## Tuesday March 26

Topics for this Lecture:

- Simple Harmonic Motion
- Hooke's Law
- Mass \& spring system

Midterm Exam 2 is in 6 days.

## Study!

Me (studied) \& my friend (didinnt study after the PMYS2001 Exam

- Assignment 10 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 226, M\&Tu 6:20-7:10pm
-SI Review: Thurs 6:20-8:10 Morton 115
- Dr. Piccard Review: 6-9pm Walter 245 Sunday March 31
- Office Hours: 204 EAL, 3-4pm Thurs or by appointment (meisel@ohio.edu)

$$
\begin{gathered}
F_{\text {applied }}=k x \\
\bullet \mathrm{~F}_{\text {applied }}=\mathrm{k}^{*} \mathrm{x}=\mathrm{F}_{\text {gravity }}=\mathrm{mg}
\end{gathered}
$$

If you cannot make the regular exam time due to an official university excuse, email me ASAP.

Simple Harmonic Motion
What length pendulum should I choose to make a clock that ticks once per second? ... what about on the moon?

Why did this happen at Tacoma Narrows?
How do you select a spring for a race car vs sedan suspension?


What are the units for the spring constant, $k$ ?
A. $N$
B. $m / N$
C. $N * m$
D. $\mathrm{N} / \mathrm{m}$
E. $\mathrm{kg}^{*} \mathrm{~m}$
F. $\mathrm{kg}^{\star} \mathrm{N}$

1. Spring constant, $\mathrm{k}=\mathrm{F} / \mathrm{x}$. $\mathrm{F}=$ force applied, $\mathrm{x}=$ displacement of spring.
2. SI units for $\mathrm{F}: \mathrm{N}$
3. SI units for $x$ : $m$
4. SI units for $k$ : $N / m$

- The displacement of a stretchy-object (e.g. spring) from it's equilibrium position (where it is when you're not touching it) requires a force linearly proportional a property of the spring, $k$.
- $\mathrm{F}_{\text {applied }}=\mathrm{k}^{*} \mathbf{x}$
- F = applied force
-x = displacement
- $\mathrm{k}=$ spring constant (SI units: $\mathrm{N} / \mathrm{m}$ )
- This is "Hooke's Law"


Cutnell \& Johnson, Physics, $6^{\text {th }}$ ed, Fig. 10.1

* The sign of $x$ depends on the direction of displacement! You choose which way is $+x$ \& the other way is $-x$.

Springs: Applied Force \& Restoring Force

- The force you apply to stretch
(or compress) a spring is

Unstrained length
of the spring balanced by a "restoring force", a.k.a. the spring force.

$$
\begin{aligned}
& -F_{\text {applied }}=-F_{\text {spring }}=k^{*} x \\
& -F_{\text {spring }}=-k^{*} x
\end{aligned}
$$

Springs: Anything that stretches \& tries to return to equilibrium!

- Hooke's law applies to anything with "elasticity"
- I.e. objects which can stretch \& tend to return to the position from which they were stretched
- E.g. Springs, rubber bands, taught guitar strings, bridges in the wind, etc.
- Complicated solids held together by molecular bonds are well represented by coupled spring systems.


This graph represents the behavior of three springs.
Which one is the "stiffest"? (i.e. has the largest spring constant k)
A. 1
B. 2

| C. 3 |
| :--- |
| D. All the same |



1. "Stiffness" indicates how large a spring constant is.
2. "Stiff" = large k.
3. $F=k^{*} x$
4. $k=F / x$
5. The slope of $F$ vs $x$ gives $k$.
6. Spring 3 has the largest slope, therefore the largest $k$, and so is the stiffest.

Suppose you attach a mass to three identical springs and stretch them by 1 cm . If you attached the same mass to one spring and displaced it by 1 cm , how much force would the single spring require in comparison?
A. Less than the 3-spring system
B. The same as the 3-spring system
C. More than the 3-spring system


1. Spring constant, $\mathrm{k}=\mathrm{F} / \mathrm{x}$.
2. This applies to each of the 3 springs, since they are being stretched in parallel.
3. Therefore, it takes $3 X$ the force to stretch all 3 springs at once (in parallel) than it does to stretch one.

