#### Tuesday April 7 This is a link. Click it!

See the <u>course webpage</u> for slides, video links, and google docs Q&A links

## Topics for this Lecture:

- Fluids
  - Density, Pressure
  - Pressure vs height
- Pressure = Force/Area (P = F/A)
- Density = Mass/Volume ( $\rho = M/V$ )
- $P_2 = P_1 + \rho gh$ •  $P_{gauge} = P - P_{atm}$



- •Assignment 11 due Friday
- Pre-class due 15min before class
- •Help Room: via Teams, 6-9pm Wed/Thurs, also via email on Wed/Thurs
- •SI: via Teams
- •Office Hours: via email (meisel@ohio.edu)

\*<u>If your circumstances</u> <u>hinder your completion of</u> <u>any of this class's</u> <u>assignments, please let</u> <u>me know</u>

\*These slides are recorded and posted in advance for your convenience. Administrative details may be out of date. Keep an eye on your email please.

#### Fluids

How do hose nozzles change the water speed?



How do planes generate lift?

How do you know if something will float? How do you know how high it will sit above the water?



How do you use a vacuum cleaner to make a hover pad?



## Fluids

- "Fluid": anything that takes the shape of its container.
  - Can be a liquid or a gas.
- A fluid is a collection of many microscopic particles.
- This is a new way for us to think about matter.
- Instead of considering each (e.g.) molecule individually, we consider the collective properties of all molecules in the fluid
  - Common collective properties of interest:
    - Density: How compact the fluid is
    - Temperature: How much internal energy the fluid has
    - Pressure: How much force the fluid can exert



What is the mass of an object which has a density of 800 kg/m<sup>3</sup> and a volume of 0.4m<sup>3</sup>?



(A) 800kg (B) 320kg (C) 0.003kg (D) 4kg

1. Density = Mass/Volume  
2. 
$$\rho = M/V$$
 Greek letter rho

3.  $M = \rho^* V$ 

4. 
$$M = (800 \text{ kg/m}^3)^*(0.4 \text{m}^3)$$

5. M = 320kg

The density of water at room temperature is ~1000kg/m<sup>3</sup>
The density of air at room temperature is ~1.3kg/m<sup>3</sup>
Most solids are within ~1,000-20,000kg/m<sup>3</sup>!

#### Pressure

- Fluids consist of many individual particles moving around, bouncing off of each other and the container walls.
- The collective effect of these collisions is the pressure.
  - If there are no molecules, the pressure is zero.

This is why a grocery store balloon loses pressure. It leaks molecules through small holes in the balloon.

- Many "common" units: Pascals, atmospheres (atm), Torr, mmHg, lb/in<sup>2</sup> (psi)
  - The SI unit is Pascal:  $1Pa = 1N/m^2 = 1kgm^{-1}s^{-2}$
  - Air pressure at sea level =  $1atm = 1.013x10^5Pa = 14.7lb/in^2 = 14.7psi$
- Pressure = Force/Area
  - Force is the force perpendicular to the surface
  - Area is the area over which the pressure is distributed
- P = F/A
- F = the force that is perpendicular to the surface





#### Pressure: Example

- Suppose you want to make a hover pad out of a vacuum cleaner, how much pressure would it take?
- P=F/A
- Need to counteract force of gravity:
  - F=mg
- Have some practical area, probably a circle with some radius r
  - $A=\pi r^2$
- To lift a 40kg kid with a r=0.6m hoverpad
  - F=(40kg\*9.8m/s<sup>2</sup>)/(π(0.6m)<sup>2</sup>)
  - F= (392N)/(1.13m<sup>2</sup>) = 347N/m<sup>2</sup> = 347Pa = 0.05psi



Two pistons each have fluid just beneath them, and each fluid is at the same pressure just below the piston. Piston B has four times the surface area as piston A. The density of the fluid under piston A is four times the density of the fluid under piston B. (A) (B) Which piston can support the most weight?

