Tuesday April 4

Topics for this Lecture:

- *Fluids*
  - *Density, Pressure*
  - *Pressure vs height*
  - *Buoyancy*

- Pressure = Force/Area $(P = F/A)$
- Density = Mass/Volume $(\rho = M/V)$
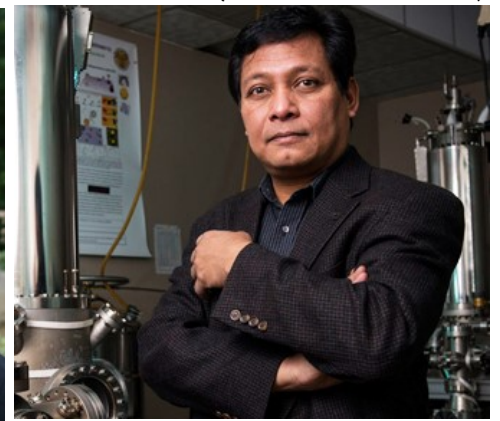- $P_2 = P_1 + \rho g h$
- $P_{gauge} = P - P_{atm}$
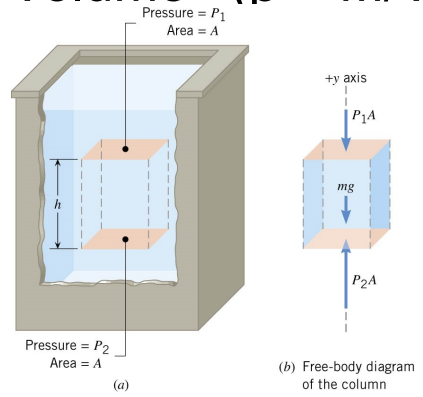
- $F_{buoyancy} = \rho_{fluid} V_{fluid\text{-}displaced} g$

Next week's lectures, guest starring:

Shiv Subedi (PHYS2001 TA)   Prof. Hla (Section 102 Instructor)

Tuesday   Thursday

Fluids

How do hose nozzles change the water speed?

How do know if something will float? How do you know how high it will sit above the water?


How do planes generate lift?

How do you use a vacuum cleaner to make a hover pad?

Fluids

- "Fluid": anything that takes the shape of its container.
    - Can be a liquid or a gas.

- A fluid is a collection of many microscopic particles.
- This is a new way for us to think about matter.
- *Instead* of considering each (e.g.) molecule individually, we consider the **collective properties** of all molecules in the fluid
    - Common collective properties of interest:
        - Density: How compact the fluid is
        - Temperature: How much internal energy the fluid has
        - Pressure: How much force the fluid can exert
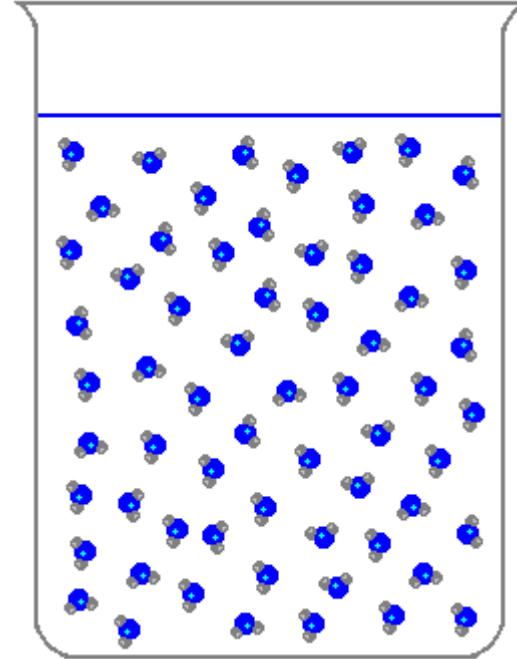
What is the mass of an object which has a density of 800 kg/m³ and a volume of 0.4m³?

(A) 800kg   (B) 320kg   (C) 0.003kg   (D) 4kg

1. Density = Mass/Volume
2. ρ = M/V *Greek letter rho*

3. M = ρ*V
4. M = (800 kg/m³)*(0.4m³)
5. M = 320kg

- The density of water at room temperature is ~1000kg/m³
- The density of air at room temperature is ~1.3kg/m³
- Most solids are within ~1,000–20,000kg/m³!

1

Pressure

- Fluids consist of many individual particles moving around, bouncing off of each other and the container walls.
- The collective effect of these collisions is the pressure.
    - If there are no molecules, the pressure is zero.

