Tuesday March 28

Topics for this Lecture:

- *Simple Harmonic Motion*
  - *Periodic (a.k.a. repetitive) motion*
  - *Hooke's Law*
  - *Mass & spring system*

- Assignment 11 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed or by appointment (meisel@ohio.edu)

Key take-aways:

spring force

$$F_{applied} = kx$$

angular frequency

$$\omega_{spring} = \sqrt{\frac{k}{m}}$$

frequency

$$f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

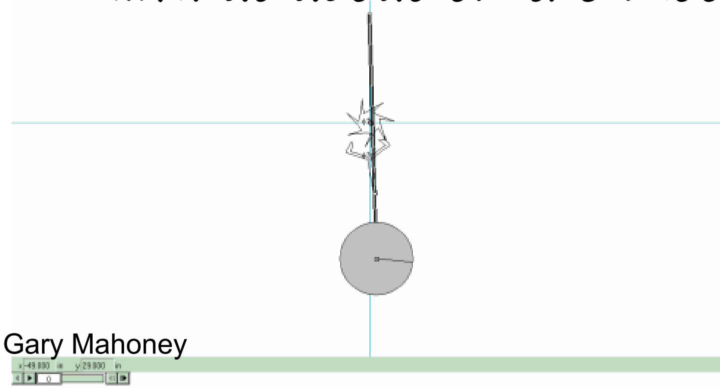
oscillation period

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$

Note: T, f, ω are ***independent of amplitude!***

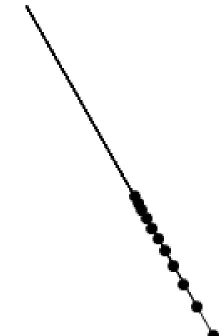
Simple Harmonic Motion

What length pendulum should I choose to make a clock that ticks once per second? ...what about on the moon?



Gary Mahoney

How do I predict when these pendula of different lengths will re-align?



Sol Lederman

Why did this happen at Tacoma Narrows?

How do you select a spring for a race car vs sedan suspension?

What are the units for the spring constant, k?

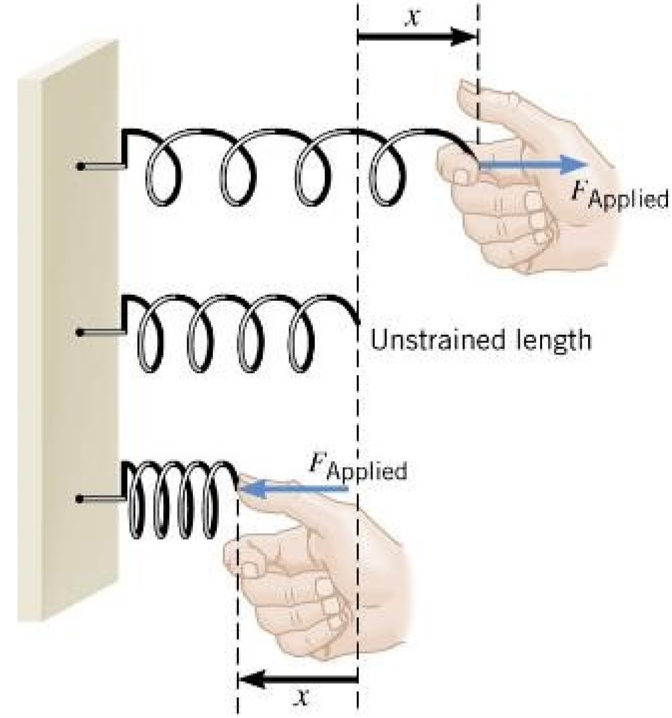
A. N
B. m/N
C. N*m
D. N/m
E. kg*m
F. kg*N

1. Spring constant, k = F/x. F= force applied, x = displacement of spring.
2. SI units for F: N
3. SI units for x: m
4. SI units for k: N/m

Springs: Hooke's Law

Sect 16.1

- The displacement of a stretchy-object (e.g. spring) from it's equilibrium position (where it is when you're not touching it) requires a force linearly proportional a property of the spring, k.
  - **$F_{applied}$ = k*x**
  - F = applied force
  - x = displacement
  - k = spring constant (SI units: N/m)
- This is "Hooke's Law"

