

Thursday March 7

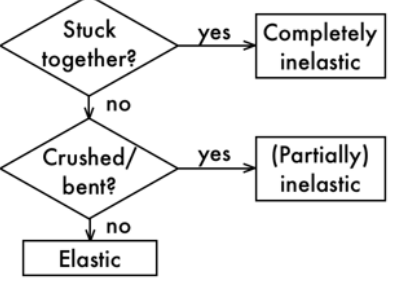
Topics for this Lecture:

- Momentum

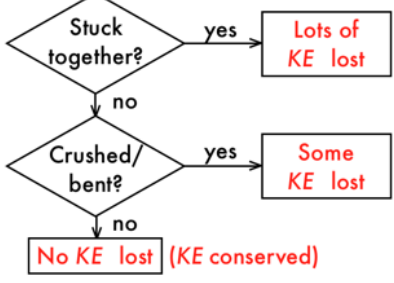
$$\sum \vec{F}\Delta t = \Delta\vec{p}$$

Impulse = $\vec{J} = \Delta\vec{p}$

Collision Type Flowchart



Kin. Energy Conservation



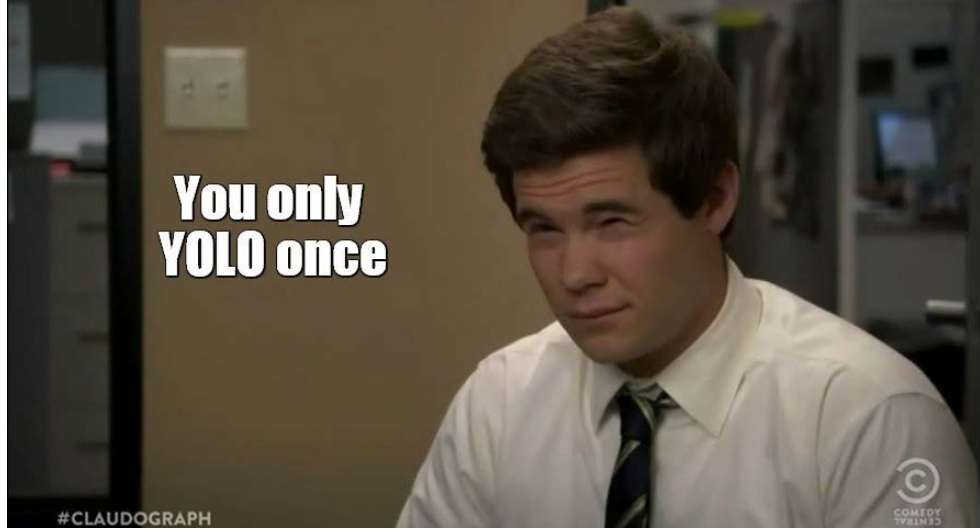
Patrick M. Len

- Assignment 8 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 226, M&Tu 6:20-7:10pm
- Office Hours: 204 EAL, 3-4pm Thurs or by appointment (meisel@ohio.edu)

Feel free to study over spring break.
Exam 2 is 3 weeks away.

Momentum is a vector and is conserved:

$$\sum p_{x,i} = \sum p_{x,f} \quad \sum p_{y,i} = \sum p_{y,f}$$





A tennis ball cannon is mounted on sled in the middle of a frozen pond.

The total mass of the unit (unloaded) is 50kg.

The speed of each ball after being fired is 30m/s and the mass of each ball is 0.10 kg.

How fast is the unit moving after it shoots 10 tennis balls to the right?

Presume it starts at rest.

(A) 0.06 m/s left

(B) 0.06 m/s right

(C) 0.6 m/s left

(D) 0.6 m/s right

(E) 6 m/s left

(F) 6 m/s right

1. $p_{\text{initial}} = p_{\text{final}}$
2. $p = mv$ & $v_i = 0$..so $p_i = 0$
3. $0 = 10 * p_{\text{ball},f} + p_{\text{cannon},f}$
4. $p_{\text{cannon},f} = -10 p_{\text{ball},f}$
5. $m_c v_{c,f} = -10 m_b v_{b,f}$
6. $v_{c,f} = -10 m_b v_{b,f} / m_c$
7. $v_{c,f} = (-10 * 0.1 \text{kg} * 30 \text{m/s}) / 50 \text{kg}$
8. $v_{c,f} = (-30 / 50) \text{m/s} = 0.6 \text{m/s left}$

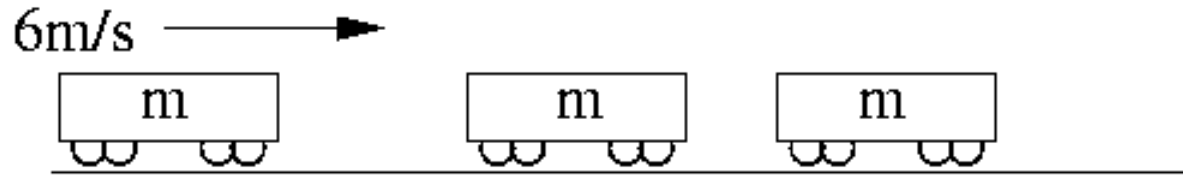


Three railroad cars of equal mass M are on a frictionless track.

The one on the left is approaching at $6m/s$.

At the end, the two left cars are linked and traveling at $1m/s$, but the right-most car didn't connect properly and is free.

What is the speed of the separate un-linked car?



(A) 0 m/s

(B) 2 m/s

(C) 3 m/s

(D) 4 m/s

(E) 5 m/s

(F) 6 m/s

(G) 12 m/s

(H) 18 m/s

1. $p_{\text{initial}} = p_{\text{final}}$

2. $p_{1,i} + p_{2,i} + p_{3,i} = p_{1+2,f} + p_{3,f}$

3. $(m)6m/s + 0 + 0 = (2m)(1m/s) + mv_{3,f}$

4. $(6m)m/s = (2m)m/s + mv_{3,f}$

5. $6m/s = 2m/s + v_{3,f}$

6. $v_{3,f} = 4m/s$

You're driving home from a bug-spray convention, moving along the highway at 30m/s in your 1000kg wagon.