        This is why a grocery store balloon loses pressure.
   *It leaks molecules through small holes in the balloon.*
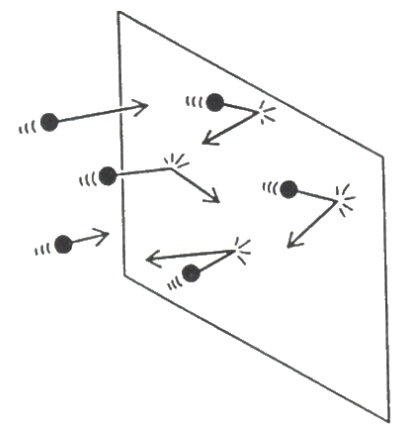
- Many "common" units: Pascals, atmospheres (atm), Torr, mmHg, lb/in^2 (psi)
    - The SI unit is Pascal: $1Pa = 1N/m^2 = 1kgm^{-1}s^{-2}$
    - Air pressure at sea level = $1atm = 1.013 \times 10^5 Pa = 14.7 lb/in^2 = 14.7 psi$

- Pressure = Force/Area
    - Force is the force perpendicular to the surface
    - Area is the area over which the pressure is distributed
- $P = F/A$
- F = the force that is perpendicular to the surface
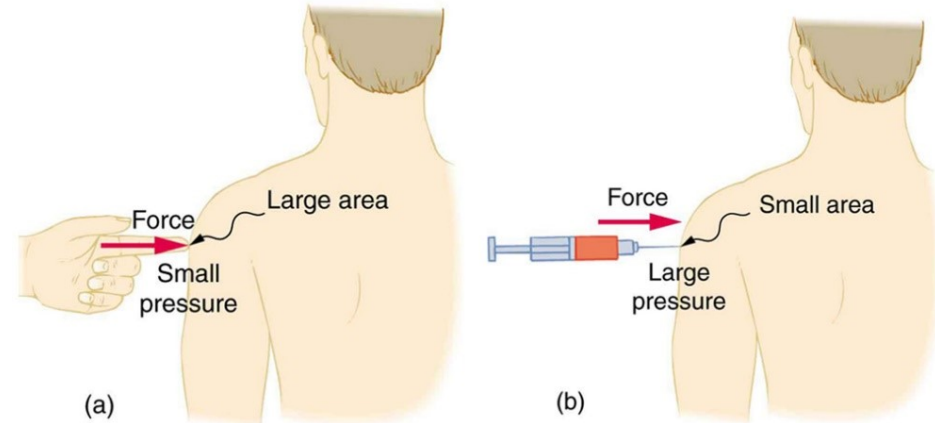
Pressure: Example

- Suppose you want to make a hover pad out of a vacuum cleaner, how much pressure would it take?
- $P=F/A$
- Need to counteract force of gravity:
  - $F=mg$
- Have some practical area, probably a circle with some radius r
  - $A=\pi r^2$
- To lift a 40kg kid with a r=0.6m hoverpad
  - $F=(40kg*9.8m/s^2)/(\pi(0.6m)^2)$
  - $F= (392N)/(1.13m^2) = 347N/m^2 = 347Pa = 0.05psi$

Two pistons each have fluid just beneath them, and each fluid is at the same pressure just below the piston. Piston B has four times the surface area as piston A. The density of the fluid under piston A is four times the density of the fluid under piston B.
Which piston can support the most weight?

(A) A   (B) B   (C) Both support the same

(A)   (B)

P   P
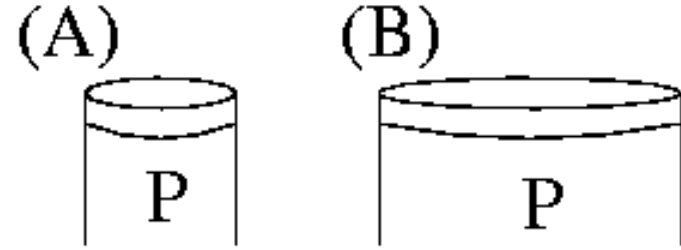
1. P=F/A
2. F=P*A
3. Both pistons A & B have the same pressure.
4. The area for piston B is larger.
5. So F is larger for piston B,
6. therefore this force can support a larger weight

Pressure: Depth Dependence

Pressure = P_1
Area = A

$+y$ axis

P_1A

mg

P_2A

Pressure = P_2
Area = A

(a)

