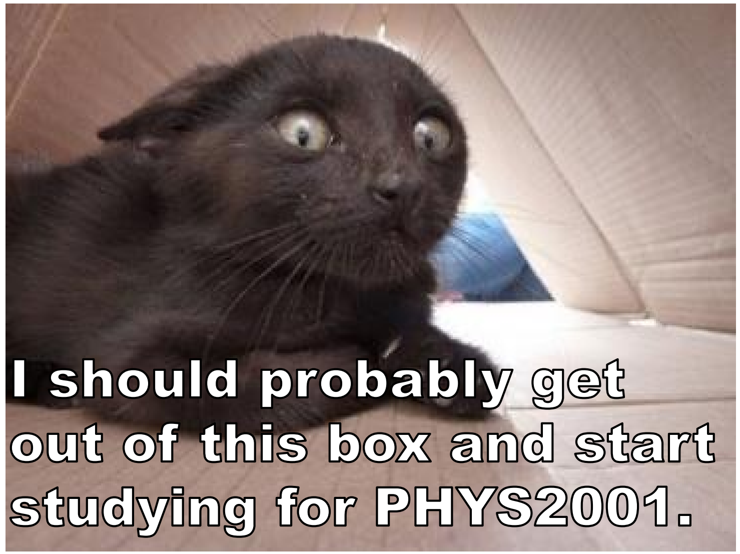
Thursday March 28

Topics for this Lecture:

- *Simple Harmonic Motion*
  - *Periodic (a.k.a. repetitive) motion*
  - *Pendulum systems*
  - *Resonances & Damping*

- Assignment 10 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- **SI Review: Tonight 6:20-8:10 Morton 115**
- **Dr. Piccard Review: Sunday 6-9pm Walter 245**

Midterm 2 is Monday April 1st
7-9pm in Morton 201



Mass-Spring system:

$$\omega_{spring} = \sqrt{\frac{k}{m}} \quad T_{spring} = 2\pi\sqrt{\frac{m}{k}} \quad f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Note: T, f, ω are ***independent of amplitude!***

$$x = A\cos(\omega t) \quad x_{max} = A$$
$$v = -A\omega\sin(\omega t) \quad v_{max} = A\omega$$
$$a = -A\omega^2\cos(\omega t) \quad a_{max} = A\omega^2$$

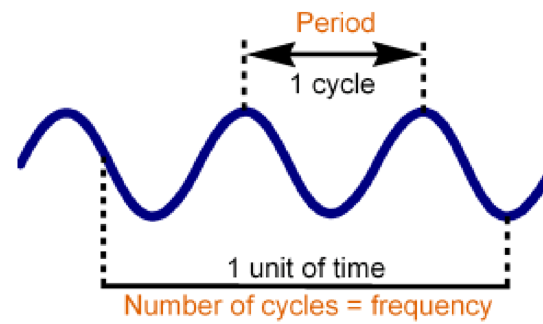
Pendulum system:

F ≈ -mg θ

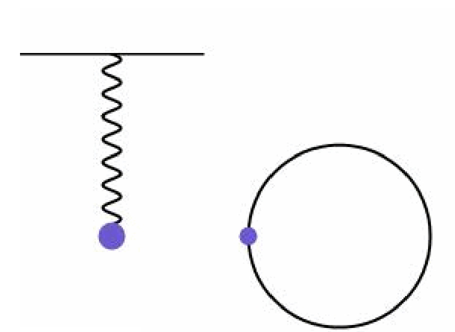
F ≈ -(mg/L)s

$$\omega_{pendulum} = \sqrt{\frac{g}{l_{pendulum}}} = \sqrt{\frac{g}{l}}$$

Frequency & Period: Two sides of the same coin

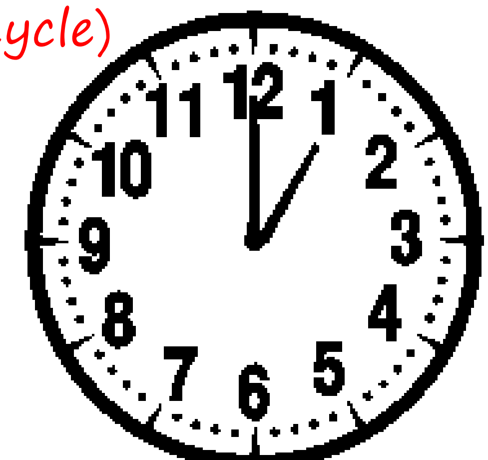


- Frequency, f = cycles per unit time.
  - cycle can be one rotation, or oscillation, or one of whatever it is that is repeating
  - SI units are inverse seconds s^{-1}, also called Hertz: Hz
- Period, T = time for one cycle
  - SI units are seconds: s



- f = 1/T
- T = 1/f

The second-hand on a clock completes one full circle (cycle) in 60 seconds.
So, T_{hand} = 60s.
f_{hand} = 1/60s = 0.017Hz.