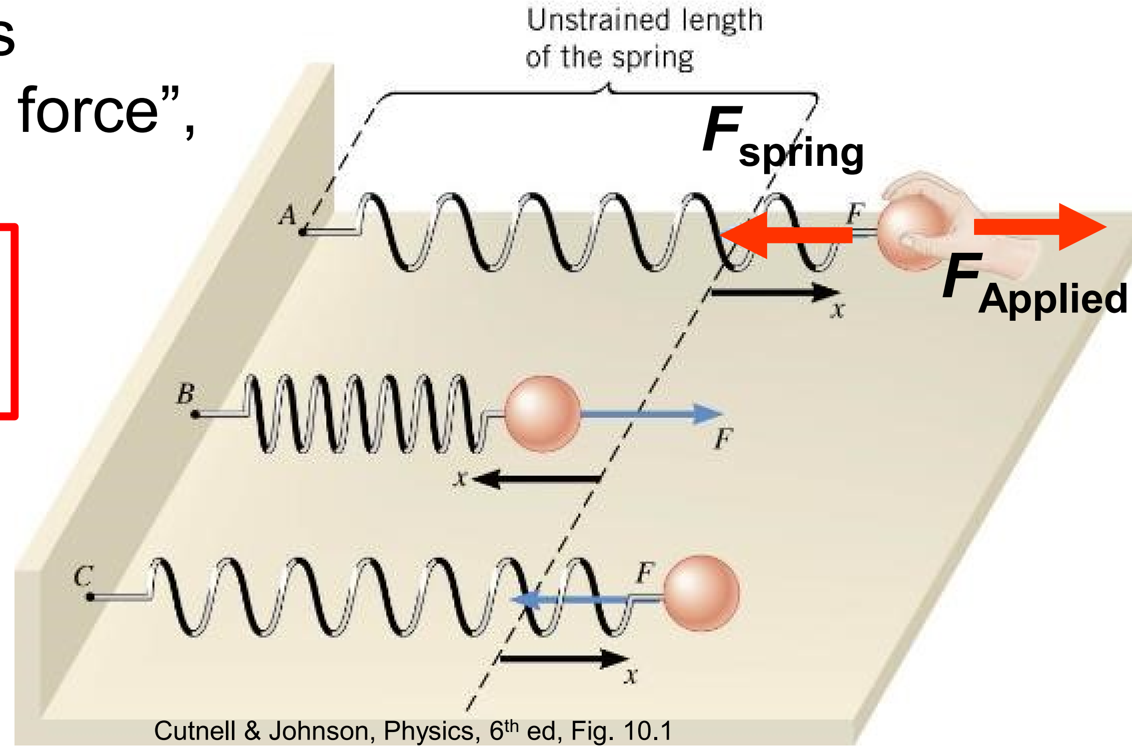


Cutnell & Johnson, Physics, 6th ed, Fig. 10.1

*The sign of x depends on the direction of displacement!
You choose which way is +x & the other way is -x.

Springs: Applied Force & Restoring Force

- The force you apply to stretch (or compress) a spring is balanced by a "restoring force", a.k.a. the spring force.
  - $F_{applied} = -F_{spring} = k*x$
  - $F_{spring} = -k*x$



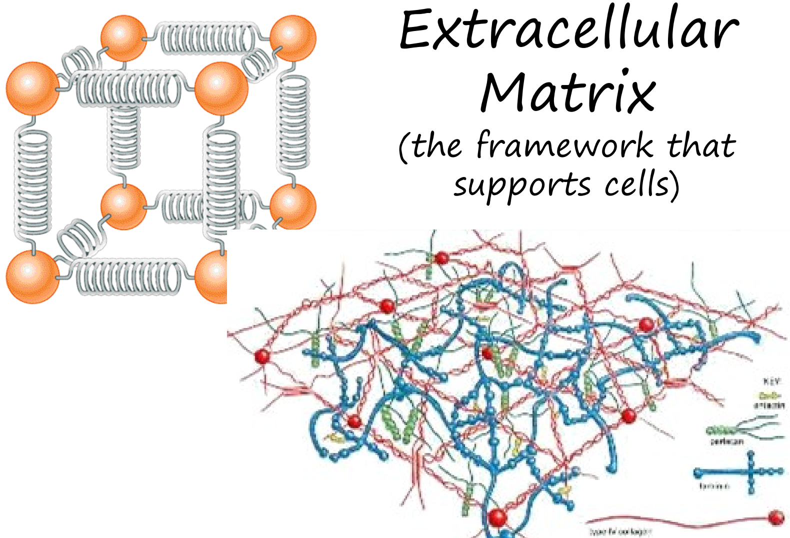
Cutnell & Johnson, Physics, 6th ed, Fig. 10.1

Springs: Anything that stretches & tries to return to equilibrium!

- Hooke's law applies to anything with "elasticity"
  - I.e. objects which can stretch & tend to return to the position from which they were stretched
  - E.g. Springs, rubber bands, taught guitar strings, bridges in the wind, etc.

- Complicated solids held together by molecular bonds are well represented by coupled spring systems.

