Tuesday March 14

<u>Topics for this Lecture</u>:

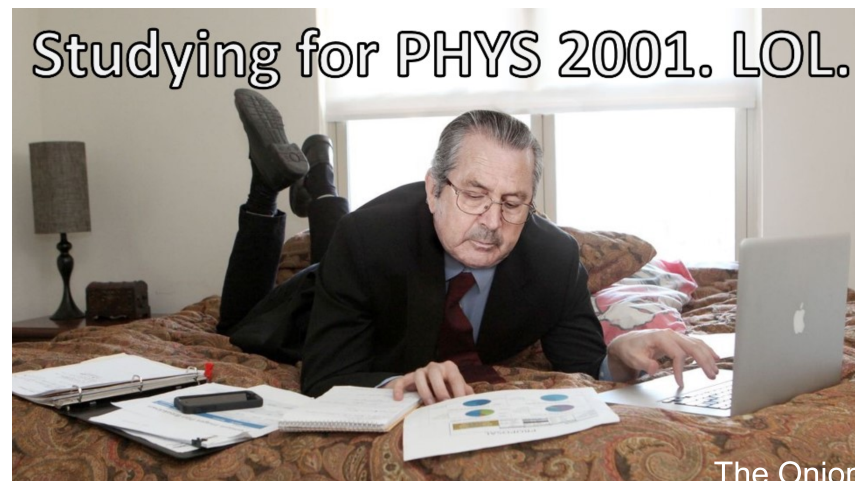
- *Circular Motion*
  - *Angular frequency*
  - *Centripetal force/acceleration*
  - *"Fictitious" (a.k.a. Inertial) forces:*
    - *Centrifugal force*
    - *Coriolis effect*
  - *Gravity & orbits*

*Anything undergoing circular motion is experiencing acceleration.

So, this motion is "non-inertial".

- Assignments <u>**8 & 9**</u> due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed or by appointment (meisel@ohio.edu)
- Midterm 2: Monday March 27th 7:15-9:15pm Morton Hall Room 201

Midterm Exam 2 is in two weeks. ***Study!***



Circular Motion

Why do hurricanes swirl?

How do you simulate weightlessness on earth?

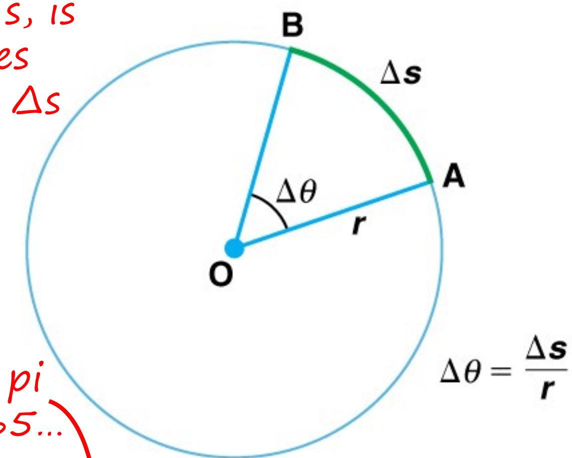
How fast do I have to spin my car tires to achieve a given linear speed?

How do satellites stay in orbit?

Circular Motion: The Basics

📖 Sect 6.1

arc length, s, is sometimes written as Δs



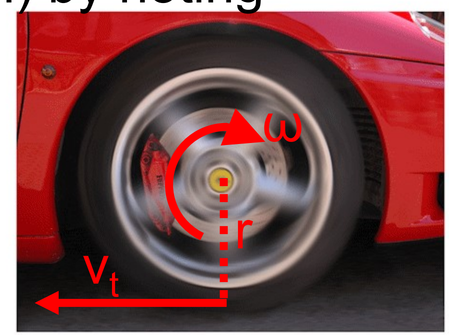
- Radians
  - Unit of angle
  - Defined as: $\boxed{\theta(\text{rad}) = (\text{arc length})/(\text{radius}) = s/r}$
  - "radian" is really just a place-holder, since a length over a length is unit-less

    Greek letter pi = 3.14159265…
  - Arc length of a full circle, the circumference, is $s = c = 2\pi r$ … so for a full circle, $\theta(\text{rad}) = 2\pi r/r = \boxed{2\pi = 360°}$

- Angular velocity:

  Greek letter omega
  - Rate of rotation can be described by angular frequency: $\boxed{\omega = \Delta\theta/\Delta t}$
  - We can convert this to revolutions per unit time (e.g. RPM) by noting that there are 2π radians per revolution
  - We can get a transverse velocity at a given radius by multiplying the angular frequency by the radius:

    $\boxed{v_t = r{*}\omega}$



A wheel undergoes an angular displacement of $\pi/3$ radians. What is this displacement in degrees?