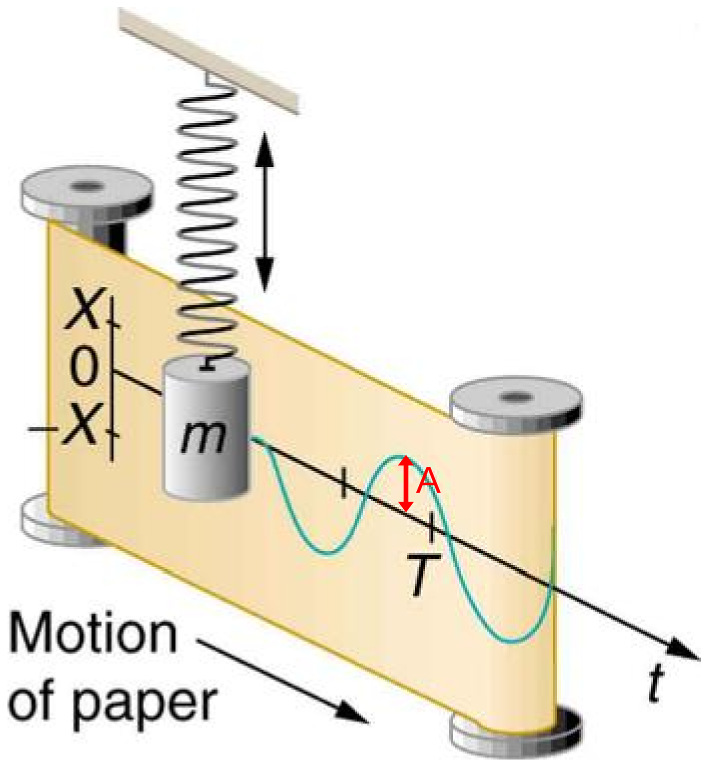


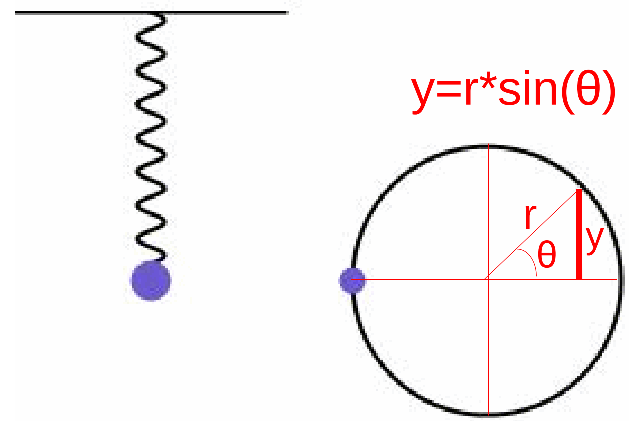
A mass is oscillating up and down on a spring.
It completes 10 complete oscillations in 15 seconds.
What is the frequency of oscillation?

(A) 0.67 Hz (B) 1.5 Hz (C) 10 Hz (D) 15 Hz

1. One oscillation = one cycle.
2. The time for one cycle, the period, T.
3. T = (time)/(# cycles)
4. Frequency, f =1/T
5. $f = 1/T = 1/(15s/10) = 10/(15s) \approx 0.67s^{-1} = 0.67Hz$

Oscillations: Displacement vs Time is described by the sinusoid function



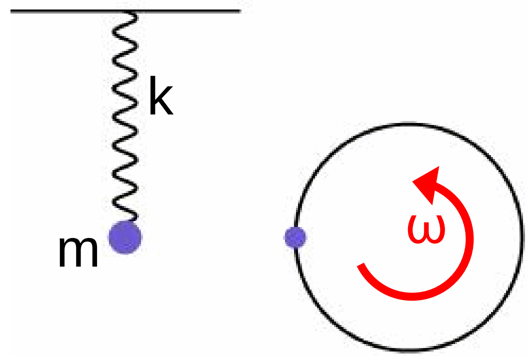


Note that y/r = sin(θ).
So how far an oscillator is through one cycle is independent of the amplitude.
I.e. the ***oscillation period is independent of the oscillation amplitude.***

The largest displacement during the oscillation is called the ***amplitude***.
The book uses “X” for amplitude … “A” is more common.

Oscillations: Physical properties

- How does an oscillator's properties depend on the system that is oscillating?
- Quantities of interest are:
  - How many cycles are completed per second?
    - Frequency: f [unit: Hz]
  - If we map the oscillation into cycles on a circle, how many radians do we trace per second?
    - Angular frequency: ω = 2π*f [unit: rad/s]
  - How long does it take to complete one cycle,
    - Period: T [unit: s]



- With calculus & patience, it can be shown that, for an oscillating spring with spring constant k and attached weight of mass m:

- $\omega_{spring} = \sqrt{\frac{k_{spring}}{m_{weight}}} = \sqrt{\frac{k}{m}}$
- $f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- $T_{spring} = 2\pi\sqrt{\frac{m}{k}}$

*Since f = 1/T = ω/2π ...you really only need to remember one of the above formulas.

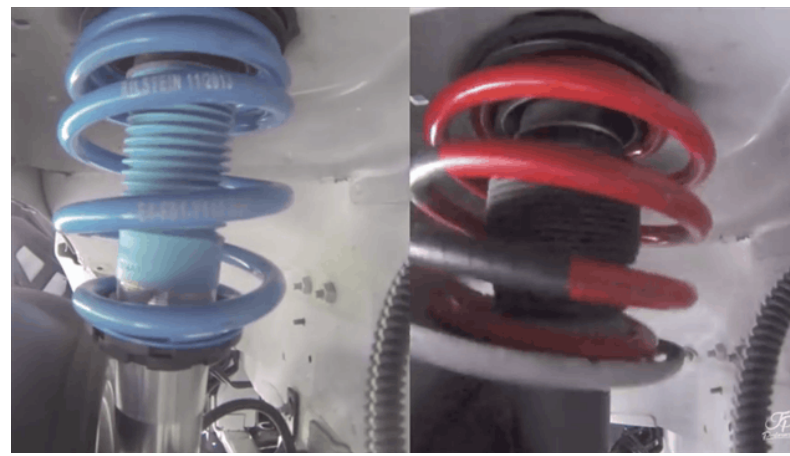
*Note: Frequency, angular frequency, and period are independent of amplitude!

