Tuesday March 19

Topics for this Lecture:

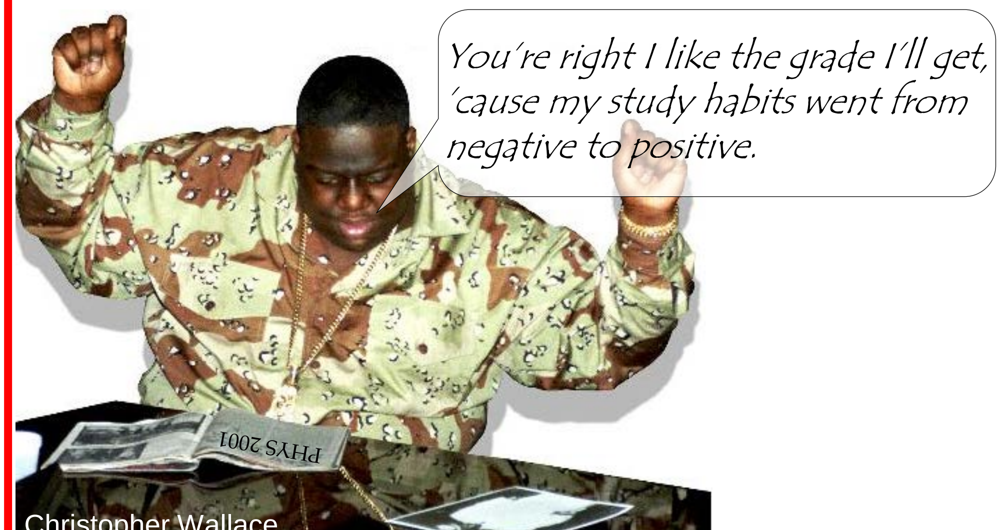
- *Rotational Motion*
  - *Angular acceleration*
  - *Rotational-to-linear quantities*
  - *Moment of Inertia*
  - *Rotational kinetic energy*

- Assignment 9 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 226, M&Tu 6:20-7:10pm
- Office Hours: 204 EAL, 3-4pm Thu
  or by appointment (meisel@ohio.edu)
- Midterm 2: Monday April 1st
  7:15-9:15pm Morton Hall Room 201

- *Write these equations in your notes if they're not already there.*
- *You will want them for Exam 2 & the Final.*

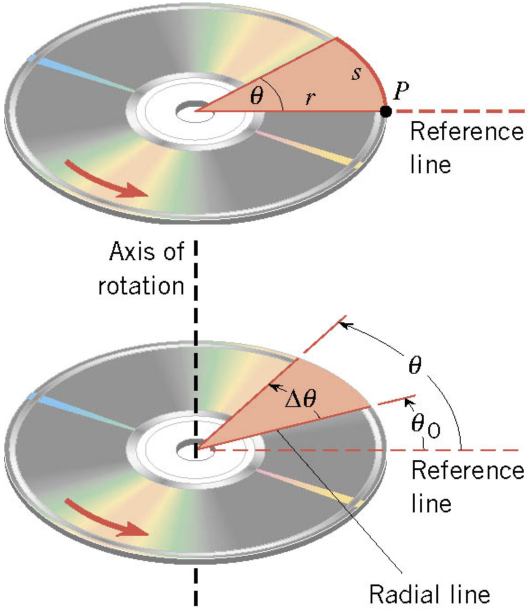
- $\theta = s/r$
- $\omega = \Delta\theta/\Delta t$
- $v_t = r*\omega$
- $\alpha = \Delta\omega/\Delta t$
- π rad = 180°
- $a_c = v_t^2/r = r*\omega^2$
- $F_c = mv^2/r$
- $F_g = G\frac{m_1 m_2}{r^2}$

Midterm Exam 2 is in 2 weeks. ***Study!***



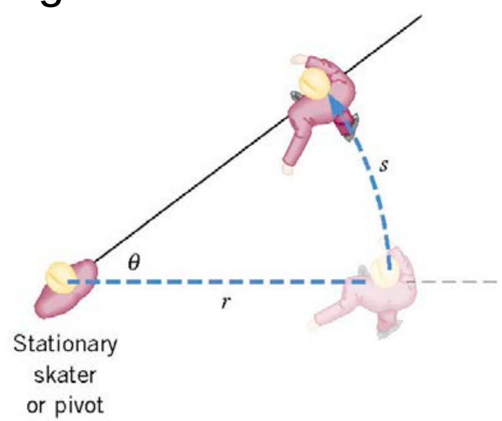
Relationship between Rotational & Linear Kinematics