Serway, College Physics, 5/e
Text Figure 9.1

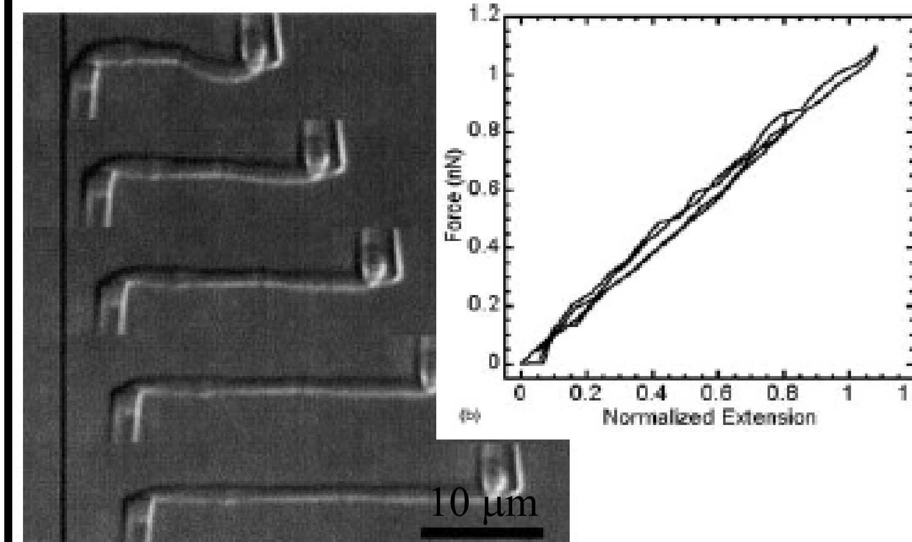


Extracellular Matrix

(the framework that supports cells)

Alberts et al, Molecular Biology of the Cell, 2002

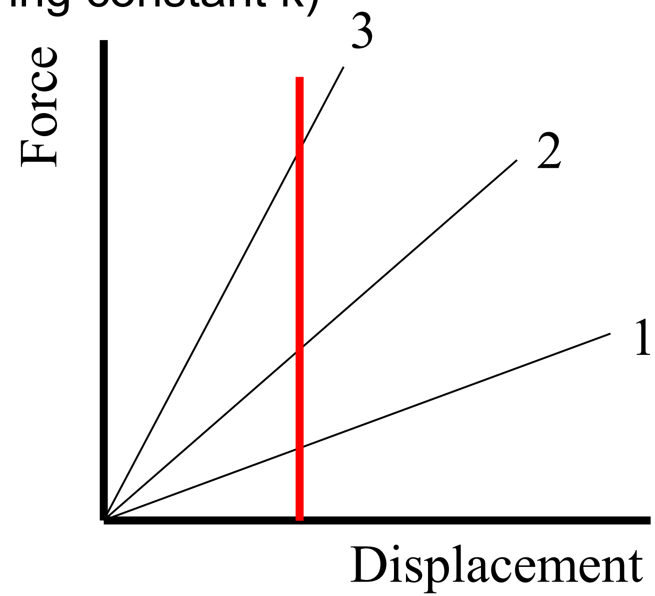
*Newt Chromosome: F=k*x*



Poirier & Marko, *J. Muscle Res. Cell Mot.*, 23:409, 2002

This graph represents the behavior of three springs.
Which one is the "stiffest"? (i.e. has the largest spring constant k)

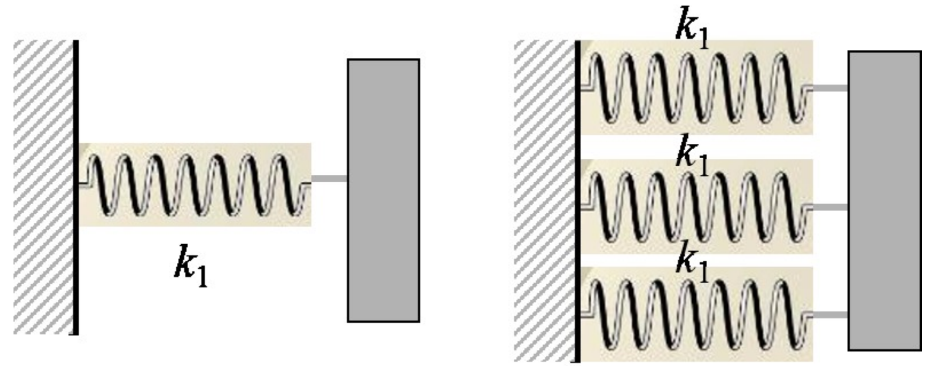
A. 1
B. 2
C. 3
D. All the same



1. "Stiffness" indicates how large a spring constant is.
2. "Stiff" = large k.
3. F = k*x
4. k = F/x
5. The slope of F vs x gives k.
6. Spring 3 has the largest slope, therefore the largest k, and so is the stiffest.