Springs: Measuring the spring constant with mass

- A spring will have an intrinsic stiffness, described by the spring constant k .
- Since $F_{\text {applied }}=k^{*} x$, we can determine $k$ if we measure x and know $\mathrm{F}_{\text {applied }}$.
- For a mass hanging from a spring, the applied force is just the force due to gravity
- $F_{\text {applied }}=k^{\star} x=F_{\text {gravity }}=m g$
-Therefore, the spring constant will be:
- $k=(m g) / x$


An 0.20kg mass hangs from a spring, stretching it 0.10 m from its equilibrium position. What is the spring constant of the spring?

A. $0.051 \mathrm{~N} / \mathrm{m}$<br>B. $0.50 \mathrm{~N} / \mathrm{m}$<br>C. $2.0 \mathrm{~N} / \mathrm{m}$<br>D. $19.6 \mathrm{~N} / \mathrm{m}$

1. $F=k x=m g$
2. $\mathrm{k}=(\mathrm{mg}) / \mathrm{x}$
3. $\mathrm{k}=\left\{(0.20 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\right\} /(0.10 \mathrm{~m})$
4. $k=19.6 \mathrm{~N} / \mathrm{m}$

Springs A through D are shown in their unstretched positions and stretched by some hanging masses. Using the grid as a reference for displacement, rank the spring constants.

A. $\mathrm{k}_{\mathrm{A}}>\mathrm{k}_{\mathrm{B}}>\mathrm{k}_{\mathrm{C}}>\mathrm{k}_{\mathrm{D}}$
B. $\mathrm{k}_{\mathrm{A}}=\mathrm{k}_{\mathrm{B}}>\mathrm{k}_{\mathrm{C}}=\mathrm{k}_{\mathrm{D}}$
C. $k_{C}=k_{D}>k_{A}=k_{B}$
D. $k_{A}=k_{D}>k_{B}=k_{C}$
E. $k_{A}>k_{B}=k_{D}>k_{C}$
F. $k_{B}=k_{C}>k_{A}=k_{D}$

1. $\mathrm{F}=\mathrm{kx}=\mathrm{Mg}$
2. $\mathrm{k}=(\mathrm{Mg}) / \mathrm{x}$
3. For a fixed mass, larger displacement, $x$, means a smaller spring constant, $k$.
4. For equal displacements but larger applied mass, the spring constant must be larger.
5. Spring A stretches less than spring $B$ for the same applied mass, so $k_{A}>k_{B}$.
6. Spring $D$ stretches less than spring $C$ for the same applied mass, so $k_{D}>k_{C}$.
7. $M / x$ for $A$ equals $M / x$ for $D$ and $M / x$ for $B$ equals $M / x$ for $C$, so $k_{A}=k_{D}>k_{B}=k_{C}$.

All of the content preceding this point is fair game for midterm 2

## Quick review of material for midterm 2



Advice for studying:

1. Review notes
2. Try typical practice problems
(a couple per topic). You can use
TopHat to review problems from class.
3. Look at relevant notes section \& read part of book related to problems you struggled on
4. Repeat
5. Try practice exam (\& then see 1).

Time yourself and grade yourself for an honest assessment.

## Torque:



- Torque: $\tau=r \times F=|F||r| \sin (\theta) \quad$ - Two ways to think about this:
(1) Perpendicular distance:

$\tau=\mathrm{F}^{*} \mathrm{r}_{\perp}=\mathrm{F}^{*}\left[\mathrm{~d}^{*} \cos (\theta)\right]$
(2) Perpendicular force:

$\tau=\mathrm{F}_{\perp} \mathrm{d}=[\mathrm{F} * \cos (\theta)]^{*} \mathrm{~d}$

1. $F_{t}=m^{*} a_{t}$
2. $F_{t}=m\left(r^{\star} \alpha\right)$
3. $\left(F_{t}^{*} r\right)=m^{*} r^{\star} r^{*} \alpha$
4. $\tau=\left(m^{\star} r^{2}\right) \alpha$
5. $\tau=I^{*} \alpha$