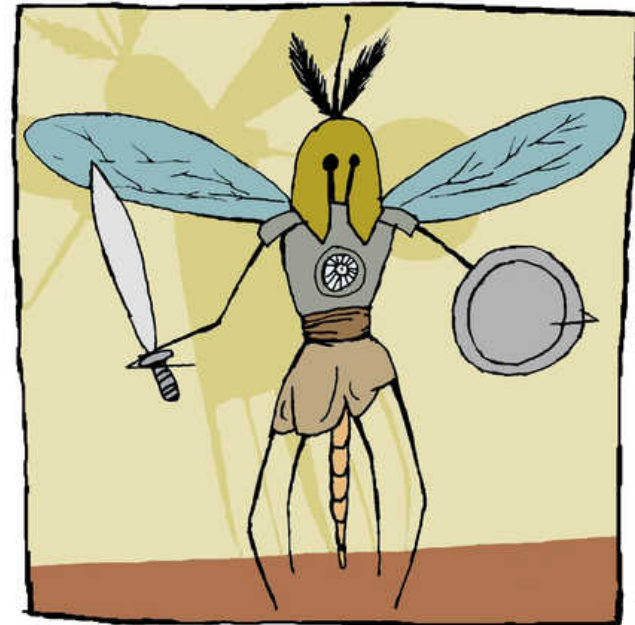
An army of mosquitos ($m_{\text{mosquito}} = 5\text{milligrams}$) has decided enough is enough and they ram your vehicle, ultimately sticking themselves to your windshield.

If the mosquito army charges your car at 2m/s, how many mosquitos would it take to slow you down by 0.001m/s?

Note that the army will not significantly increase the mass of the wagon.

- (A) 10^3 (B) 10^4 (C) 10^5
 (D) 10^6 (E) 10^7 (F) 10^8

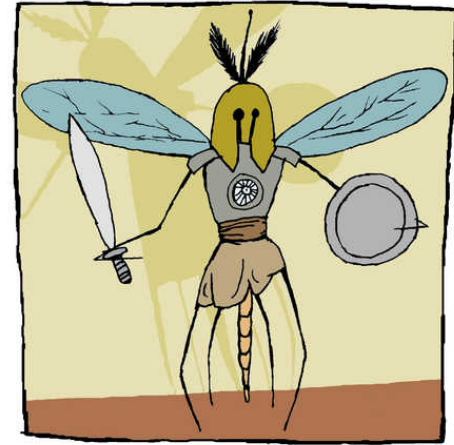
1. $p_{\text{initial}} = p_{\text{final}}$
2. $p_{\text{wagon},i} + p_{\text{army},i} = p_{\text{wagon+army},f}$
3. $m_w v_{w,i} + N_m m_m v_{m,i} = (m_w + N_m m_m) v_f$
4. "army will not significantly increase the mass of the wagon": $m_w + N_m m_m \rightarrow m_w$
5. $m_w v_{w,i} + N_m m_m v_{m,i} = m_w v_f$
6. $N_m = (m_w v_f - m_w v_{w,i}) / (m_m v_{m,i})$
7. "slow you down by 1m/s": $v_f - v_{w,i} = -0.001\text{m/s}$
8. $N_m = 1000\text{kg}(-0.001\text{m/s}) / (5\text{e-}6\text{kg}(-2\text{m/s}))$
9. $N_m = (1\text{kgm/s}) / (1\text{e-}5\text{kgm/s}) = 100,000 = 10^5$



Learning his lesson, a lone mosquito survivor with a mass of 5mg is retreating at 2m/s. Your wagon, which has a mass of 1000kg, hits him at 30m/s, sticking the single mosquito to your windshield and slightly increasing the mass of your vehicle.

By how much does your wagon slow down?

- (A) 10^{-3} m/s (B) 10^{-4} m/s (C) 10^{-5} m/s
 (D) 10^{-6} m/s (E) 10^{-7} m/s (F) 10^{-8} m/s



1. $p_{\text{initial}} = p_{\text{final}}$
2. $p_{\text{wagon},i} + p_{\text{mosquito},i} = p_{\text{wagon+mosquito},f}$
3. $m_w v_{w,i} + m_m v_{m,i} = (m_w + m_m) v_f$
4. $v_f = \{m_w v_{w,i} + m_m v_{m,i}\} / \{m_w + m_m\}$
5. $v_f = \{1000\text{kg} \cdot 30\text{m/s} + (5 \times 10^{-6}\text{kg}) 2\text{m/s}\} / \{1000\text{kg} + 5 \times 10^{-6}\text{kg}\}$
6. $v_f = 29.99999986\text{m/s}$
7. $v_i - v_f = 1.4 \times 10^{-7} \text{ m/s} \sim 100\text{nm/s}$

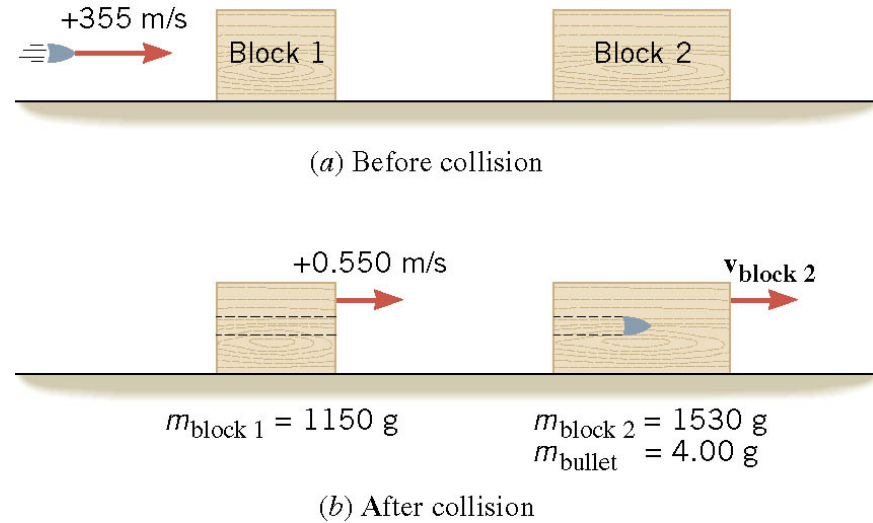
Hair grows at $\sim 15\text{cm/yr}$...which is $\sim 5\text{nm/s}$

A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

What is the initial momentum of the blocks+bullet system?