*(b)* Free-body diagram of the column

- The pressure of a fluid depends on the depth
- This is because deeper depths of the fluid have the shallower layers stacked on top of it
- For a static fluid (pictured to the right), the internal forces will be balanced.
- So, the force of layer 1 pushing down will match the force of layer 2 pushing up.
- But the fluid in between layer 1 and layer 2 have mass and their weight will put a downward force on layer 2.
    - $P_2A = P_1A + m_{fluid}g$
    - Since the mass of the fluid depends on the amount you consider, instead can consider density*volume
    - $P_2 = P_1 + \rho g h$

In SI: [P] = Pa = $1 kgm^{-1}s^{-2}$
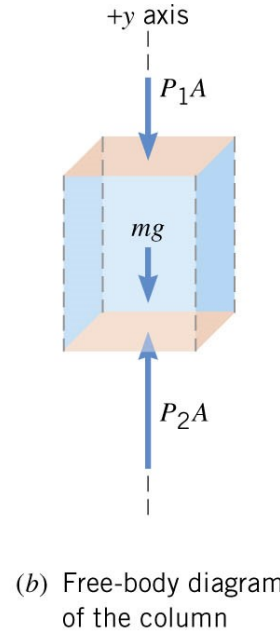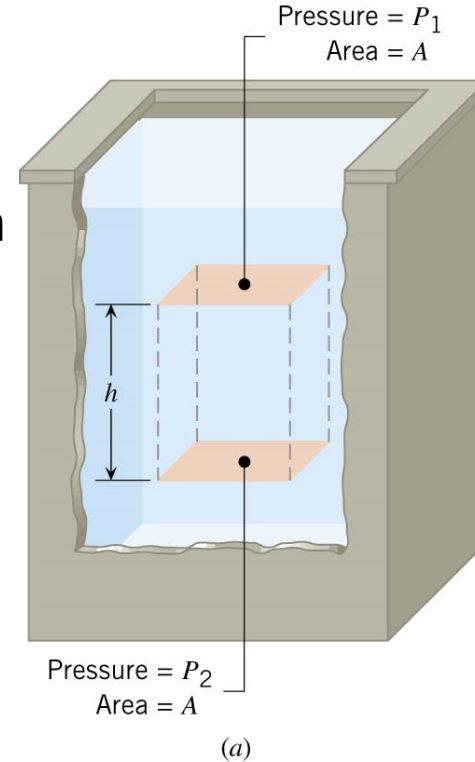
For a given fluid, at a given depth the pressure will be the same at all points.

The deepest fish seen was at the bottom of the Mariana Trench at a depth of roughly 11,500m below the atmosphere.
What is the pressure there?
Note that the water density is mostly constant.

(A) 10^5Pa   (B) 10^7Pa   (C) 10^8Pa   (D) 10^{10}Pa

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1. $P_2 = P_1 + \rho gh$
2. Know pressure at surface: $P_1 = 1.013 \times 10^5$Pa
3. Know density of water: $\rho = 1000$kg/m^3
4. Given depth: h=11,500m
5. $P_{trench} = P_2 = P_1 + \rho gh = 1.013 \times 10^5$Pa + (1000kg/m^3)(9.8m/s^2)(11,500m)
6. $P_{trench} = 1.013 \times 10^5$Pa + 1.12×10^8Pa $\approx 10^8$Pa

The deepest fish seen was at the bottom of the Mariana Trench at a depth of roughly 11,500m below the atmosphere.
What would the force be on a 0.2m-dimeter submarine window down there?
Note that the water density is mostly constant.

(A) 10^6N (B) 10^7N (C) 10^8N (D) 10^{10}N

1. $P_2 = P_1 + \rho gh$
2. Know pressure at surface: $P_1 = 1.013 \times 10^5$Pa
3. Know density of water: $\rho = 1000$kg/m^3
4. Given depth: h=11,500m
5. $P_{trench} = P_2 = P_1 + \rho gh = 1.013 \times 10^5$Pa $+ (1000$kg/m$^3)(9.8$m/s$^2)(11,500$m$)$
6. $P_{trench} = 1.013 \times 10^5$Pa $+ 1.12 \times 10^8$Pa $\approx 10^8$Pa
7. P=F/A
8. F=P*A
9. F= $(10^8$Pa$)*(\pi(0.1$m$)^2) \sim 10^6$N

Equivalent to ~400 tons!

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Consider the four points in the lake. The points are all at the same depth below the surface, but the depth of the bottom of the lake underneath the points varies. At which point is the pressure the greatest?