A mass is hung from a spring and set into oscillation.
It oscillates with a given frequency f_1.
Now a second identical spring is also attached to the mass (same k, same length).
How does the new frequency, f_2, compare to the old frequency, f_1?

A. $f_2 = 2*f_1$
B. $f_2 = \text{sqrt}(2)*f_1$
C. $f_2 = f_1$
D. $f_2 = (1/\text{sqrt}(2))*f_1$
E. $f_2 = (1/2)f_1$

A larger spring constant will give a stiffer ride. This is great for handling/responsiveness ...but bad for comfort. So, larger k is good for race cars, but smaller k is good for sedans.

1. The second spring doubles the spring force, effectively doubling the spring constant.
2. $k_2 = 2*k_1$
3. The new spring is stiffer & so will have a greater oscillation frequency.
4. $f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}}$
5. $f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k_1}{m}} = \sqrt{2}f_1$

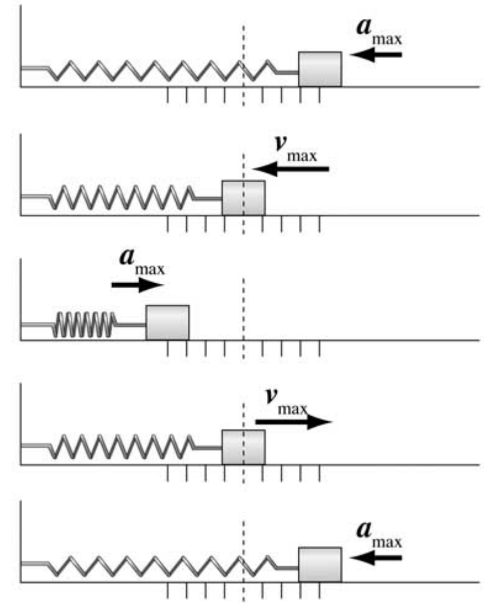


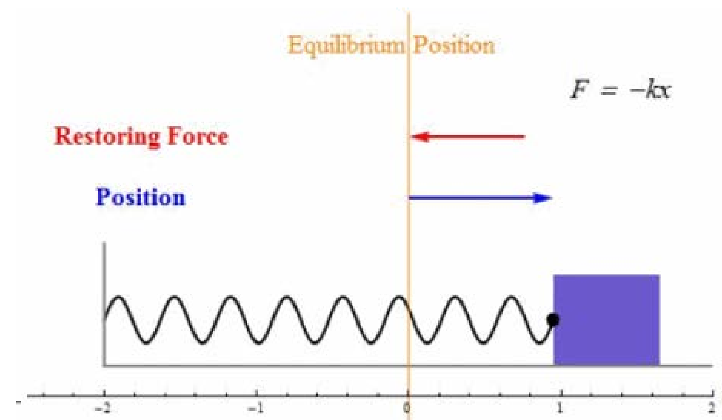
OPTIMUM G

- 0.5 - 1.5 Hz for passenger cars
- 1.5 - 2.0 Hz for sedan racecars and moderate downforce formula cars
- 3.0 - 5.0+ Hz for high downforce racecars

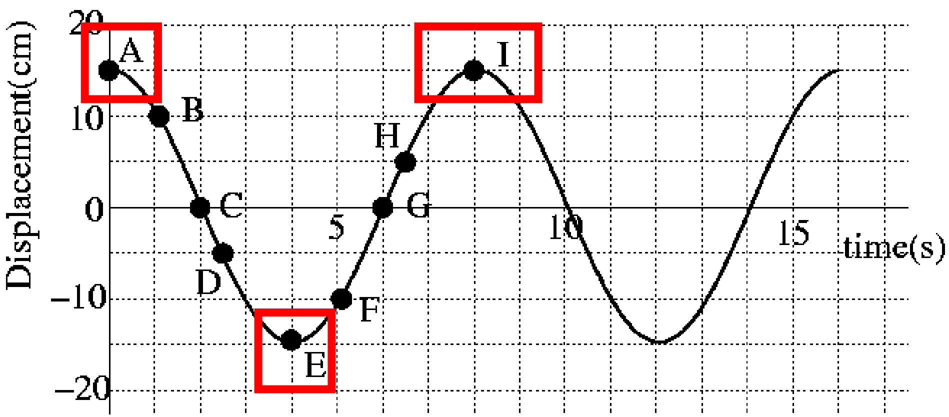
Mass & Spring system: Velocity & Acceleration

- $F_{spring} = -k*x$
- Therefore, force is greatest at place where the displacement is the largest (the amplitude)
- Since, F = m*a, ma = -k*x,
  - a = -k*x/m
  - **acceleration is greatest at the amplitude & velocity is zero at the amplitude**
- The mass accelerates from zero velocity at one amplitude (the extreme point in its motion) to a **maximum velocity at zero displacement** and then experiences large negative acceleration approaching the other extreme.
  - At zero displacement, x = 0, so F = k*x = m*a = 0. i.e. **acceleration is zero at zero displacement**





This graph represents the displacement of a mass in a mass-spring system. At which point(s) is the magnitude of the force at a maximum?



