Thursday March 30

Topics for this Lecture:

- *Simple Harmonic Motion*
  - *Kinetic & Potential Energy*
  - *Pendulum systems*
  - *Resonances & Damping*

- Assignment 11 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed
  or by appointment (meisel@ohio.edu)

Spring:

$$F_{applied} = kx \qquad \omega_{spring} = \sqrt{\frac{k_{spring}}{m_{weight}}} = \sqrt{\frac{k}{m}}$$

Note: T, f, ω are ***independent of amplitude!***

$$x = A\cos(\omega t) \qquad x_{max} = A$$

$$v = -A\omega\sin(\omega t) \qquad v_{max} = A\omega$$

$$a = -A\omega^2\cos(\omega t) \qquad a_{max} = A\omega^2$$

Note: $f = 1/T$
$\omega = 2\pi f$

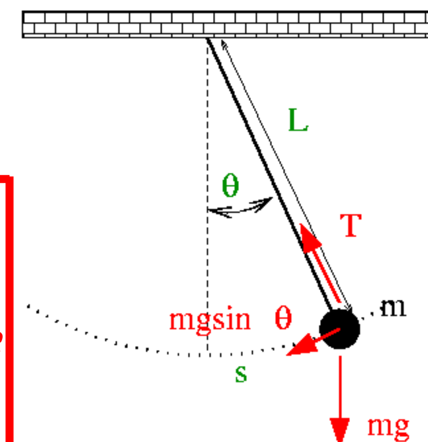
T, f, ω independent of oscillation amplitude.

Pendulum:

$$\omega_{pendulum} = \sqrt{\frac{g}{l_{pendulum}}} = \sqrt{\frac{g}{l}}$$

$F \approx -mg\,\theta$

$F \approx -(mg/L)s$



Which of the following situations will have the greatest amplitude of oscillation?

A. A mass M is attached to a spring with a spring constant k, pulled back a distance d and released.

B. A mass 2M is attached to a spring with a spring constant (1/2)k, pulled back a distance 2d and released.

C. A mass (1/2)M is attached to a spring with a spring constant 2k, pulled back a distance (1/2)d and released.

D. A & B

Focus on the relevant information!

1. The amplitude is the amplitude.
2. Once stretched to some position, the spring will oscillate back to that position as the maximum.

Which of the following situations will have the greatest **maximum force**?