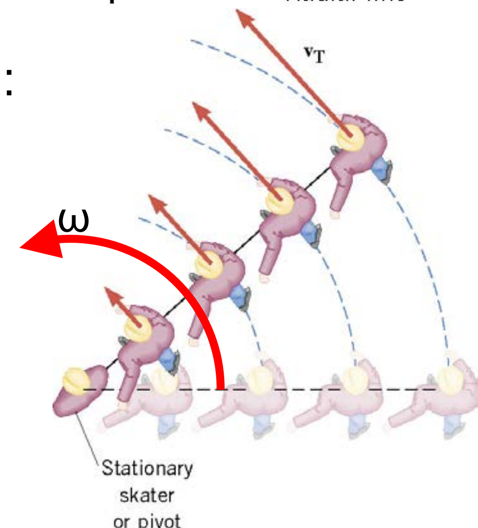
Equations for kinematics in 1D & Newton's Laws apply to rotational motion as well, by subsituting the appropriate quantities:					
Linear Quantity			**Corresponding Rotational Quantity**		
Quantity	***Variable***	***SI units***	***Quantity***	***Variable***	***SI units***
length	x	m	angle	θ = s/r	rad
velocity	v = Δx/Δt	m/s	angular velocity	ω = Δθ/Δt	rad/s
acceleration	a = Δv/Δt	m/s^2	angular acceleration	α = Δω/Δt	rad/s^2
mass	m	kg	Fig. 10.12 moment of inertia	I (formula depends on object shape)	$kg*m^2$
force	F = ma	N	torque	τ = Iα	N*m
momentum	p = mv	kg*m/s	angular momentum	L = Iω	$kg*m^2/s$



To find tangential motion corresponding to a given rotational motion:

- $s = r*\Delta\theta$ "arc length"
- $v_t = r*\omega$ "tangential velocity"
- $a_t = r*\alpha$ "tangential acceleration"





An object starts at rest and undergoes an average angular acceleration of 0.5 rad/s^2 for 10 seconds.
What is the angular speed after 10 seconds?

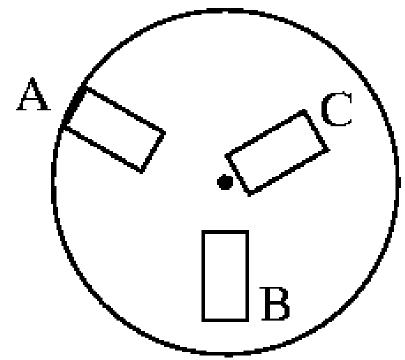
(A) 0.05 rad/s (B) 0.5 rad/s (C) 5 rad/s

(D) 10 rad/s (E) 20 rad/s (F) 50 rad/s

1. $\alpha = \Delta\omega/\Delta t$
2. $\Delta\omega = \alpha*\Delta t = (0.5 \text{ rad/s}^2)*10 \text{ s} = 5 \text{ rad/s}$
3. Since $\omega_0 = 0$, $\omega_f = 5$ rad/s

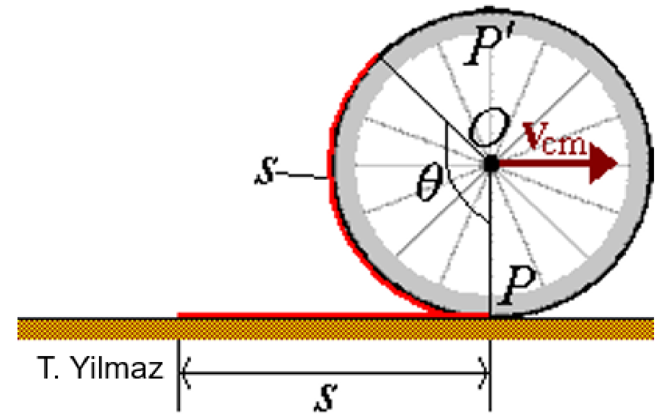
Three erasers are on a turntable, as pictured.
Eraser A is near the edge, eraser C is the closest to the center, and eraser B is in the middle.
The surface is not frictionless. Starting from rest, the turntable slowly accelerates.
Which eraser flies off of the edge first?

A. A
B. B
C. C
D. All at the same time



1. $a_c = r*\omega^2$
2. ω is the same for A, B, & C, because on a rigid turn-table.
3. r is greatest for A, so a_c is greatest there.
4. A would therefore require the greatest force to stay moving in a circle.
5. Absent that force, A will fly-off the quickest.

Rolling (without slipping)



- The linear distance traveled for a given rotation is equal to the arc length between the initial & final points of contact on the surface of the wheel
  - $x = s = r*\Delta\theta$ *here θ must be in radians!*

- The translational velocity is related to the rotation rate by dividing the relationship for distance by time:
  - $v_t = r*\omega$

- Similarly, for acceleration divide by time again to get:
  - $a_t = r*\alpha$

You could use the 2nd relationship + your tachometer & speedometer readings to find the gear ratio between your crankshaft & tires, so long as you know your tire radius.