(A) 1.04°   (B) 104°   (C) 15°   (D) 60°

1. 2π radians = 360°
2. So, π radians = 180°, i.e. (π rad)/180° = 1
3. (π/3 rad)*(180°/ π rad) = 180°/3 = 60°

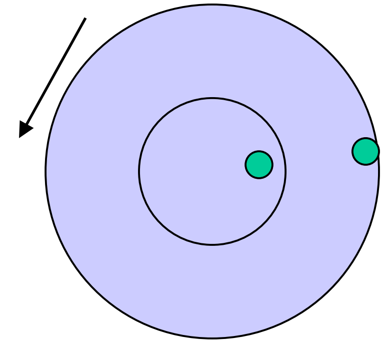
An object is rotating at an angular velocity of 0.5rad/s.
What is the total angular displacement after rotating 10s?

(A) 0.5rad (B) 5rad (C) 10rad (D) 50rad

1. $\omega=\Delta\theta/\Delta t$
2. $\Delta\theta = \omega * \Delta t$
3. $\Delta\theta = (0.5rad/s)*10s = 5rad$

Two objects are attached to a rotating turntable. One is much farther out from the axis of rotation. Which one has the larger angular velocity?

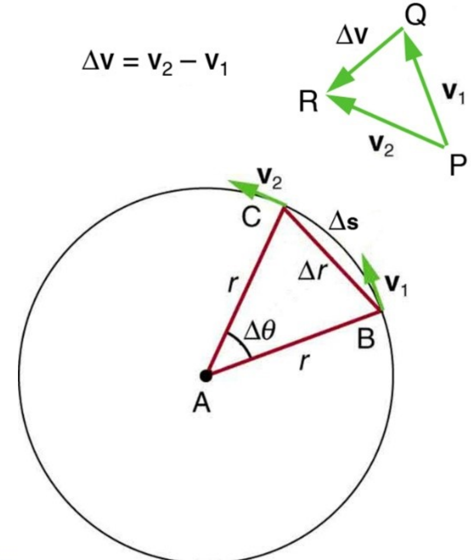
(A) the one nearer the disk center
(B) the one nearer the disk edge
(C) they both have the same angular velocity



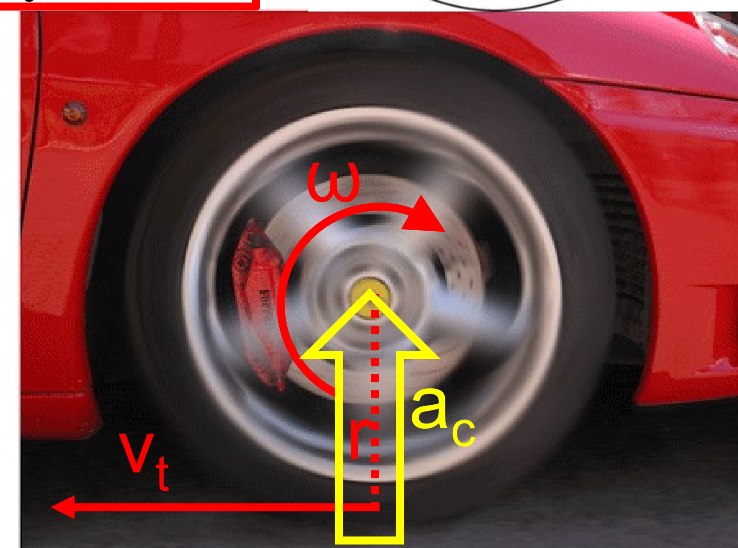
1. $\omega = \Delta\theta / \Delta t$
2. Both objects are located at the same angle θ and they will sweep through the same angular range $\Delta\theta$ in the same amount of time Δt.
3. So they have the same angular velocity. (& same angular acceleration $\boxed{\alpha = \Delta\omega / \Delta t}$)