(A) A

(B) C

(C) E

(D) G

(E) I

(F) A and I

(G) A, E, and I

(H) C and G

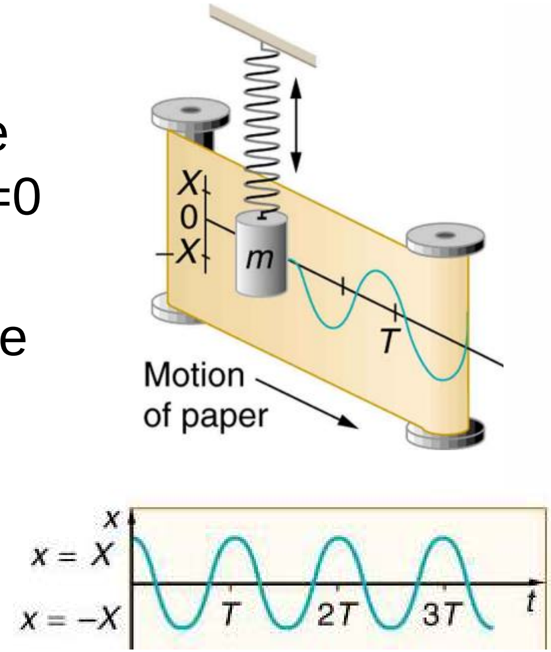
(I) A,C,G, and I

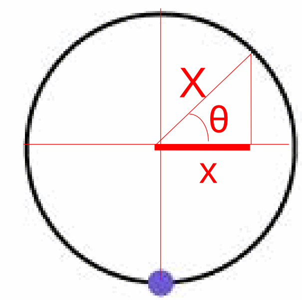
1. $F_{spring} = -k*x$
2. The maximum force is at the maximum displacement (the amplitude).
3. This is at points A, E, & I.

Oscillation Temporal Properties: Analogy to circular motion

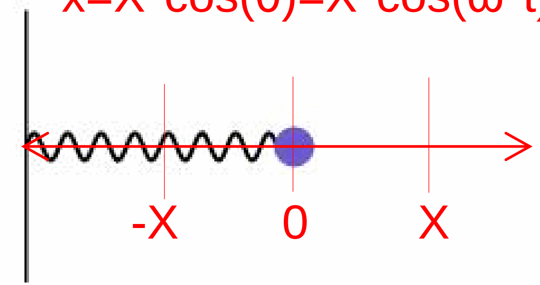
- If we start a spring at the position x =X, where X is the positive amplitude, at time t=0
- Then the spring position vs time oscillates at a rate of the angular frequency, ω=2π*f
- x(t) = X*cos(ω*t)
- a.k.a.

  x(t) = A*cos(ω*t)





x=X*cos(θ)=X*cos(ω*t)



- Recall that velocity v=Δx/Δt and acceleration a=Δv/Δt = (Δ(Δx))/(Δ(Δt)).
- In calculus, these are the 1^{st} and 2^{nd} derivatives of position in time, respectively.
- The 1^{st} derivative of cos(ω*t) is -ω*sin(ω*t) and the 2^{nd} derivative is $-\omega^2$*cos(ω*t).
- So, the velocity and acceleration of an oscillator are described by:

v(t) = -ω*A*sin(ω*t) $a(t) = -\omega^2 * A * \cos(\omega * t)$

Formula summary for mass-spring system:

$$\omega_{spring} = \sqrt{\frac{k}{m}}$$

$$f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$

angular frequency = ω = 2πf [SI units: rad/s]
frequency = f = 1/T   [SI units: Hz]

period = T = time per oscillation cycle [SI units: s]

T, f, ω are independent of oscillation amplitude!

$$x = A\cos(\omega t)$$

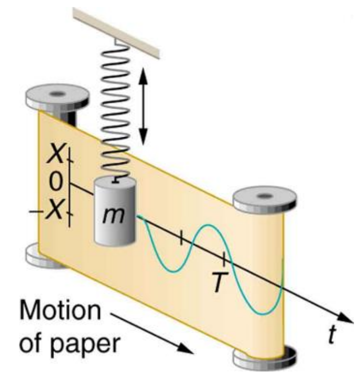
$$x_{max} = A$$

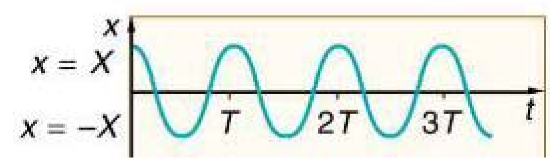
$$v = -A\omega\sin(\omega t)$$

$$v_{max} = A\omega$$

$$a = -A\omega^2\cos(\omega t)$$

$$a_{max} = A\omega^2$$





*Sine and cosine can only take-on values between -1 and 1.

Which of the following situations will have the greatest **maximum force**?