A. A mass M is attached to a spring with a spring constant k, pulled back a distance d and released.

B. A mass 2M is attached to a spring with a spring constant (1/2)k, pulled back a distance 2d and released.

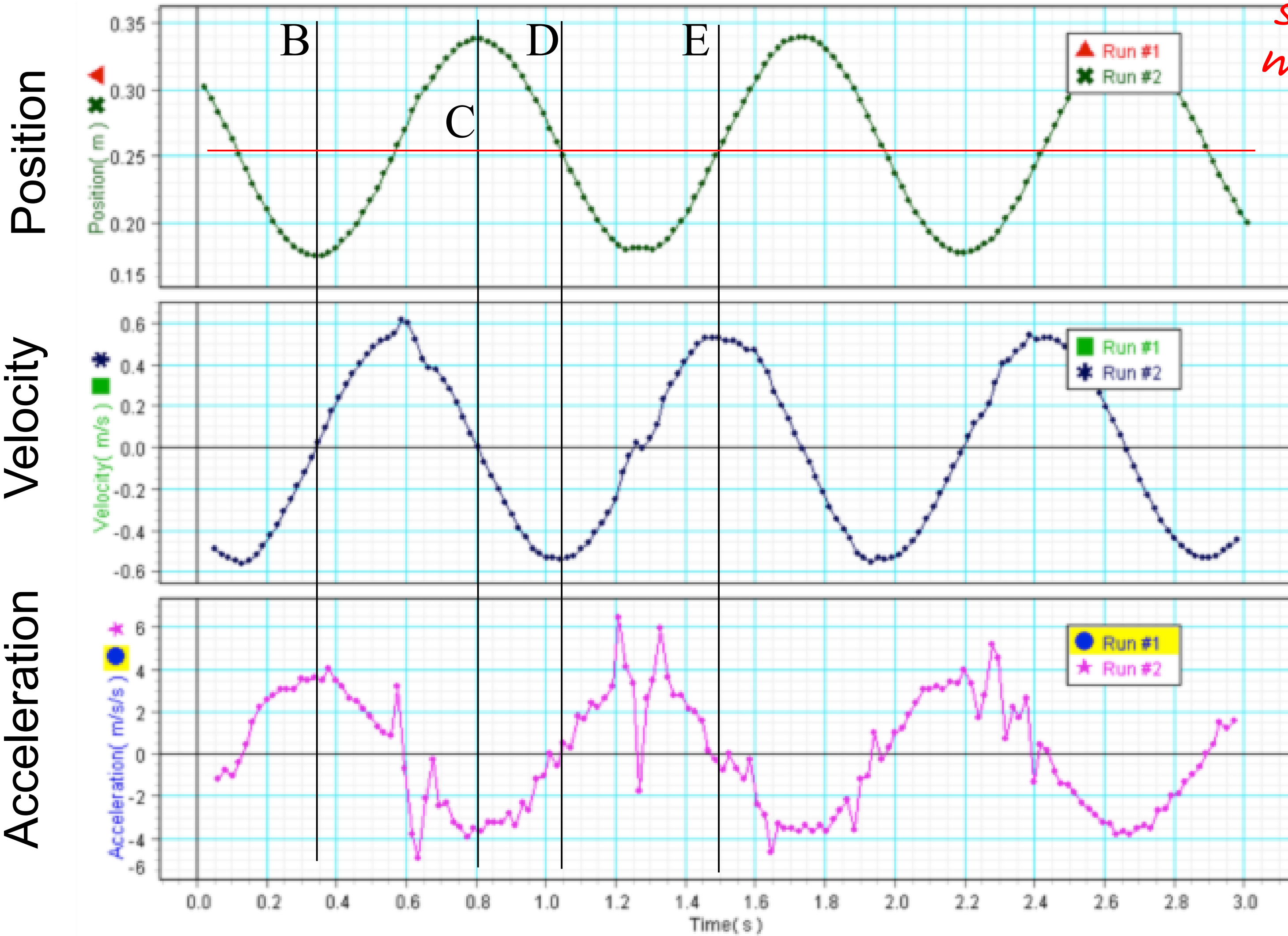
C. A mass (1/2)M is attached to a spring with a spring constant 2k, pulled back a distance (1/2)d and released.

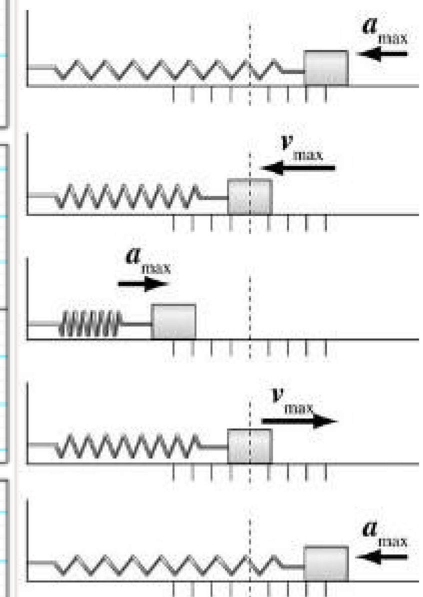
D. A, B, & C

1. $F_{spring} = -k*x$
2. The maximum force will be at the amplitude, x = A.
3. $F_{max} = k_{spring}*A$
4. For A: $F_{max,A} = k*d$
5. For B: $F_{max,B} = (1/2)k*(2d) = k*d$
6. For C: $F_{max,C} = (2k)*(1/2)d = k*d$
7. $F_{max,A} = F_{max,B} = F_{max,C}$