Torque: Force applied to a lever some perpendicular distance from an axis

- Simplest case, Force perpendicular to a lever arm:

Greek letter 'tau'
都


Axis

- $\tau=F^{*} r_{\perp}$ where $r_{\perp}$ is the distance from the 'axis of rotation'
- Torque is always with respect to an axis of rotation (in fact, it's direction is along that axis of rotation). If it's a static problem, you can choose the axis of rotation.
- Units: $\mathrm{N}^{*} \mathrm{~m}$ (enter this way in LON-CAPA)

A 1500 N beam is attached to a wall by a hinge. The beam is 4.0 m long, and the mass of the beam is distributed evenly along the beam. The beam is supported by a cable that is attached 1.0 m to the beam from the wall and makes an angle of $35^{\circ}$ from the vertical.

What is the tension in the cable?

(A) 1500 N
(B) 2460 N
(C) 3000 N
(D) 3660 N
(E) 5230 N
(F) 6000 N

1. Choose the hinge as the axis of rotation, so that we can ignore its force, and then balance the torques about there.
2. $\sum \tau=\tau_{\text {cable }}-\tau_{\text {gravity }}=0$
3. $\tau_{\text {cable }}=\tau_{\text {gravity }}=\mathrm{m}_{\text {board }} \mathrm{F}^{*}\left(\mathrm{~L}_{\text {board }} / 2\right)=(1500 \mathrm{~N})(2 \mathrm{~m})=3000 \mathrm{Nm}$
4. $\tau_{\text {cable }}=F_{\text {cable }, \perp} r_{\text {cable }}=F_{\text {cable }} \cos (\theta) r_{\text {cable }}$
5. $\mathrm{F}_{\text {cable }}=\tau_{\text {cable }} /\left[\mathrm{r}_{\text {cable }} \cos (\theta)\right]=(3000 \mathrm{Nm}) /\left(1 \mathrm{~m}^{*} \cos \left(35^{\circ}\right)\right) \approx 3660 \mathrm{~N}$

## Energy

- Kinetic Energy $=K E=\frac{1}{2} m v^{2}$
- Potential energy $=P E=m g h$
- Work:
- From a force: $\mathrm{W}=\mathrm{F}^{\star} \mathrm{d}^{\star} \cos (\theta)$
- From change in energy:
- $W_{\text {net }}=K E_{\text {final }}-K E_{\text {initial }}$
- $W_{\text {net }}=P E_{\text {initial }}-P E_{\text {final }}$
- Energy conservation:
- $\left(K_{f}+P E_{f}\right)=\left(K E_{i}+P E_{i}\right)+W_{N C}$
- ...if $\mathrm{W}_{\mathrm{NC}}=0$, then $\left(\mathrm{KE}_{\mathrm{f}}+P E_{\mathrm{f}}\right)=\left(\mathrm{KE}_{\mathrm{i}}+P E_{\mathrm{i}}\right)$

An 80-kg stunt-person starts at rest and slides down a roof, flies through the air, and lands on a large pad, which compresses 1.2 m in order to bring the stuntperson to a stop. Assume it is an icy day and the roof is frictionless. Ignore air resistance. What is the average force on the stunt-person due to the pad? (Hint: pick the top of the landing pad as the $\mathrm{h}=0$ reference level.)

(A) 3400 N
(B) 3140 N
(C) 4080 N
(D) 0 N
(E) 940 N
(F) 2350 N

$$
\left(\mathrm{KE}_{\mathrm{f}}+P E_{\mathrm{f}}\right)=\left(\mathrm{KE}_{\mathrm{i}}+P E_{\mathrm{i}}\right)+\mathrm{W}_{\mathrm{NC}}
$$