Suppose you attach a mass to three identical springs and stretch them by 1cm. If you attached the same mass to one spring and displaced it by 1cm, how much force would the single spring require in comparison?

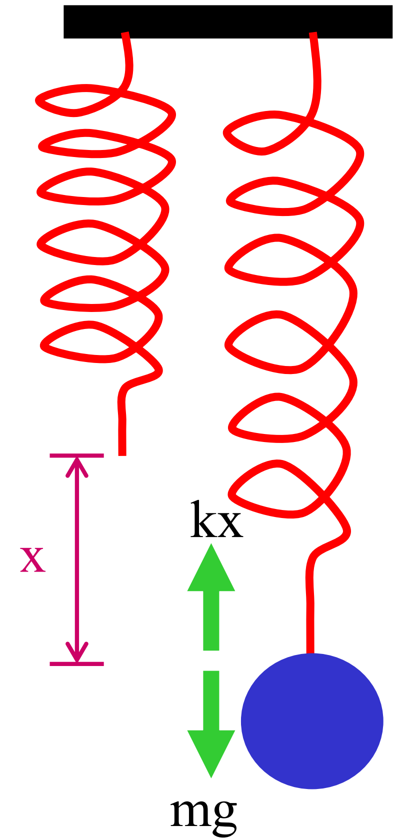
A. Less than the 3-spring system
B. The same as the 3-spring system
C. More than the 3-spring system



1. Spring constant, k = F/x.
2. This applies to each of the 3 springs, since they are being stretched **in parallel**.
3. Therefore, it takes 3X the force to stretch all 3 springs at once (***in parallel***) than it does to stretch one.

Springs: Measuring the spring constant with mass

- A spring will have an intrinsic stiffness, described by the spring constant k.
- Since $F_{applied} = k*x$, we can determine k if we measure x and know $F_{applied}$.
- For a mass hanging from a spring, the applied force is just the force due to gravity
    - $F_{applied} = k*x = F_{gravity} = mg$
- Therefore, the spring constant will be:
    - $k = (mg)/x$

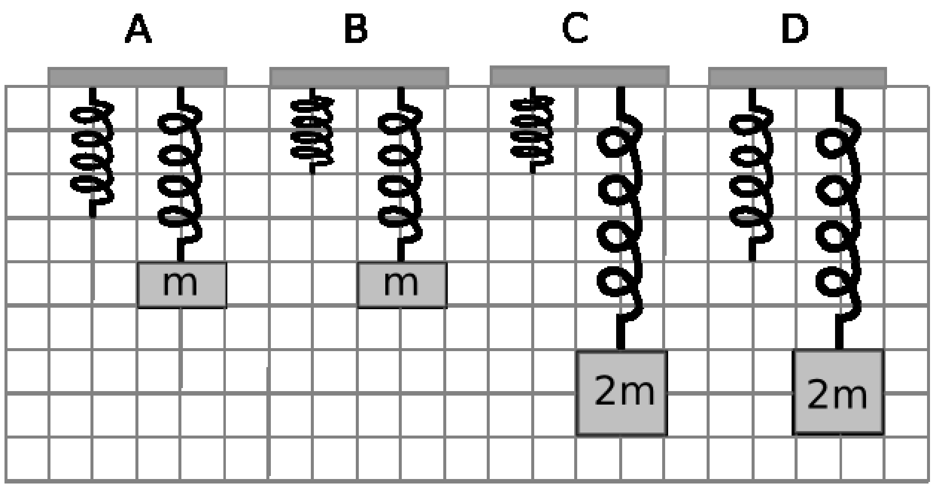


An 0.20kg mass hangs from a spring, stretching it 0.10m from its equilibrium position.
What is the spring constant of the spring?

A. 0.051 N/m
B. 0.50 N/m
C. 2.0 N/m
D. 19.6 N/m

1. $F = kx = mg$
2. $k = (mg)/x$
3. $k = \{(0.20kg)(9.8m/s^2)\}/(0.10m)$
4. $k = 19.6$ N/m

Springs A through D are shown in their unstretched positions and stretched by some hanging masses. Using the grid as a reference for displacement, rank the spring constants.



