

Thursday March 23

Topics for this Lecture:

- *Review of Section 2*
- *Torque*
- *Energy*
- *Momentum/Collisions*
- *Circular Motion*

**PHYS2001 GREATEST
HITS**

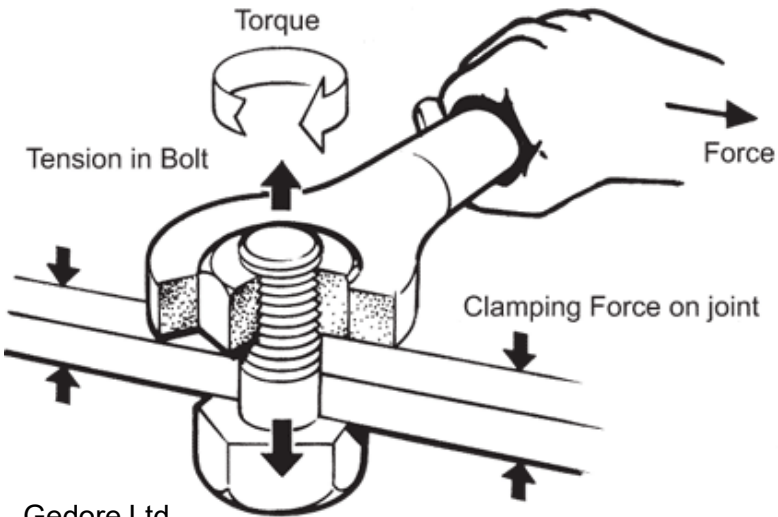


VOLUME II

- Assignment 10 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed or by appointment (meisel@ohio.edu)
- **Midterm 2: Monday March 27th**
7:15-9:15pm Morton Hall Room 201

- *Advice for studying:*
 1. *Review notes*
 2. *Try typical practice problems (a couple per topic...like this lecture)*
 3. *Read part of book related to problems you struggled on & look at relevant notes section*
 4. *Repeat*
 5. *Try practice exam (& then see 1)*

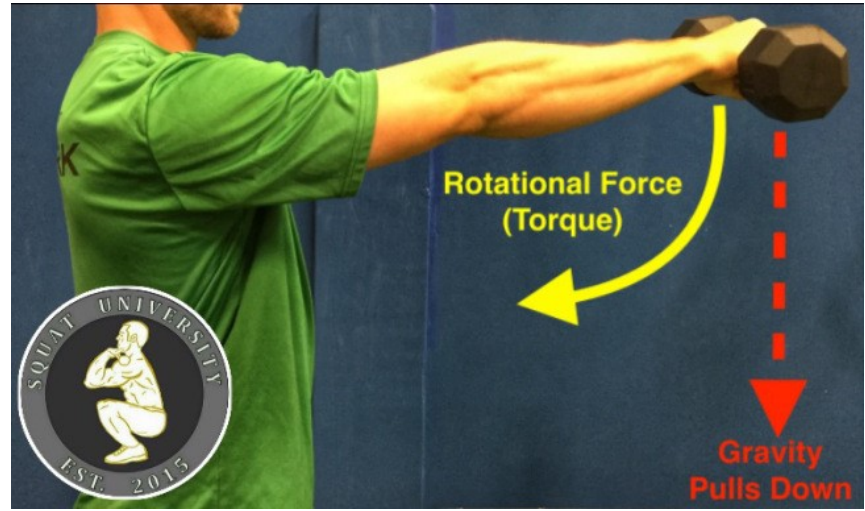
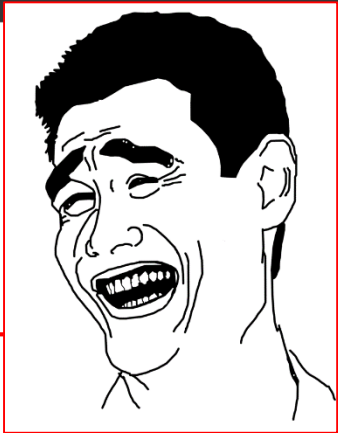
Torque:



Gedore Ltd.

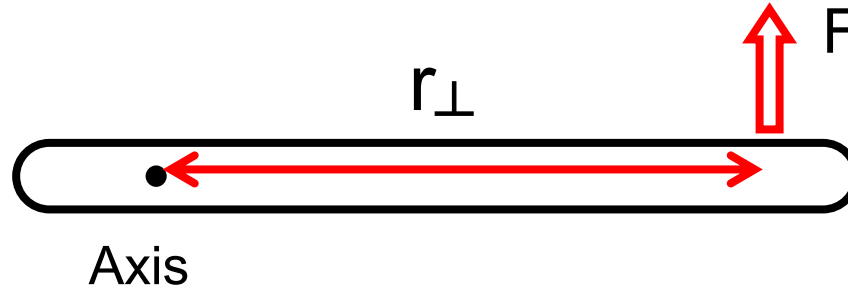
TORQUE: The Biggest Weightlifting Secret You've Probably Never Heard Of

Volt HQ / Christye Estes



Torque: Force applied to a lever some perpendicular distance from an axis

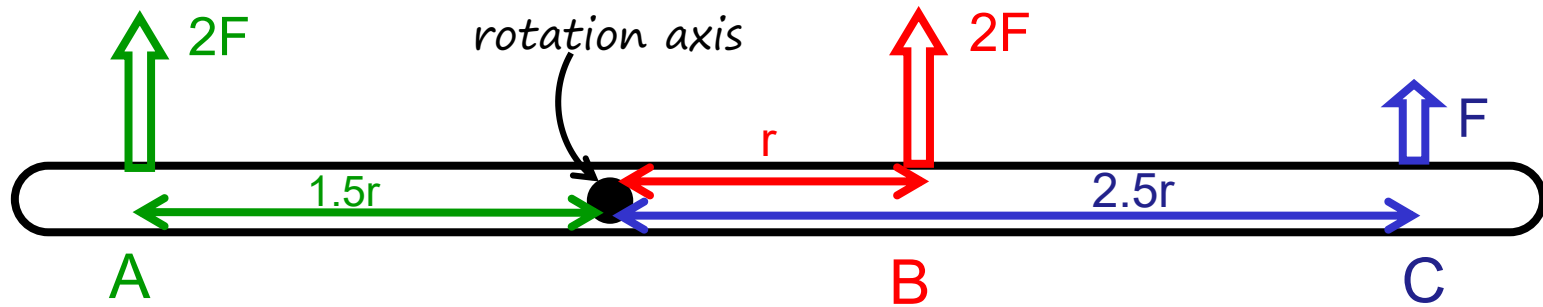
- Simplest case, Force perpendicular to a lever arm:



Greek letter 'tau'

- $\tau = F \cdot r_{\perp}$ where r_{\perp} is the distance from the 'axis of rotation'
 - Torque is always with respect to an axis of rotation (in fact, it's direction is along that axis of rotation).
If it's a static problem, you can choose the axis of rotation.
 - Units: $N \cdot m$ [enter this way in LON-CAPA)

Which of the following forces provides the largest magnitude for torque about the axis (indicated by the solid circle)?



(A) A

(B) B

(C) C

(D) All same

- All forces are perpendicular to lever arm, so
 - $\tau = r \times F = |F||r| \sin(\theta) = |F||r| \sin(90^\circ) = |F||r|$
- Just need to compare product of lever arm length & force:
 - $|\tau_A| = 2F \cdot 1.5r = 3F \cdot r$ τ_A : Clockwise (CW): (-)
 - $|\tau_B| = 2F \cdot r = 2F \cdot r$ τ_B : Counterclockwise (CCW): (+)
 - $|\tau_C| = F \cdot 2.5r = 2.5F \cdot r$ τ_C : CCW: (+)

Torque: Force applied to a lever some perpendicular distance from an axis

- Torque is a “cross product” between the lever arm length r and the force applied F
- Mathematically speaking,