$$
\begin{aligned}
& \text { 1. }\left(\mathrm{KE}_{F}+P E_{F}\right)=\left(\mathrm{KE}_{0}+P E_{0}\right)+W_{N C} \\
& \text { 2. }(0+(-1.2) \mathrm{mg})=(0+4 \mathrm{mg})+\mathrm{F}^{\star} \mathrm{d}^{\star} \cos \left(180^{\circ}\right) \\
& \text { 3. }-1.2 \mathrm{mg}=4 \mathrm{mg}+\mathrm{F}^{\star}(1.2)^{\star}(-1) \\
& \text { 4. } 1.2 \mathrm{~F}=5.2 \mathrm{mg} \\
& \text { 5. } F=3400 \mathrm{~N}
\end{aligned}
$$

|  | KE | PE | $\mathrm{E}_{\mathrm{TOT}}$ |
| :--- | :--- | :--- | :--- |
| Top | 0 | $\mathrm{mg}(4 \mathrm{~m})$ | 4 mg |
| Top of <br> Pad | $1 / 2 \mathrm{mv}^{2}$ | 0 | $1 / 2 \mathrm{mv}^{2}$ |
| Rest | 0 | $\mathrm{mg}(-1.2 \mathrm{~m})$ | -1.2 mg |

An 0.2 kg ball is dropped from a height of 3.0 m above the floor. What is the speed of the ball just before it hits the floor? (ignore air resistance)

$$
\mathrm{h}=3 \mathrm{~m}
$$

(A) $3 \mathrm{~m} / \mathrm{s}$
(B) $9.8 \mathrm{~m} / \mathrm{s}$
(C) $5.9 \mathrm{~m} / \mathrm{s}$
(D) $7.7 \mathrm{~m} / \mathrm{s}$

- Could use free-fall equations...
...but using energy is easier.
- Know mass, final \& initial heights, \& initial velocity. Therefore, know:
- $P E_{i}=\mathrm{mgh}_{\mathrm{i}}=(0.2 \mathrm{~kg})^{\star}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)^{\star}(3 \mathrm{~m})=5.88 \mathrm{~J}$
- $\mathrm{PE}_{\mathrm{f}}=\mathrm{mgh}_{\mathrm{f}}=\mathrm{m}^{\star} \mathrm{g}^{\star}(0 \mathrm{~m})=0 \mathrm{~J}$
- $\mathrm{KE}_{\mathrm{i}}=(1 / 2) \mathrm{m} v_{i}^{2}=(1 / 2) \mathrm{m}(0 \mathrm{~m} / \mathrm{s})^{2}=0 \mathrm{~J}$
- $\mathrm{E}_{\mathrm{f}}=\mathrm{E}_{\mathrm{i}}+\mathrm{W}_{\mathrm{NC}}$
- No non-conservative forces, so $\mathrm{W}_{\mathrm{NC}}=0 \ldots$. so, $\mathrm{E}_{\mathrm{f}}=\mathrm{E}_{\mathrm{i}}$
- $P E_{i}+K E_{i}=P E_{f}+K E_{f}$
- $\mathrm{KE}_{\mathrm{f}}=5.88 \mathrm{~J}=(1 / 2) \mathrm{m} v_{f}^{2}$
- $v_{f}=\sqrt{2 K E / m}=\sqrt{2(5.88 J) /(0.2 \mathrm{~kg})}=\sqrt{2\left(5.88 \mathrm{kgm}^{2} \mathrm{~s}^{-2}\right) /(0.2 \mathrm{~kg})}=7.7 \mathrm{~m} / \mathrm{s}$


## Momentum

$$
m \vec{v}=\vec{p}
$$


(a) Before

(c) After

Momentum is a vector and is conserved:

$$
\sum p_{x, i}=\sum p_{x, f} \quad \sum p_{y, i}=\sum p_{y, f}
$$



$$
\begin{gathered}
\sum \vec{F} \Delta t=\Delta \vec{p} \\
\text { Impulse }=\overrightarrow{\mathrm{J}}=\Delta \vec{p}
\end{gathered}
$$

A 2000kg railroad car is traveling at 3m/s along a track.
A 5000kg railroad car is travelling on the same track, but going in the opposite direction at $1 \mathrm{~m} / \mathrm{s}$.
The two railroad cars collide and stick together.
What is the speed and direction of the smashed-together railroad cars?