E.g. if it were one, 2000rpm would equal ~200rad/s at your tires. For 0.25m-radius tires, your velocity would be 50m/s (~112mph).

'04 C5 Z06 Corvette, Manual Tr.

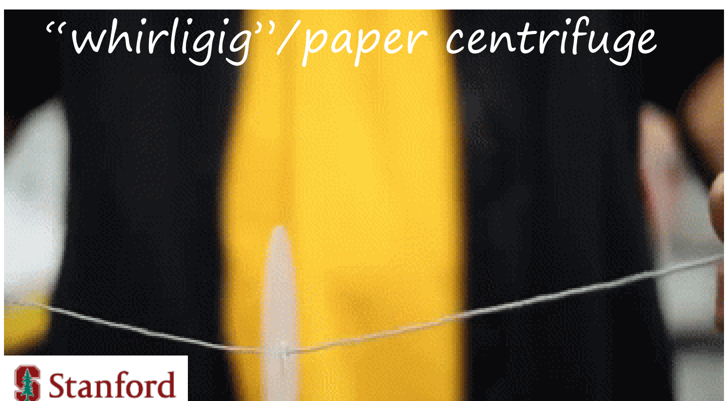
Gear	Ratio
1st gear	2.97:1
2nd gear	2.07:1
3rd gear	1.43:1
4th gear	1.00:1
5th gear	0.84:1
6th gear	0.56:1
reverse	3.28:1

A pulley of radius 0.10m has a string wrapped around the rim.
If the pulley is rotating on a fixed axis at an angular speed of 0.5rad/s,
what is the length of the string that comes off of the reel in 10 seconds?

(A) 0.005 m (B) 0.05 m $\boxed{\text{(C) 0.5 m}}$

(D) 5.0 m (E) 50 m (F) 500 m

1. $\omega = \Delta\theta / \Delta t$
2. $\Delta\theta = \omega * \Delta t = (0.5\text{rad/s})(10\text{ s}) = 5\text{rad}$
3. $\Delta\theta = \Delta s / r$
4. $\Delta s = r * \Delta\theta = (0.10\text{m})(5\text{rad}) = 0.5\text{m}$
5. Linear "distance" traveled by string is equal to the arc length.



"whirligig"/paper centrifuge


JOE'S BIG IDEA

shots HEALTH NEWS FROM NPR

npr

Children's Whirligig Toy Inspires a Low-Cost Laboratory Test

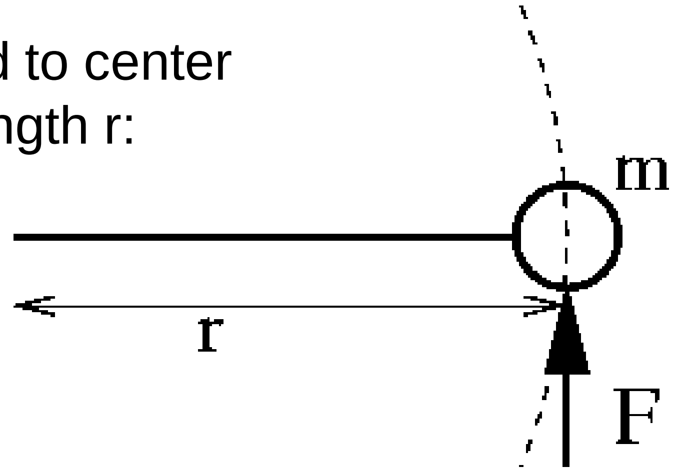
January 10, 2017 · 12:37 PM ET
Heard on Morning Edition

MADELINE K. SOFIA

Stanford bioengineering professor Manu Prakash looked to a children's toy to create a hand-powered centrifuge for processing blood tests.
Kurt Hickman /Stanford University