A B C   D

(A) A   (B) B   (C) C   (D) D   (E) All the same
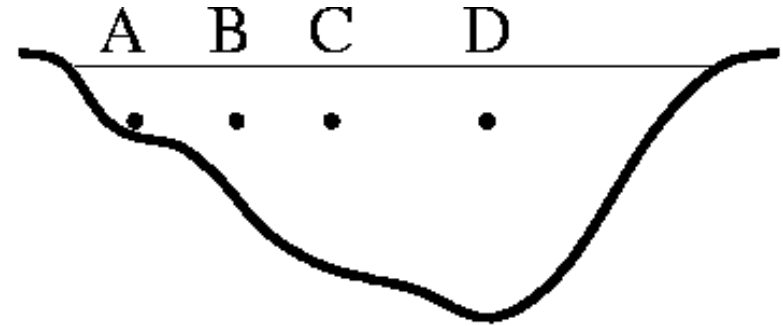
1. $P_2 = P_1 + \rho g h$
2. All are at the same height, h, relative to the surface (or any other vertical position).
3. So, the same amount of fluid will be pushing down on each point, contributing the same amount to the pressure there.
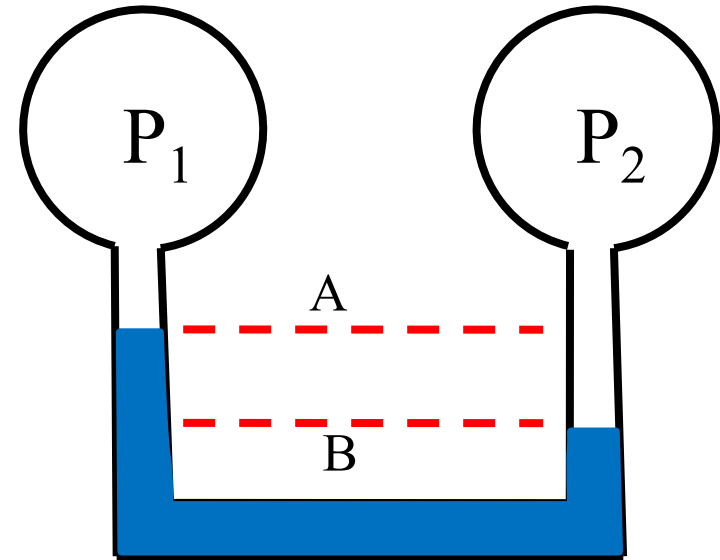4. Therefore all will have the same pressure.

Pressure at a given depth **in a static fluid** only depends on the depth of the point where the pressure is being measured!

Two pressurized bulbs connected by a tube with blue liquid in it.
How do the pressures in the two bulbs compare?

A. $P_1 > P_2$

B. $P_1 = P_2$

C. $P_1 < P_2$

P_1 P_2

A

B

1. If we call the height of the top of the fluid below bulb 2 "B", and the height of the top of the fluid below bulb 1 "A", we see that B is below A.
2. The pressure of a fluid at some height is the same for all points at that height.
3. On the left side, height B is below A-B of fluid and so the pressure will be higher there.
4. The fluid is not flowing, so the pressure of the fluid at B on the right must be equal to the pressure P_2 pushing down ($P_B = P_2$). Similarly at A ($P_A = P_1$).
5. Since $P_B > P_A$: $P_2 > P_1$.
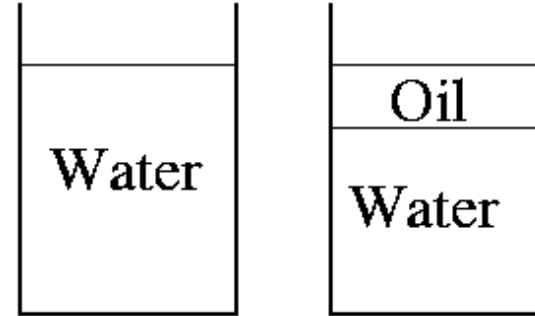6. The pressure difference is ρgh, where h=A-B.

Two beakers are filled to the same level with fluid. The left beaker is filled with water. The right beaker is filled with some water and then some oil, where the oil is on top because it floats on water.
Which beaker will have the greatest pressure at the bottom?

