Thursday March 21

<u>Topics for this Lecture</u>:

- *Rotational*
  - *Angular momentum*

***Actual class-content for lecture likely to be somewhat different from this. Guest lecture given by Dr. Piccard.**

- Assignment 9 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 226, M&Tu 6:10-7:20pm
- Office Hours: This week by appointment (meisel@ohio.edu)
- Midterm 2: Monday April 1st
  7:15-9:15pm Morton Hall Room 201

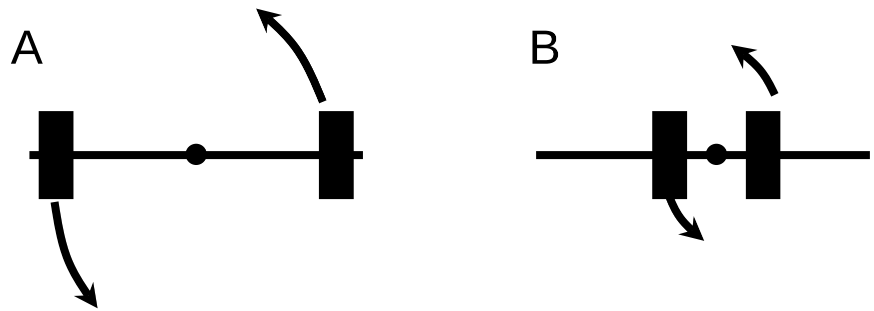
- *Fun facts to know & tell:*

Equations for kinematics in 1D & Newton's Laws apply to rotational motion as well, by subsituting the appropriate quantities:

Linear Quantity			**Corresponding Rotational Quantity**		
Quantity	***Variable***	***SI units***	***Quantity***	***Variable***	***SI units***
length	x	m	angle	$\theta = s/r$	rad
velocity	$v = \Delta x/\Delta t$	m/s	angular velocity	$\omega = \Delta\theta/\Delta t$	rad/s
acceleration	$a = \Delta v/\Delta t$	m/s^2	angular acceleration	$\alpha = \Delta\omega/\Delta t$	rad/s^2
mass	m	kg	moment of inertia	I (formula depends on object shape)	$kg*m^2$
force	$F = ma$	N	torque	$\tau = I\alpha$	N*m
momentum	$p = mv$	kg*m/s	angular momentum	$L = I\omega$	$kg*m^2/s$

Consider two masses on a rod rotating as shown.
In case A, the masses are at the end of the rod.
In case B the same 2 masses are closer to the center.
If the same torque is applied to each situation, which arrangement of masses has the lowest angular acceleration?

A. A
B. B
C. Both have the same

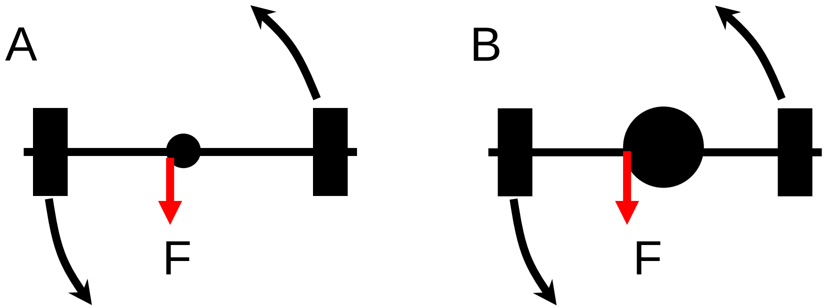


1. $\tau = I\alpha = F*r*\sin(\theta)$ = same for all
2. $\alpha = \tau / I$
3. The mass for case A is less concentrated toward the axis of rotation.
   So, case A has a larger moment of inertia, I.
4. From (2), larger I means smaller α. …So case A will have the lowest α.

Consider two masses on a rod that is attached to an axle, rotating as shown. The masses are placed identically in each situation.

In each case the rod (massless) is turned by pulling a string wrapped around the center axle (also massless). The center axle in case B has a much larger radius.