(A) A

(B) B

(C) Both support the same



1. P=F/A

- 3. Both pistons A & B have the same pressure.
- 4. The area for piston B is larger.
- 5. So F is larger for piston B,
- 6. therefore this force can support a larger weight

#### **Pressure**: Depth Dependence

- The pressure of a fluid depends on the depth
- This is because deeper depths of the fluid have the shallower layers stacked on top of it
- For a static fluid (pictured to the right), the internal forces will be balanced.
- So, the force of layer 1 pushing down will match the force of layer 2 pushing up.
- But the fluid in between layer 1 and layer 2 have mass and their weight will put a downward force on layer 2.

• 
$$P_2A = P_1A + m_{fluid}g$$

• Since the mass of the fluid depends on the amount you consider, instead can consider density\*volume

• 
$$P_2 = P_1 + \rho g h$$

For a given fluid, at a given depth the pressure will be the same at all points.



- The deepest fish seen was near the bottom of the Mariana Trench at a depth of roughly 11,500m below the atmosphere. What is the pressure there?
- Note that the water density is mostly constant.

(A) 10<sup>5</sup>Pa (B) 10<sup>7</sup>Pa

(C) 10<sup>8</sup>

- 1.  $P_2 = P_1 + \rho g h$
- 2. Know pressure at surface:  $P_1 = 1.013 \times 10^5 Pa$
- 3. Know density of water:  $\rho$ =1000kg/m<sup>3</sup>
- 4. Given depth: h=11,500m
- 5.  $P_{trench} = P_2 = P_1 = 1.013 \times 10^5 Pa + (1000 kg/m^3)(9.8 m/s^2)(11,500 m)$
- 6.  $P_{trench} = 1.013 \times 10^5 Pa + 1.12 \times 10^8 Pa \approx 10^8 Pa$

 $P_{atm} = 1.013 \times 10^5 Pa$  $\rho_{water} = 1000 \text{ kg/m}^3$ 





The deepest fish seen was near the bottom of the Mariana Trench at a depth of roughly 11,500m below the atmosphere.



What would the force be on a 0.2m-dimeter submarine window down there?

Note that the water density is mostly constant.

## (A) $10^6$ N (B) $10^7$ N (C) $10^8$ N (D) $10^{10}$ N

- 1.  $P_2 = P_1 + \rho g h$
- 2. Know pressure at surface:  $P_1 = 1.013 \times 10^5 Pa$
- 3. Know density of water:  $\rho = 1000 \text{kg/m}^3$
- 4. Given depth: h=11,500m
- 5.  $P_{trench} = P_2 = P_1 = 1.013 \times 10^5 Pa + (1000 \text{kg/m}^3)(9.8 \text{m/s}^2)(11,500 \text{m})$
- 6. P<sub>trench</sub> = 1.013x10<sup>5</sup>Pa + 1.12x10<sup>8</sup>Pa ≈ 10<sup>8</sup>Pa
- 7. P=F/A
- 8. F=P\*A
- 9.  $F = (10^8 Pa)^* (\pi (0.1m)^2) \sim 10^6 N$

Equivalent to ~400 tons!





Consider the four points in the lake. The points are all at the same depth below the surface, but the depth of the bottom of the lake underneath the points varies. At which point is the pressure the greatest?



- 1.  $P_2 = P_1 + \rho gh$
- 2. All are at the same height, h, relative to the surface (or any other vertical position).
- 3. So, the same amount of fluid will be pushing down on each point, contributing the same amount to the pressure there.
- 4. Therefore all will have the same pressure.

Pressure at a given depth **in a static fluid** only depends on the depth of the point where the pressure is being measured! Two pressurized bulbs connected by a tube with blue liquid in it. How do the pressures in the two bulbs compare?