Oscillations: Position, Velocity, and Acceleration vs. Time

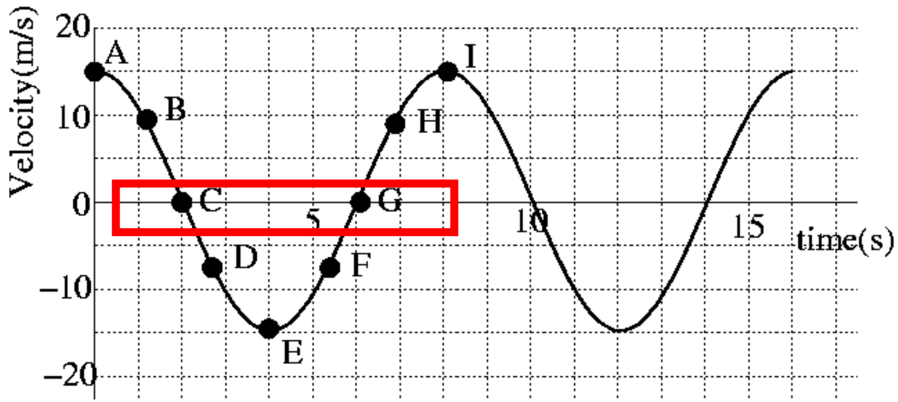
Upcoming lab on simple harmonic motion





The graph represents **velocity** of a mass in a mass-spring system.
At what point(s) is the magnitude of the force at a maximum?

3



(A) A
(B) C
(C) E
(E) G
(F) A and I
(G) A, E, and I
(H) C and G
(I) A,C,G, and I

1. $F_{spring} = -k*x$
2. The force is maximum for largest displacement, $x = x_{max} = A$
3. At the amplitude, the mass is turning around and so is stopped for an instant.
4. Therefore, velocity is zero at the amplitude.
5. C and G correspond to zero velocity, therefore the amplitude, where the force is maximum.

Which of the following are true about a mass spring system?

(A) Increasing the mass increases the period
(B) A stiffer spring increases the frequency
(C) Decreasing the mass increases the frequency
(D) All of the above

$$\omega_{spring} = \sqrt{\frac{k}{m}} = 2\pi f = \frac{2\pi}{T}$$

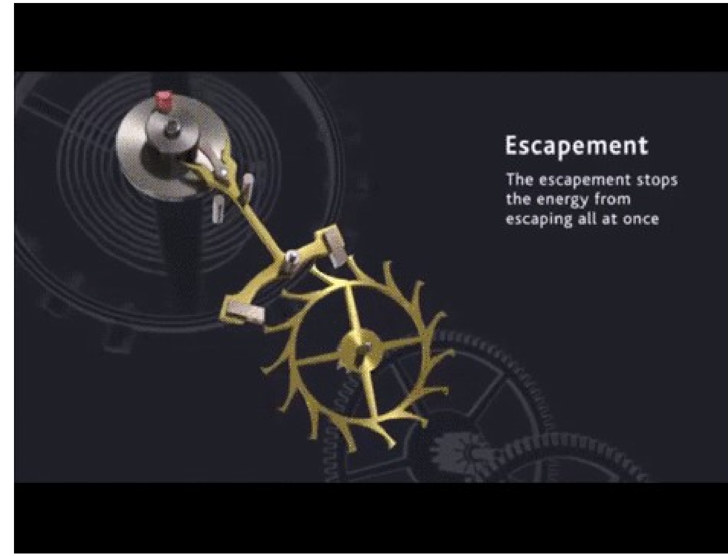
A mass attached to a spring is sliding back and forth on a horizontal frictionless table.
The kinetic energy is a maximum:

1. At maximum compression
2. At maximum extension
3. At equilibrium
4. At both extremes (compression & extension)

1. KE = $(1/2)mv^2$
2. Maximum v will lead to maximum KE.
3. v is maximum at equilibrium

Mass-spring system: Energy

Spring-powered watch mechanism



- Energy can be stored in a spring and released later as kinetic energy
- After some calculus (derivative of the force):
  - $PE = \frac{1}{2}kx^2$
- Work done by a spring: W = ΔPE
  - stretching or compressing will do negative or positive work
- Since no non-conserving forces are present, Total Energy (KE + PE) is constant
  - We can use this to find velocity at a given displacement: v(x)