If the force is the same, which situation has the smallest angular acceleration?



A. A

B. B

C. Both have the same

1. $\tau = I\alpha = F*r*\sin(\theta)$
2. $\alpha = \tau/I$
3. The masses are distributed identically, so both have the same moment of inertia, I.
4. However, the force F in case A is applied at a shorter distance r from the axis of rotation. So, the applied torque is smaller in case A.
5. From (2), smaller τ means smaller α. …So case A will have the lowest α.

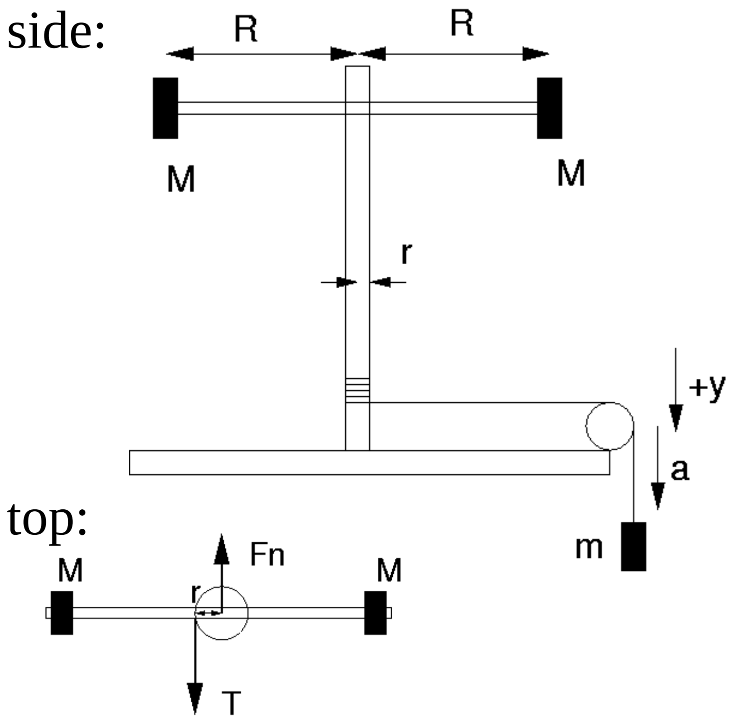
A T-shaped rotating post, with a mass M and radius r, has two point-like masses of mass M positioned a distance R from the rotation axis, at the end of the T, as pictured. A string that is wrapped around the base of the post runs through a pulley and is attached to a hanging mass m. M=1.0kg, R=0.5m, m=0.1kg, and r=0.1m. What is the total moment of inertia of the post (cylinder) and its masses?

(A) $5\text{x}10^{-3}$ kg*m^2

(B) $5\text{x}10^{-2}$ kg*m^2

(C) $5\text{x}10^{-1}$ kg*m^2

(D) 5.0 kg*m^2



1. The moment of inertia for a mutli-component object is just the sum of the moment of inertia for the components: $I_{total} = \Sigma I_i$
2. The multi-component object here is a cylinder plus two point masses.
3. $I = I_{cyl} + I_{point,1} + I_{point,2}$
4. $I = (1/2)Mr^2 + MR^2 + MR^2$
5. $I = (1/2)(1.0\text{kg})(0.1\text{m})^2 + 2*(1.0\text{kg})(0.5\text{m})^2$
6. $I = 0.005\text{kg*m}^2 + 0.5\ \text{kg*m}^2 \approx 0.5\ \text{kg*m}^2$

$I_{point} = MR^2$

$I_{cylinder} = (1/2)Mr^2$

Angular Momentum, L

- Recall:

Equations for kinematics in 1D & Newton's Laws apply to rotational motion as well, by subsituting the appropriate quantities:

Linear Quantity			**Corresponding Rotational Quantity**		
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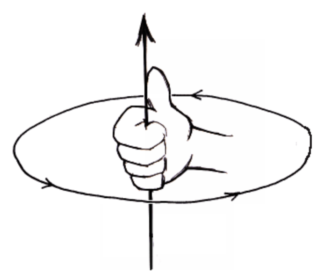
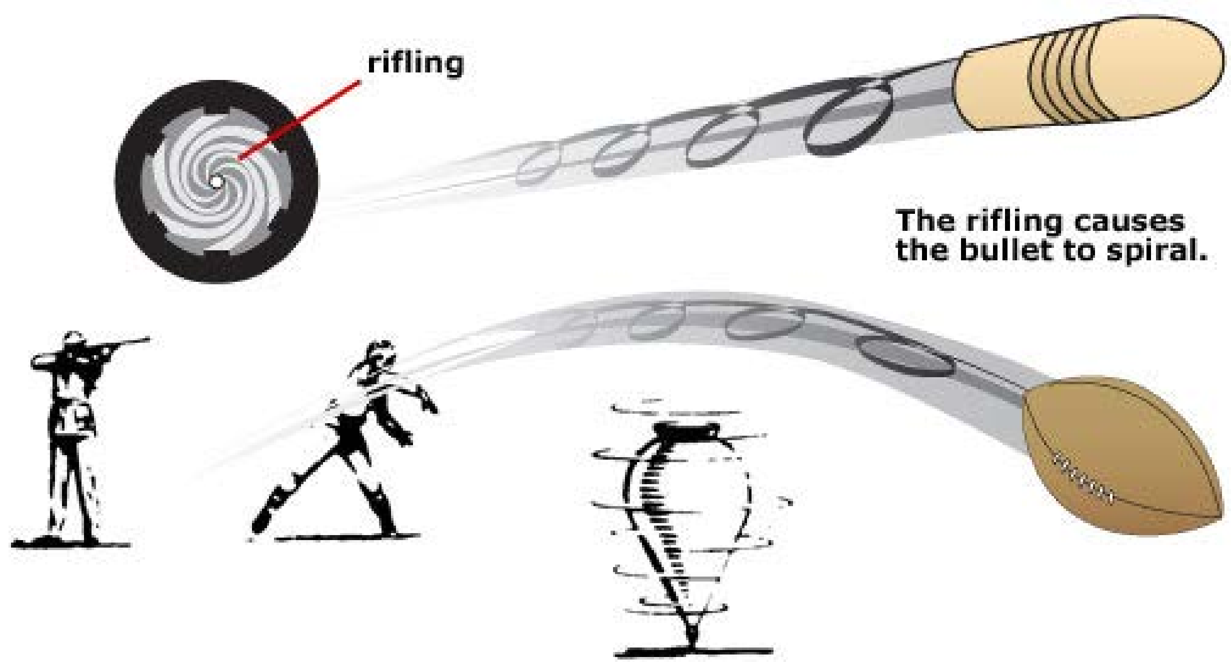
mass	m	kg	moment of inertia	I (formula depends on object shape)	$kg*m^2$
force	$F = ma$	N	torque	$\tau = I\alpha$	N*m
momentum	$p = mv$	kg*m/s	angular momentum	$L = I\omega$	$kg*m^2/s$

- Just as linear momentum (p) is conserved in the absence of an external force, angular momentum (L) is conserved in the absence of an external torque.
  - $L_{initial} = L_{final}$
  - $I_{initial} * \omega_{initial} = I_{final} * \omega_{final}$
- Angular momentum, L, is a vector whose direction corresponds to the spin-axis.



Angular Momentum Conservation

- Consequence 1:
  A spinning object keeps pointing in the same direction
  - Spinning objects require an external torque to change their direction
  - So, a spinning object will tend to stay pointed the way it is originally pointed