A. $k_A > k_B > k_C > k_D$

B. $k_A = k_B > k_C = k_D$

C. $k_C = k_D > k_A = k_B$

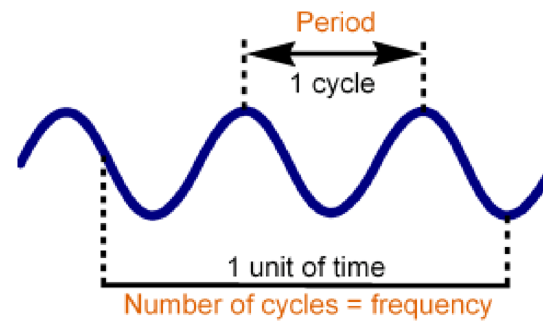
D. $k_A = k_D > k_B = k_C$

E. $k_A > k_B = k_D > k_C$

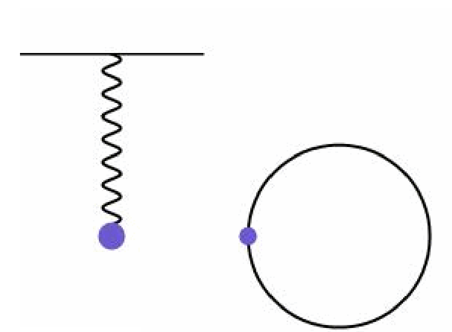
F. $k_B = k_C > k_A = k_D$

1. $F = kx = Mg$
2. $k = (Mg)/x$
3. For a fixed mass, larger displacement, x, means a smaller spring constant, k.
4. For equal displacements but larger applied mass, the spring constant must be larger.
5. Spring A stretches less than spring B for the same applied mass, so $k_A > k_B$.
6. Spring D stretches less than spring C for the same applied mass, so $k_D > k_C$.
7. M/x for A equals M/x for D and M/x for B equals M/x for C, so $k_A = k_D > k_B = k_C$.

Frequency & Period: Two sides of the same coin

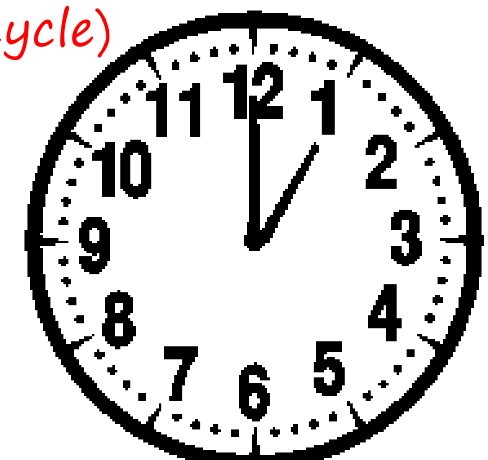


- Frequency, f = cycles per unit time.
  - cycle can be one rotation, or oscillation, or one of whatever it is that is repeating
  - SI units are inverse seconds s^{-1}, also called Hertz: Hz
- Period, T = time for one cycle
  - SI units are seconds: s



- $f = 1/T$
- $T = 1/f$

The second-hand on a clock completes one full circle (cycle) in 60 seconds.
So, $T_{hand} = 60s$.
$f_{hand} = 1/60s = 0.017Hz$.