A. 
$$P_1 > P_2$$
  
B.  $P_1 = P_2$   
C.  $P_1 < P_2$ 

- 1. If we call the height of the top of the fluid below bulb 2 "B", and the height of the top of the fluid below bulb 1 "A", we see that B is below A.
- 2. The pressure of a fluid at some height is the same for all points at that height.
- 3. On the left side, height B is below A-B of fluid and so the pressure will be higher there.
- 4. The fluid is not flowing, so the pressure of the fluid at B on the right must be equal to the pressure P<sub>2</sub> pushing down (P<sub>B</sub> = P<sub>2</sub>). Similarly at A (P<sub>A</sub>=P<sub>1</sub>).
- 5. Since  $P_B > P_A$ :  $P_2 > P_1$ .
- 6. The pressure difference is  $\rho$ gh, where h=A-B.





Two beakers are filled to the same level with fluid. The left beaker is filled with water. The right beaker is filled with some water and then some oil, where the oil is on top because it floats on water.

Which beaker will have the greatest pressure at the bottom?



B. Both the same

C.Oil + Water beaker



- 1.  $P_2 = P_1 + \rho g h$
- 2. For both cases, at the surface  $P_1 = P_{atmosphere}$  and they have the height from the surface of the fluid down to the bottom of the beaker.
- 3. However, some of the oil+water beaker has a portion of the height that is oil.
- 4. Oil is floats on water, so it is less dense than it.
- 5. Therefore, the pressure contribution from the oil section will be less than from the water section for that same height range in the other beaker.
- 6. Thus, the water beaker will have a greater pressure at the bottom.



A container is filled with oil and fitted on both ends with pistons. The area of the left piston is 10mm<sup>2</sup>. The area of the right piston is 10,000mm<sup>2</sup>. What force must be exerted on the left piston to keep the 10,000N car on the right at the same height?



- 1. The pressure must be equal at the small left piston and the large right piston for the system to stay balanced.
- 2.  $P_{left} = P_{right}$
- 3. P=F/A
- 4.  $F_{left}/A_{left} = F_{right}/A_{right}$
- 5.  $F_{left} = A_{left}(F_{right}/A_{right})$
- 6.  $F_{left} = (10 \text{ mm}^2)(10,000 \text{ N})/(10,000 \text{ mm}^2)$
- 7.  $F_{left} = 10N$

#### Absolute Pressure vs "Gauge Pressure"

- Pressure describes the force particles within a fluid exerts over a given area
- However, it is often more convenient to refer to the pressure relative to the pressure of the outside environment.
  - This is referred to as the "gauge pressure"
- Absolute pressure:
  - includes atmospheric pressure
  - $P_{absolute} = P_{atomphere} + \rho gh$
- Gauge pressure:
  - relative to outside pressure

• 
$$P_{gauge} = P_{absolute} - P_{atmosphere}$$
  
• At sea level,  $P_{atmosphere} \equiv P_0 = 1.01 \times 10^5 \text{ Pa}$ ,  
so  $P_{gauge} = \rho gh$ 

• E.g. Tire pressure gauges report the gauge pressure.

A deflated tire is at atmospheric pressure, but has zero gauge pressure

#### **Gauge Pressure Measurement**

- $P_{gauge} = \rho g h$
- If you know the density of a fluid & measure the height, you know the pressure
- Can measure the pressure of a sealed container with a *manometer*
- Can measure the atmospheric pressure with a *barometer*
- The change in fluid height for a given change in pressure is maximized by using a high-density fluid, e.g. mercury (Hg): ρ ~ 13,600 kg/m<sup>3</sup>
  - Hence the pressure unit "mm of Hg" (≈ Torr)
  - 1atm ≈ 760mmHg

Blood pressure is a gauge pressure & depends on the height in your body at which it is measured. "120/80" means max./min. pressures are 120/80 mmHg gauge.



When you drive up into the mountains to higher altitudes, your ears occasionally "pop" (if you're lucky).



Which direction are your ear-drums bowing in between pops?