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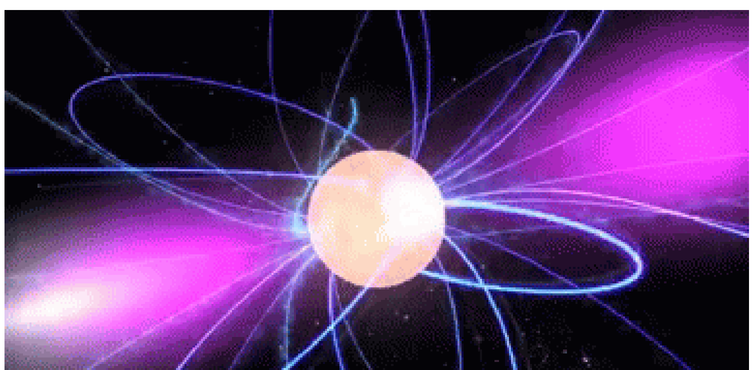
Angular Momentum Conservation

- Consequence 2:
  An object whose mass distribution changes will change rotation rate & vice versa
  - E.g. Figure skater:

  - E.g. Pulsar (ultra-dense,ultra-fast spinning remnant from a supernova)



https://www.youtube.com/watch?v=YzbTfC0tyxg

~mass of sun compressed to ~size of city, spinning at ~60-60,000 RPM,
Left over from a massive stellar explosion.

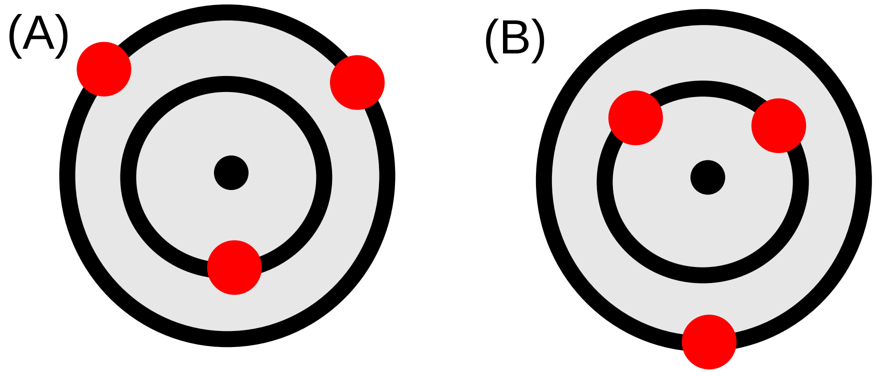
A professor sits on a turntable holding two 2-kg masses in their hands. Their arms are folded close to their body. They are rotating.
When the professor stretches their arms out (with the masses in-hand), the angular velocity of the professor will:

A. Increase
B. Decrease
C. Stay the Same

1. Angular momentum is conserved.
2. So, $L = I^*\omega$ = constant.
3. When the masses are moved outward, the mass distribution is less concentrated, and so the moment of inertia, I, increases.
4. To maintain L, ω must decrease.

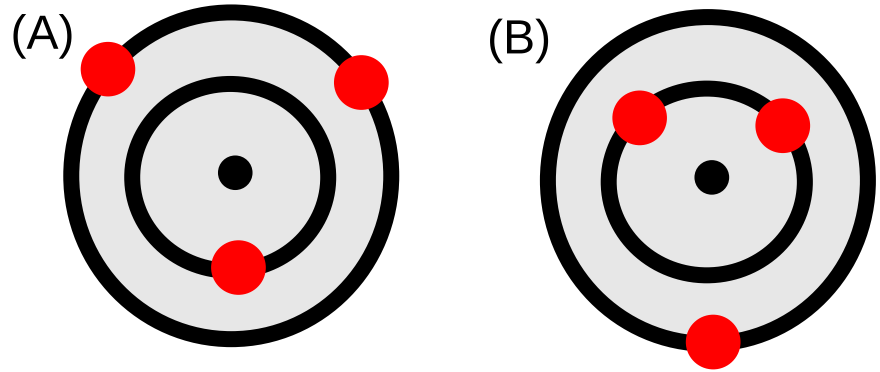
Each Red dot represents a 1kg mass attached to a massless turntable.
The inner circle has a radius half that of the outer circle.
If the masses start in Configuration A and move to Configuration B while the turntable is spinning, how does the final moment of inertia of the turntable compare to the initial? (assume no external torque)