Newton's Second Law *…for Rotational Motion*

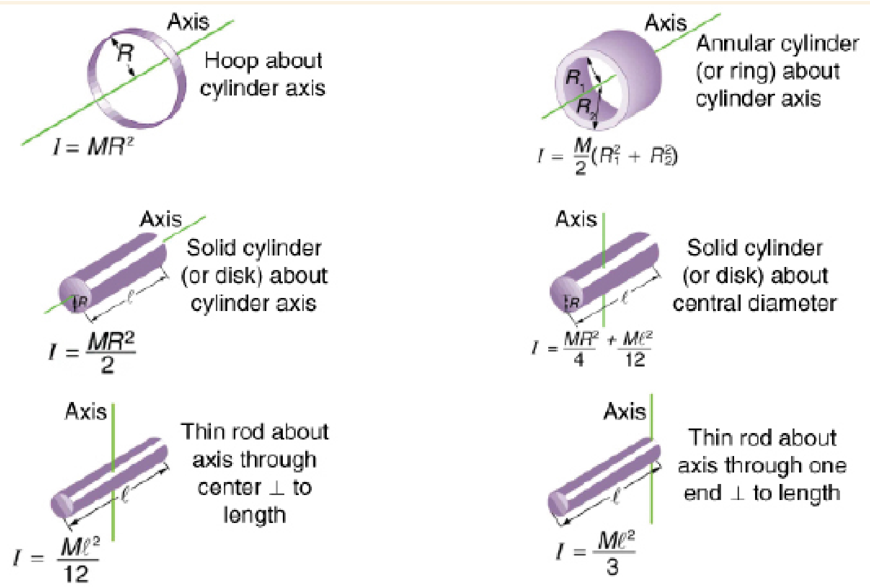
Consider a puck on a frictionless tabletop attached to center post (about which it rotates) by massless rod of length r:

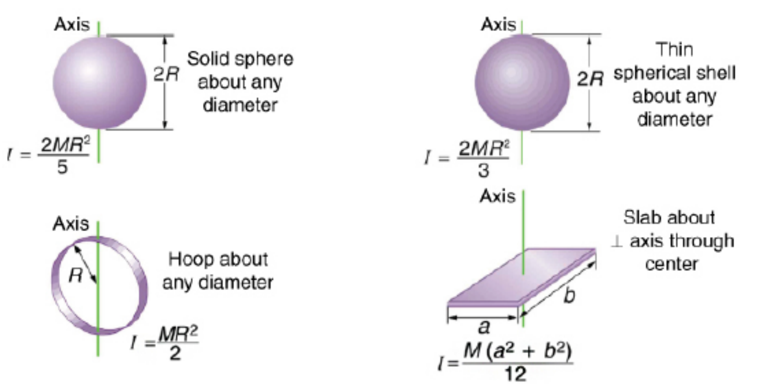


1. $F_t = m*a_t$
2. $F_t = m(r*\alpha)$
3. $(F_t*r) = m*r*r*\alpha$
4. $\tau = (m*r^2)\alpha$
5. $\boxed{\tau = I*\alpha}$

- I = moment of inertia
- $I_{POINT} = m\, r^2$

Fig. 10.12

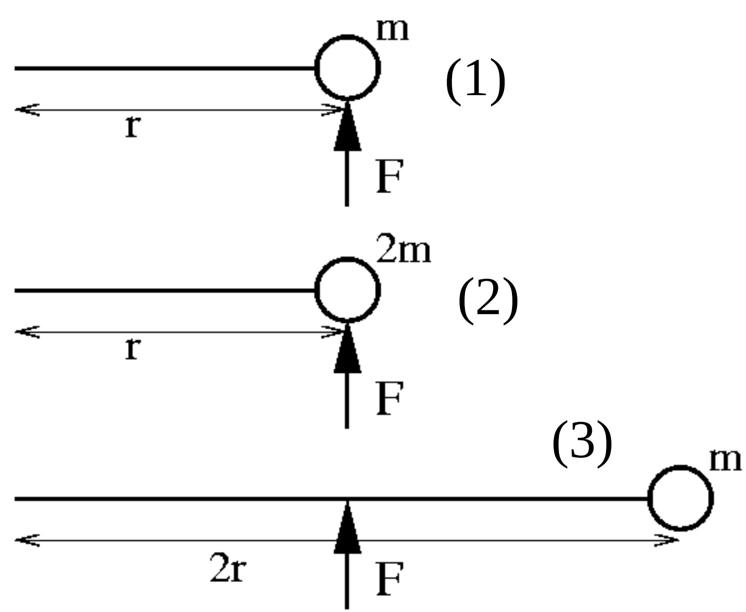




Three pucks with various masses are attached to massless rods of various length, as shown.
Which system has the greatest moment of inertia?