Greek letter alpha

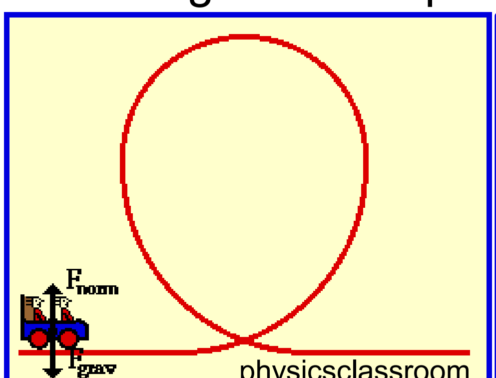
Circular Motion: Acceleration

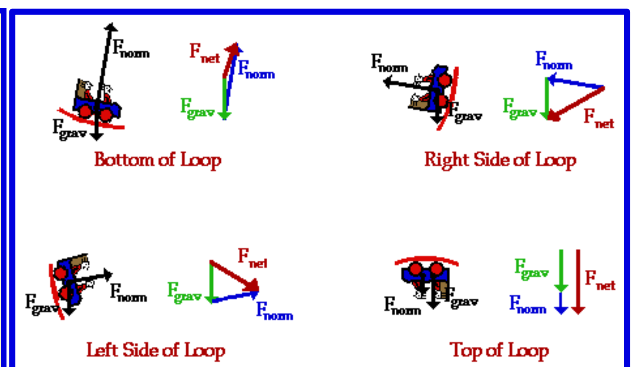


- Centripetal acceleration:
  - Velocity is a vector, i.e. it has an associated direction.
  - Changing direction while moving means changing velocity.
  - Therefore an object moving in a circle is accelerating.
  - This acceleration is called "centripetal acceleration".
  - Centripetal acceleration is pointed inwards (toward the rotation axis) with a magnitude: $\boxed{a_c = v_t^2/r = r^*\omega^2\,.}$



- To move in a circle, there must be a ***net*** inward force ("centripetal force") causing the centripetal acceleration.





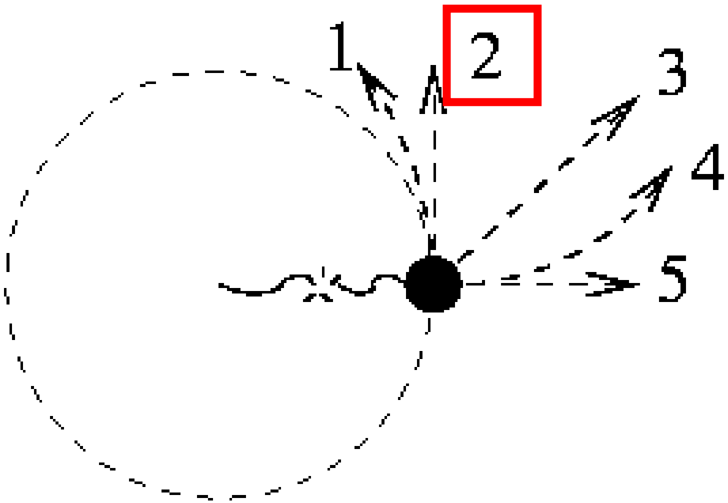
Would not put centripetal force on a free-body diagram, as it is from other forces.

An 800kg car travels a distance of 20m around a turn with a radius of curvature of 40m at a speed of 25m/s in a time of 0.80s. What is the centripetal acceleration of the car?

(A) 12.5 m/s^2 (B) 15.6 m/s^2 (C) 25.0 m/s^2 (D) 12500 m/s^2

1. $a_c = v_t^2/r$
2. $a_c = \{(25\text{m/s})^2\}/(40\text{m})$
3. $a_c = (625\ \text{m}^2/\text{s}^2)/(40\text{m}) \approx 15.6\ \text{m/s}^2$

You are whirling a tennis ball on a string around in circles, when the string suddenly snaps.
What direction does the tennis ball fly? (the figure below is a top view).



1. Circular motion requires a centripetal force to provide the centripetal acceleration.
2. Once the string snaps, there is no longer an inward force, so circular motion will cease.
3. The tennis ball will continue onward in the direction of the tangential velocity it had just prior to the string snapping.