#### A. Outward

- B. Inward
- C. They don't bow

- 1.  $P = \rho g h$
- 2. At higher altitudes, less of the atmosphere is stacked on top of you, so the fluid-height above you, h, decreases.
- 3. Therefore the outside pressure decreases.
- 4. Until your ear "pops", the pressure inside stays the same.
- 5. So, until your ear pops, the gas inside your ear is pushing harder on your ear drum tha the atmosphere outside.
- 6. As such, your ear drum will bow outward until the pressure is equalized (by "popping").

## Fluids in Motion: Continuity

- Continuity:
  - Mass leaves a system at the same rate that it enters (if in a steady state, for an incompressible fluid)
  - For example, the same amount of stuff flows in a hose as flows out. The hose doesn't gain mass over time.
    m/Δt = constant
- Considering fluid flowing through a pipe:

• m =  $\rho V = \rho^* A^* \Delta x$ 

• m/ $\Delta t = \rho^* A^* \Delta x / \Delta t = \rho^* A^* v = constant$ 

• So, my pipe decreases in diameter, the fluid will speed up at that location

 $\Delta x_1 = v_1 \Delta t_1$ 

- For a fluid flowing through a pipe of changing area:
  - $\rho_1 * A_1 * v_1 = \rho_2 * A_2 * v_2$
  - i.e. the mass-flow rate is constant
  - if p=constant ("incompressible)
  - volume flow is constant:  $Q = A_1 * v_1 = A_2 * v_2$





 $\Delta x_2 = v_2 \Delta t_2$ 

An incompressible fluid is flowing through a pipe. At which point is the fluid traveling the fastest?





- 1.  $\rho^* A^* v = constant$
- 2. Incompressible:  $\rho$  = constant
- 3. Cross sectional area A is smallest at 3, so v must be largest there.

An incompressible fluid is flowing through a pipe. At which point is the fluid traveling the slowest?



- 1.  $\rho^* A^* v = constant$
- 2. Incompressible:  $\rho$  = constant
- 3. Cross sectional area A is largest at 5, so v must be smallest there.

#### Fluids in Motion: Conservation of Energy & Bernoulli's equation

- Energy in a system is conserved, except for energy removed by nonconserving forces.
- A moving fluid has:
  - -kinetic energy, from motion
  - -potential energy, from height
- Changes in pressure for a moving fluid correspond to work by a non-conserving force
- Therefore,
  - 1.  $E_i = E_f + W_{NC}$
  - 2.  $PE_i + KE_i = PE_f + KE_f + \Delta E$
  - 3.  $mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2 + Fd$
  - 4. Since  $\rho$  is constant for incompressible fluid, divide everything by volume
  - 5.  $(m/V)gh_i + \frac{1}{2}(m/V)v_i^2 = (m/V)gh_f + \frac{1}{2}(m/V)v_f^2 + F(d/V)$
  - 6.  $\rho g h_i + \frac{1}{2} \rho v_i^2 = \rho g h_f + \frac{1}{2} \rho v_f^2 + \Delta P$
  - 7.  $P_i + \rho g h_i + \frac{1}{2} \rho v_i^2 = P_f + \rho g h_f + \frac{1}{2} \rho v_f^2$  Bernoulli's Equation

An incompressible fluid flows through a pipe. Which point is the pressure greater at?







- 1.  $\rho^* A^* v = constant$
- 2. Incompressible:  $\rho$  = constant
- 3. Cross sectional area A is largest at 1, so v must be smallest there.
- 4. P +  $\rho gh$  +  $\frac{1}{2}\rho v^2$  = constant
- 5. Since pgh=constant, if v gets smaller, P must get larger.
- 6. Therefore, the pressure P is largest at point 1.

An incompressible fluid flows through a pipe. At two different points, small tubes are attached above the pipe to allow for the observation of the pressure in the pipe. How does the height of the fluid in the second pipe compare to the height of the fluid in the first pipe?