Mass Spring with Vectors

Mass-spring system: Energy

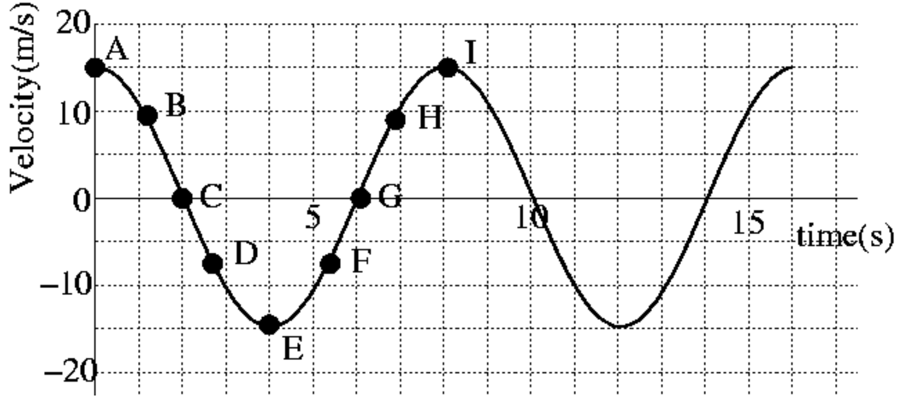
- When there are rotating bits (e.g. pendulum), the total energy is: $E_{tot} = PE_{height} + PE_{spring} + KE_{trans} + KE_{rot}$
  - $E = mgh + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

- For a horizontal mass & spring, life is simpler:
  - No changes in height, so can ignore PE_{height}
  - No rotating pieces, so can ignore rotational energy
  - $E_{horiz-spring} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
  - This gives us a way to relate velocity & position!
    - For x = A, total energy is: $\frac{1}{2}kA^2$, since velocity is zero. This is conserved.
    - So, $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
    - …after some algebra: $v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$

*For the mass-spring system

The graph represents **velocity** of a mass in a mass-spring system.
At what point(s) is the magnitude of the **Potential Energy** at a maximum?

6

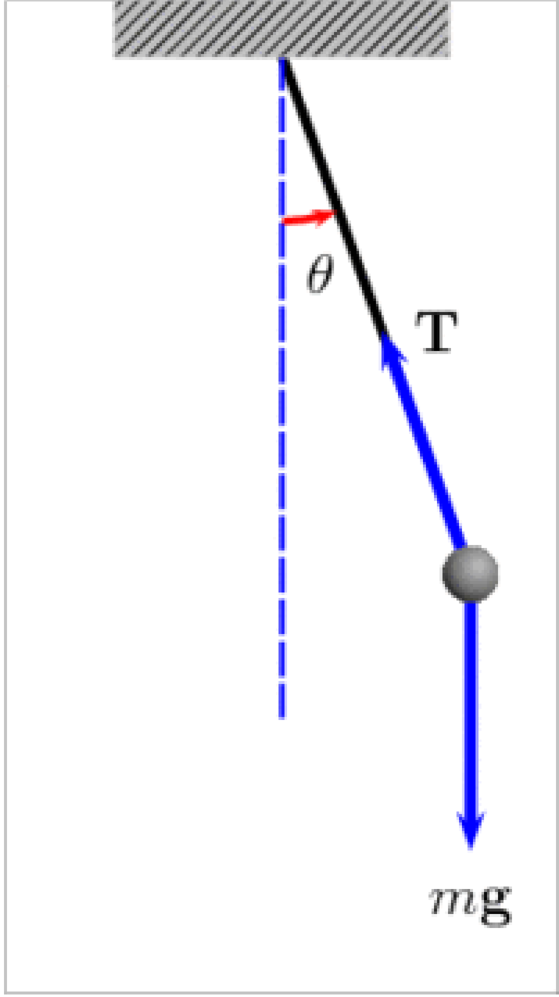


(A) A
(B) C
(C) E
(E) G
(F) A and I
(G) A, E, and I
(H) C and G
(I) A,C,G, and I

1. E = PE + KE = (1/2)kx^2+(1/2)mv^2
2. Energy will be conserved throughout motion, so KE will be converted to PE and back.
3. Minimum KE means maximum PE.
4. When v=0 (at the amplitude), KE =0
5. v=0 at C and G, so PE is maximized there.