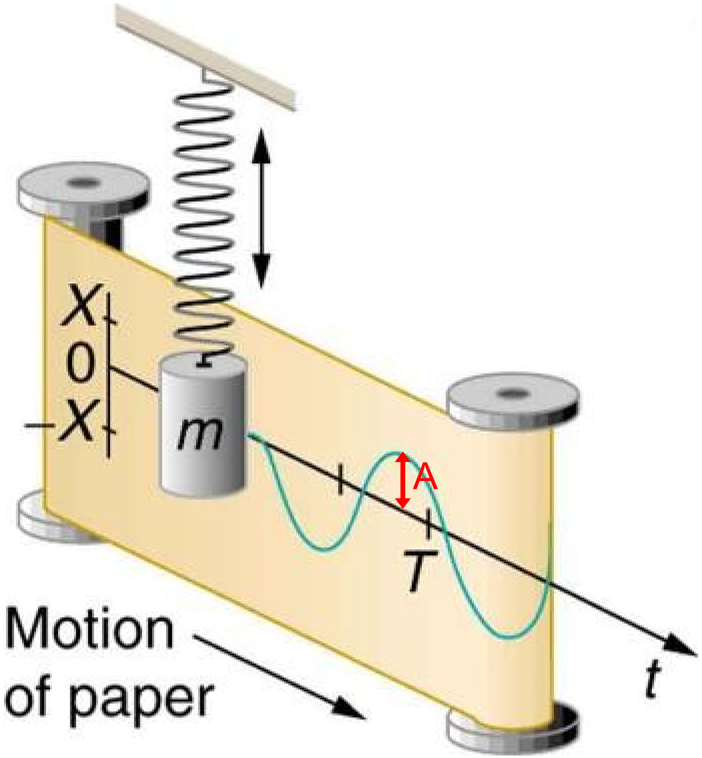


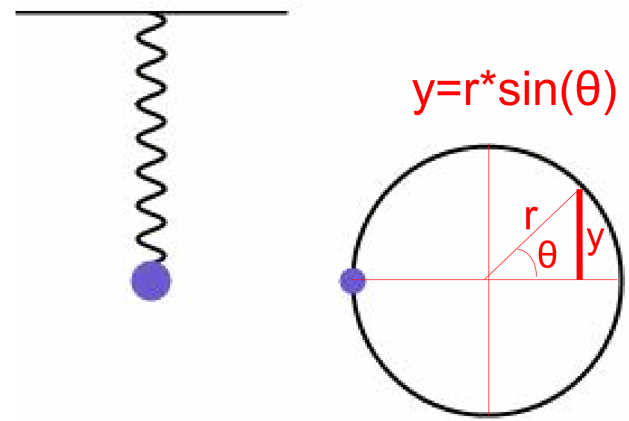
A mass is oscillating up and down on a spring.
It completes 10 complete oscillations in 15 seconds.
What is the frequency of oscillation?

(A) 0.67 Hz (B) 1.5 Hz (C) 10 Hz (D) 15 Hz

1. One oscillation = one cycle.
2. The time for one cycle, the period, T.
3. T = (time)/(# cycles)
4. Frequency, f =1/T
5. $f = 1/T = 1/(15s/10) = 10/(15s) \approx 0.67 s^{-1} = 0.67 Hz$

Oscillations: Displacement vs Time is described by the sinusoid function





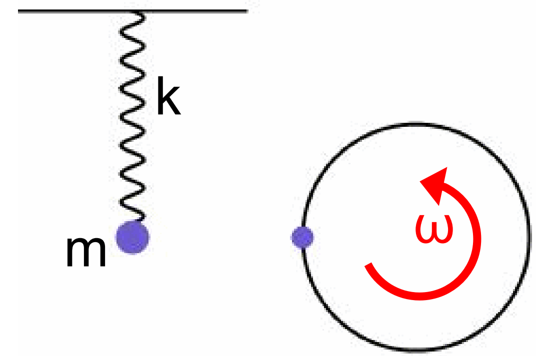
Note that $y/r = \sin(\theta)$.
So how far an oscillator is through one cycle is independent of the amplitude.
I.e. the **oscillation period is independent of the oscillation amplitude.**

The largest displacement during the oscillation is called the ***amplitude***.
The book uses “X” for amplitude … “A” is more common.

Oscillations: Physical properties

📖 Sect 16.3

- How does an oscillator's properties depend on the system that is oscillating?
- Quantities of interest are:
  - How many cycles are completed per second?
    - Frequency: f [unit: Hz]
  - If we map the oscillation into cycles on a circle, how many radians do we trace per second?
    - Angular frequency: ω = 2π*f [unit: rad/s]
  - How long does it take to complete one cycle,
    - Period: T [unit: s]



- With calculus & patience, it can be shown that, for an oscillating spring with spring constant k and attached weight of mass m:

- $\omega_{spring} = \sqrt{\frac{k_{spring}}{m_{weight}}} = \sqrt{\frac{k}{m}}$
- $f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- $T_{spring} = 2\pi\sqrt{\frac{m}{k}}$

*Since f = 1/T = ω/2π ...you really only need to remember one of the above formulas.

*Note: Frequency, angular frequency, and period are independent of amplitude!

A mass is hung from a spring and set into oscillation.
It oscillates with a given frequency f_1.
Now a second identical spring is also attached to the mass (same k, same length).
How does the new frequency, f_2, compare to the old frequency, f_1?

A. $f_2 = 2*f_1$

B. $f_2 = \text{sqrt}(2)*f_1$

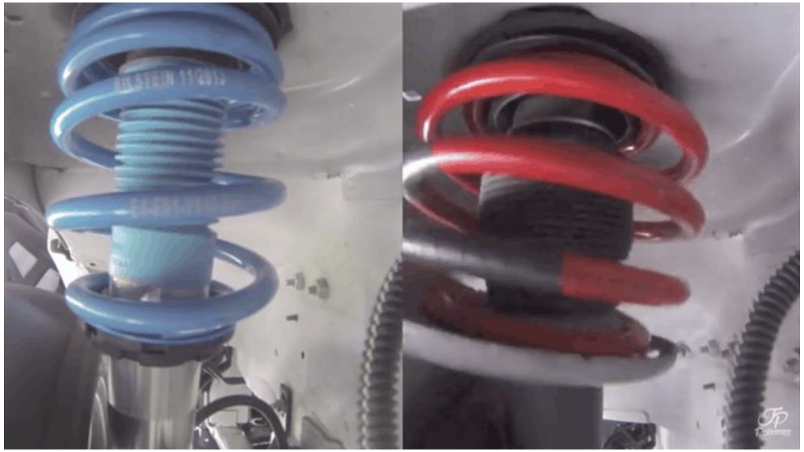
C. $f_2 = f_1$

D. $f_2 = (1/\text{sqrt}(2))*f_1$

E. $f_2 = (1/2)f_1$

A larger spring constant will give a stiffer ride. This is great for handling/responsiveness ...but bad for comfort. So, larger k is good for race cars, but smaller k is good for sedans.