(A) 1 (B) 2 (C) 3 (D) All the same

(E) 1 & 2 tied (F) 1 & 3 tied

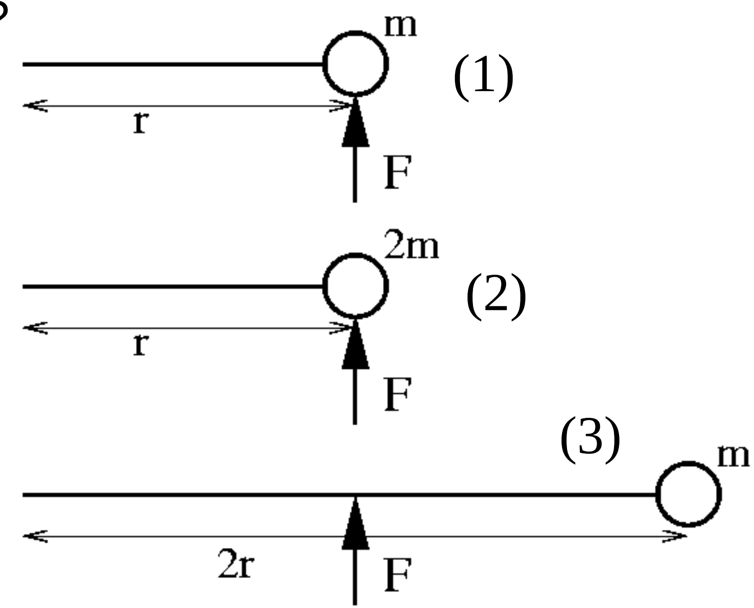


1. Point object: $I_{point} = mr^2$
2. $I_1 = (m)(r)^2 = mr^2$
3. $I_2 = (2m)(r)^2 = 2mr^2$
4. $I_3 = (m)(2r)^2 = 4mr^2$
5. $I_3 > I_2 > I_1$

Moment of inertia depends on the mass & the mass distribution/location.

Three pucks with various masses are attached to massless rods of various lengths, as shown.

Which system has the greatest angular acceleration?



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4. $I_3 = (m)(2r)^2 = 4mr^2$
5. $I_3 > I_2 > I_1$
6. $\tau = I\alpha = F*r*\sin(\theta)$ = same for all
7. $\alpha = \tau/I$
8. Since I is smallest for (1), α is greatest.

Moment of inertia is resistance to rotation given some applied torque.
(Just like how inertia is resistance to motion given an applied force.)

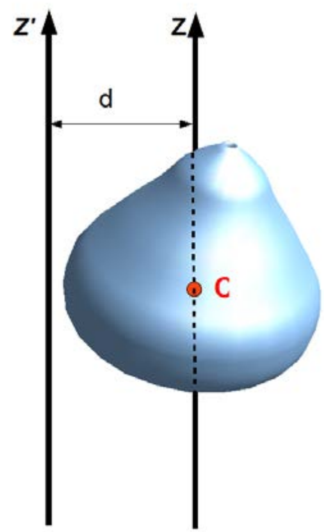
Moment of Inertia

Found by integrating formula for point mass over mass distribution.

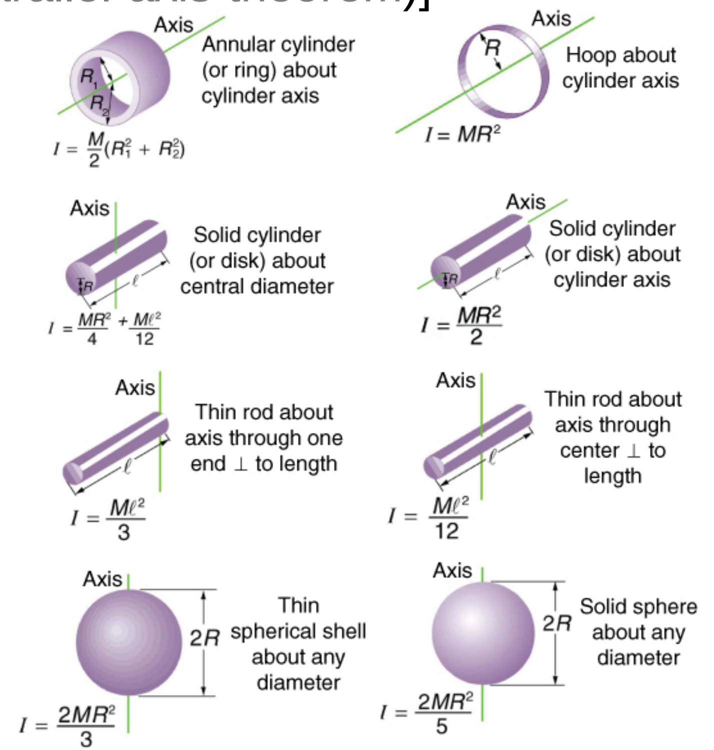
- "Moment of inertia", I: resistance to rotational motion
- Depends on mass distribution ***with respect to the rotation axis***
  - e.g. A cylinder's I depends on where you're rotating it about
- Can be combined for compound objects
  - e.g: Golf club ~ [rod] + [shell at distance (using parallel-axis theorem)]

In general, more concentrated mass is to rotation axis, lower I.

📖 Fig. 10.12



$$I_c' = I_c + m_c d^2$$



Moment of Inertia

The "swing weight index" of a baseball bat is just an indicator of the moment-of-inertia, I.