Pendulum

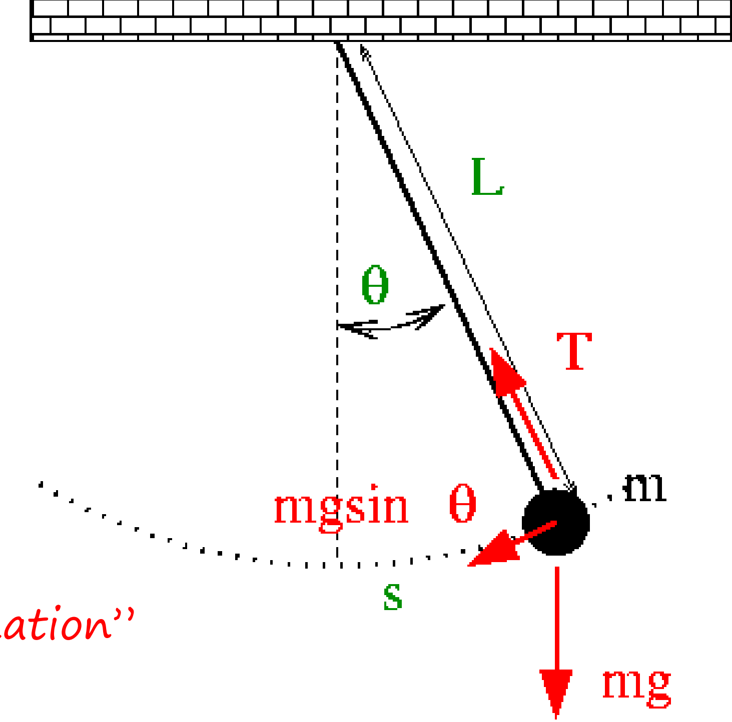
- Pendulum: a periodic swinging thing
  - physical pendulum: arm with weight on the end
  - "simple pendulum": most mass concentrated at one point
    - we'll mostly stick to this, because it's simpler since you can ignore the moment of inertia of the pendulum arm



- Like the mass on the spring, the pendulum oscillates.
- *For small angles*, the pendulum undergoes simple harmonic motion, analogous to the mass & spring
  - We'll work with small angle cases
  - "Small" means ~ <0.244radians, which is ~ <14°
  - (Justification comes from when sin(θ) ~ θ

Pendulum: Forces

- The pendulum weight experiences a downward force from gravity and an upward tension from the pendulum cable.
- The net force experienced in the swinging direction is:
    - $F_{net} = -mg*\sin(\theta)$
    - For $\theta < 0.244$ radians (~14 degrees), $\sin(\theta) \approx \theta$, so *This is the "small angle approximation"* $F_{net} \approx -mg*\theta$
    - Since θ=(arc length)/radius = s/r, and r is the pendulum length L, $F \approx -(mg/L)s$
- Recalling that for a spring, F=-kx, it is apparent that pendulums for small swinging-angles undergo simple harmonic motion!

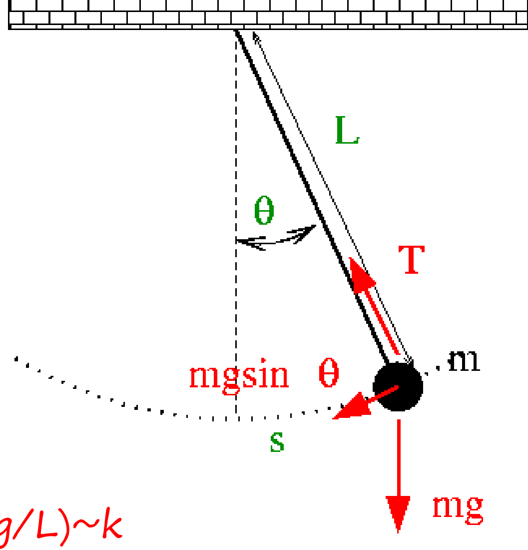


Pendulum-Spring Comparison

Quantity	Mass-Spring	Pendulum
Stiffness	k	mg/L
inertia	m	m
ω	$\sqrt{k/m}$	$\sqrt{g/L}$

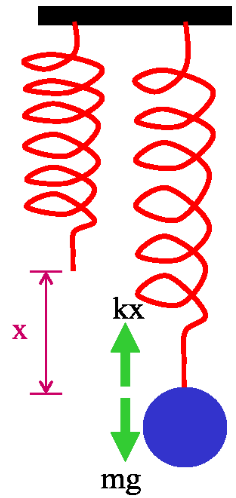
Note: ω=2πf and f =1/T

from the fact that (mg/L)~k



The pendulum period only depends on the pendulum length & the local gravity!

$$\omega_{pendulum} = \sqrt{g/L}$$



Two identical pendulum clocks are set in motion, one on the Earth and one on the Moon. The acceleration due to gravity on the Moon is 1.67 m/s^2 (1/6 that of Earth).
Which of the following statements is true?