A. 
$$h_2 < h_1$$
  
B.  $h_2 = h_1$   
C.  $h_2 > h_1$ 



- 1.  $\rho^* A^* v = constant$
- 2. Incompressible:  $\rho$  = constant
- 3. Cross sectional area A is larger at 2, so v must be smaller there.
- 4. P +  $\rho gh$  +  $\frac{1}{2}\rho v^2$  = constant
- 5. Since pgh=constant, if v is smaller, P is larger.
- 6. Therefore, the pressure P is larger on the centerline at point 2.
- 7. The larger pressure can support a larger (heavier) column of fluid, so  $h_2 > h_1$ .



Consider a blood vessel which has a weakened wall, allowing the vessel to expand. What happens to the pressure in the vessel?

4

A. Pressure increases

- B. Pressure decreases
- C. Pressure stays the same
- 1. From continuity:  $\rho^* A^* v = constant$
- 2. Incompressible:  $\rho$  = constant
- 3. Cross sectional area A increases, so v decrease.
- 4. Bernoulli equation:  $P + \rho g h + \frac{1}{2} \rho v^2$  = constant
- 5. Since pgh=constant, if the velocity decreases, the pressure must increase.



MedicalNewsToday

As you can imagine, the increased pressure can cause the weakened blood vessel to expand even more. This is known as an aneurysm. (If it is in the brain & bursts, that causes a stroke.) You have a giant cask of water with a spigot some height below the water surface. The surface of the water, which is essentially at rest, is exposed to atmosphere (~10<sup>5</sup>Pa). The water density is ~1000kg/m<sup>3</sup>. The water pours out of the spigot at 3m/s.

How far below the water surface is the spigot positioned?



 P<sub>atmosphere</sub> + ρgh<sub>surface</sub> + ½ρv<sup>2</sup><sub>surface</sub> = P<sub>atmosphere</sub> + ρgh<sub>spigot</sub> + ½ρv<sup>2</sup><sub>spigot</sub>
 Atmospheric pressure is going to be negligible compared to the water pressure near the spigot and the water is nearly at rest at the surface. Also, we can define the spigot height h<sub>spigot</sub> = 0. So,

3. 
$$\rho g h_{surface} = \frac{1}{2} \rho v_{spigot}^2$$
  
4.  $h_{surface} = \frac{v_{spigot}^2}{2g} = 0.46m$ 

You'll note this is the height you got on homework & exam questions when you convert an object with only kinetic energy to a situation with only potential energy.

#### The Magnus Effect: A consequence of forces from Bernoulli's principle



https://www.youtube.com/watch?v=QtP\_bh2lMXc



#### Hader's pitches start low and make their way up fast

Average vertical release points vs. average vertical movement on four-seam fastballs for left-handed MLB pitchers\* in 2019



\*With a minimum of 50 four-seam fastballs thrown, through July 14.

FiveThirtyEight

SOURCES: BASEBALL PROSPECTUS, BASEBALL SAVANT

https://fivethirtyeight.com/features/josh-haders-fastball-is-baseballs-most-mysterious-pitch/

# You will <u>NOT</u> be asked about the material below on any homework or exam. Therefore you <u>CAN</u> <u>IGNORE IT</u>, if you like.

We had to trim the following from this year's course content due to the pandemic. However, I think some of you may be curious. Basic questions this would help you answer are, "how/why does this thing float?" and "how does water flow change when I change a pipe/nozzle's diameter?

Two beakers are filled with oil to the same level. The left beaker is much wider than the right beaker. Which beaker will have the greatest pressure at the bottom?

A. Wide beaker

B. Both the same

C. Narrow beaker



- 1.  $P_2 = P_1 + \rho g h$
- 2. For both cases, at the surface  $P_1 = P_{atmosphere}$  and they have the height from the surface of the fluid down to the bottom of the beaker, and they have the same density.
- 3. Therefore, the pressure will be the same at the bottom of both beakers.
- P=F/A
- F is larger for the left beaker, because more mass above the bottom
- But,  $F=mg=(\rho^* \vee)g=(\rho^*h^*A)g$
- So the larger A for the left case will cancel out when calculating P

Buoyancy & Archimedes' Principle You will NOT be asked about this on any homework or exam. You CAN IGNORE IT, if you like.

