector

$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 5^2} = 11.2 \text{ m/s}$$