A. Greater
B. Smaller
C. Same



1. The masses are moved inward, the mass distribution is more concentrated, and so the moment of inertia, I, decreases.

Each Red dot represents a 1kg mass attached to a massless turntable.
The inner circle has a radius half that of the outer circle.
If the masses start in Configuration A and move to Configuration B while the turntable is spinning, how does the final angular momentum of the turntable compare to the initial? (assume no external torque)



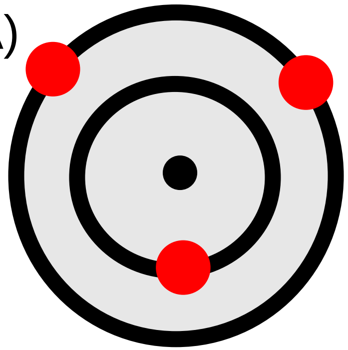
A. Greater
B. Smaller
C. Same

1. Angular momentum is conserved in the absence of external torques.
2. There are no external torques, so $L_{initial} = L_{final}$

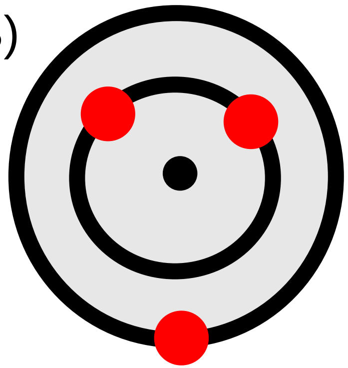
Each Red dot represents a 1kg mass attached to a massless turntable.
The inner circle has a radius half that of the outer circle.
If the masses start in Configuration A and move to Configuration B while the turntable is spinning, how does the final angular velocity of the turntable compare to the initial? (assume no external torque)

A. Greater
B. Smaller
C. Same

(A)



(B)



1. Angular momentum is conserved.
2. So, $L = I*\omega$ = constant.
3. When the masses are moved inward, the mass distribution is more concentrated, and so the moment of inertia, I, decreases.
4. To maintain L, ω must increase.

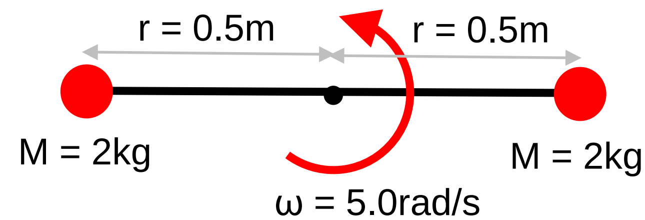
Suppose a rod of negligible mass has a 2.0kg object on each end. The rod is 1.0m long and is spinning at an angular speed of 5.0rad/s. What is the angular momentum?

(A) 2 kg*m^2/s

(B) 4 kg*m^2/s

(C) 5 kg*m^2/s

(D) 10 kg*m^2/s



1. $L_i = I_i \omega_i$
2. $L_i = (MR_i^2 + MR_i^2)\omega_i$
3. $L_i = 2(2.0\text{kg})(0.50\text{m})^2(5.0\text{rad/s}) = 5\text{kg*m}^2/\text{s}$

$I_{point} = MR^2$

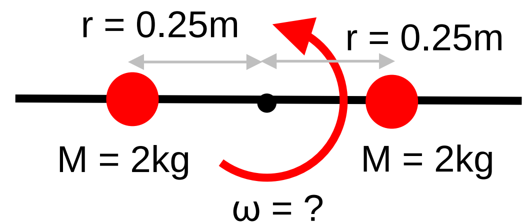
Suppose a rod of negligible mass has a 2.0kg object on each end.
The rod is 1.0m long and is spinning at an angular speed of 5.0rad/s.
Little motors move the masses inward by 0.25m.
What is the new angular momentum?