- (A) 1.42 kgm/s
- (B) 1420 kgm/s
- (C) -1.42 kgm/s
- (D) -1420 kgm/s
- (E) 142 kgm/s
- (F) -142kg m/s



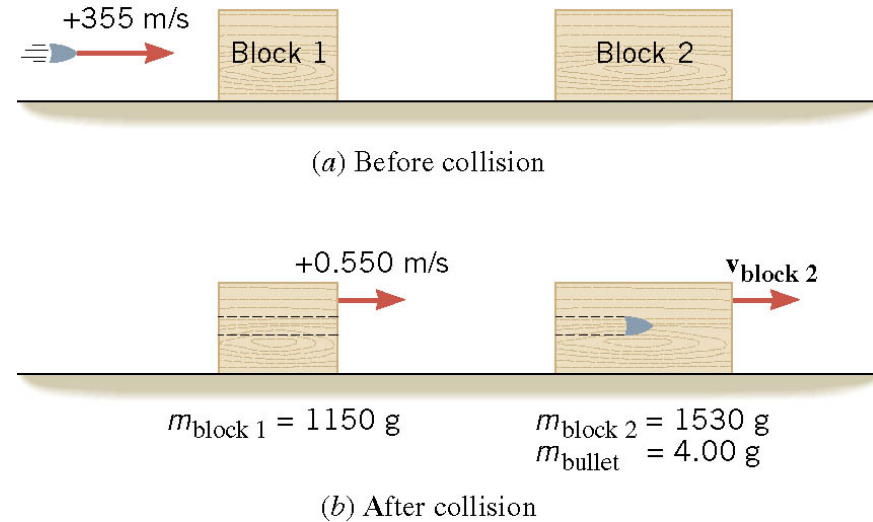
1. $p_{\text{initial}} = p_{\text{bullet},i} + p_{\text{block1},i} + p_{\text{block2},i}$
2. $p = mv$
3. only the bullet has non-zero initial velocity
4. $p_{\text{initial}} = p_{\text{bullet},i} = m_{\text{bullet}} v_{\text{bullet},i}$
5. $p_{\text{initial}} = (4 \times 10^{-3} \text{ kg})(355 \text{ m/s})$
6. $p_{\text{initial}} = 1.42 \text{ kgm/s}$

A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

What is the final momentum of the blocks+bullet system?

- (A) 0 kgm/s
 (B) 0.142 kgm/s
 (C) 1.42 kgm/s
 (D) -1.42 kgm/s
 (E) 0.613 kgm/s
 (F) 2.03 kg m/s



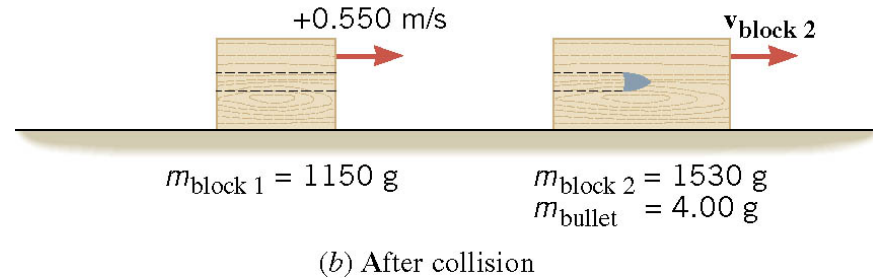
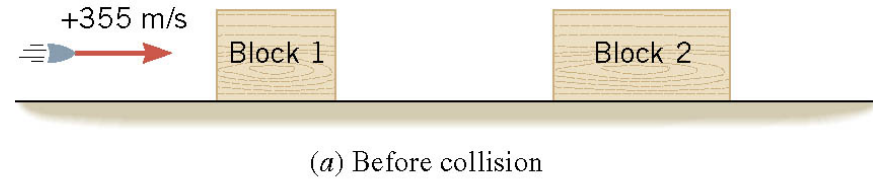
- Momentum is conserved [no external forces]
- $p_{\text{final}} = p_{\text{initial}} = p_{\text{bullet},i} = (4 \times 10^{-3} \text{ kg})(355 \text{ m/s}) = 1.42 \text{ kgm/s}$

A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

What is the final velocity of the second block?

- (A) 0.00 m/s
- (B) 0.550 m/s
- (C) 1.42 m/s
- (D) 355 m/s
- (E) 0.920 m/s
- (F) 0.513 m/s**



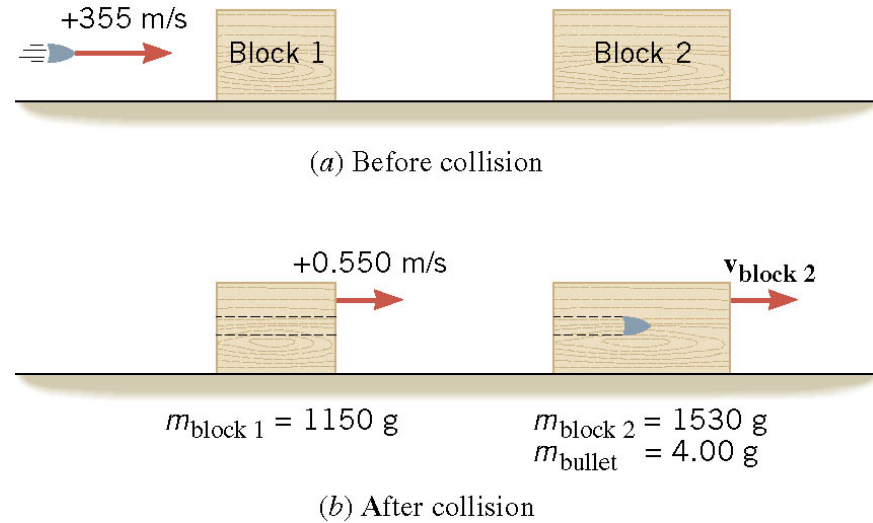
1. $p_{\text{initial}} = p_{\text{final}}$
2. $p_{\text{bullet},i} = p_{\text{bullet},f} + p_{\text{block2},f} + p_{\text{block1},f}$
3. $m_{\text{bullet}} v_{\text{bullet},i} = (m_{\text{bullet}} + m_{\text{block2}}) v_f + m_{\text{block1}} v_{\text{block1},f}$
4. $1.42 \text{ kgm/s} = (1.534 \text{ kg}) v_f + 0.632 \text{ kgm/s}$
5. $v_f = (1.42 \text{ kgm/s} - 0.632 \text{ kgm/s}) / (1.534 \text{ kg})$
6. $v_f = 0.513 \text{ m/s}$

A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

What is the ratio of the final kinetic energy to the initial kinetic energy (KE_f/KE_i)?