You are whirling a tennis ball on a string around in circles, when the string suddenly snaps. This was because the centripetal force exceeded the maximum tension of the string.
What would you need to know to find the maximum string tension?

slinging.org

A. Mass of the tennis ball
B. Transverse velocity of the tennis ball
C. Length of the string
D. A & B
E. B & C
F. A, B, & C

1. Tension T is a force F.
2. $F = m*a$
3. $a = a_c = v_t^2/r$
4. $T + F_{grav} = ma_c = mv_t^2/r$

$\boxed{F = mv^2/r}$ is the "centripetal force".
It points inwards towards the rotation axis.
**Gravity is adding to the tension in this case to provide centripetal force!

An 800kg car wants to travel a distance of 20m around a turn with a radius of curvature of 40m at a speed of 25m/s in a time of 0.80s. What is the frictional force required to make this turn?

(A) 15.6 N (B) 31.3 N (C) 10000N (D) 12500 N

1. Friction provides centripetal force for cornering.
2. $F_{friction} = F_c = ma_c = mv_t^2/r$
3. $F_f = \{(800kg)(25m/s)^2\}/40m = 12500N$

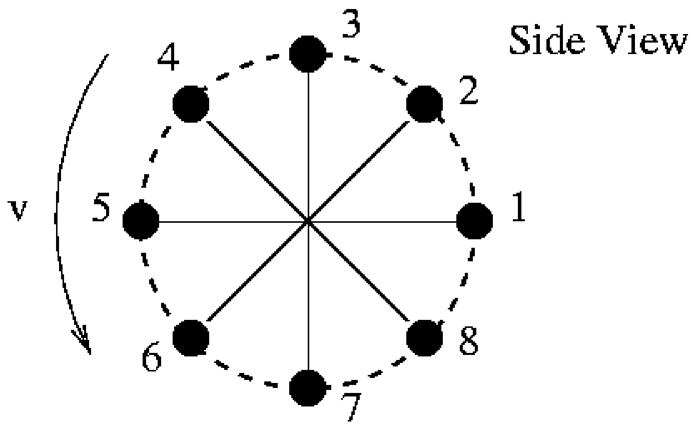
If this is on a flat track, what is the coefficient of friction required?

(A) 0.6 (B) 1.6 (C) 15.6 (D) 1.0

1. $F_f = \mu F_{normal}$
2. The normal force opposes forces normal to the surface. Here, that's just gravity.
3. $F_f = \mu F_{normal} = \mu F_{gravity} = \mu mg$
4. $\mu = F_f/mg = (12500N)/(800kg*9.8m/s^2) \approx 1.6$

Typical values for rubber-to-concrete are $\mu \sim 1$.

You are swinging a tennis ball on a string, where the axis of rotation is parallel to the surface of the earth.
At what point is the tension in the string the smallest?

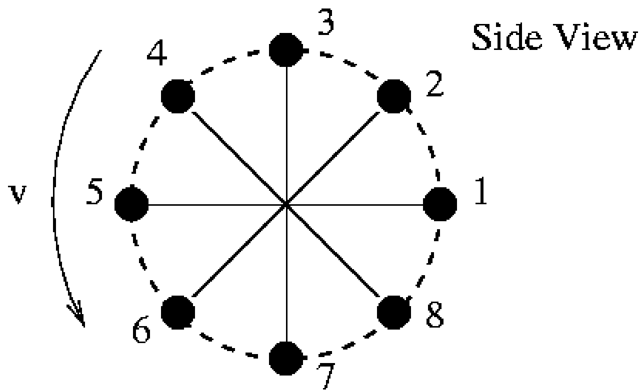


(A) 1 (B) 2 (C) 3 (D) 4
(E) 5 (F) 6 (G) 7 (H) 8
(I) Same tension at all points

1. Circular motion implies a constant inward centripetal force.
2. But, centripetal force is the ***net*** inward force.
3. At the top of the swing, the force of gravity is working with the tension to pull inward.
4. So, at the top of the swing, the tension doesn't have to be as large to maintain the constant centripetal force.

10

You are swinging a 0.20kg tennis ball on a 1.2m-long string, where the axis of rotation is parallel to the surface of the earth.
What is the slowest transverse speed you can swing the ball & keep it moving in a circle?