(A) 2 kg*m^2/s

(B) 4 kg*m^2/s

(C) 5 kg*m^2/s

(D) 10 kg*m^2/s



1. There are no external torques, so angular momentum is conserved.
2. $L_f = L_i = 5 kg*m^2/s$

$I_{point} = MR^2$

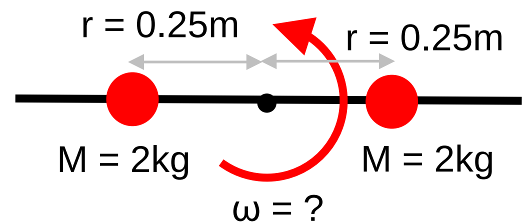
Suppose a rod of negligible mass has a 2.0kg object on each end.
The rod is 1.0m long and is spinning at an angular speed of 5.0rad/s.
Little motors move the masses inward by 0.25m.
What is the new angular speed?

(A) 2.5 rad/s

(B) 5 rad/s

(C) 10 rad/s

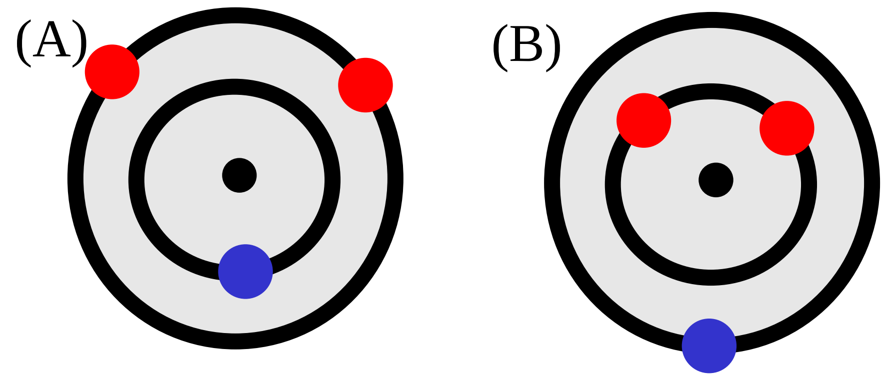
(D) 20 rad/s



1. $L_i = 5 kg*m^2/s = I_i \omega_i = L_f = I_f \omega_f$
2. $\omega_f = L_i/I_f$
3. $I_f = 2*(Mr^2) = 2*(2kg)(0.25m)^2 = 0.25\ kg*m^2$
4. $\omega_f = L_i/I_f = (5kg*m^2/s)/(0.25\ kg*m^2) = 20\ rad/s$

$I_{point} = MR^2$

Each Red dot represents a 1kg mass on a massless turntable.
The blue dot is 2kg.
The inner circle has a radius half that of the outer circle.
If the masses start in Configuration A and move to Configuration B while the turntable is spinning, how does the final angular speed of the turntable compare to the initial? (assume no external torque)