A. Water beaker

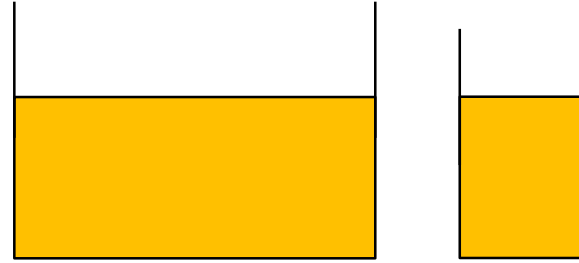B. Both the same

C. Oil + Water beaker

1. $P_2 = P_1 + \rho gh$
2. For both cases, at the surface $P_1 = P_{atmosphere}$ and they have the height from the surface of the fluid down to the bottom of the beaker.
3. However, some of the oil+water beaker has a portion of the height that is oil.
4. Oil is floats on water, so it is less dense than it.
5. Therefore, the pressure contribution from the oil section will be less than from the water section for that same height range in the other beaker.
6. Thus, the water beaker will have a greater pressure at the bottom.

Two beakers are filled with oil to the same level.
The left beaker is much wider than the right beaker.
Which beaker will have the greatest pressure at the bottom?

A. Wide beaker

B. Both the same

C. Narrow beaker

1. $P_2 = P_1 + \rho gh$
2. For both cases, at the surface $P_1 = P_{atmosphere}$ and they have the height from the surface of the fluid down to the bottom of the beaker, and they have the same density.
3. Therefore, the pressure will be the same at the bottom of both beakers.

- $P=F/A$
- F is larger for the left beaker, because more mass above the bottom
- But, $F=mg=(\rho*V)g=(\rho*h*A)g$
- So the larger A for the left case will cancel out when calculating P

A container is filled with oil and fitted on both ends with pistons. The area of the left piston is 10mm². The area of the right piston is 10,000mm². What force must be exerted on the left piston to keep the 10,000N car on the right at the same height?

F   10,000N

A=10mm²   A=10,000mm²

(A) 10 N   (B) 10^2 N

(C) 10^3 N   (D) 10^4 N
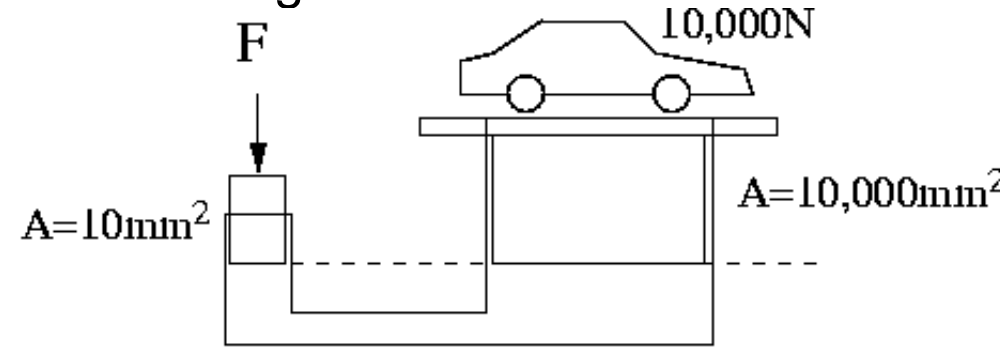
1. The pressure must be equal at the small left piston and the large right piston for the system to stay balanced.
2. $P_{left} = P_{right}$
3. $P = F/A$
4. $F_{left}/A_{left} = F_{right}/A_{right}$
5. $F_{left} = A_{left}(F_{right}/A_{right})$
6. $F_{left} = (10mm^2)(10,000N)/(10,000mm^2)$
7. $F_{left} = 10N$
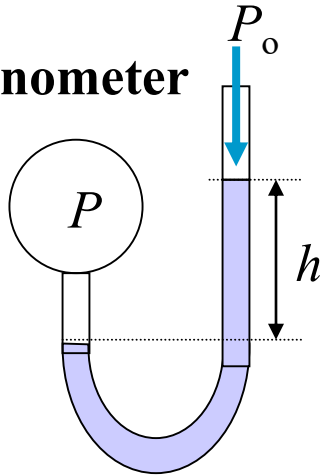
Absolute Pressure vs "Gauge Pressure"