A. The clock on the Moon runs faster than the clock on Earth
B. The clock on the Moon runs the same speed as the clock on Earth
C. The clock on the Moon runs slower than the clock on Earth

1. $\omega = \sqrt{g/L}$
2. On the moon, “g” is smaller, so ω will be smaller.
3. $T=2\pi/\omega$
4. So, smaller ω means larger T, i.e. a longer period.
5. The clock will take longer for one “tick” on the Moon & therefore runs slower.

Two identical pendulum clocks are set in motion, one on the Earth and one on the Moon. The acceleration due to gravity on the Moon is 1.67 m/s^2 (1/6 that of Earth).
If we want the clock on the Moon to run at the same speed, what would we have to do?

A. Shorten the pendulum
B. Keep the pendulum the same length
C. Lengthen the pendulum

1. $\omega = \sqrt{g/L}$
2. On the moon, “g” is smaller, so ω will be smaller.
3. Decreasing L will increase ω and compensate for the decrease in g.

Two identical pendulum clocks are set in motion, one on the Earth and one on the Moon. The acceleration due to gravity on the Moon is 1.67 m/s^2 (1/6 that of Earth).

How much do we have to shorten the Moon clock pendulum to get it to tick at the same rate as the Earth clock?

A. 1/6
B. $1/\sqrt{6}$
C. 0.6
D. 6/(2π)

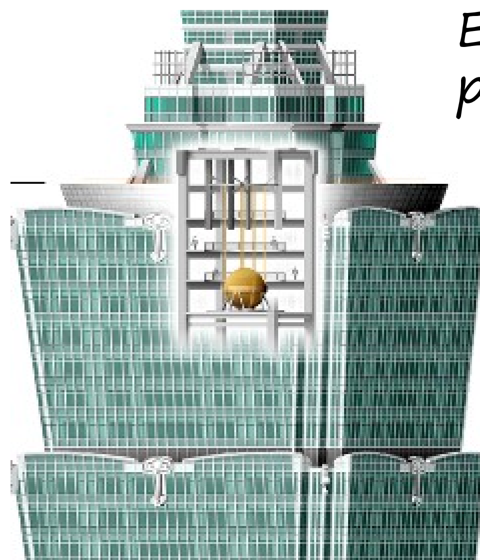
1. $\omega = \sqrt{g/L}$
2. $g_{Moon} = (1/6)g_{Earth}$,
3. So, need $L_{Moon} = (1/6)L_{Earth}$
4. $\omega = \sqrt{(\frac{1}{6}g)/(\frac{1}{6}L)} = \sqrt{g/L}$

Resonances

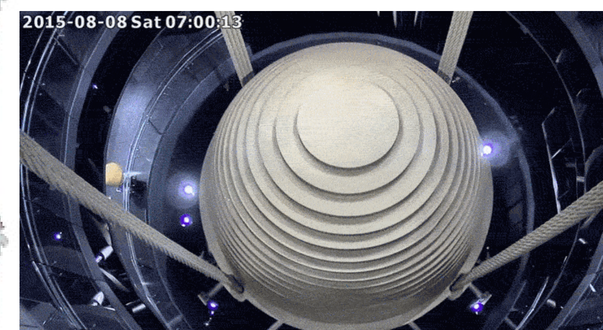
- Most oscillatory systems have a frequency at which they easily oscillate.
- This is referred to as the “natural” or “resonant” frequency.
- If a force is applied periodically, matching the resonant frequency, the system will be driven “on the resonance” and energy will build-up.
- This can be good (musical instruments) or bad (Tacoma Narrows bridge)