A. A mass M is attached to a spring with a spring constant k, pulled back a distance d and released.

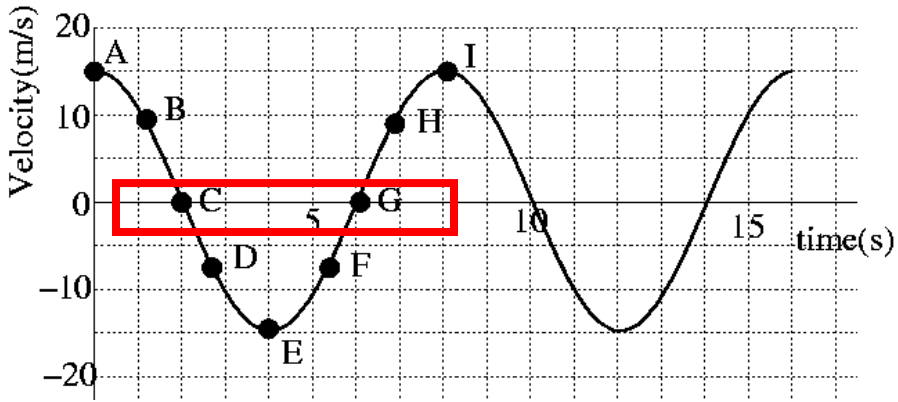
B. A mass 2M is attached to a spring with a spring constant (1/2)k, pulled back a distance 2d and released.

C. A mass (1/2)M is attached to a spring with a spring constant 2k, pulled back a distance (1/2)d and released.

D. A, B, & C

1. $F_{spring} = -k*x$
2. The maximum force will be at the amplitude, x = A.
3. $F_{max} = k_{spring}*A$
4. For A: $F_{max,A} = k*d$
5. For B: $F_{max,B} = (1/2)k*(2d) = k*d$
6. For C: $F_{max,C} = (2k)*(1/2)d = k*d$
7. $F_{max,A} = F_{max,B} = F_{max,C}$

The graph represents **velocity** of a mass in a mass-spring system.
At what point(s) is the magnitude of the force at a maximum?



(A) A
(B) C
(C) E
(D) G ← no;

(E) G
(F) A and I
(G) A, E, and I
(H) C and G
(I) A,C,G, and I

1. $F_{spring} = -k*x$
2. The force is maximum for largest displacement, $x = x_{max} = A$
3. At the amplitude, the mass is turning around and so is stopped for an instant.
4. Therefore, velocity is zero at the amplitude.
5. C and G correspond to zero velocity, therefore the amplitude, where the force is maximum.

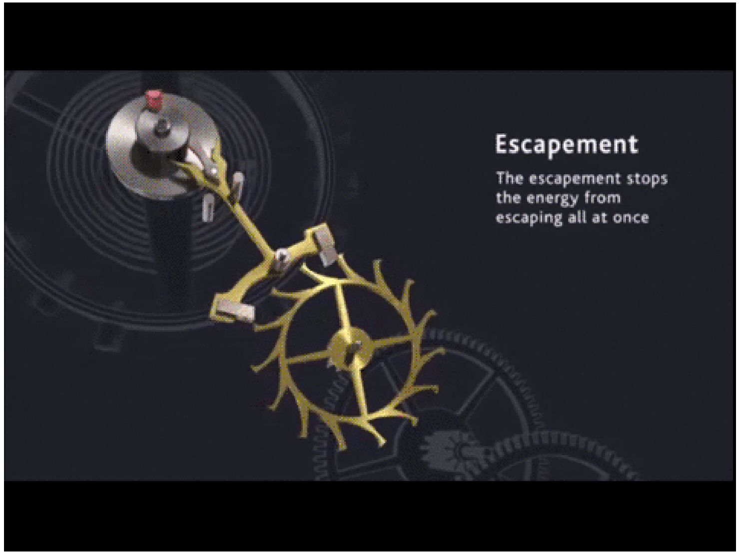
A mass attached to a spring is sliding back and forth on a horizontal frictionless table.
The kinetic energy is a maximum:

1. At maximum compression
2. At maximum extension
3. At equilibrium
4. At both extremes (compression & extension)

1. KE = (1/2)mv^2
2. Maximum v will lead to maximum KE.
3. v is maximum at equilibrium

Mass-spring system: Energy

Spring-powered watch mechanism



- Energy can be stored in a spring and released later as kinetic energy
- After some calculus (derivative of the force):
  - $PE = \frac{1}{2}kx^2$
- Work done by a spring: W = ΔPE
  - stretching or compressing will do negative or positive work
- Since no non-conserving forces are present, Total Energy (KE + PE) is constant
  - We can use this to find velocity at a given displacement: v(x)

Mass Spring with Vectors

Mass-spring system: Energy

- When there are rotating bits (e.g. pendulum), the total energy is: $E_{tot} = PE_{height} + PE_{spring} + KE_{trans} + KE_{rot}$
  - $E = mgh + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

- For a horizontal mass & spring, life is simpler:
  - No changes in height, so can ignore PE_{height}
  - No rotating pieces, so can ignore rotational energy
  - $E_{horiz-spring} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
  - This gives us a way to relate velocity & position!
    - For x = A, total energy is: $\frac{1}{2}kA^2$, since velocity is zero. This is conserved.
    - So, $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ *For the horizontal mass-spring system
    - …after some algebra: $v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$

Mass Spring with Vectors

The graph represents **velocity** of a mass in a mass-spring system.
At what point(s) is the magnitude of the **Potential Energy** at a maximum?