- Pressure describes the force particles within a fluid exerts over a given area
- However, it is often more convenient to refer to the pressure relative to the pressure of the outside environment.
    - This is referred to as the "gauge pressure"
- *Absolute* **pressure**:
    - includes atmospheric pressure
    - $P_{absolute} = P_{atomphere} + \rho gh$
- *Gauge* **pressure**:
    - relative to outside pressure
    - $P_{gauge} = P_{absolute} - P_{atmosphere}$
    - At sea level, $P_{atmosphere} \equiv P_0 = 1.01 \times 10^5$ Pa,
      so $P_{gauge} = \rho gh - P_0$
    - E.g. Tire pressure gauges report the gauge pressure.
      A deflated tire is at atmospheric pressure, but has zero gauge pressure
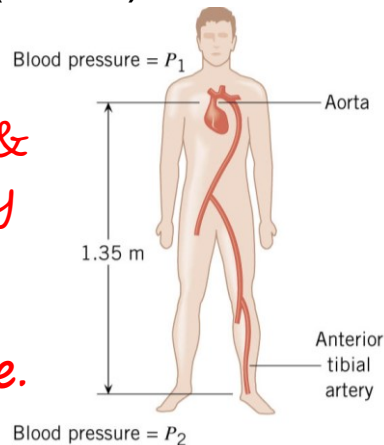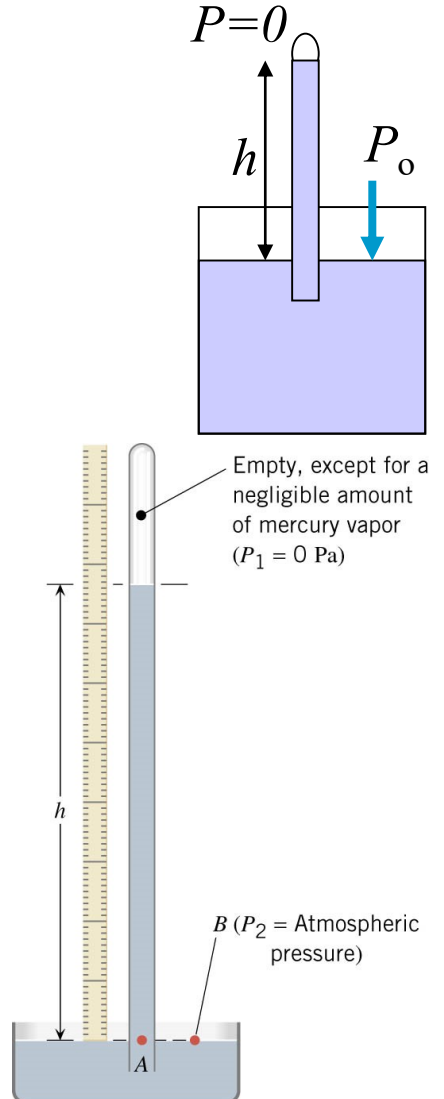
Gauge Pressure Measurement

- $P_{gauge} = \rho g h - P_0$
- If you know the density of a fluid & measure the height, you know the pressure
- Can measure the pressure of a sealed container with a *manometer*
- Can measure the atmospheric pressure with a *barometer*
- The change in fluid height for a given change in pressure is maximized by using a high-density fluid, e.g. mercury (Hg): $\rho \sim 13,600$ kg/m^3
  - Hence the pressure unit "mm of Hg" ($\approx$ Torr)
  - 1atm $\approx$ 760mmHg

*Blood pressure is a gauge pressure & depends on the height in your body at which it is measured. "120/80" means max./min. pressures are 120/80 mmHg **gauge**.*

Manometer

P_o

P

h

Barometer

$P=0$

h P_o

Empty, except for a negligible amount of mercury vapor ($P_1 = 0$ Pa)

h

B (P_2 = Atmospheric pressure)

A

Blood pressure = P_1

Aorta

h

1.35 m

Anterior tibial artery

Blood pressure = P_2

When you drive up into the mountains to higher altitudes, your ears occasionally "pop" (if you're lucky).
Which direction are your ear-drums bowing in between pops?

A. Outward

B. Inward

C. They don't bow

1. $P = \rho g h$
2. At higher altitudes, less of the atmosphere is stacked on top of you, so the fluid-height above you, h, decreases.
3. Therefore the outside pressure decreases.
4. Until your ear "pops", the pressure inside stays the same.
5. So, until your ear pops, the gas inside your ear is pushing harder on your ear drum tha the atmosphere outside.
6. As such, your ear drum will bow outward until the pressure is equalized (by "popping").