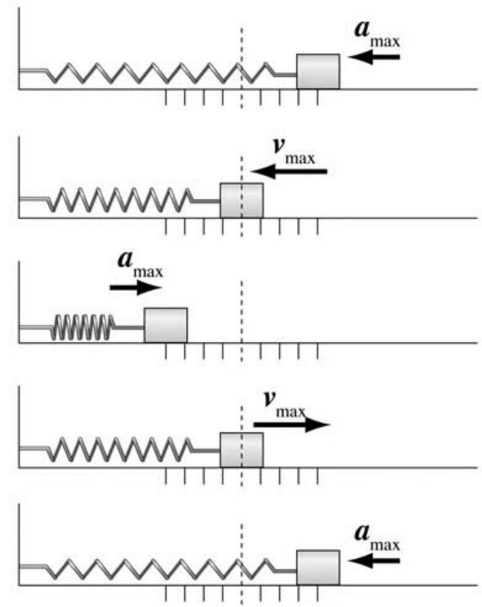
1. The second spring doubles the spring force, effectively doubling the spring constant.
2. $k_2 = 2*k_1$
3. The new spring is stiffer & so will have a greater oscillation frequency.
4. $f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}}$
5. $f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k_1}{m}} = \sqrt{2}f_1$



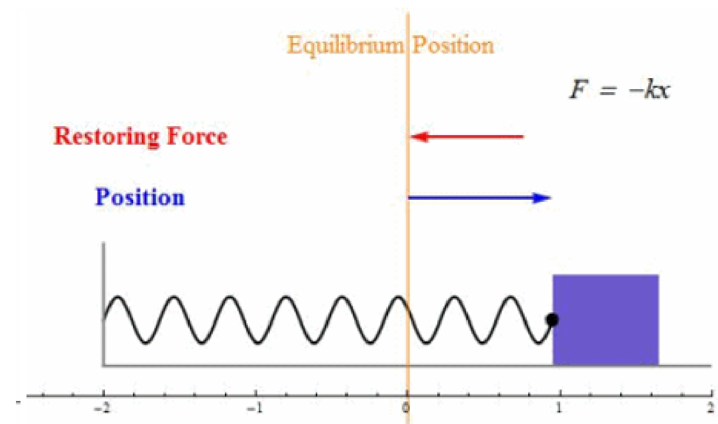
OPTIMUM G

- 0.5 - 1.5 Hz for passenger cars
- 1.5 - 2.0 Hz for sedan racecars and moderate downforce formula cars
- 3.0 - 5.0+ Hz for high downforce racecars

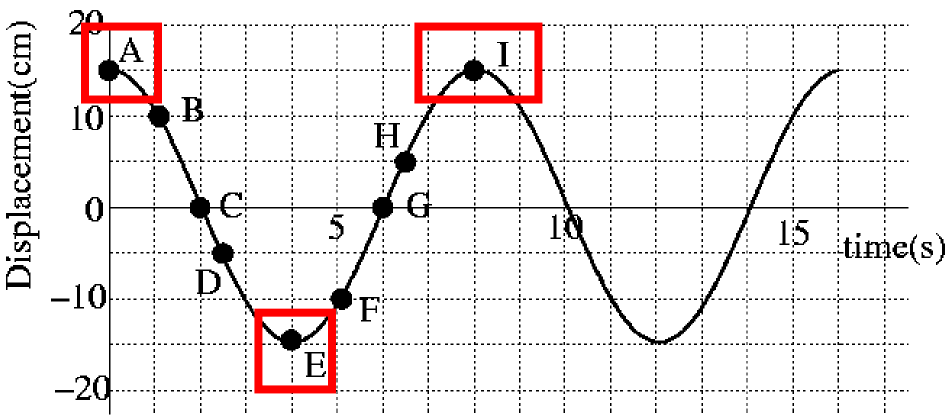
Mass & Spring system: Velocity & Acceleration



- $F_{spring} = -k*x$
- Therefore, force is greatest at place where the displacement is the largest (the amplitude)
- Since, $F = m*a$, $ma = -k*x$,
  - $a = -k*x/m$
  - **acceleration is greatest at the amplitude & velocity is zero at the amplitude**
- The mass accelerates from zero velocity at one amplitude (the extreme point in its motion) to a **maximum velocity at zero displacement** and then experiences large negative acceleration approaching the other extreme.
  - At zero displacement, $x = 0$, so $F = k*x = m*a = 0$. i.e. **acceleration is zero at zero displacement**



This graph represents the displacement of a mass in a mass-spring system. At which point(s) is the magnitude of the force at a maximum?