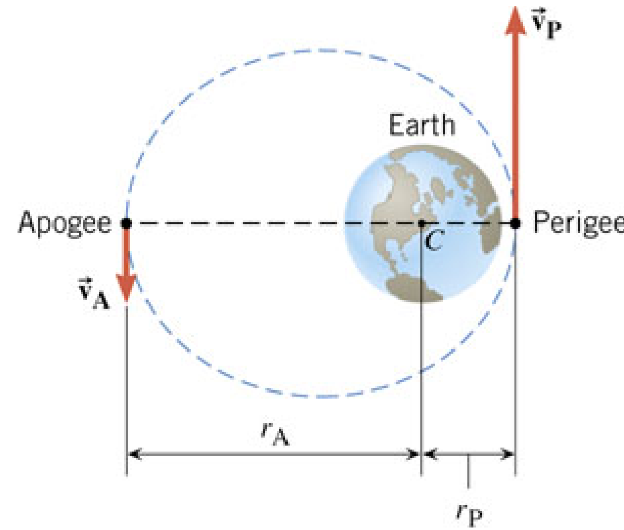
A. Greater
B. Smaller
C. Same

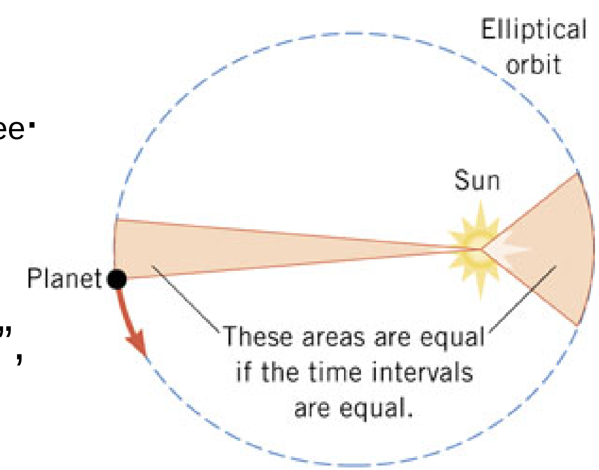
1. $L_i = I_i \omega_i = L_f = I_f \omega_f$
2. $I_i = 2*(M_{red})(R_{red,i})^2 + (M_{blue})(R_{blue,i})^2$
3. $R_{blue,i} = (1/2)*R_{red,i}$ and $M_{blue} = 2*M_{red}$, so … $I_i = (5/2)*(M_{red})(R_{red,i})^2$.
4. $I_f = 2*(M_{red})(R_{red,f})^2 + (M_{blue})(R_{blue,f})^2$
5. Since $R_{red,f} = (1/2)*R_{blue,f}$ and $M_{blue} = 2*M_{red}$ … $I_f = (5/2)*(M_{red})(R_{blue,f})^2$
6. Since $R_{blue,f} = R_{red,i}$ … $I_i = I_f$
7. Since L = constant and I = constant, ω= constant.

$I_{point} = MR^2$

Conservation of Angular Momentum for Elliptical Orbits

- Angular momentum is conserved in any system without an external torque.
- This includes objects in elliptical orbits, like planets in the solar system.
- In an elliptical orbit, the orbiting objects moves further from & closer to the more massive object it is orbiting as it orbits.
- The furthest distance is called apogee; the closest distance is called perigee.
- Since angular momentum is conserved, $L_{apogee} = L_{perigee}$.
- In both cases, $L = I\omega$, and $I = M_{satellite}R^2$, but $R_{apogee} > R_{perigee}$.
- This means $\omega_{perigee} > \omega_{apogee}$ (since L is the same).
- Consequently, the satellite moves faster as it approaches the object it slows down as it moves away.
- This means, “planets sweep out equal areas in equal times”, which is known as Kepler’s 2nd law of planetary motion





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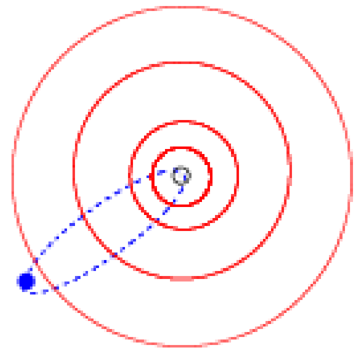
Haley's comet completes a full orbit of the sun every ~75 years.
At closest approach to the sun (perihelion), it is moving at 54m/s 0.57AU from the sun.
How fast is Haley's comet moving when it is farthest from the sun (aphelion) at 35AU?

(A) 0.02m/s

(B) 0.88m/s

(C) 27m/s

(D) 54m/s



1. $L_{perihelion} = I_{perhelion}\,\omega_{perhelion} = L_{aphelion} = I_{aphelion}\,\omega_{aphelion}$
2. $L_{perihelion} = \left(M_{Haley} R^2_{perihelion}\right)\left({}^{v_{perihelion}}/_{R_{perihelion}}\right) = M_H r_p v_p$
3. $L_{aphelion} = M_H r_a v_a$
4. $M_H r_a v_a = M_H r_p v_p$ … so: $r_a v_a = r_p v_p$
5. $v_a = \frac{r_p v_p}{r_a} = \frac{(0.57AU)(54^m/_s)}{35AU} = 0.88m/s$

$I_{point} = MR^2$
$v = r\omega$

1AU = 1 "astronomical unit" = 1 Earth-Sun distance