SWING WEIGHT INDEX SWI

		30"	31"	32"	33"	34"
BALANCED	MAKO	•	6.9	7.7	8.6	9.7
BALANCED	MAKO TORQ	•	7.0	7.9	8.9	9.9
BALANCED	Z-CORE TORQ	•	7.3	8.1	9.0	10.1
SWING SPEED	Z-CORE	6.7	7.4	8.2	9.2	10.2
SWING SPEED	Z-CORE HYBRID TORQ	•	7.4	8.3	9.3	10.3
SWING SPEED	Z-CORE HYBRID	•	7.6	8.5	9.5	10.5
LOADED	MAKO TORQ XL	•	•	8.7	9.6	10.6
LOADED	MAKO XL	•	•	8.7	9.6	10.6
LOADED	Z-CORE HYBRID XL	•	•	8.9	9.9	10.9
LOADED	Z-CORE XL	•	•	8.9	9.9	10.9

Less weight for a given length means a lower I. But moving the center-of-mass in closer to the rotation axis also lowers I.

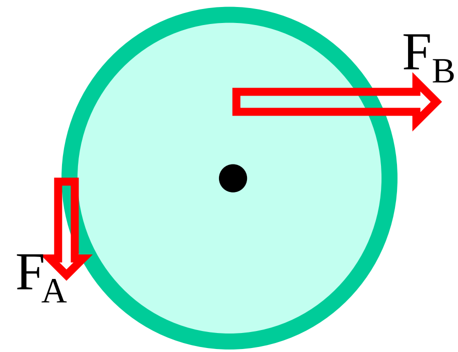
Two forces are exerted on a wheel which has a fixed axle at the center. Force A is applied at the rim. Force B is applied halfway between the axle and the rim. $|F_A| = ½|F_B|$.

Which best describes the direction of the angular acceleration?

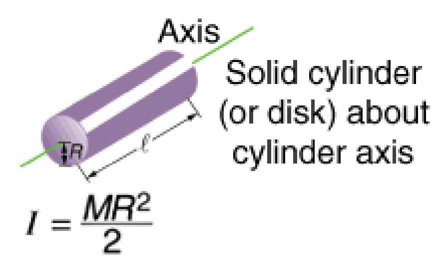
A. Counterclockwise

B. Clockwise

C. Zero



1. $\tau = I\alpha$
2. $\tau_B = -F_B(r_{rim}/2)$ $\tau_A = F_A(r_{rim}) = (F_B/2)(r_{rim}) = -\tau_B$
3. So, the torques cancel & the net torque is zero.
4. Therefore, the angular acceleration is zero.



A solid wood cylinder and an aluminum ring are released at the same time from the top of a ramp.
Which will roll to the bottom first?

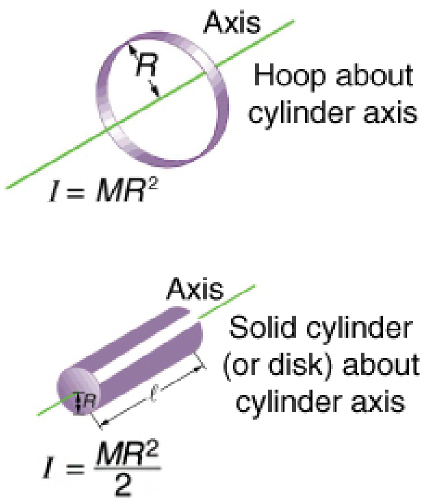
- A. Solid cylinder
- B. Ring
- C. They will tie
- D. Whichever is heavier
- E. Whichever is lighter
- F. Not enough information

https://www.youtube.com/watch?v=CHQOctEvtTY

1. The ring's mass is concentrated further from the rotation axis than the cylinder.
2. Therefore, the ring has a larger rotational inertia.
3. This means the ring is more resistant to rolling.
4. Therefore, the cylinder will start rolling earlier and make it to the bottom of the ramp first.

This is the same reason a broom is easier to balance brush-end up. It is more resistant to rotating that way.

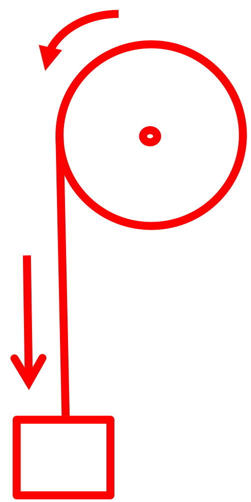
youthwork-pratice



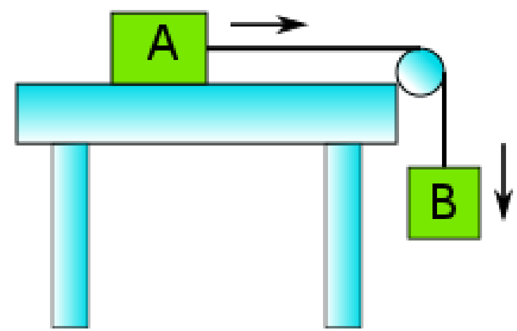
Pulleys & Rotational Inertia

- Previously, when dealing with pulleys, we noted they were massless….Why?
- Because the pulley has rotational inertia! It will resist rolling.
- Hanging a weight from a rope wrapped around a ***massive*** pulley will behave similarly to a weight hung from a rope attached to a mass on a frictionless table.