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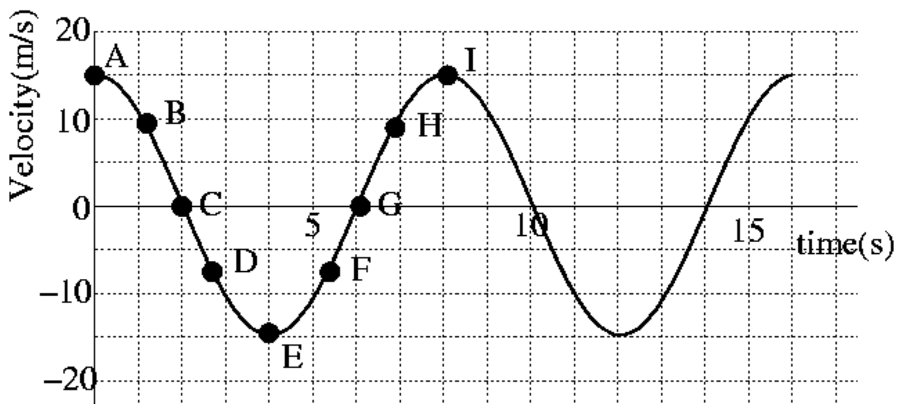


(A) A

(B) C

(C) E

(E) G

(F) A and I

(G) A, E, and I

(H) C and G

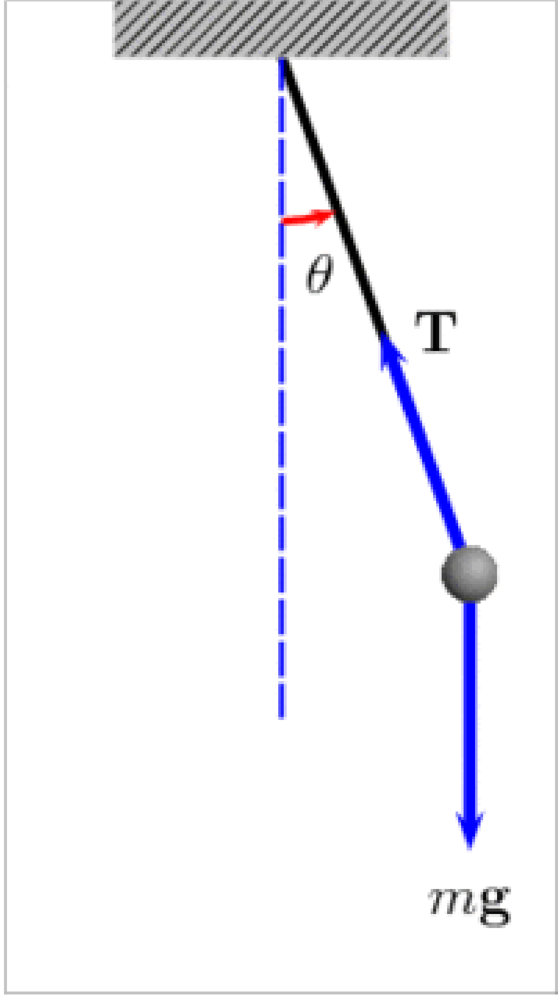
(I) A,C,G, and I

1. E = PE + KE = $(1/2)kx^2+(1/2)mv^2$
2. Energy will be conserved throughout motion, so KE will be converted to PE and back.
3. Minimum KE means maximum PE.
4. When v=0 (at the amplitude), KE =0
5. v=0 at C and G, so PE is maximized there.

Pendulum

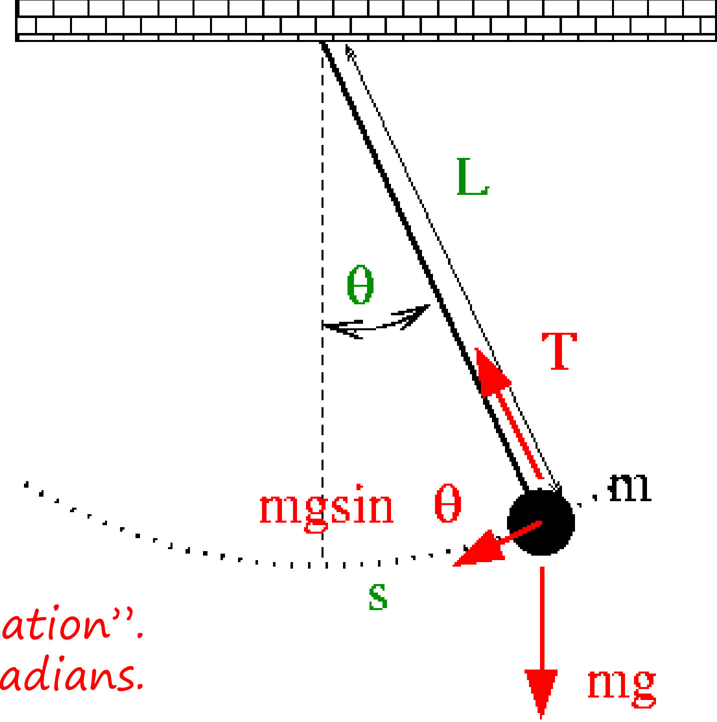
- Pendulum: a periodic swinging thing
  - physical pendulum: arm with weight on the end
  - “simple pendulum”: most mass concentrated at one point
    - we’ll mostly stick to this, because it’s simpler since you can ignore the moment of inertia of the pendulum arm