- (A) 0.0 (B) 1.0×10^{-3}
 (C) 0.38 (D) 1.5×10^{-3}
 (E) 5.0×10^{-2} (F) 1.0



$$1. KE_i = \frac{1}{2} m_{\text{bullet}} v_{\text{bullet},i}^2 = 252 \text{ J}$$

$$2. KE_f = \frac{1}{2} m_{\text{block 1}} v_{\text{block 1},f}^2 + \frac{1}{2} (m_{\text{block 2}} + m_{\text{bullet}}) v_f^2 = 0.376 \text{ J}$$

$$3. KE_f / KE_i = 1.5 \times 10^{-3}$$

Since the bullet & block are stuck together, this is perfectly inelastic. For perfectly inelastic collisions, "lots" of kinetic energy is lost. Now you see how much "lots" means.



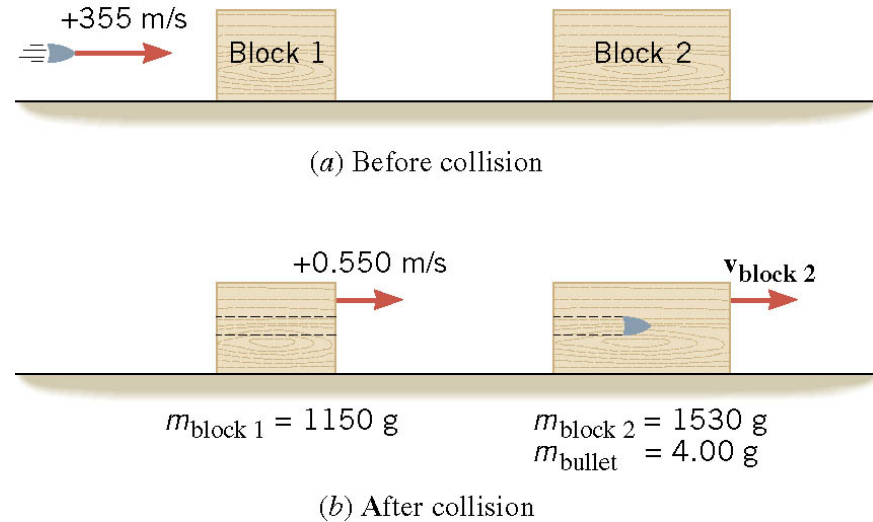
A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

What is the speed of the bullet after going through block 1, but before going into block 2?

- (A) 355 m/s
- (C) 354 m/s
- (E) 142 m/s

- (B) 197 m/s**
- (D) 0 m/s
- (F) 19.7 m/s



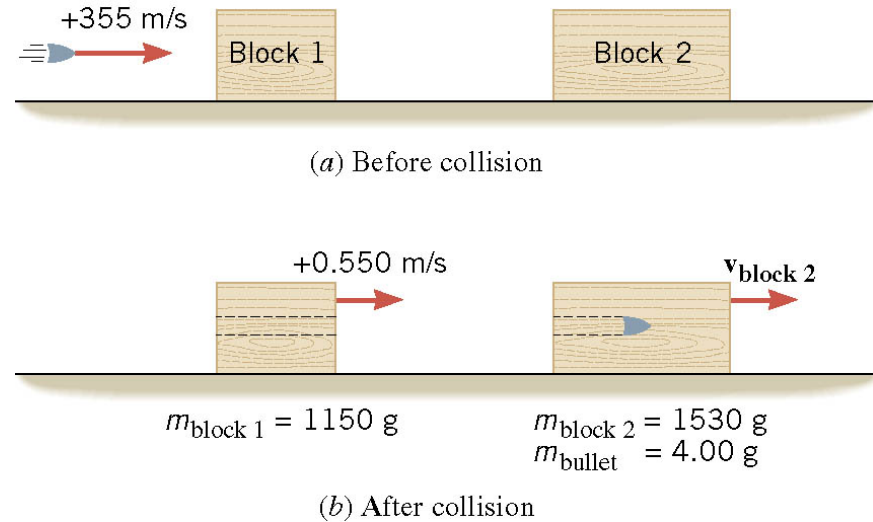
1. $p_{\text{initial}} = p_{\text{final}}$
2. $p_{\text{bullet},i} = p_{\text{bullet},f} + p_{\text{block2},f} + p_{\text{block1},f}$
3. $m_{\text{bullet}} v_{\text{bullet},i} = m_{\text{bullet}} v_{\text{bullet},f} + m_{\text{block2}} (0) + m_{\text{block1}} v_{\text{block1},f}$
4. $1.42 \text{ kgm/s} = (0.004\text{kg})v_f + 0.632\text{kgm/s}$
5. $v_f = (1.42\text{kgm/s} - 0.632\text{kgm/s})/(0.004\text{kg}) = 197 \text{ m/s}$

A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

If the bullet takes 0.4ms to go through block 1, what is the force of the bullet on the block?