This situation: *…is quite a bit like this situation:*

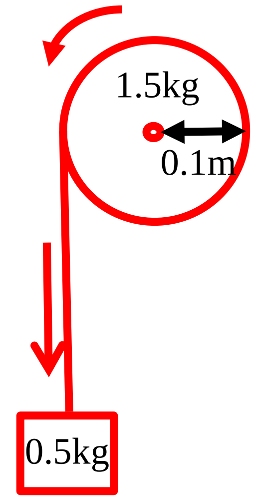


The pulley applies resistance through rotational inertia.

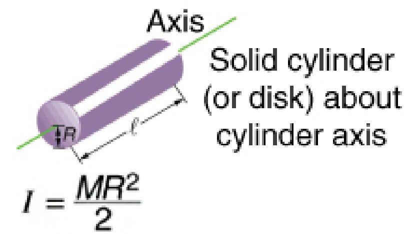


Block A applies resistance through inertia.

A 0.50kg weight is hung from a frictionless pulley which has a mass of 1.5kg and radius of 0.10m. How long does it take for the mass to fall 1m?

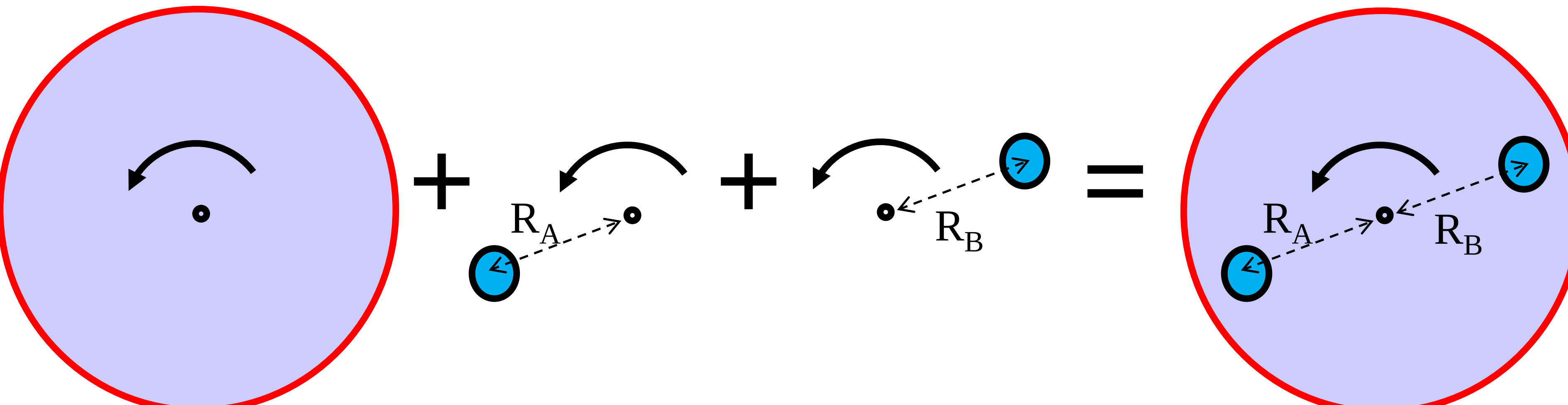


- Ultimately need the acceleration of the mass to turn this into a 1D kinematics problem
- Can use $\sum F = ma$
  - $\sum F = mg - T = ma$
  - So $a = g - \frac{T}{m}$ …but we don't know T
- Can work to a through the relation to rotational motion, $a = R\alpha$
  - $\tau = I\alpha = RT$, where $I = \frac{1}{2}MR^2$
  - So $\alpha = \frac{RT}{\frac{1}{2}MR^2} = 2\mathrm{T}/(\mathrm{MR})$
  - And therefore $a = R\alpha = \frac{2T}{M}$
- Set our a equal, $g - T/m = \frac{2T}{M}$
- Then $\frac{g}{\mathrm{T}} = \frac{1}{\mathrm{m}} + \frac{2}{\mathrm{M}} = \frac{\mathrm{M+2m}}{\mathrm{mM}}$, meaning $T = \frac{gmM}{(M+2m)}$
- Substituting in, $a = \frac{2gm}{(M+2m)}$
- Recall $y = y_i + v_i t + \frac{1}{2}at^2$.
- Since $y_i = 0$ and $v_i = 0$, $t = \sqrt{2y/a} = \sqrt{\frac{2y}{2gm}(M+2m)} = 0.71s$



Moment of Inertia: Multiple Objects

- Moments of inertia add together for a compound object.
- Example:
  Disk with two point masses on top, all spinning about the center of the disk
    - $I_{total} = I_{cylinder} + I_{point,A} + I_{point,B}$
    - $I_{total} = (½)M_{disk}(R_{disk})^2 + M_AR_A^2 + M_BR_B^2$



Each red dot represents a 1kg mass on a turntable.
Which of the three turntables requires the least torque to achieve a given angular velocity after some fixed amount of time?