- Like the mass on the spring, the pendulum oscillates.
- *For small angles*, the pendulum undergoes simple harmonic motion, analogous to the mass & spring
  - We’ll work with small angle cases
  - “Small” means ~ <0.244radians, which is ~ <14°
  - (Justification comes from when sin(θ) ~ θ

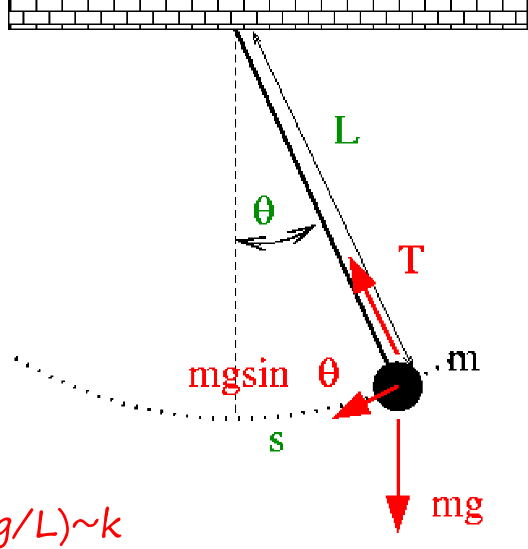


Pendulum: Forces

- The pendulum weight experiences a downward force from gravity and an upward tension from the pendulum cable.
- The net force experienced in the swinging direction is:
    - $F_{net} = -mg*\sin(\theta)$
    - For $\theta < 0.244$ radians (~14 degrees), $\sin(\theta) \approx \theta$, so $F_{net} \approx -mg*\theta$ *This is the "small angle approximation". It only works when using radians.*
    - Since θ=(arc length)/radius = s/r, and r is the pendulum length L, $F \approx -(mg/L)s$
- Recalling that for a spring, F=-kx, it is apparent that pendulums for small swinging-angles undergo simple harmonic motion!



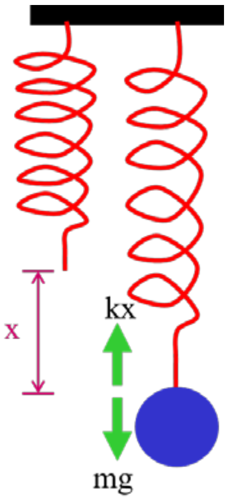
Pendulum-Spring Comparison



Quantity	Mass-Spring	Pendulum
Stiffness	k	mg/L
inertia	m	m
ω	$\sqrt{k/m}$	$\sqrt{g/L}$

from the fact that (mg/L)~k

Note: $\omega=2\pi f$ and $f=1/T$



The pendulum period only depends on the pendulum length & the local gravity!

$$\omega_{pendulum} = \sqrt{g/L}$$

Two identical pendulum clocks are set in motion, one on the Earth and one on the Moon. The acceleration due to gravity on the Moon is 1.67 m/s^2 (1/6 that of Earth).
Which of the following statements is true?

A. The clock on the Moon runs faster than the clock on Earth
B. The clock on the Moon runs the same speed as the clock on Earth
C. The clock on the Moon runs slower than the clock on Earth

1. $\omega = \sqrt{g/L}$
2. On the moon, “g” is smaller, so ω will be smaller.
3. $T=2\pi/\omega$
4. So, smaller ω means larger T, i.e. a longer period.
5. The clock will take longer for one “tick” on the Moon & therefore runs slower.

Two identical pendulum clocks are set in motion, one on the Earth and one on the Moon. The acceleration due to gravity on the Moon is 1.67 m/s^2 (1/6 that of Earth).

How much do we have to shorten the Moon clock pendulum to get it to tick at the same rate as the Earth clock?

A. 1/6

B. $1/\sqrt{6}$

C. 0.6

D. 6/(2π)

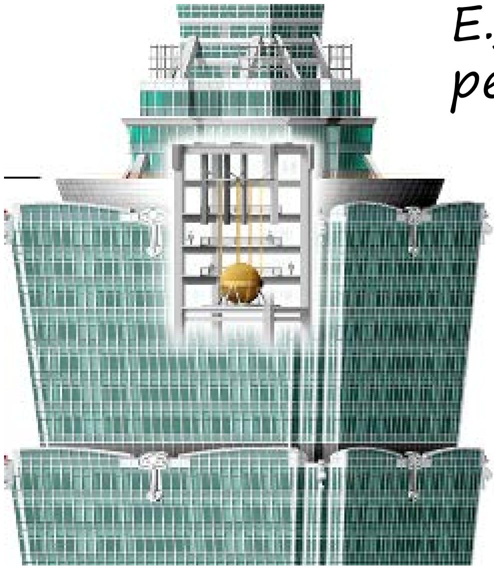
1. $\omega = \sqrt{g/L}$
2. $g_{Moon} = (1/6)g_{Earth}$,
3. So, need $L_{Moon} = (1/6)L_{Earth}$
4. $\omega = \sqrt{(\frac{1}{6}g)/(\frac{1}{6}L)} = \sqrt{g/L}$