- (A) 0.158 N (B) 158 N
 (C) 1580 N (D) 0 N
 (E) -1580 N (F) -158 N



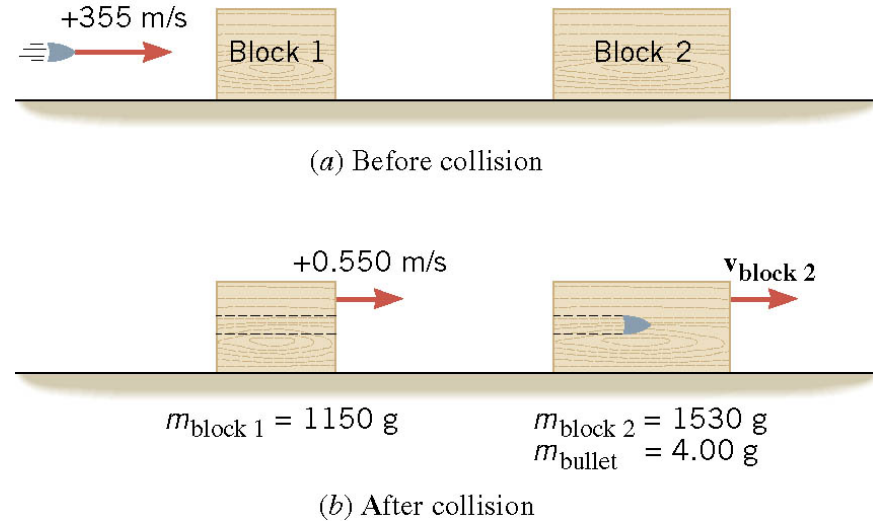
1. $F_{\text{on block 1}} \Delta t = \Delta p_{\text{block 1}}$
2. $F_{\text{on block 1}} (4 \times 10^{-4} \text{ s}) = p_{\text{block 1, f}} - p_{\text{block 1, i}}$
3. $F (4 \times 10^{-4} \text{ s}) = m_{\text{block 1}} v_{\text{block 1, f}} - 0$
4. $F = (1.150 \text{ kg} \cdot 0.550 \text{ m/s}) / (4 \times 10^{-4} \text{ s}) = 1580 \text{ kgm/s}^2 = 1580 \text{ N}$

A 4.00-g bullet is moving horizontally with a velocity of +355 m/s, where the + sign indicates that it is moving to the right.

The mass of the first block is 1150 g, and its velocity is +0.550 m/s after the bullet passes through it. The mass of the second block is 1530 g.

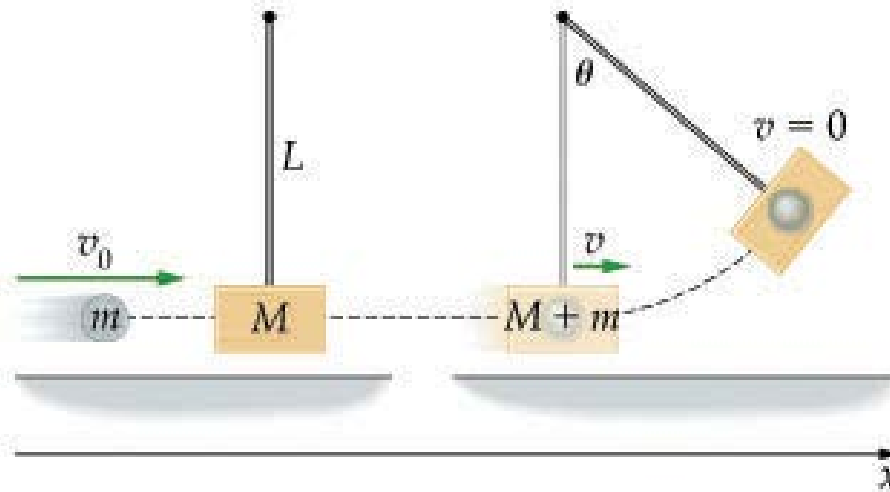
If the bullet takes 0.4ms to go through block 1, what is the force of the **block on the bullet**?

- (A) 0.158 N (B) 158 N
 (C) 1580 N (D) 0 N
 (E) -1580 N (F) -158 N



1. An action force will have an equal & opposite reaction force.
2. $F_{\text{bullet-on-block}} = -F_{\text{block-on-bullet}} = 1580\text{N}$
3. $F_{\text{block-on-bullet}} = -1580\text{N}$

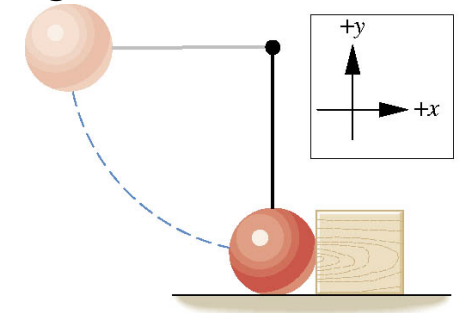
“Ballistic Pendulum” (lab after wooo spring break)



- Perfectly inelastic collision: Ball launched via spring mechanism into pendulum
- Will determine change in KE
- Will get final KE using conservation of energy (measuring final PE) & conservation of momentum
- Will get initial KE by measuring initial velocity (using 1D kinematics)

A ball is attached to a wire, held horizontally, and dropped. It strikes a block that is sitting on a horizontal, frictionless surface. Air resistance is negligible and the collision is elastic. The block is more massive than the ball. Which of the following are conserved as the ball swings down?

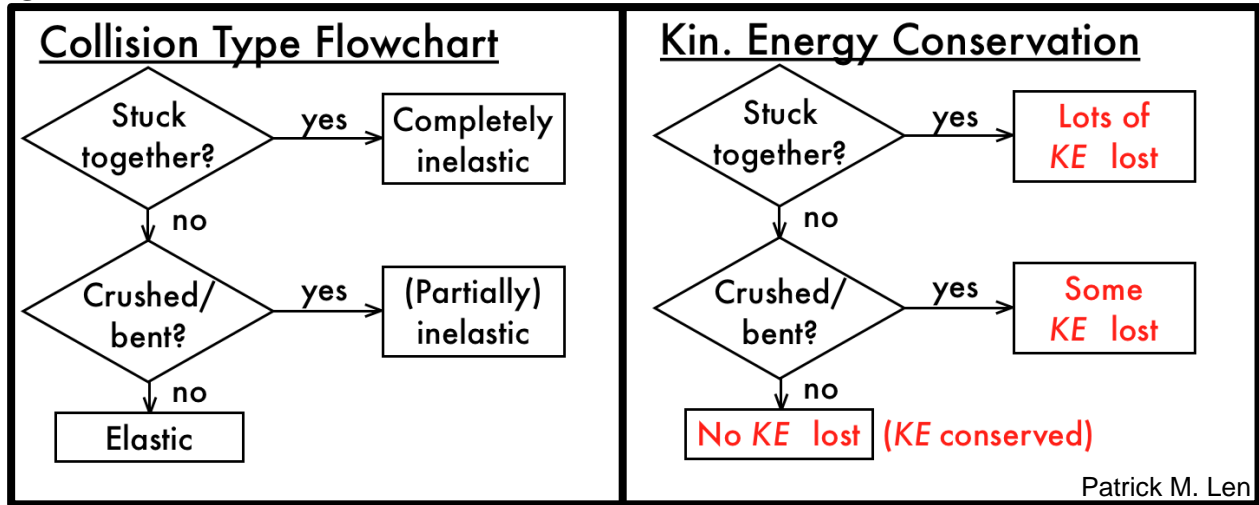
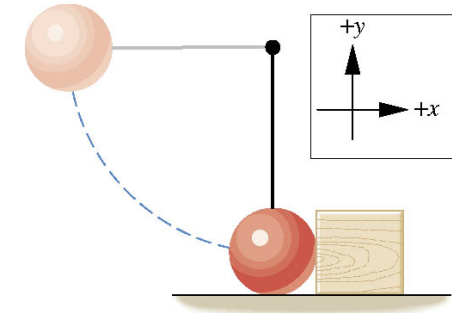
- (A) Ball's Kinetic Energy
- (B) Ball's Momentum
- (C) Ball's Total Mechanical Energy**
- (D) A & B (E) A & C (F) B & C
- (G) A, B, & C