Buoyancy & Archimedes' Principle

- The tendency for an object to float in a fluid is **buoyancy**.
- If an object is floating, there must be some upward buoyant force.
  - The direction of the buoyant force is towards a decrease in pressure of the fluid. For terrestrial scenarios, this is "up".
  - **The magnitude of the buoyant force depends on the weight of the fluid that was displaced.**
  - $F_{buoyancy} = \rho_{fluid} V_{fluid\text{-}displaced} g$

This is "Archimedes' Principle".
It can be used to measure the volume of an irregular object.
By measuring the mass, you can get the density of the object.

- Whether or not an object floats just takes a comparison between the upwards buoyancy force and the downward force of gravity.
  - Floating:
    - $F_{buoyancy} = F_{gravity}$ and $V_{displaced} < V_{object}$
    - $\rho_{fluid} V_{displaced} g = \rho_{object} V_{object} g$
    - …therefore, must have $\rho_{object} < \rho_{fluid}$
  - Completely submerged:
    - $F_{buoyancy} < F_{gravity}$ and $V_{displaced} = V_{object} = V$
    - $\rho_{fluid} V g < \rho_{object} V_t g$
    - …therefore, must have $\rho_{object} > \rho_{fluid}$
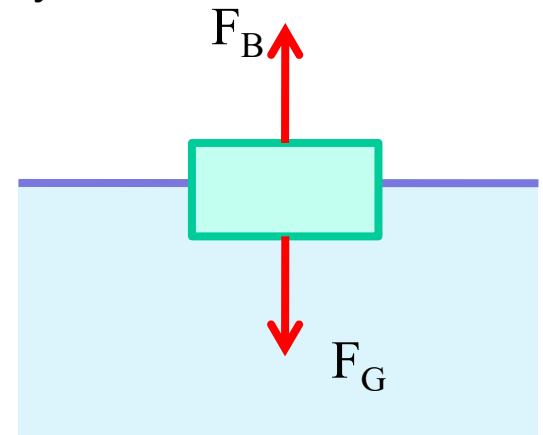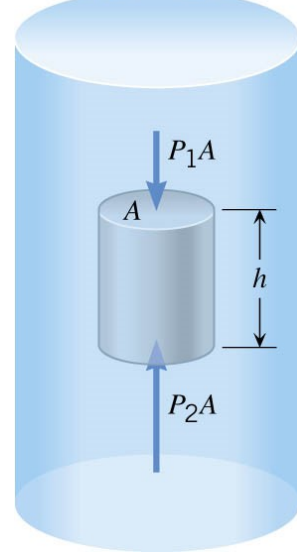
Cylinders A, B, and C have the listed weights and volumes and are completely submerged in a fluid.
Rank the buoyant force exerted on the cylinders by the fluid.

A. $B_A < B_B < B_C$
B. $B_A = B_B < B_C$
C. $B_C < B_B < B_A$
D. $B_A < B_C < B_B$

Cylinder	Weight	Volume
A	2N	V_A
B	7N	$V_B = V_A$
C	7N	$V_C > V_A$

1. $F_{buoyancy} = \rho_{fluid} V_{fluid\text{-}displaced} g$
2. The buoyancy force only depends on the amount of fluid displaced.
3. The buoyancy force does not depend on the mass of the object doing the displacing.

Ice has a density of ~920kg/m³, compared to liquid water's ~1000kg/m³. What fraction of an iceberg's volume is above the surface of the water?