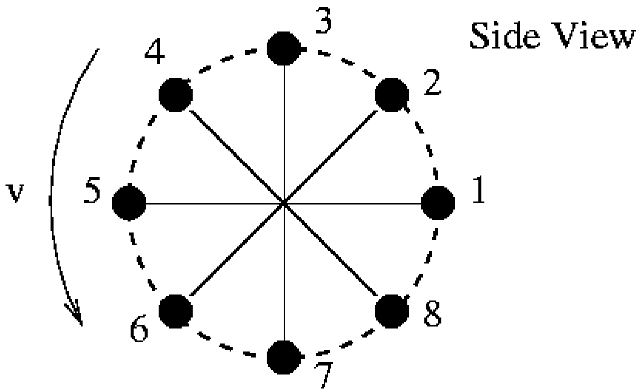


(A) 9.80m/s (B) 11.8m/s (C) 1.20m/s
(D) 1.96m/s $\boxed{\text{(E) 3.43m/s}}$ (F) 0.00m/s

1. Both gravity & the tension are supplying centripetal force.
2. At the slowest possible speed, the tension of the string will be zero at the top of the swing (position 3).
3. So $F_{c,min} = F_{gravity} = mg = mv_t^2/r$
4. $g = v_t^2/r$
5. $v_t = \sqrt{gr} = \sqrt{(9.8\frac{m}{s^2})(1.2m)} = 3.43m/s$

You are swinging a 0.20kg tennis ball on a 1.20m-long string, where the axis of rotation is parallel to the surface of the earth.

At the minimum speed, what is the tension in the string at the bottom of the circle (position 7) ?



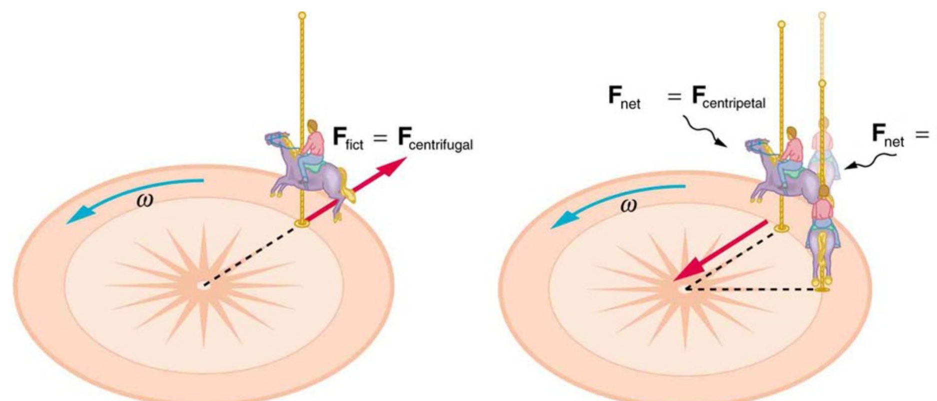
(A) 0.00N (B) 0.98N (C) 1.96N $\boxed{\text{(D) 3.92N}}$

1. Both gravity & the tension are supplying centripetal force.
2. Gravity is pulling outward, tension is pulling inward (in the direction of the net centripetal force).
3. $F_c = T - F_g$
4. $T = F_c + F_g = mv_t^2/r + mg$
5. $T = \{(0.20kg)(3.43m/s)^2\}/(1.2m) + (0.20kg)(9.8m/s^2)$
6. $T = 1.96N + 1.96N = 3.92N$

Fictitious (a.k.a. Inertial) **Forces**

- Newton’s laws apply to a non-accelerating (“inertial”) frame of reference.
- If you’re observing from a rotating frame of reference (“non-inertial”), such as the earth’s surface or on a merry-go-round, then fictitious forces must be introduced for Newton’s laws to work in that frame of reference.
- Fictitious forces are called “fictitious” because they are not real.
- Ficticious forces are apparent forces. I.e. they do not arise from the physical interaction between two objects (like friction & gravity do), but instead from acceleration of your reference frame.

- For example, the “centrifugal force” is what you feel on a merry-go-round.
- For an outside observer, there is no such force.