- The tendency for an object to float in a fluid is *buoyancy*.
- If an object is floating, there must be some upward buoyant force.
  - The direction of the buoyant force is towards a decrease in pressure of the fluid. For terrestrial scenarios, this is "up".
  - The magnitude of the buoyant force depends on the weight of the fluid that was displaced.

•  $F_{buoyancy} = \rho_{fluid} V_{fluid-displaced} g$  This is "Archimedes' Principle". It can be used to measure the volume of an irregular object.



- By measuring the mass, you can get the density of the object. Whether or not an object floats just takes a comparison between the upwards buoyancy force and the downward force of gravity.
  - Floating:
    - $F_{buoyancy} = F_{gravity}$  and  $V_{displaced} < V_{object}$
    - $\rho_{\text{fluid}} V_{\text{displaced}} g = \rho_{\text{object}} V_{\text{object}} g$
    - ...therefore, must have  $\rho_{object} < \rho_{fluid}$
  - Completely submerged:
    - $F_{buoyancy} < F_{gravity}$  and  $V_{displaced} = V_{object} = V$
    - $\rho_{\text{fluid}} Vg < \rho_{\text{object}} Vg$
    - ...therefore, must have  $\rho_{object} > \rho_{fluid}$



Cylinders A, B, and C have the listed weights and volumes and are completely submerged in a fluid.

Rank the buoyant force exerted on the cylinders by the fluid.



Cylinder	Weight	Volume
А	2N	V <sub>A</sub>
В	7N	$V_{\rm B} = V_{\rm A}$
С	7N	$V_{\rm C} > V_{\rm A}$

1.  $F_{buoyancy} = \rho_{fluid} V_{fluid-displaced} g$ 

- 2. The buoyancy force only depends on the amount of fluid displaced.
- 3. The buoyancy force does not depend on the mass of the object doing the displacing.

You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like. Ice has a density of ~920kg/m<sup>3</sup>, compared to liquid water's ~1000kg/m<sup>3</sup>. What fraction of an iceberg's volume is above the surface of the water?

0.08% Α. B. 0.8% 8% 80% 1.  $F_{buoyancy} = F_{gravity}$ 2.  $\rho_{water}V_{water-displaced}g = \rho_{ice}V_{iceberg}g$ 3.  $V_{water-displaced}/V_{iceberg}$  is the volume below water. 4.  $V_{water-displaced}/V_{iceberg} = \rho_{ice}/\rho_{water} = 0.92$ 5. I.e. 92% of the iceberg is below the surface, meaning 8% is above the water surface.



You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like. You want to salvage your yacht. You don't have traditional salvaging equipment, but you did recently inherit a ping pong ball empire. You decide to pump ping pong balls ( $V_{ball} \sim 3x10^{-4}m^3$ ) into your yacht ( $M_{yacht}$ =1500kg) in order to raise it from the sea floor. How many ping pong balls do you need to pump into the yacht? Consider the yacht buoyancy to be negligible and the ball mass to be small.



(C) 50,000 (D) 500,000

- 1. To make the ship float:  $F_{buoyancy} = F_{gravity}$
- 2. Since the ping-pong balls make-up most of the displaced volume and the ship makes-up most of the mass:

 $\rho_{water}V_{balls}g=M_{yacht}g$ 

- 3.  $V_{\text{balls}} = (M_{\text{yacht}})/\rho_{\text{water}}$
- 4.  $V_{\text{balls}} = (1500 \text{kg})/(1000 \text{kg/m}^3) = 1.5 \text{m}^3$
- 5. One ping-pong ball has a volume of ~  $3x10^{-4}m^3$ ,
- 6. So, the number of ping-pong balls is:  $V_{\text{balls}}/V_{\text{ball}} = 1.5 \text{m}^3/(3 \times 10^{-5} \text{m}^3) \sim 50,000 \text{ balls}$
- 7. ...it turns out a fiberglass hull would provide almost half that much buoyancy, so you would only need ~25K ping-pong balls.
  You will NOT be asked about this on any homework or exam. You CAN IGNORE IT, if you like.