- $\tau = r \times F$

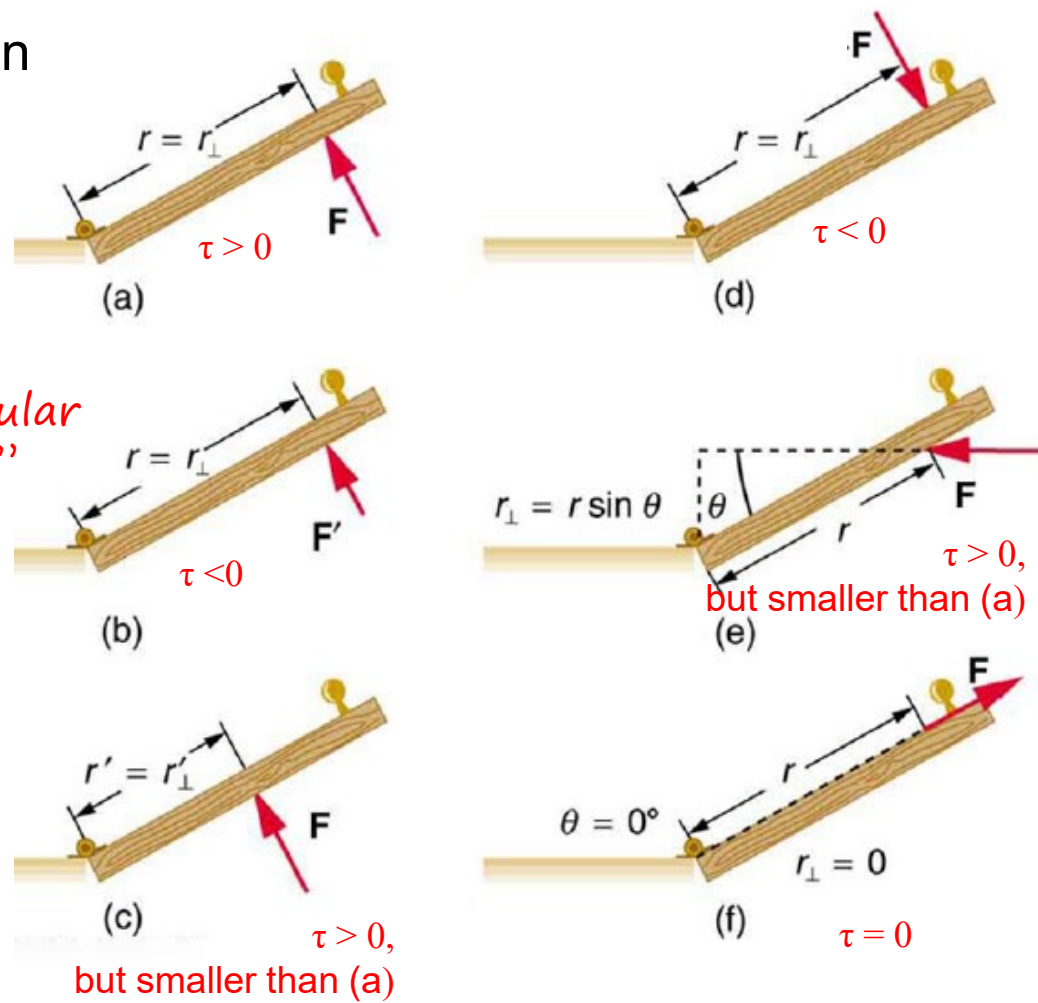
- Here, this means,

$$\tau = |F| |r| \sin(\theta) \text{ “perpendicular distance”}$$

- Often need trig to find the perpendicular distance.

- The sign convention is (due to the cross-product):

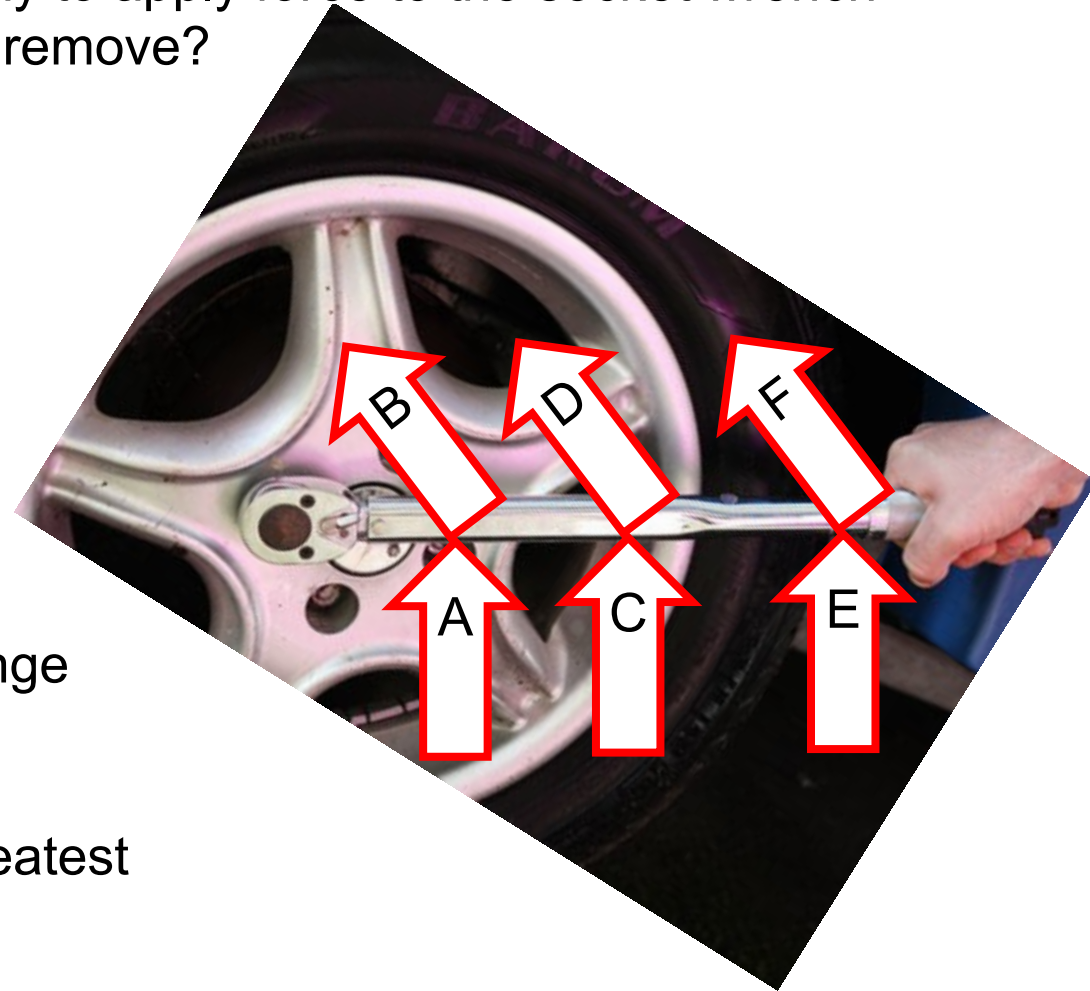
- + for counter-clockwise
- - for clockwise



You just got some sweet new rims for your car and so you need to take off the old ones.

Which arrow indicates the best way to apply force to the socket wrench so that the lug nuts are easiest to remove?

- (A) A (G) A & B
(B) B (H) C & D
(C) C (I) E & F
(D) D
(E) E
(F) F

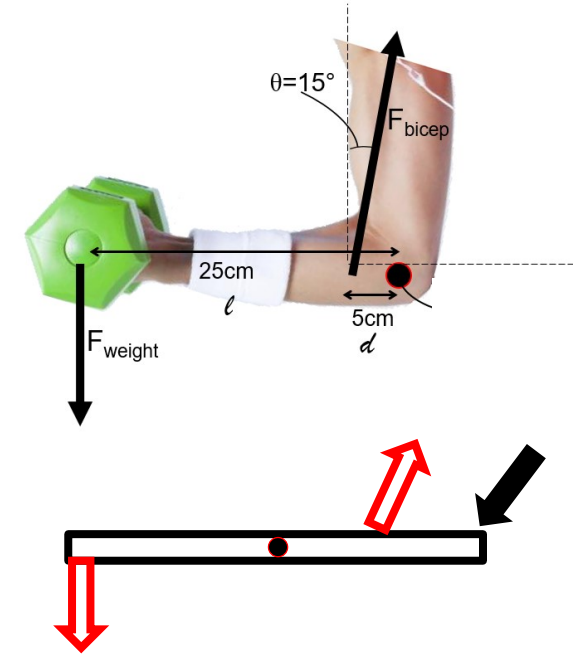


- Want to *maximize* **perpendicular distance** from hinge (in this case the lug nut).
- “E” has the largest perpendicular distance, so the torque will be greatest for a force applied there.

Tips for solving static equilibrium problems

no translational or rotational motion

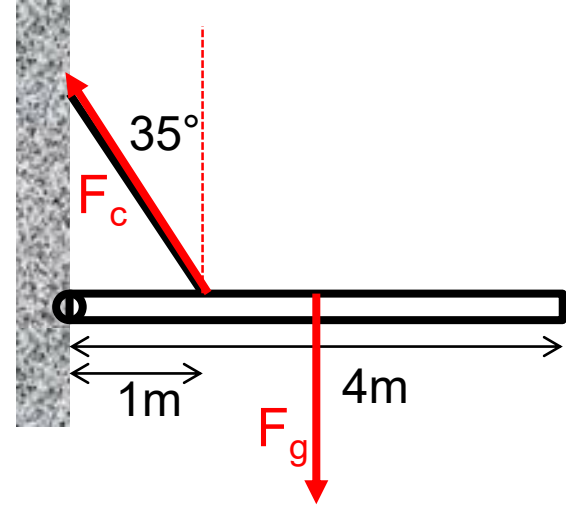
1. Draw a simplified version of your situation, including the forces in a free-body diagram
2. Select your axis of rotation
 - For static equilibrium, there is no rotation, so the rotation axis can be anywhere.
 - Choose your rotation axis so that a force you do not know provides zero torque.
3. Write expressions for the balanced forces & torques
 1. $\sum F_x = 0$
 2. $\sum F_y = 0$
 3. $\sum \tau = 0$
4. Work through the algebra & arithmetic



A 1500N beam is attached to a wall by a hinge.

The beam is 4.0m long, and the mass of the beam is distributed evenly along the beam.

The beam is supported by a cable that is attached 1.0m to the beam from the wall and makes an angle of 35° from the vertical.



What is the tension in the cable?