1. Momentum is conserved: $p_{\text {final }}=p_{\text {initial }}$
2. $p_{\text {initial }}=m_{1} v_{1}+m_{2} v_{2}=(2000 \mathrm{~kg})(3 \mathrm{~m} / \mathrm{s})-(5000 \mathrm{~kg})(1 \mathrm{~m} / \mathrm{s})=1,000 \mathrm{~kg} * \mathrm{~m} / \mathrm{s}$
3. They stick together after the collision: $\mathrm{m}_{\text {final }}=2000 \mathrm{~kg}+5000 \mathrm{~kg}=7000 \mathrm{~kg}$
4. $p_{f}=m_{\text {final }} v_{\text {final }}=p_{\text {initial }}=1,000 \mathrm{~kg} * \mathrm{~m} / \mathrm{s}$
5. $v_{\text {final }}=p_{\text {final }} / m_{\text {final }}=\left(1,000 \mathrm{~kg}{ }^{\star} \mathrm{m} / \mathrm{s}\right) /(7000 \mathrm{~kg})=0.14 \mathrm{~m} / \mathrm{s}$ to the right

## A 1.60 kg ball is attached to a 1.20 m -long wire,

 held horizontally, and dropped.It strikes a 2.40 kg block that is sitting on a horizontal, frictionless surface. Air resistance is negligible and the collision is elastic. What is the velocity of the ball just before the collision?
(A) $4.33 \mathrm{~m} / \mathrm{s}$ (B) $18.8 \mathrm{~m} / \mathrm{s}$
(C) $23.5 \mathrm{~m} / \mathrm{s}$
(D) $4.85 \mathrm{~m} / \mathrm{s}$
(E) $3.96 \mathrm{~m} / \mathrm{s}$


1. Total energy is conserved: $K E_{i}+P E_{i}=K E_{f}+P E_{f}$
2. Initial state: just before ball is released
3. At rest: $\mathrm{KE}_{\mathrm{i}}=0$.
4. $\mathrm{PE}_{\mathrm{i}}=\mathrm{mgh}$
5. Final: just before ball hits block
6. At reference height $h=0: P E_{f}=0$
7. $\mathrm{KE}_{\mathrm{f}}=(1 / 2) m v_{f}^{2}$
8. $\mathrm{KE}_{\mathrm{f}}=(1 / 2) m v_{f}^{2}=P E_{\mathrm{i}}=\mathrm{mgh}$
9. $\mathrm{v}_{\mathrm{f}}=\sqrt{\frac{2}{m} m g h}=\sqrt{2 g h}=\sqrt{2\left(9.80 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(1.20 \mathrm{~m})}=4.85 \mathrm{~m} / \mathrm{s}$

## Rotational Motion

$\cdot \theta=s / r$
$\cdot \omega=\Delta \theta / \Delta t$

- $V_{t}=r^{*} \omega$
- $\alpha=\Delta \omega / \Delta t$
- $\pi \mathrm{rad}=180^{\circ}$
- $\mathrm{a}_{\mathrm{c}}=\mathrm{v}_{\mathrm{t}}^{2} / r=\mathrm{r}^{*} \omega^{2}$
- $\mathrm{F}_{\mathrm{c}}=\mathrm{mv}^{2} / \mathrm{r}$
- $F_{g}=G \frac{m_{1} m_{2}}{r^{2}}$

Equations for kinematics in 1D \& Newton's Laws apply to rotational motion as well, by subsituting the appropriate quantities:

| Linear Quantity |  |  | Corresponding Rotational Quantity |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Quantity | Variable | SI units | Quantity | Variable | SI units |
| length | x | m | angle | $\theta=\mathrm{s} / \mathrm{r}$ | rad |
| velocity | $\mathrm{v}=\Delta \mathrm{x} / \Delta \mathrm{t}$ | $\mathrm{m} / \mathrm{s}$ | angular velocity | $\omega=\Delta \theta / \Delta \mathrm{t}$ | $\mathrm{rad} / \mathrm{s}$ |
| acceleration | $\mathrm{a}=\Delta \mathrm{v} / \Delta \mathrm{t}$ | $\mathrm{m} / \mathrm{s}^{2}$ | angular acceleration | $\alpha=\Delta \omega / \Delta \mathrm{t}$ | $\mathrm{rad} / \mathrm{s}^{2}$ |
| mass | m | kg | moment of inertia |  |  |