Mythbusters, E21

You're a modern artist and your newest installation will consist of an air-filled balloon under water in a fish tank, held there by a single cooked spaghetti noodle that is attached to the bottom of the tank. Don't ask why, it's art. Given that a single cooked noodle can withstand a tension of ~0.5N, what is the maximum volume the air (~1kg/m<sup>3</sup>) filled balloon can be? Ignore the balloon material mass.

INFLUENCE OF EXTRUSION-COOKING PROCES PARAMETERS ON SELECTED MECHANICAL PROPERTIES OF PRECOOKED MAIZE PASTA PRODUCTS

Agnieszka Wójtowicz

You CAN IGNORE IT, if you like.



(B) 50 cm<sup>3</sup>

(C) 500 cm<sup>3</sup> (D) 5000 cm<sup>3</sup>

(A) 5  $cm^3$ 

2.  $F_{buoyancy} = F_{gravity} + T_{noodle}$ 3.  $\rho_{water}V_{displaced}g = \rho_{air}V_{balloon}g + T_{noodle}$ 4. The displaced volume of water is the same as the volume of the balloon 5.  $\rho_{water}V_{balloon}g = \rho_{air}V_{balloon}g + T_{noodle}$ 6.  $V_{balloon}(\rho_{water} - \rho_{air})g = T_{noodle} \approx V_{balloon}(\rho_{water})g$  ...since  $\rho_{water} >> \rho_{air}$ 7.  $V_{balloon} = T_{noodle}/(\rho_{water}g)$ 8.  $V_{balloon} = (0.5 \text{kgm/s}^2)/\{(1000 \text{kg/m}^3)^*(9.8 \text{m/s}^2)\}$ 9.  $V_{balloon} \sim 5x10^{-5}\text{m}^3 = 50\text{cm}^3$ 

 $1m^3 = 10^6 cm^3$ 



## Fluids in Motion: Flow

- Typically consider "ideal fluid":
  Incompressible: has a constant density
  Non-viscous: no friction between layers of fluid
- Flow: How components of a fluid move relative to each other
  Describe the direction of travel with "streamline"s
  - Classify types of flow based on streamlines:
    - Steady: velocity constant at a point.
      - i.e. the fluid properties aren't changing
    - Unsteady: velocity changes magnitude
      fluid may be speeding up or slowing down
    - Turbulent: erratic changes in velocity
      technically a type of unsteady flow
- Fluids exhibiting turbulence are "turbulent"
- Fluids not exhibiting turbulence are "laminar" You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like.









#### Turbulence

You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like.



Werner Heisenberg

"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first."

![](_page_35_Picture_5.jpeg)

#### Viscosity - η

- Frictional forces between layers in a fluid.
- Causes zero speed at the boundaries...
- and means speed is greatest at the flow center.
- The SI unit is: (Pa)s
- The more resistant to flow, the greater the viscosity.

![](_page_36_Picture_6.jpeg)

![](_page_36_Picture_7.jpeg)

You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like.