- (A) 1500N (B) 2460N (C) 3000 N **(D) 3660 N** (E) 5230 N (F) 6000N

1. Choose the hinge as the axis of rotation, so that we can ignore its force, and then balance the torques about there.

$$2. \quad \sum \tau = \tau_{\text{cable}} - \tau_{\text{gravity}} = 0$$

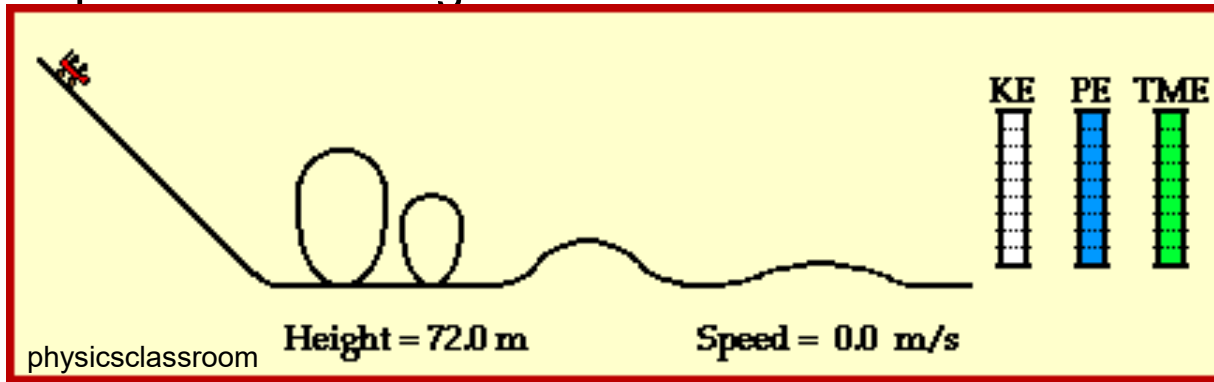
$$3. \quad \tau_{\text{cable}} = \tau_{\text{gravity}} = m_{\text{board}} g (L_{\text{board}}/2) = (1500\text{N})(2\text{m}) = 3000\text{Nm}$$

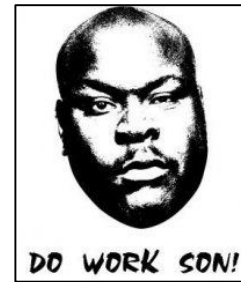
$$4. \quad \tau_{\text{cable}} = F_{\text{cable}, \perp} r_{\text{cable}} = F_{\text{cable}} \cos(\theta) r_{\text{cable}}$$

$$5. \quad F_{\text{cable}} = \tau_{\text{cable}} / [r_{\text{cable}} \cos(\theta)] = (3000\text{Nm}) / (1\text{m} * \cos(35^\circ)) \approx 3660\text{N}$$

Energy & Work

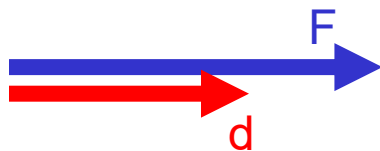
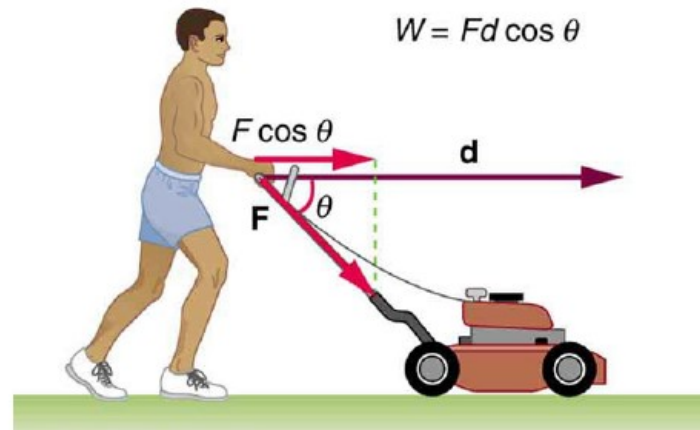
- Energy is a useful way to look at the world
 - Often can ignore time information
 - Energy is conserved, except when taken away by ‘non-conservative forces’ (which we’ll cover later)
- Energy due to motion: Kinetic Energy (KE) [Units: Joules (J)]
- Stored energy (e.g. due to gravity): Potential Energy (PE) [Units: Joules (J)]
- Can convert between KE & PE
- Changes in energy can lead to Work (W) [Units: Joules (J)]
 - Work is also defined as the product of the force applied along some axis and the displacement along that axis



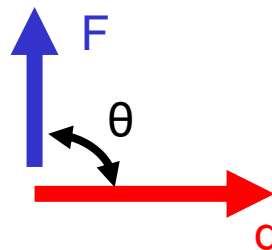


Work: Work done by a force or forces

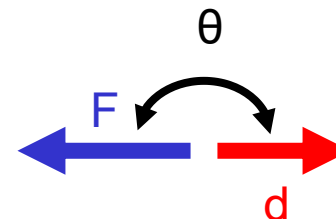
- Work done by a force or group of forces
 - Work = (mag. of force) (mag. of displacement) (cos θ)
 - $W = F \cdot d \cdot \cos(\theta)$
- Units of Work and Energy: Joules (J)
- Sign: θ angle between force and displacement
 - same direction (+)
 - opposite direction (-)
 - perpendicular (0)



$\theta = 0^\circ$
W positive



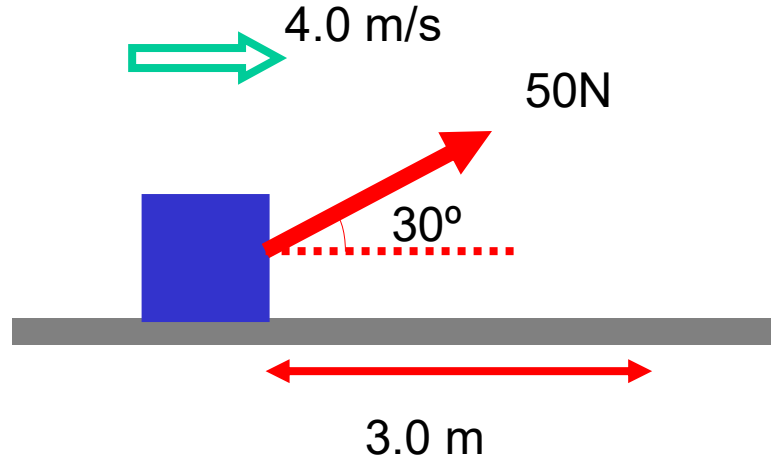
$\theta = 90^\circ$
W zero



$\theta = 180^\circ$
W negative

A person is pulling a crate with a force of 50N (as shown) over a distance of 3.0m at a constant speed of 4.0 m/s. What is the work done on the object by the person?

- A. 16.7J
- B. 75 J
- C. 130 J
- D. 150 J

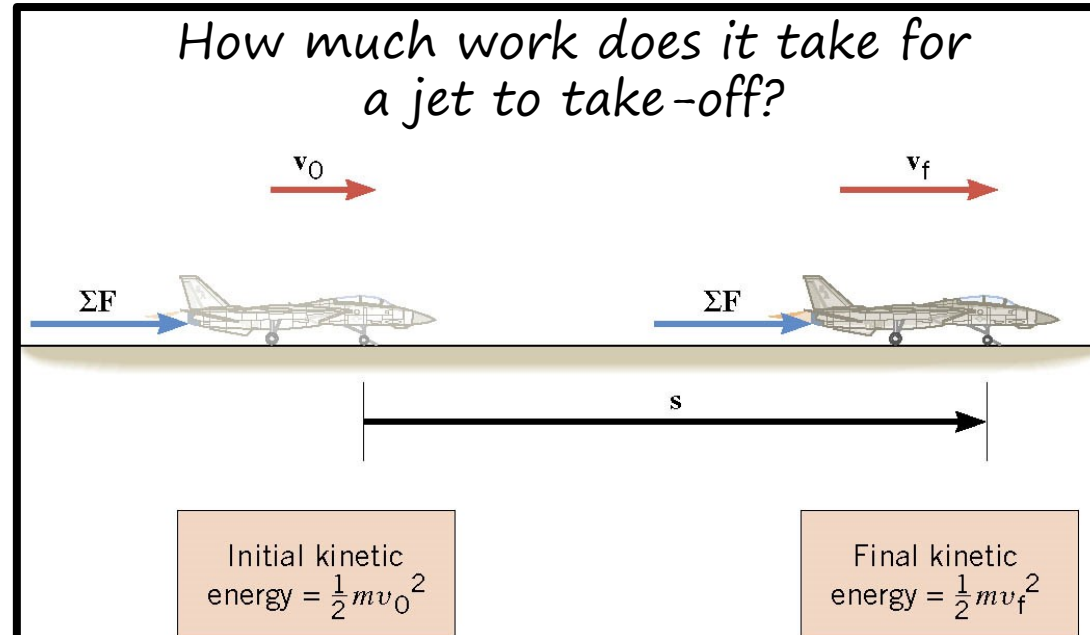


1. $W = F \cdot d \cdot \cos(\theta)$
2. $W = (50\text{N})(3.0\text{m})\cos(30^\circ) = 130 \text{ J}$

$$W = F \cdot d \cdot \cos(\theta)$$
$$1\text{J} = 1\text{kg} \cdot \text{m}^2/\text{s}^2$$

Work: Net work from changes in kinetic energy

1. $W_{net} = F_{net} \cdot d = ma \cdot d$
2. From 1D-kinematics, recall: $v_f^2 = v_i^2 + 2a(x - x_0) = 2ad$
 1. So: $2ad = v_f^2 - v_i^2$
3. We are free to multiply both sides of (2.1) by mass, m , and divide by 2
 1. $mad = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
4. Then, we recall that $W_{net} = F_{net} \cdot d = ma \cdot d$
 1. $W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
5. Kinetic Energy = $KE = \frac{1}{2}mv^2$
 1. $W_{net} = KE_{final} - KE_{initial}$
 2. $W_{net} = \Delta KE$



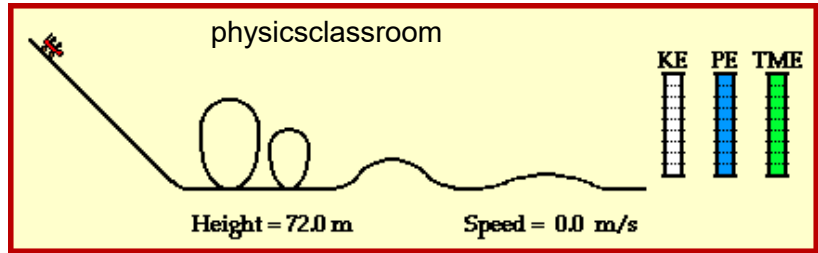
Work: Work done by gravity, considering **potential** energy

- We established: $W_g = m \cdot g \cdot h$
- But, keep in mind, here "h" = $y_f - y_i$
- So: $W_g = mgy_i - mgy_f = mgh_i - mgh_f$
- Gravitational potential energy: $PE = mgh$
- $W_g = PE_i - PE_f = -\Delta PE$

"h" is positive, because, in our derivation, $a = +g$, so lower heights are larger y.