(A) A

(B) C

(C) E

(D) G

(E) I

(F) A and I

(G) A, E, and I

(H) C and G

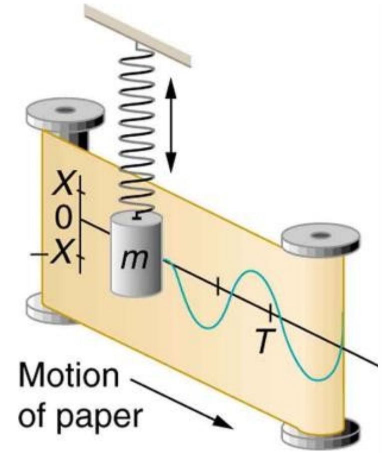
(I) A,C,G, and I

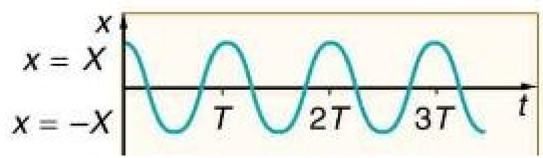
1. $F_{spring} = -k*x$
2. The maximum force is at the maximum displacement (the amplitude).
3. This is at points A, E, & I.

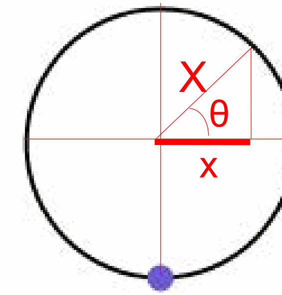
Oscillation Temporal Properties: Analogy to circular motion

- If we start a spring at the position x =X, where X is the positive amplitude, at time t=0
- Then the spring position vs time oscillates at a rate of the angular frequency, $\omega=2\pi*f$
- $x(t) = X*\cos(\omega*t)$
- a.k.a.

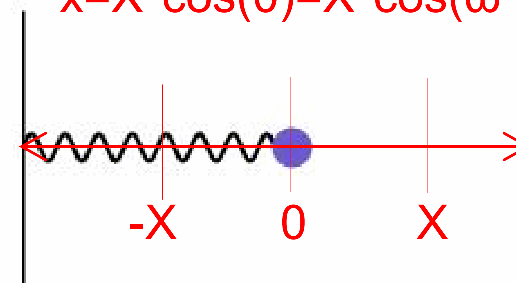
$$x(t) = A*\cos(\omega*t)$$







$x=X*\cos(\theta)=X*\cos(\omega*t)$



- Recall that velocity $v=\Delta x/\Delta t$ and acceleration $a=\Delta v/\Delta t = (\Delta(\Delta x))/(\Delta(\Delta t))$.
- In calculus, these are the 1st and 2nd derivatives of position in time, respectively.
- The 1st derivative of $\cos(\omega*t)$ is $-\omega*\sin(\omega*t)$ and the 2nd derivative is $-\omega^2*\cos(\omega*t)$.
- So, the velocity and acceleration of an oscillator are described by:

$$v(t) = -\omega*A*\sin(\omega*t)$$

$$a(t) = -\omega^2*A*\cos(\omega*t)$$

Formula summary for mass-spring system:

$$\omega_{spring} = \sqrt{\frac{k}{m}}$$

$$f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$

angular frequency = ω = 2πf [SI units: rad/s]
frequency = f = 1/T [SI units: Hz]
period = T = time per oscillation cycle [SI units: s]

T, f, ω are independent of oscillation amplitude!

$$x = A\cos(\omega t)$$

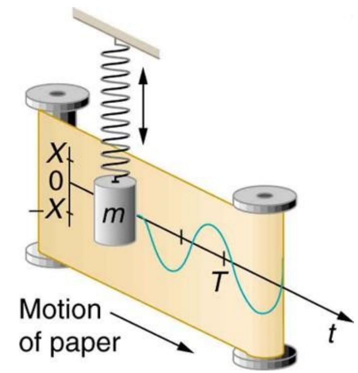
$$x_{max} = A$$

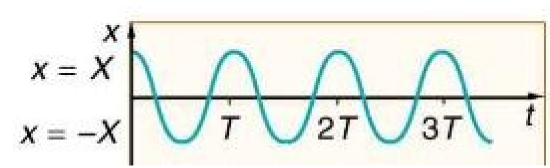
$$v = -A\omega\sin(\omega t)$$

$$v_{max} = A\omega$$

$$a = -A\omega^2\cos(\omega t)$$

$$a_{max} = A\omega^2$$





*Sine and cosine can only take-on values between -1 and 1.