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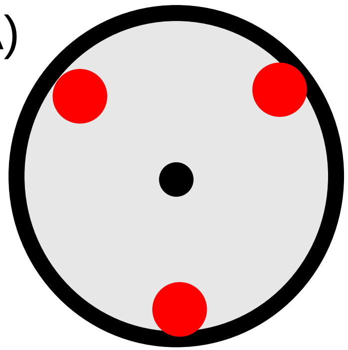
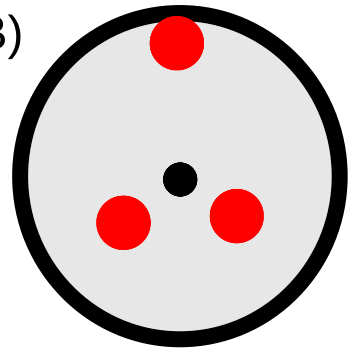
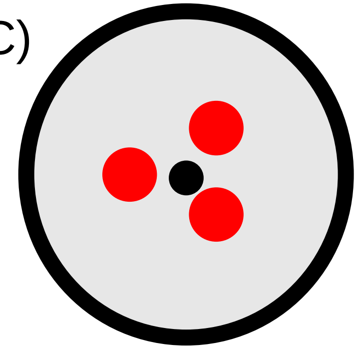
(A)



(B)



(C)



(D) All the same

1. $\tau = I\alpha$
2. $\alpha = \tau/I$
3. So, smallest moment of inertia will require least torque to achieve the same angular acceleration.
4. The mass in situation (C) is the most concentrated near the rotation axis, so it will have the smallest moment of inertia & therefore will accelerate the quickest.
5. Therefore, (C) will achieve a given rotational velocity first.

Could calculate: $I_{total} = (1/2)M_{cylinder}R^2 + M_1R_1^2 + M_2R_2^2 + M_3R_3^2$

Rotational Kinetic Energy

- Rotation has an associated kinetic energy
- As noted previously, by swapping-in the appropriate variables, our equations describing motion in 1D work for rotation as well:
  - $KE_{rotation} = \frac{1}{2}I\omega^2$
- For example, a rolling object has both translational & rotational kinetic energy
  - $KE_{total} = KE_{translation} + KE_{rotation} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- Can be used to store energy!

  …often used to smooth-out power availability for intermittent power sources

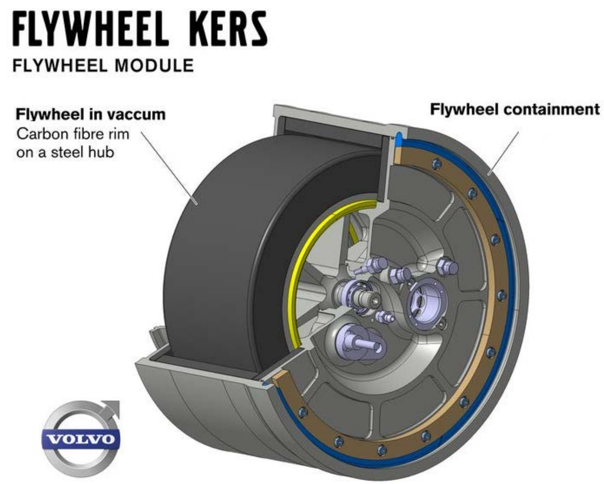
Pottery Wheel

Engine Flywheel

https://www.youtube.com/watch?v=reh79gYsaQE

An engineer is designing a flywheel for a hybrid vehicle which will be used to store energy by engaging the engine while braking. The maximum rotational velocity of the flywheel will be limited by the strength of the chosen material. The engineer wants to maximize the power storage capability of the flywheel. Compared to a nominal design, should they choose a material with twice as much mass or twice as high of a maximum rotational velocity?

A. Double the mass
B. $\boxed{\text{Double the maximum rotational velocity}}$
C. Either will give the same result



1. $KE_{rotation} = \frac{1}{2} I\omega^2$
2. The flywheel inertia, I, will be proportional to the mass:
   $I_{disk} = (1/2)MR^2$
3. Doubling M will double I and therefore double KE.
4. But, doubling ω will quadruple KE.