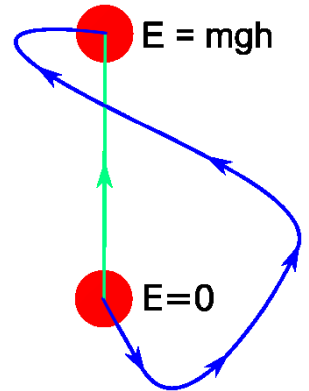
Must choose a reference level for h to calculate Δy and get ΔPE .

- Gravity can be used to store energy:
 - Raise object = convert energy **into** PE
 - Lower object = convert energy **from** PE



- If we were to lower an object & raise it back to the same spot: $W_g = 0$

- The work done by gravity is independent of the path taken!
 - Therefore, it is a "**Conservative Force**".

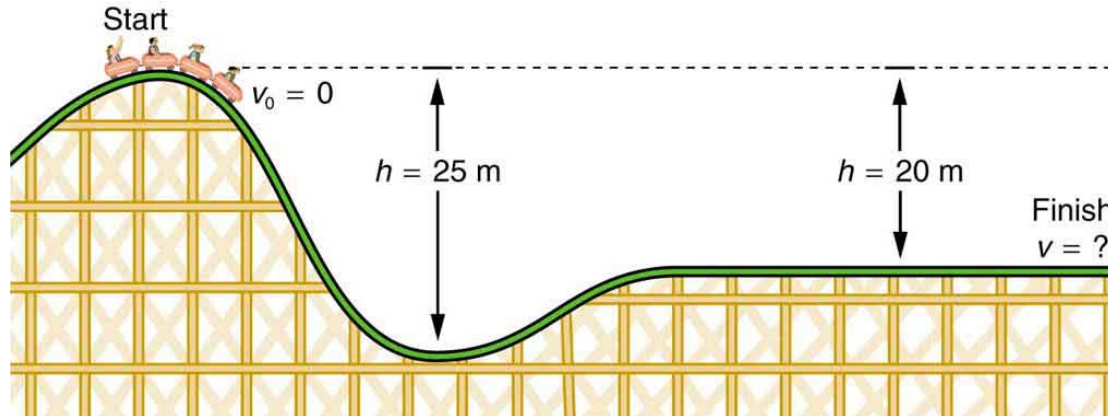


Conservation of Energy

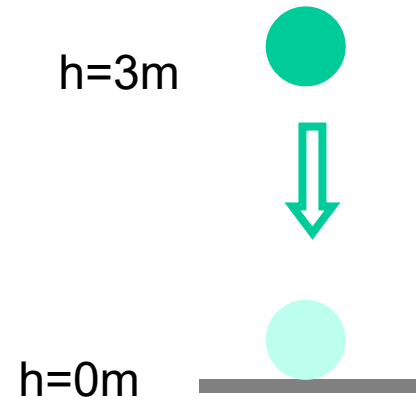
Total energy of a system stays the same **unless** there is energy being added or taken away by non-conservative forces.

$$(KE_f + PE_f) = (KE_i + PE_i) + W_{NC}$$

If $W_{NC} = 0$, then the total mechanical energy (TME a.k.a. E) stays the same. Energy may change between types, but the total stays same.



An 0.2kg ball is dropped from a height of 3.0m above the floor.
 What is the speed of the ball just before it hits the floor?
 (ignore air resistance)



- (A) 3m/s (B) 9.8m/s (C) 5.9m/s **(D) 7.7m/s**

- Could use free-fall equations...
 ...but using energy is easier.
- Know mass, final & initial heights, & initial velocity.

Therefore, know:

- $PE_i = mgh_i = (0.2\text{kg}) \cdot (9.8\text{m/s}^2) \cdot (3\text{m}) = 5.88\text{J}$
- $PE_f = mgh_f = m \cdot g \cdot (0\text{m}) = 0\text{J}$
- $KE_i = (1/2)mv_i^2 = (1/2)m(0\text{m/s})^2 = 0\text{J}$

- $E_f = E_i + W_{NC}$
 - No non-conservative forces, so $W_{NC} = 0$ so, $E_f = E_i$
 - $PE_i + KE_i = PE_f + KE_f$

- $KE_f = 5.88\text{J} = (1/2)mv_f^2$

- $v_f = \sqrt{2KE_f/m} = \sqrt{2(5.88\text{J})/(0.2\text{kg})} = \sqrt{2(5.88\text{kgm}^2\text{s}^{-2})/(0.2\text{kg})} = 7.7\text{m/s}$

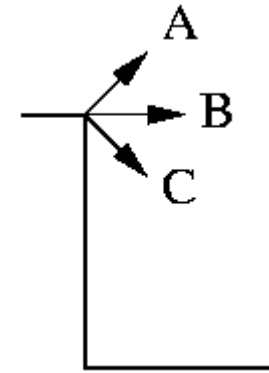
3 balls of the same mass are thrown from a cliff, all with a speed of 25m/s.

A is thrown upward at an angle of 45° .

B is thrown horizontally.

C is thrown downward at an angle of 45° .

Which one is traveling fastest just before it hits the ground?



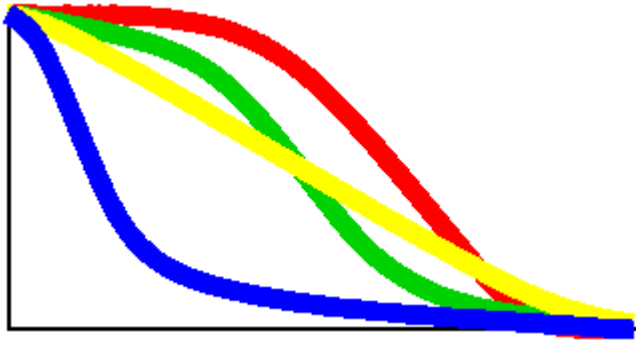
- (A) A (B) B (C) C **(D) All have the same speed**

1. No non-conservative forces, so $E_f = E_i$
2. $E_i = mgh + (1/2)mv^2$
 1. Same mass & same initial height
 2. "Same speed"; i.e. same v
 3. ...So same E_i
3. $E_f = KE_f + PE_f$
 1. $PE_f = mgh = mg*(0m) = 0J$
 2. $E_f = KE_f$
4. $KE_f = (1/2)mv^2 = E_i$, which is the same for all
5. Since m is the same, v_f must therefore be the same

Four identical balls roll off four tracks.

The tracks are of the same height but different shapes, as shown.

For which track is the ball moving the fastest when it leaves the track?



A. Yellow

B. Red

C. Green

D. Blue

E. All the same

1. No non-conservative forces, so $E_f = E_i$

2. $E_i = mgh + (1/2)mv^2$

1. Same mass, same initial height, all starting from rest ($KE_i=0$)

2. So same E_i

3. $E_f = KE_f + PE_f$

1. $PE_f = mgh = mg*(0m) = 0J$

2. $E_f = KE_f$

4. $KE_f = (1/2)mv^2 = E_i$, which is the same for all (b/c all leaving at same height)

5. Since m is the same, v_f must therefore be the same

Power

- Rate at which work is done ... which is the same as:
- Rate at which energy is expended
 - Units: J/s \equiv watt (W)
 - E.g. A 60W lightbulb uses 60J of energy every second
 - *not the same as “watt equivalent”

$$P = \frac{\text{Work}}{\text{Time}}$$

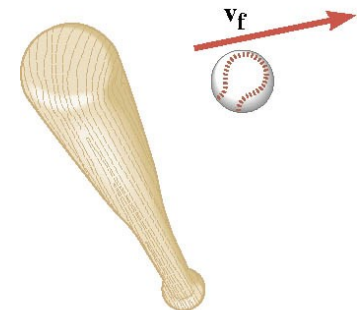
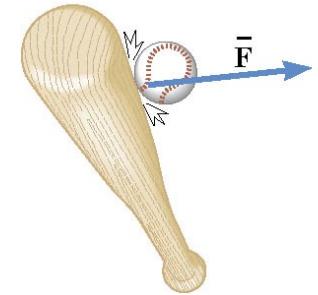
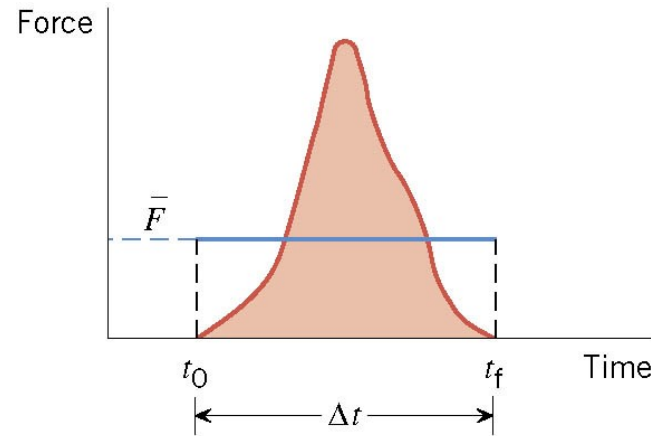
$$P = \frac{\text{Change in Energy}}{\text{Time}}$$

- 1 Mechanical Horsepower \approx 746W



Force applied over time: “Impulse”

How do we describe the change in velocity of a baseball hit by a bat?