## Bernoulli's Equation: Implications

- $P_i + \rho g h_i + \frac{1}{2} \rho v_i^2 = P_f + \rho g h_f + \frac{1}{2} \rho v_f^2$
- What happens if the velocity increases?
   (by decreasing the pipe area [p\*A\*v = constant])
  - KE term ( $\frac{1}{2}\rho v^2$ ) will increase
  - height & density are unchanged, so PE term (pgh) will stay the same
  - But energy is conserved, so pressure must change
  - Pressure decreases to:  $P_f = P_i \frac{1}{2}\rho(v_f^2 v_i^2)$
  - Higher fluid speed leads to a lower pressure!
- If a fluid is made to flow at different speeds on opposite sides of a barrier:
  - there will be different pressures on the opposite sides.
  - therefore, the force from pressure will be different
  - The difference in forces is:  $F_{net} = F_1 F_2 = P_1 A P_2 A = (\Delta P)A = F_{lift}$

#### **NOTE:** angle-of-attack is far more important for determining a wing's lift

![](_page_37_Figure_13.jpeg)

![](_page_37_Picture_14.jpeg)

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## Hagen-Poiseuille Equation:

Volumetric flow rate due to a pressure gradient for a viscous non-turbulent fluid.

- You can only flow a limited amount of fluid through a pipe in a given amount of time.
- For a non-turbulent viscous liquid, this limiting flow rate is described by the Hagen-Poiseuille Equation

•  $Q = \frac{\pi R^4 (P_{upstream} - P_{downstream})}{8\eta L}$ 

- where Q is the volumetric-flow rate, R is the pipe radius, L is the pipe length,  $\eta$  is the fluid viscosity, and P<sub>u</sub>-P<sub>d</sub> is the pressure difference before & after the pipe (i.e. at positions x=0 and x=L).
- Since the units for viscosity are commonly (Pa)s, it is usually easiest to specify pressure in Pa, so the pressure unit cancels.
- Hagen-Poiseulle is somewhat intuitive:
  - More flow for a larger radius pipe
  - More flow for a larger pressure difference  $((\Delta P^*A = F))$
  - Less flow for a more viscous fluid.
  - Less flow for a longer pipe (More friction between the fluid & the walls).
- This can be inverted to get the pressure drop along a pipe for a given volume-flow rate.
  - $\Delta P = \frac{8\eta L}{\Omega \pi R^4}$

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Atherosclerosis is a process in which fats (& other junk) build-up on your blood vessel walls, decreasing the effective radius for blood to flow through. If your effective blood vessel radius decreases to 0.8x its original radius (i.e. R --> 0.8\*R),

by how much does your blood flow decrease?

![](_page_39_Picture_3.jpeg)

1.  $Q = \frac{\pi R^4 (P_{upstream} - P_{downstream})}{8\eta L}$ 2. If R --> 0.8\*R, then R<sup>4</sup> --> (0.8\*R)<sup>4</sup>. 3. So, Q --> (0.8)4\*Q ≈ 0.41Q. 4. This corresponds to a reduction by ~60%.

You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like. A 20% reduction in blood vessel radius leads to a 60% reduction in blood flow rate. A 50% reduction in vessel radius leads to a 94% decrease in blood flow rate!

#### Hagen-Poiseuille Equation: Practical example

- Consider several tanks of fluid with some volume V.
- The tanks are each drained by a different pipe with different radii R and lengths L, and the pressure at the upstream side of each pipe is the same.
- The volume is proportional to the pipe length, so V is proportional to L<sup>3</sup>.
- Then flow rate for fluid to drain a given tank will be:  $Q = \frac{\pi R^4 \Delta P}{8\eta L} = \frac{\Delta V}{\Delta t}$
- If the upstream pressure (in the tank) is much higher than the pressure at the exit of the pipe, then ΔP ≈ P<sub>upstream</sub>.
- Then, ignoring the scaling factors and doing some algebra:  $\Delta t \propto \frac{L^4}{R^4}$
- If each of the different tanks have a fixed L/R ratio, then they would take the same time to drain.
   Duration of urination does not change with bod

![](_page_40_Picture_8.jpeg)

#### You will <u>NOT</u> be asked about this on any homework or exam. You <u>CAN IGNORE IT</u>, if you like.

![](_page_40_Figure_10.jpeg)