Resonances

- Most oscillatory systems have a frequency at which they easily oscillate.
- This is referred to as the "natural" or "resonant" frequency.
- If a force is applied periodically, matching the resonant frequency, the system will be driven "on the resonance" and energy will build-up.
- This can be good (musical instruments) or bad (Tacoma Narrows bridge)

Example:
Kid being pushed on a swing

Object designs often try to counteract motion at the resonant frequency



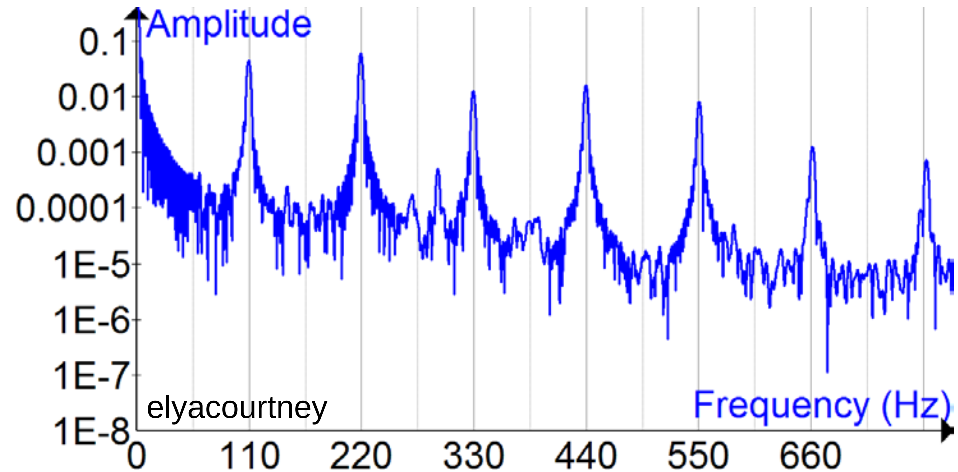
E.g. the 728-ton pendulum in Taipei 101

Resonances

Shattering a glass with a high-pitched noise

https://www.youtube.com/watch?v=L2VoR1lorqQ

Notes for a base guitar



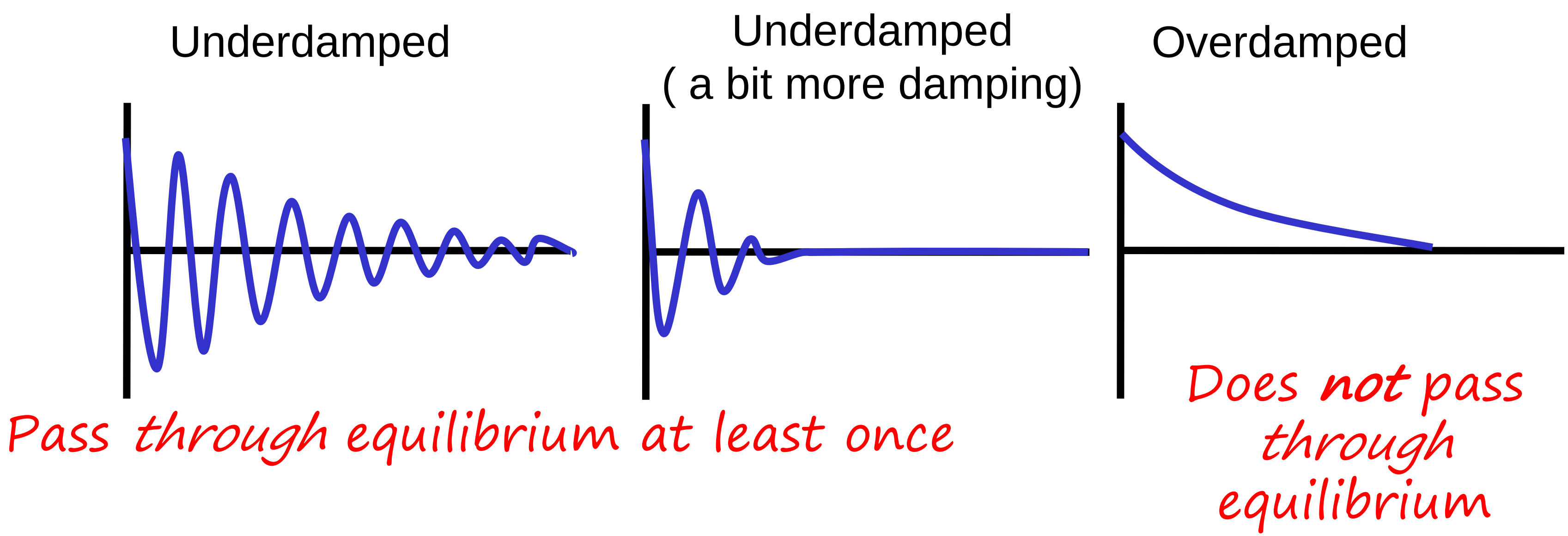
A car rests on springs and has a certain resonant frequency.
If a driver picks up passengers, how will the resonant frequency be affected?

A. Increase with passengers in the car.
B. Stay the same.
C. Decrease with passengers in the car.

1. $\omega = \sqrt{k/m}$
2. Increasing m, means decreasing ω

Damped Motion

"Damping" is removing energy from a system (e.g. via friction, air resistance,…), removing energy from a system & therefore decreasing the oscillation amplitude.



"Critically damped":

The least amount of damping where, when applied, a system will not oscillate.

This will return the system to equilibrium as quickly as possible.