- Impulse: Integral of $F(t)$, i.e. area under the F vs t graph

$$\vec{J} = \int F(t)dt = \sum \vec{F}\Delta t$$

- Impulse describes the change in direction and is therefore a vector quantity
- Impulse: Change in momentum

$$\vec{J} = \vec{F}\Delta t = m\vec{a}\Delta t = m\frac{\Delta\vec{v}}{\Delta t}\Delta t = m\Delta v = m\vec{v}_f - m\vec{v}_i = \Delta\vec{p}$$

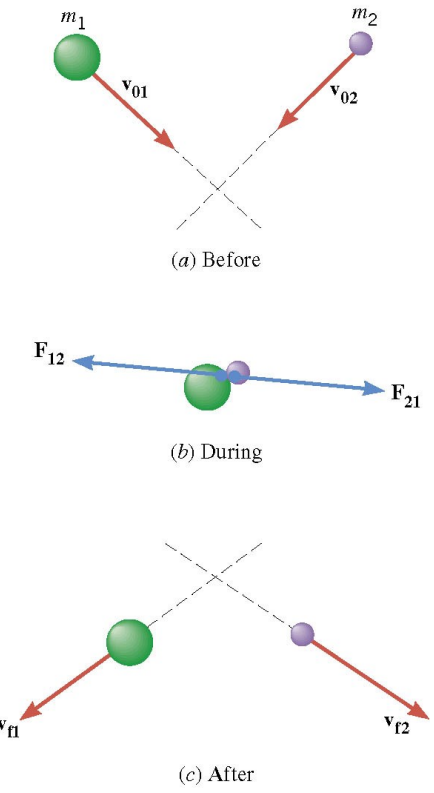
- *Impulse in a given direction only affects momentum in that direction

Collisions

1. Consider two spheres colliding.
2. When they impact (i.e. are touching) the force of sphere 1 on sphere 2 is equal & opposite to the force of sphere 2 on sphere 1
 - $F_{12} = -F_{21}$



For every action, there is an equal and opposite reaction.



3. Since the contact time is the same,
 - $|\vec{J}_{12}| = |\vec{F}_{12}|\Delta t = |\vec{F}_{21}|\Delta t = |\vec{J}_{21}|$
 - The impulse on each is the same magnitude, but in opposite directions
4. Therefore, each sphere has an equal change in momentum, but in the opposite direction
 - $\vec{p}_{1,f} = \vec{p}_{1,i} + \Delta\vec{p}$
 - $\vec{p}_{2,f} = \vec{p}_{2,i} + \Delta\vec{p}$
 - Because of directionality, $\Delta\vec{p}$ will increase $\vec{p}_{1,f}$ or $\vec{p}_{2,f}$ and decrease the other ...so the total momentum will be unchanged!
5. Therefore, **momentum is conserved** [i.e. Total momentum stays the same]

An old cannon with a mass of 600kg is sitting on a frozen lake. The cannon fires a 5kg cannon ball at a speed of 300m/s to the right, where the cannon starts at rest.

What is the velocity of the cannon after it has been fired?

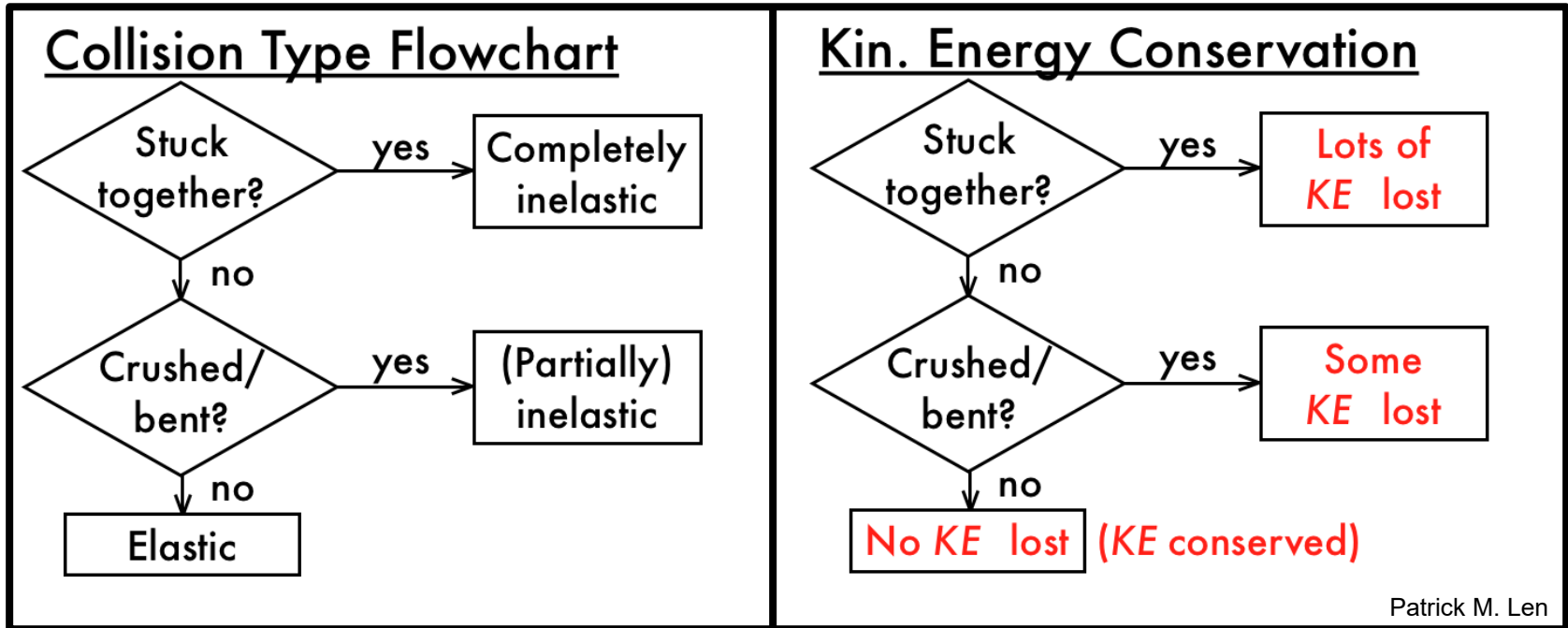


- A. 0.5 m/s to the left
- B. 0.5 m/s to the right
- C. 2.5 m/s to the left
- D. 2.5 m/s to the right
- E. 300 m/s to the left
- F. 5.0 m/s to the left

1. Momentum is conserved: $p_{\text{final}} = p_{\text{initial}}$
2. Initially, everything is at rest: $p_{\text{initial}} = 0$
3. $p_{\text{final}} = 0 = p_{c,f} + p_{b,f}$
4. $p_{c,f} = -p_{b,f}$
5. $m_c v_{c,f} = -m_b v_{b,f}$
6. $v_{c,f} = -(5\text{kg} \cdot 300\text{m/s}) / 600\text{kg} = -2.5\text{m/s}$

Collision Classification

- **All** collisions conserve momentum.
- Collisions that conserve kinetic energy KE are called: “*elastic*” *e.g. billiards*
- Collisions that **do not** conserve KE are called: “*inelastic*” *e.g. car crash*
- When colliding objects stick together, their collision is: “perfectly inelastic” *e.g. mosquito on windshield*

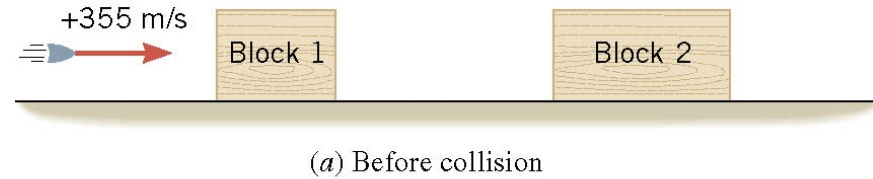


A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

What is the final velocity of the second block?

- (A) 0.00 m/s
- (B) 0.550 m/s
- (C) 1.42 m/s
- (D) 355 m/s
- (E) 0.920 m/s
- (F) 0.513 m/s



$$m_{\text{block 1}} = 1150 \text{ g}$$

$$m_{\text{block 2}} = 1530 \text{ g}$$

$$m_{\text{bullet}} = 4.00 \text{ g}$$

$$1. \quad p_{\text{initial}} = p_{\text{final}}$$

$$2. \quad p_{\text{bullet},i} = p_{\text{bullet},f} + p_{\text{block2},f} + p_{\text{block1},f}$$

$$3. \quad m_{\text{bullet}} v_{\text{bullet},i} = (m_{\text{bullet}} + m_{\text{block2}}) v_f + m_{\text{block1}} v_{\text{block1},f}$$