A. 0.08%
B. 0.8%
C. 8%
D. 80%

Ralph Clevenger

1. $F_{buoyancy} = F_{gravity}$
2. $\rho_{water}V_{water-displaced}g = \rho_{ice}V_{iceberg}g$
3. $V_{water-displaced}/V_{iceberg}$ is the volume below water.
4. $V_{water-displaced}/V_{iceberg} = \rho_{ice}/\rho_{water} = 0.92$
5. I.e. 92% of the iceberg is below the surface, meaning 8% is above the water surface.
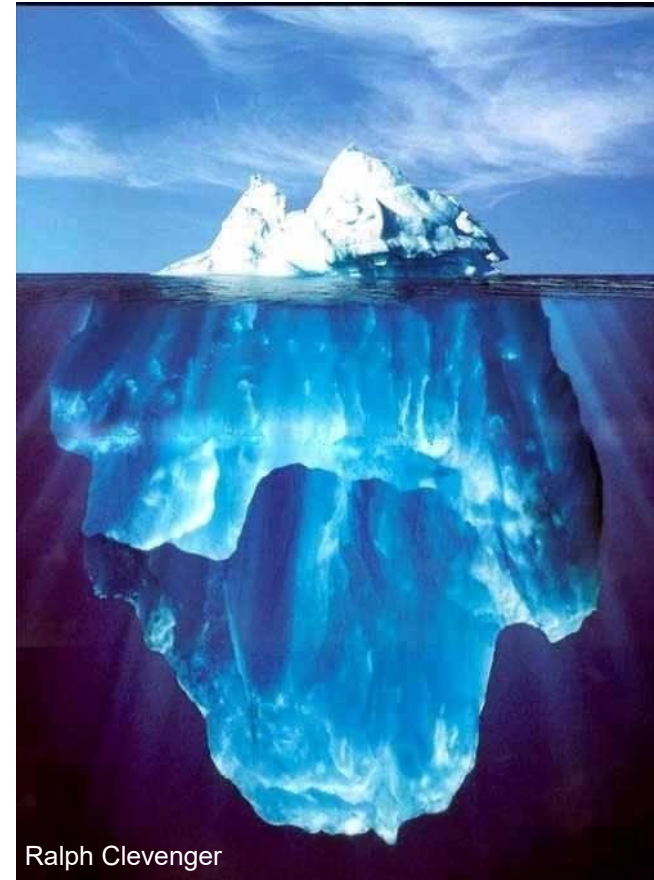
You want to salvage your yacht. You don't have traditional salvaging equipment, but you did recently inherit a ping pong ball empire.
You decide to pump ping pong balls (V_{ball} ~$3\times10^{-4}m^3$) into your yacht ($M_{yacht}=1500kg$) in order to raise it from the sea floor.
How many ping pong balls do you need to pump into the yacht?
Consider the yacht buoyancy to be negligible and the ball mass to be small.

(A) 500   (B) 5,000

(C) 50,000   (D) 500,000

1. To make the ship float: $F_{buoyancy} = F_{gravity}$
2. Since the ping-pong balls make-up most of the displaced volume and the ship makes-up most of the mass:
   $\rho_{water}V_{balls}g = M_{yacht}g$
3. $V_{balls} = (M_{yacht})/\rho_{water}$
4. $V_{balls} = (1500kg)/(1000kg/m^3) = 1.5m^3$
5. One ping-pong ball has a volume of ~ $3\times10^{-4}m^3$,
6. So, the number of ping-pong balls is: $V_{balls}/V_{ball} = 1.5m^3/(3\times10^{-5}m^3)$ ~ 50,000 balls
7. …it turns out a fiberglass hull would provide almost half that much buoyancy, so you would only need ~25K ping-pong balls.

Mythbusters, E21

You're a modern artist and your newest installation will consist of an air-filled balloon under water in a fish tank, held there by a single cooked spaghetti noodle that is attached to the bottom of the tank. Don't ask why, it's art. Given that a single cooked noodle can withstand a tension of ~0.5N, what is the maximum volume the air (~1kg/m³) filled balloon can be? Ignore the balloon material mass.

(A) 5 cm^3 (B) 50 cm^3
(C) 500 cm^3 (D) 5000 cm^3
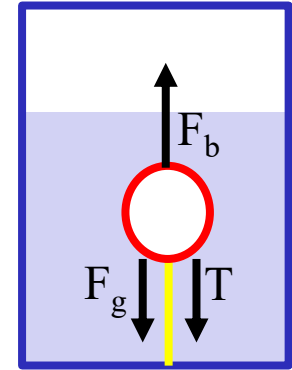
1. The downward force of gravity and tension of the noodle must balance the upward buoyancy force
2. $F_{buoyancy} = F_{gravity} + T_{noodle}$
3. $\rho_{water} V_{displaced} g = \rho_{air} V_{balloon} g + T_{noodle}$
4. The displaced volume of water is the same as the volume of the balloon
5. $\rho_{water} V_{balloon} g = \rho_{air} V_{balloon} g + T_{noodle}$
6. $V_{balloon}(\rho_{water} - \rho_{air})g = T_{noodle} \approx V_{balloon}(\rho_{water})g$ ...since $\rho_{water} >> \rho_{air}$
7. $V_{balloon} = T_{noodle}/(\rho_{water}g)$
8. $V_{balloon} = (0.5 \text{kgm/s}^2)/\{(1000\text{kg/m}^3)*(9.8\text{m/s}^2)\}$
9. $V_{balloon} \sim 5\times10^{-5}\text{m}^3 = 50\text{cm}^3$

$1\text{m}^3 = 10^6\text{cm}^3$