1. Initial kinetic energy is zero (at rest) ...but clearly moving at the bottom of the swing. So KE clearly not conserved (Nor need it be!)
2. Similarly, momentum clearly zero (at rest) but moving at bottom of swing. Why is momentum not conserved? Ball acted-on by external forces! (tension+gravity)
3. Even if the pendulum were initially swinging, velocity would be downward at first, then horizontal. Tension is an outside force acting on the ball, converting p_y to p_x .
4. Energy is always conserved. Here PE is converted into KE.

A ball is attached to a wire, held horizontally, and dropped. It strikes a block that is sitting on a horizontal, frictionless surface. Air resistance is negligible and the **collision is elastic**. The block is more massive than the ball. Which of the following are conserved during the collision?

- (A) Horizontal component of momentum for ball+block system
- (B) Total KE of ball+block system
- (C) Both A & B**
- (D) Neither A nor B

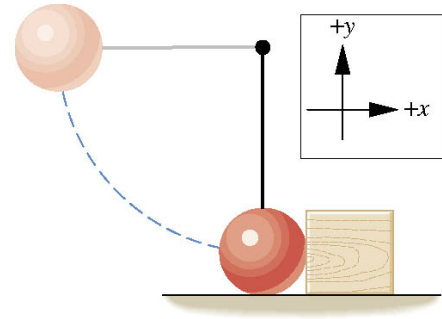


A ball is attached to a wire, held horizontally, and dropped. It strikes a block that is sitting on a horizontal, frictionless surface. Air resistance is negligible and the collision is elastic. The block is more massive than the ball. Which direction will the ball be traveling after the collision?

- (A) Left
- (B) At rest
- (C) Right



<https://www.youtube.com/watch?v=GZHJaLMnI1o>



• Say initially object 1 is moving and object 2 is at rest.
 i.e. $m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$ and $\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$
 ...using these two equations and some algebra: $v_{1,f} = \frac{1}{2} \left(1 - \frac{m_2}{m_1} \right) v_{2,f}$

- if:
- A. $m_1 > m_2$:** $\frac{m_2}{m_1} < 1$...meaning: $v_{1,f} > 0$...so both 1 and 2 move forward
 - B. $m_1 < m_2$:** $\frac{m_2}{m_1} > 1$...meaning $v_{1,f} < 0$...so 1 bounces back & 2 moves slowly forward
 - C. $m_1 = m_2$:** $v_{1,f} = 0$...meaning $v_{2,f} = v_{1,i}$...so they swap velocities!

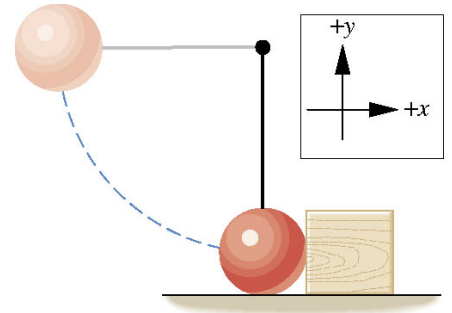
A 1.60kg ball is attached to a 1.20m-long wire, held horizontally, and dropped.

It strikes a 2.40kg block that is sitting on a horizontal, frictionless surface.

Air resistance is negligible and the collision is elastic.

What is the velocity of the ball just before the collision?

- (A) 4.33m/s (B) 18.8m/s (C) 23.5m/s (D) 4.85m/s (E) 3.96m/s



1. Total energy is conserved: $KE_i + PE_i = KE_f + PE_f$
2. Initial state: just before ball is released
 1. At rest: $KE_i = 0$.
 2. $PE_i = mgh$
3. Final: just before ball hits block
 1. At reference height $h=0$: $PE_f = 0$
 2. $KE_f = (1/2)mv_f^2$
4. $KE_f = (1/2)mv_f^2 = PE_i = mgh$
5. $v_f = \sqrt{\frac{2}{m}mgh} = \sqrt{2gh} = \sqrt{2(9.80 \frac{m}{s^2})(1.20m)} = 4.85m/s$

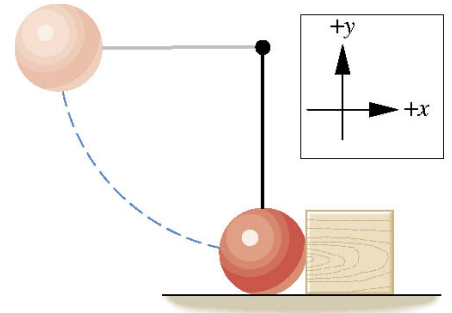
A 1.60kg ball is attached to a 1.20m-long wire, held horizontally, and dropped.

It strikes a 2.40kg block that is sitting on a horizontal, frictionless surface.

Air resistance is negligible and the collision is elastic.

What is the velocity of the ball just **after** the collision?