$$4. \quad 1.42 \text{ kgm/s} = (1.534 \text{ kg}) v_f + 0.632 \text{ kgm/s}$$

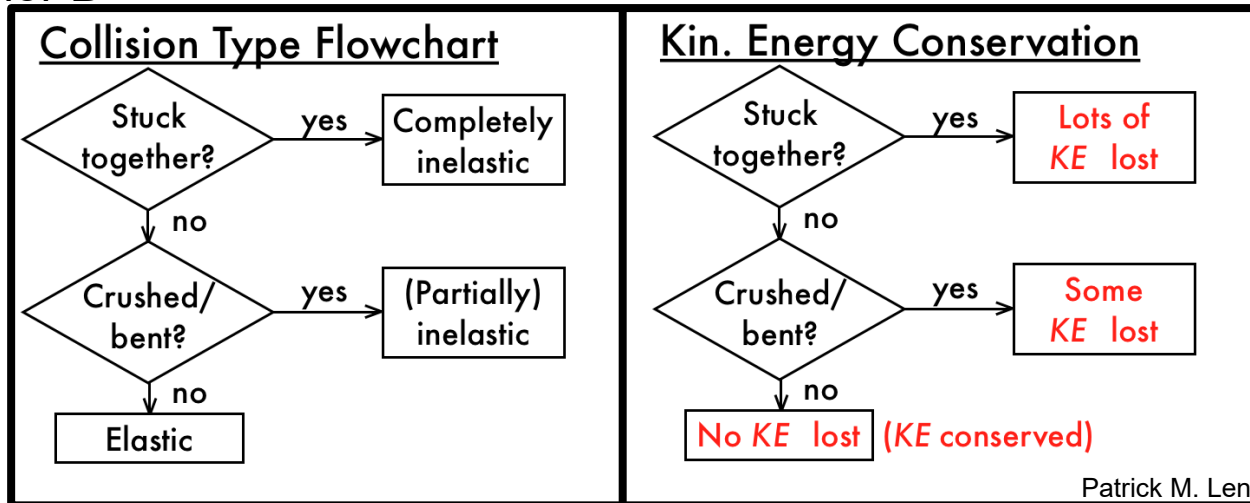
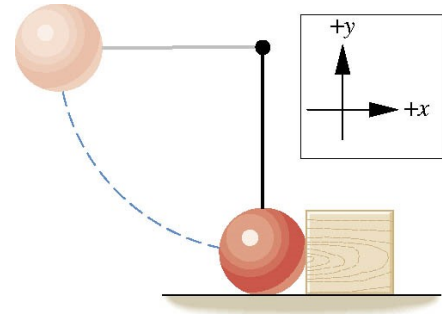
$$5. \quad v_f = (1.42 \text{ kgm/s} - 0.632 \text{ kgm/s}) / (1.534 \text{ kg})$$

$$6. \quad v_f = 0.513 \text{ m/s}$$

A ball is attached to a wire, held horizontally, and dropped. It strikes a block that is sitting on a horizontal, frictionless surface. Air resistance is negligible and the **collision is elastic**. The block is more massive than the ball.

Which of the following are conserved **during the collision**?

- (A) Horizontal component of momentum for ball+block system
 (B) Total KE of ball+block system
 (C) Both A & B
 (D) Neither A nor B



Circular Motion

Why do hurricanes swirl?



How do you simulate weightlessness on earth?



How fast do I have to spin my car tires to achieve a given linear speed?



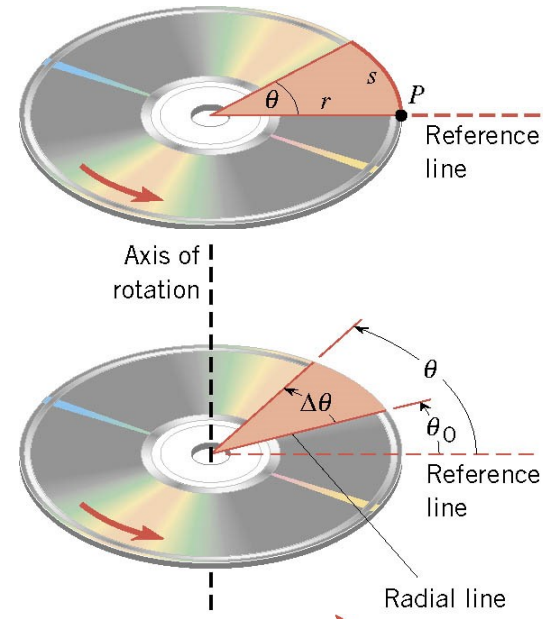
How do satellites stay in orbit?



Relationship between Rotational & Linear Kinematics

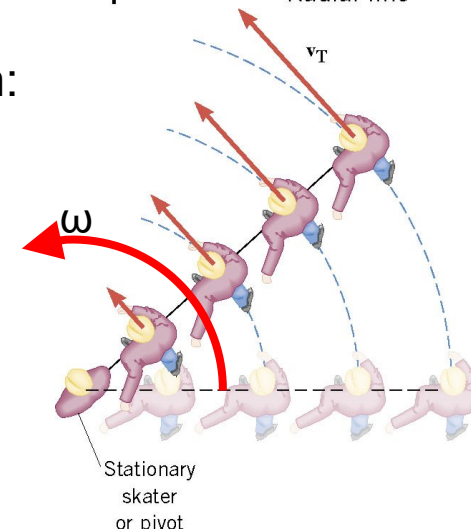
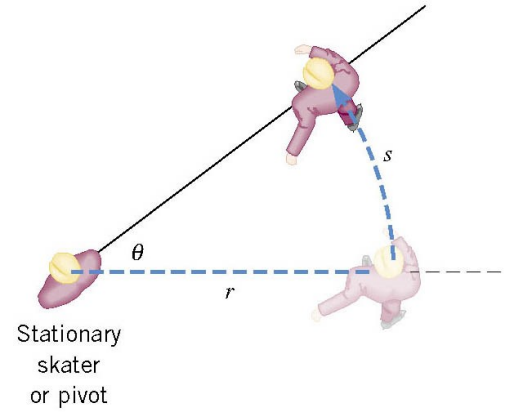
Equations for kinematics in 1D & Newton's Laws apply to rotational motion as well, by substituting the appropriate quantities:

Linear Quantity			Corresponding Rotational Quantity		
Quantity	Variable	SI units	Quantity	Variable	SI units
length	x	m	angle	$\theta = s/r$	rad
velocity	$v = \Delta x / \Delta t$	m/s	angular velocity	$\omega = \Delta \theta / \Delta t$	rad/s
acceleration	$a = \Delta v / \Delta t$	m/s ²	angular acceleration	$\alpha = \Delta \omega / \Delta t$	rad/s ²
mass	m	kg	moment of inertia	I (formula depends on object shape)	kg*m ²
force	$F = ma$	N	torque	$\tau = I\alpha$	N*m
momentum	$p = mv$	kg*m/s	angular momentum	$L = I\omega$	kg*m ² /s



To find tangential motion corresponding to a given rotational motion:

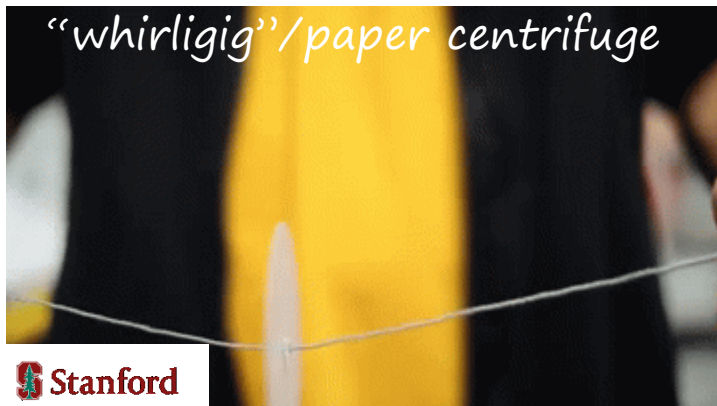
- $s = r * \Delta \theta$ "arc length"
- $v_t = r * \omega$ "tangential velocity"
- $a_t = r * \alpha$ "tangential acceleration"



A pulley of radius 0.10m has a string wrapped around the rim.
If the pulley is rotating on a fixed axis at an angular speed of 0.5rad/s,
what is the length of the string that comes off of the reel in 10 seconds?

- (A) 0.005 m (B) 0.05 m (C) 0.5 m
(D) 5.0 m (E) 50 m (F) 500 m

- $\omega = \Delta\theta / \Delta t$
- $\Delta\theta = \omega * \Delta t = (0.5 \text{ rad/s})(10 \text{ s}) = 5 \text{ rad}$
- $\Delta\theta = \Delta s / r$
- $\Delta s = r * \Delta\theta = (0.10 \text{ m})(5 \text{ rad}) = 0.5 \text{ m}$
- Linear “distance” traveled by string is equal to the arc length.



“whirligig”/paper centrifuge

JOE'S BIG IDEA **shots** HEALTH NEWS FROM NPR **npr**

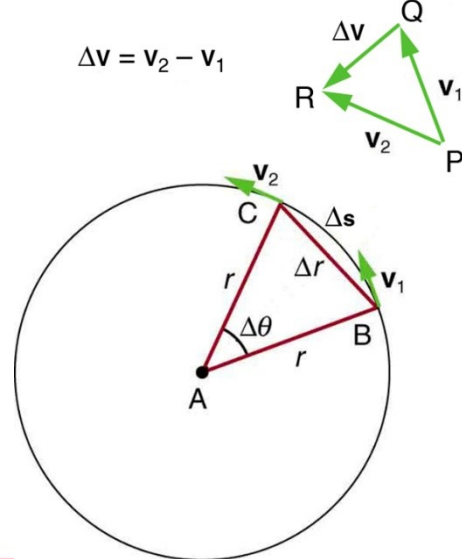
Children's Whirligig Toy Inspires a Low-Cost Laboratory Test