- I = moment of inertia
- $\mathrm{I}_{\mathrm{POINT}}=m \mathrm{r}^{2}$

You are swinging a 0.20 kg tennis ball on a 1.20 m -long string, where the axis of rotation is parallel to the surface of the earth.
At the minimum speed, what is the tension in the string at the bottom of the circle (position 7) ?

(A) 0.00 N
(B) 0.98 N
(C) 1.96 N
(D) 3.92 N

1. Both gravity \& the tension are supplying centripetal force.
2. Gravity is pulling outward, tension is pulling inward (in the direction of the net centripetal force).
3. $\mathrm{F}_{\mathrm{c}}=\mathrm{T}-\mathrm{F}_{\mathrm{g}}$
4. $T=F_{c}+F_{g}=m v_{t}^{2} / r+m g$
5. $T=\left\{(0.20 \mathrm{~kg})(3.43 \mathrm{~m} / \mathrm{s})^{2}\right\} /(1.2 \mathrm{~m})+(0.20 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$
$6 . \mathrm{T}=1.96 \mathrm{~N}+1.96 \mathrm{~N}=3.92 \mathrm{~N}$

An 800 kg car travels a distance of 20 m around a turn with a radius of curvature of 40 m at a speed of $25 \mathrm{~m} / \mathrm{s}$ in a time of 0.80 s . What is the centripetal acceleration of the car?
(A) $12.5 \mathrm{~m} / \mathrm{s}^{2}$
(B) $15.6 \mathrm{~m} / \mathrm{s}^{2}$
(C) $25.0 \mathrm{~m} / \mathrm{s}^{2}$
(D) $12500 \mathrm{~m} / \mathrm{s}^{2}$

1. $a_{c}=v_{t}^{2} / r$
2. $a_{c}=\left\{(25 \mathrm{~m} / \mathrm{s})^{2}\right\} /(40 \mathrm{~m})$
3. $a_{c}=\left(625 \mathrm{~m}^{2} / \mathrm{s}^{2}\right) /(40 \mathrm{~m}) \approx 15.6 \mathrm{~m} / \mathrm{s}^{2}$


## Spring Force

- $F_{\text {applied }}=-F_{\text {spring }}=k^{\star} X$
- $F_{\text {spring }}=-k^{*} X$

Unstrained length


- 111111
-MMN:
- $F_{\text {applied }}=k^{*} x=F_{\text {gravity }}=m g$


Springs A through D are shown in their unstretched positions and stretched by some hanging masses. Using the grid as a reference for displacement, rank the spring constants.

A. $k_{A}>k_{B}>k_{C}>k_{D}$
B. $k_{A}=k_{B}>k_{C}=k_{D}$
C. $k_{C}=k_{D}>k_{A}=k_{B}$
D. $k_{A}=k_{D}>k_{B}=k_{C}$
E. $k_{A}>k_{B}=k_{D}>k_{C}$
F. $k_{B}=k_{C}>k_{A}=k_{D}$

1. $F=k x=M g$
2. $k=(M g) / x$
3. For a fixed mass, larger displacement, $x$, means a smaller spring constant, $k$.
4. For equal displacements but larger applied mass, the spring constant must be larger.
5. Spring A stretches less than spring $B$ for the same applied mass, so $k_{A}>k_{B}$.
6. Spring $D$ stretches less than spring $C$ for the same applied mass, so $k_{D}>k_{C}$.
7. $M / x$ for $A$ equals $M / x$ for $D$ and $M / x$ for $B$ equals $M / x$ for $C$, so $k_{A}=k_{D}>k_{B}=k_{C}$.