- (A) +0.97m/s (B) 0 m/s (C) -0.97m/s (D) 4.85m/s (E) -4.85m/s



1. Elastic collision, so momentum is conserved & kinetic energy is conserved.

2. For an elastic collision, object 1 colliding into object 2:

1. $v_{ball,f} = \left\{ \frac{(m_{ball} - m_{block})}{(m_{ball} + m_{block})} \right\} v_{ball,i}$

3. $v_{ball,f} = \left\{ \frac{(1.60\text{kg} - 2.40\text{kg})}{(1.60\text{kg} + 2.40\text{kg})} \right\} 4.85\text{m/s}$

4. $v_{ball,f} = -0.97\text{m/s}$

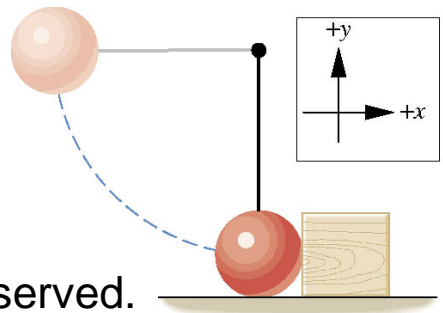
*Just write this equation down.
It's a bit of a pain to prove:*

For an elastic collision:

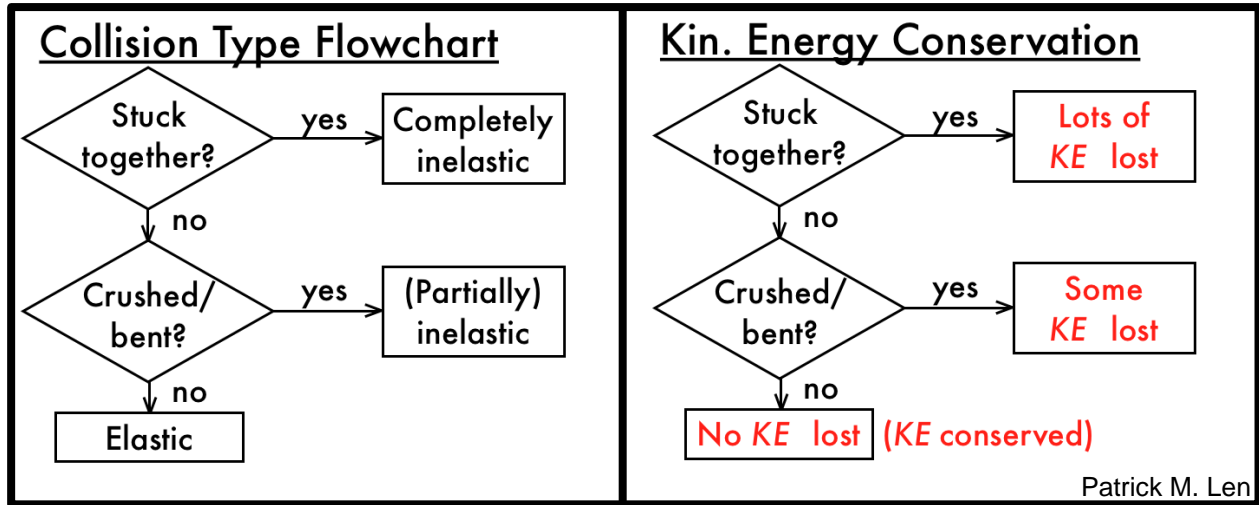
$$v_{1,f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1,i}$$

A 1.60kg ball is attached to a 1.20m-long wire, held horizontally, and dropped. It strikes a 2.40kg block that is sitting on a horizontal, frictionless surface. Air resistance is negligible and the collision is elastic. What is the kinetic energy loss of the system due to the collision?

- (A) 0.0 J (B) 19 J (C) 6.2 J (D) 7.7 J (E) 1.6 J

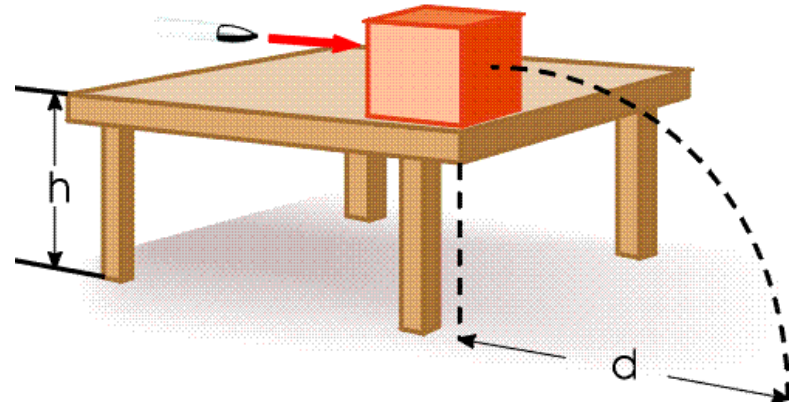


1. Elastic collision, so momentum is conserved & kinetic energy is conserved.



Multi-concept problem: *Bullet and Block* (on HW 9)

A bullet hits a block on a table, launching the block (with the bullet embedded) off of the table. How far does the block travel?



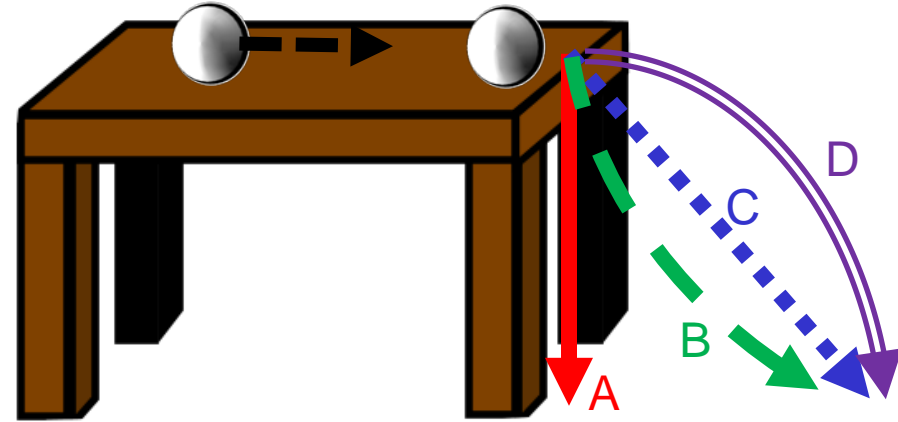
1. Use momentum conservation/collisions to find block velocity before leaving table top.
2. Use kinematics to find:
 1. Flight-time from free-fall
 2. Range traveled in flight-time

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}(v_{y,0} + v_y)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

A ball with mass m is traveling on a frictionless table-top at $2m/s$ and strikes an identical ball elastically, knocking it off of the table top. Which path best represents the path of the second ball after being struck?