January 10, 2017 - 12:37 PM ET
Heard on Morning Edition MADELINE K. SOFIA

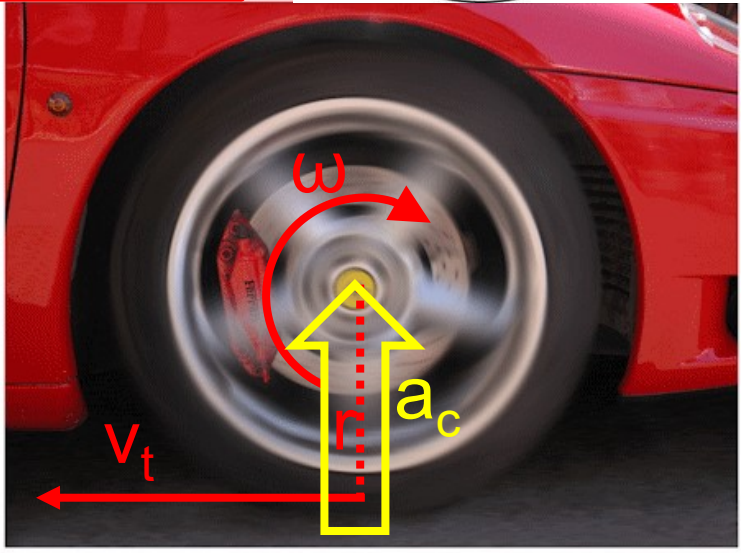
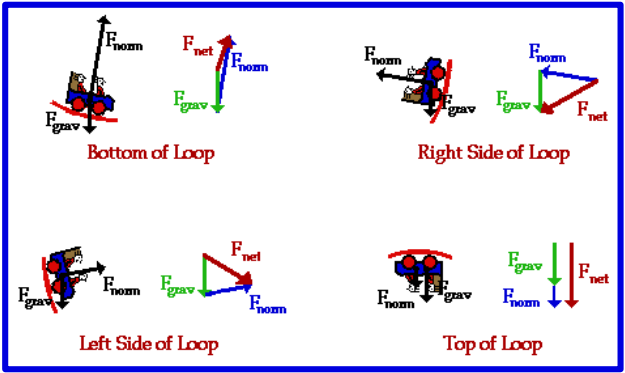
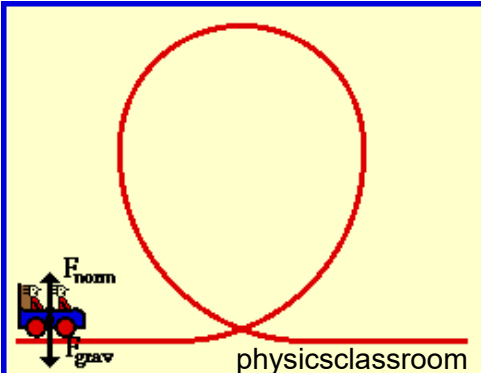
Stanford bioengineering professor Manu Prakash looked to a children's toy to create a hand-powered centrifuge for processing blood tests.
Kurt Hickman /Stanford University

Circular Motion: Acceleration

- Centripetal acceleration:
 - Velocity is a vector, i.e. it has an associated direction.
 - Changing direction while moving means changing velocity.
 - Therefore an object moving in a circle is accelerating.
 - This acceleration is called “centripetal acceleration”.
 - Centripetal acceleration is pointed inwards (toward the rotation axis) with a magnitude: $a_c = v_t^2/r = r*\omega^2$.



• To move in a circle, there must be a **net** inward force (“centripetal force”) causing the centripetal acceleration.



Would not put centripetal force on a free-body diagram, as it is from other forces.

An 800kg car travels a distance of 20m around a turn with a radius of curvature of 40m at a speed of 25m/s in a time of 0.80s. What is the centripetal acceleration of the car?

(A) 12.5 m/s²

(B) 15.6 m/s²

(C) 25.0 m/s²

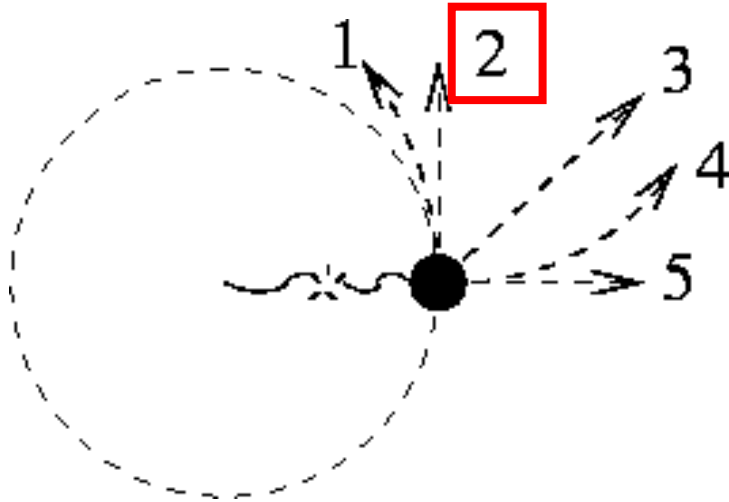
(D) 12500 m/s²

1. $a_c = v_t^2/r$
2. $a_c = \{(25\text{m/s})^2\}/(40\text{m})$
3. $a_c = (625 \text{ m}^2/\text{s}^2)/(40\text{m}) \approx 15.6 \text{ m/s}^2$



You are whirling a tennis ball on a string around in circles, when the string suddenly snaps.

What direction does the tennis ball fly? (the figure below is a top view).



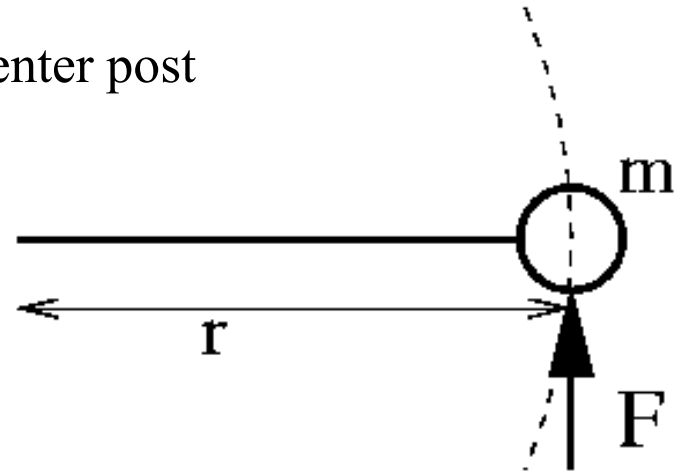
slinging.org

1. Circular motion requires a centripetal force to provide the centripetal acceleration.
2. Once the string snaps, there is no longer an inward force, so circular motion will cease.
3. The tennis ball will continue onward in the direction of the tangential velocity it had just prior to the string snapping.

Newton's Second Law ...for Rotational Motion

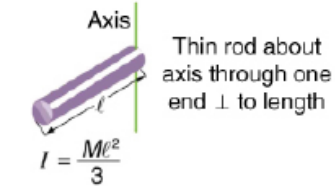
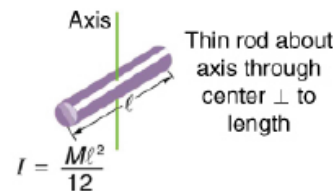
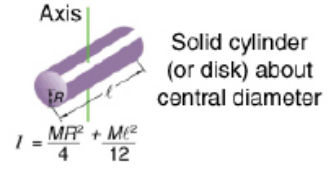
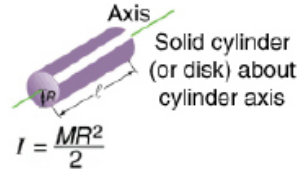
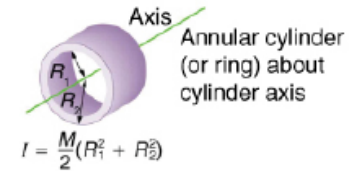
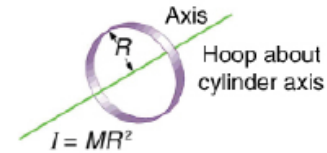
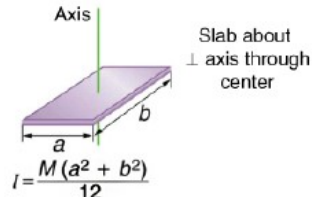
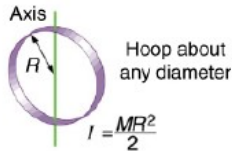
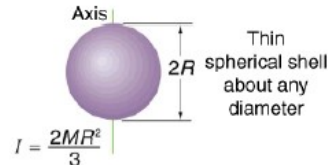
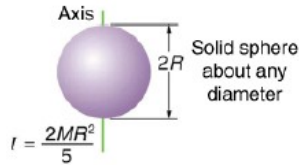
Consider a puck on a frictionless tabletop attached to center post (about which it rotates) by massless rod of length r :

1. $F_t = m \cdot a_t$
2. $F_t = m(r \cdot \alpha)$
3. $(F_t \cdot r) = m \cdot r \cdot r \cdot \alpha$
4. $\tau = (m \cdot r^2) \alpha$
5. $\tau = I \cdot \alpha$



- $I =$ moment of inertia
- $I_{\text{POINT}} = m r^2$

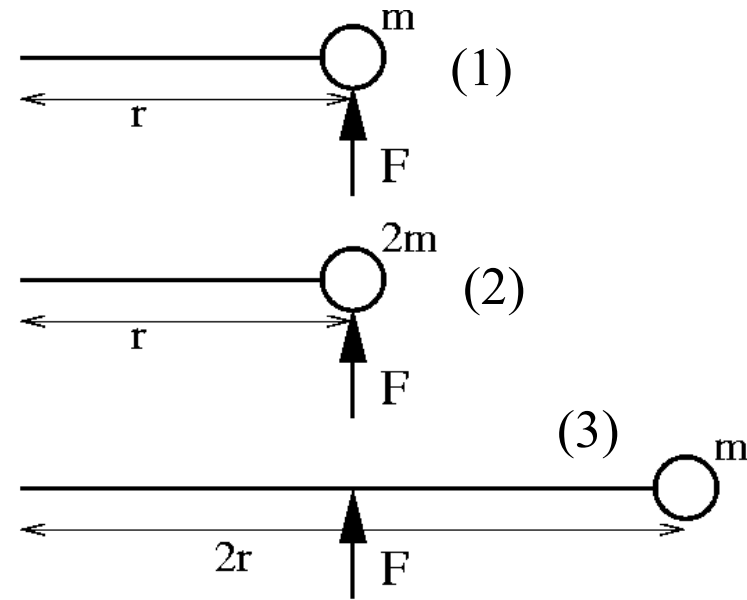
 Fig. 10.12



Three pucks with various masses are attached to massless rods of various length, as shown.