Example:
Kid being pushed on a swing

Object designs often try to counteract motion at the resonant frequency



E.g. the 728-ton pendulum in Taipei 101

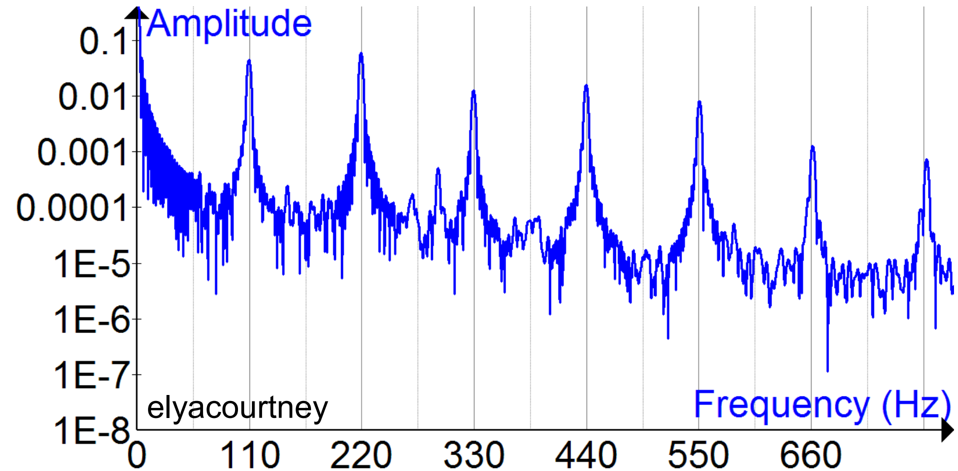


Resonances

Shattering a glass with a high-pitched noise

https://www.youtube.com/watch?v=L2VoR1lorqQ

Notes for a base guitar



A car rests on springs and has a certain resonant frequency.
If a driver picks up passengers, how will the resonant frequency be affected?

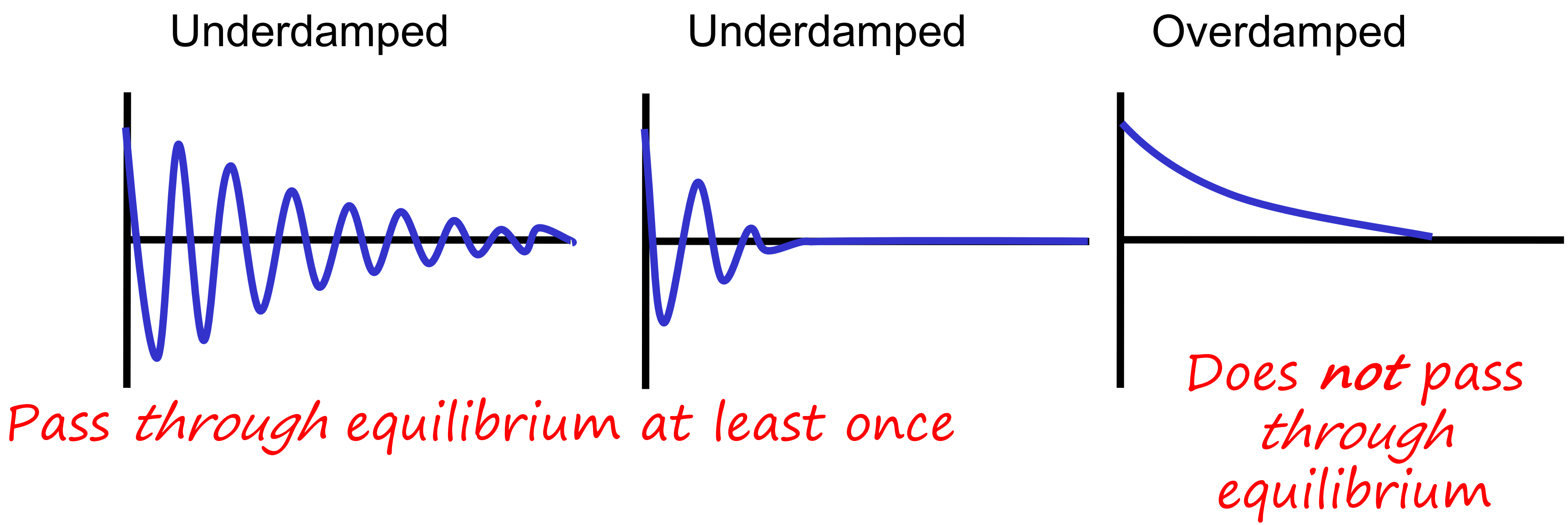
10

A. Increase with passengers in the car.
B. Stay the same.
C. Decrease with passengers in the car.

1. $\omega = \sqrt{k/m}$
2. Increasing m, means decreasing ω

Damped Motion

"Damping" is removing energy from a system (e.g. via friction, air resistance,…), removing energy from a system & therefore decreasing the oscillation amplitude.



"Critically damped":
The least amount of damping where, when applied, a system will not oscillate.
This will return the system to equilibrium as quickly as possible.