- (A) A (B) B (C) C **(D) D**
(E) None of these

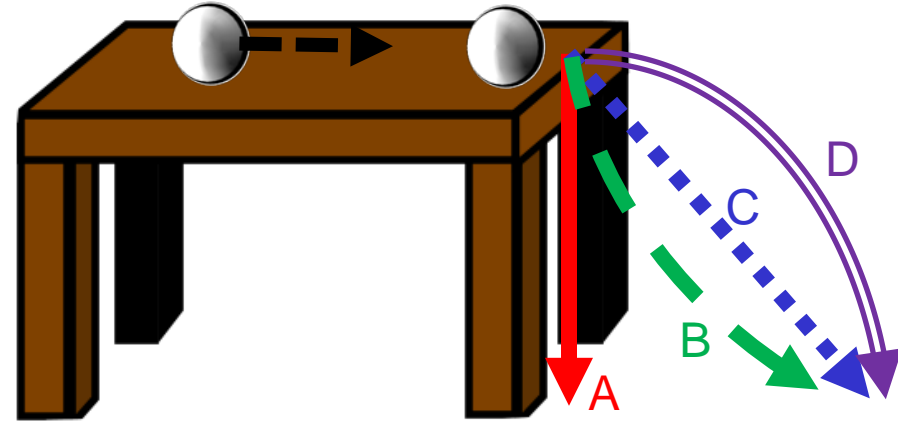


- Projectile motion looks like path D.
- Think about the fact that there is initially zero vertical velocity, but this will steadily increase due to acceleration at little g .

A ball with mass m is traveling on a frictionless table-top at $2m/s$ and strikes an identical ball elastically, knocking it off of the table top.
Which path best represents the path of the first ball, after it strikes the second ball?

(A) A (B) B (C) C (D) D

(E) None of these



1. Elastic collision with two identical masses, one initially at rest.
2. They swap velocities!
3. Therefore no path will be taken, the first ball will end at rest.

• Say initially object 1 is moving and object 2 is at rest. ① → ②
 i.e. $m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$ and $\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$
 ...using these two equations and some algebra: $v_{1,f} = \frac{1}{2} \left(1 - \frac{m_2}{m_1} \right) v_{2,f}$

If:

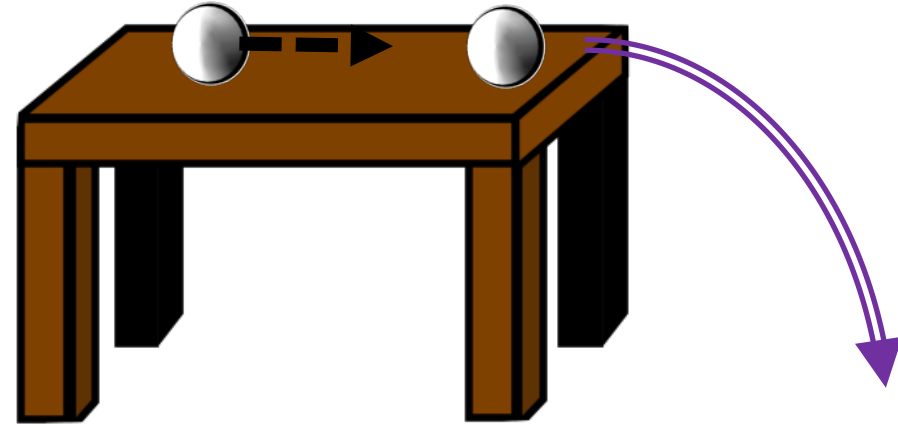
A. $m_1 > m_2$: $\frac{m_2}{m_1} < 1$...meaning: $v_{1,f} > 0$...so both 1 and 2 move forward

B. $m_1 < m_2$: $\frac{m_2}{m_1} > 1$...meaning $v_{1,f} < 0$...so 1 bounces back & 2 moves slowly forward

C. $m_1 = m_2$: $v_{1,f} = 0$...meaning $v_{2,f} = v_{1,i}$...so they swap velocities!

A ball with mass m is traveling on a frictionless table-top at $2m/s$ and strikes an identical ball elastically, knocking it off of the table top. If it takes 1 second for the second ball to hit the ground after leaving the table-top, how far away horizontally will the second ball be from the first when it hits the ground?

- (A) $0.5m$ (B) $1m$ (C) $2m$ (D) $9.8m$
(E) Not enough information



1. Elastic collision with two identical masses, one initially at rest.
2. They swap velocities!
3. Horizontal motion is separate from vertical motion.
4. So, in 1 second, the horizontal distance travelled will be $(2m/s) \cdot (1s) = 2m$

A 1000kg top fuel dragster starts at rest and reaches 44m/s in 0.8 seconds.
How much power does this take?

- (A) 90,000 W (B) 1,210,000 W
(C) 900,000 W (D) 968,000 W

1. Power = (Change in Energy)/Time
2. $P = (KE_f - KE_i) / \Delta t = KE_f / \Delta t$
3. $P = \{(1/2)mv^2\} / \Delta t$
4. $P = \{(1/2)(1000\text{kg})(1936\text{m}^2/\text{s}^2)\} / 0.8\text{s}$
5. $P = 1,210,000\text{W}$



A 1000kg top fuel dragster starts at rest and reaches 44m/s in 0.8 seconds. It burns 1 gallon of nitromethane, which has an energy content of 5×10^7 J/gallon. What is the efficiency of the dragster's engine?

- (A) 0.2%
- (B) 2%
- (C) 35%
- (D) 100%



A typical automobile engine is ~1/3 efficient.

1. (Actual Power) = (Theoretical Power)*(Efficiency)
2. Actual Power = (Change in Energy)/Time
3. $P_{act} = (KE_f - KE_i) / \Delta t = KE_f / \Delta t$
4. $P_{act} = \{(1/2)mv^2\} / \Delta t$
5. $P_{act} = \{(1/2)(1000kg)(1936m^2/s^2)\} / 0.8s$
6. $P_{act} = 1,210,000W$
7. Efficiency = $\epsilon = P_{act} / P_{theory}$
8. $P_{theory} = (1gallon * 5 \times 10^7 J/gallon) / 0.8s = 6.25 \times 10^7 W$
9. $\epsilon = (1.21 \times 10^6 W) / (6.25 \times 10^7 W) \sim 0.02 = 2\%$