Which system has the greatest moment of inertia?

- (A) 1 (B) 2 **(C) 3** (D) All the same
 (E) 1 & 2 tied (F) 1 & 3 tied



1. Point object: $I_{\text{point}} = mr^2$
2. $I_1 = (m)(r)^2 = mr^2$
3. $I_2 = (2m)(r)^2 = 2mr^2$
4. $I_3 = (m)(2r)^2 = 4mr^2$
5. $I_3 > I_2 > I_1$

Moment of inertia depends on the mass & the mass distribution/location.

Consider two masses on a rod rotating as shown.

In case A, the masses are at the end of the rod.

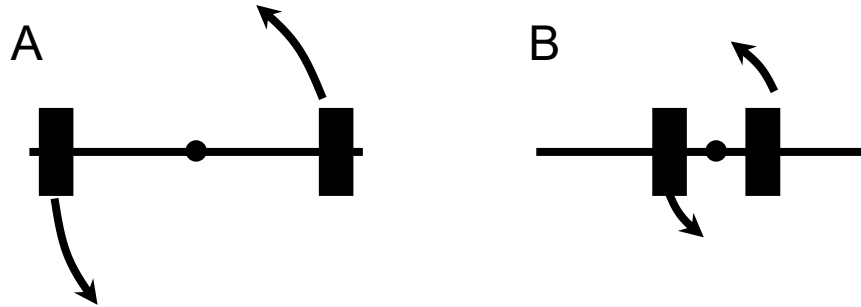
In case B the same 2 masses are closer to the center.

If the same torque is applied to each situation, which arrangement of masses has the lowest angular acceleration?

A. A

B. B

C. Both have the same



1. $\tau = I\alpha = F \cdot r \cdot \sin(\theta) = \text{same for all}$

2. $\alpha = \tau/I$

3. The mass for case A is less concentrated toward the axis of rotation.

So, case A has a larger moment of inertia, I .

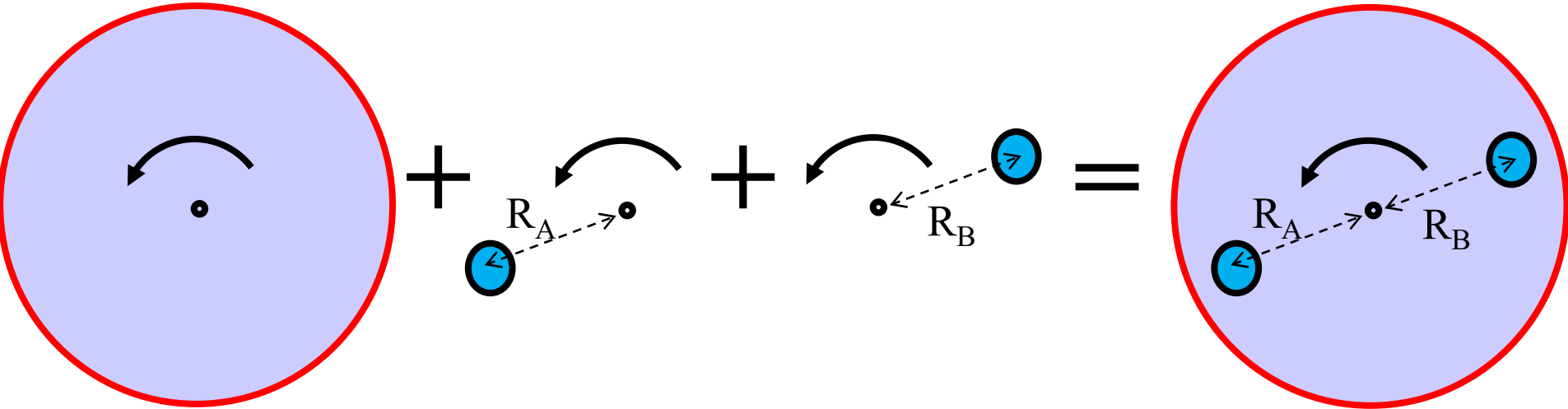
4. From (2), larger I means smaller α So case A will have the lowest α .

Moment of Inertia: Multiple Objects

- Moments of inertia add together for a compound object.
- Example:

Disk with two point masses on top, all spinning about the center of the disk

- $I_{\text{total}} = I_{\text{cylinder}} + I_{\text{point,A}} + I_{\text{point,B}}$
- $I_{\text{total}} = (1/2)M_{\text{disk}}(R_{\text{disk}})^2 + M_{\text{A}}R_{\text{A}}^2 + M_{\text{B}}R_{\text{B}}^2$



Rotational Kinetic Energy

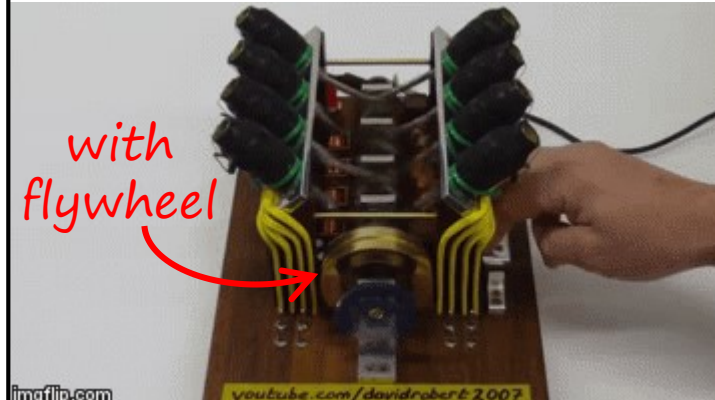
- Rotation has an associated kinetic energy
- As noted previously, by swapping-in the appropriate variables, our equations describing motion in 1D work for rotation as well:
 - $KE_{\text{rotation}} = \frac{1}{2}I\omega^2$
- For example, a rolling object has both translational & rotational kinetic energy
 - $KE_{\text{total}} = KE_{\text{translation}} + KE_{\text{rotation}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- Can be used to store energy!
 - ...often used to smooth-out power availability for intermittent power sources

Pottery Wheel



Scott Wares

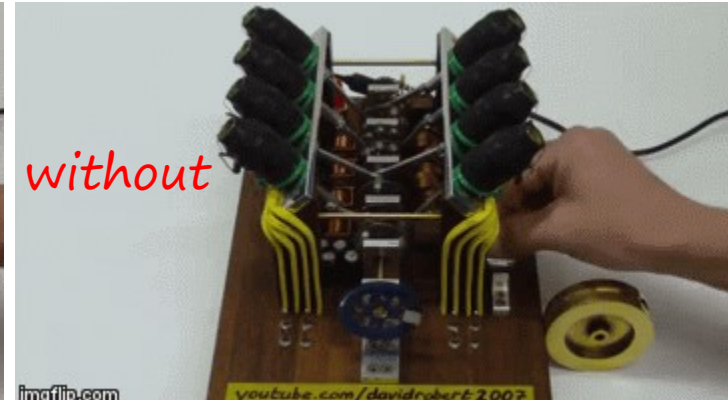
Engine Flywheel



imgflip.com

youtube.com/davidrobert2007

<https://www.youtube.com/watch?v=reh79gYsaQE>



imgflip.com

youtube.com/davidrobert2007

Angular Momentum, L

- Recall:

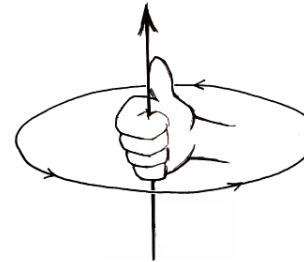
Equations for kinematics in 1D & Newton's Laws apply to rotational motion as well, by substituting the appropriate quantities:

Linear Quantity			Corresponding Rotational Quantity		
Quantity	Variable	SI units	Quantity	Variable	SI units
length	x	m	angle	$\theta = s/r$	rad
velocity	$v = \Delta x/\Delta t$	m/s	angular velocity	$\omega = \Delta\theta/\Delta t$	rad/s
acceleration	$a = \Delta v/\Delta t$	m/s^2	angular acceleration	$\alpha = \Delta\omega/\Delta t$	rad/s^2
mass	m	kg	moment of inertia	I (formula depends on object shape)	$kg \cdot m^2$
force	$F = ma$	N	torque	$\tau = I\alpha$	$N \cdot m$
momentum	$p = mv$	$kg \cdot m/s$	angular momentum	$L = I\omega$	$kg \cdot m^2/s$

- Just as linear momentum (p) is conserved in the absence of an external force, angular momentum (L) is conserved in the absence of an external torque.

- $L_{\text{initial}} = L_{\text{final}}$
- $I_{\text{initial}} \cdot \omega_{\text{initial}} = I_{\text{final}} \cdot \omega_{\text{final}}$

- Angular momentum, L, is a vector whose direction corresponds to the spin-axis.



A teacher sits on a turntable holding two 2-kg masses in their hands. Their arms are folded close to their body. They are rotating. When the teacher stretches their arms out (with the masses in-hand), the angular velocity of the teacher will:

A. Increase

B. Decrease

C. Stay the Same



1. Angular momentum is conserved.
2. So, $L = I \cdot \omega = \text{constant}$.
3. When the masses are moved outward, the mass distribution is less concentrated, and so the moment of inertia, I , increases.
4. To maintain L , ω must decrease.