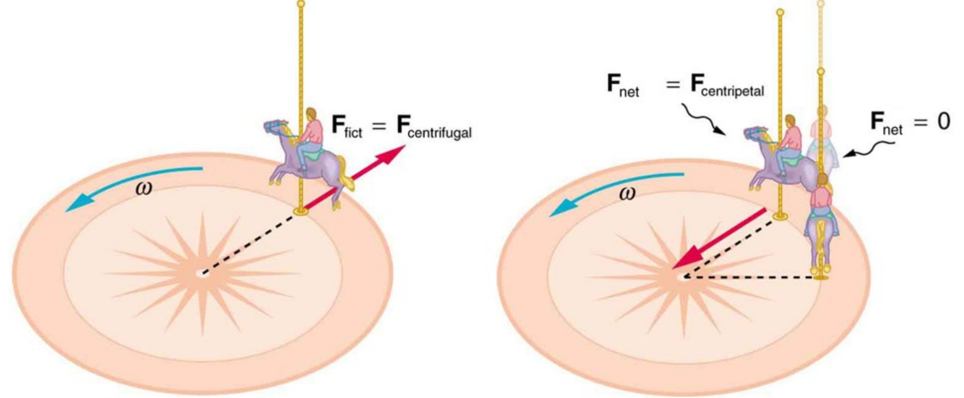
Merry-go-round’s rotating frame of reference
(a)

Inertial frame of reference
(b)

- Consider a tennis-ball on a string. You can see the string is pulling inward. But if you were riding on the tennis ball, it would feel like you’re been pulled outward.

Fictitious (a.k.a. Inertial) **Forces**

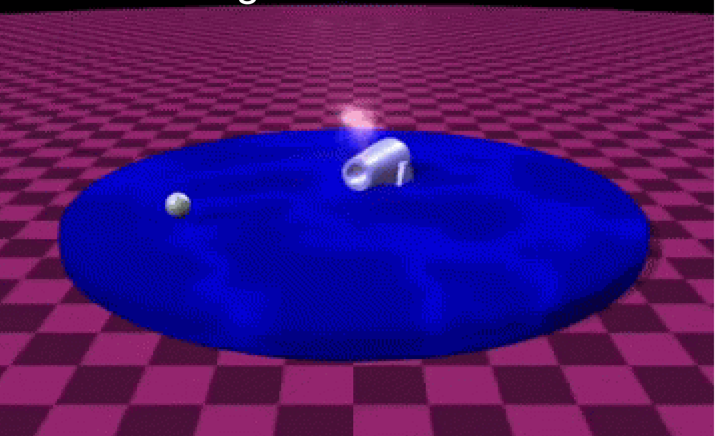
- Centrifugal force:
  - Feeling of being pulled outward in a rotating frame of reference.



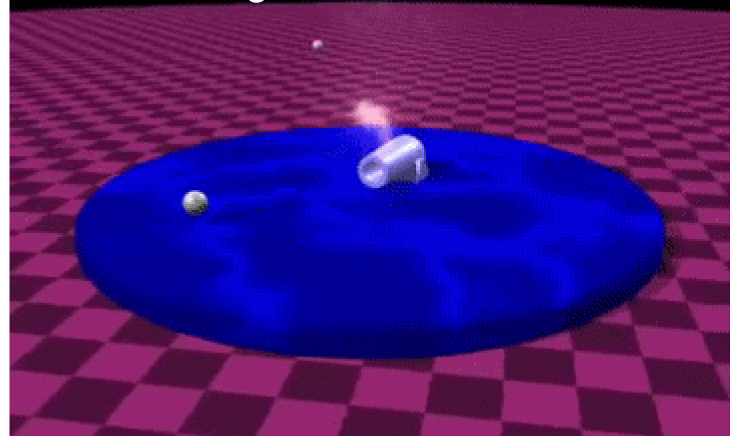
Merry-go-round's rotating frame of reference (a) | Inertial frame of reference (b)

- Coriolis force:
  - Apparent swirling motion of an object moving outward in a rotating reference frame.

Cannon firing from inertial frame:

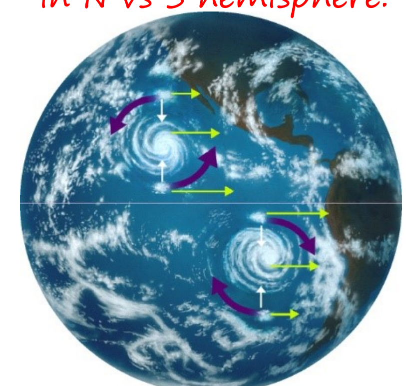


Cannon firing from non-inertial frame:



https://www.youtube.com/watch?v=49JwbrXcPjc

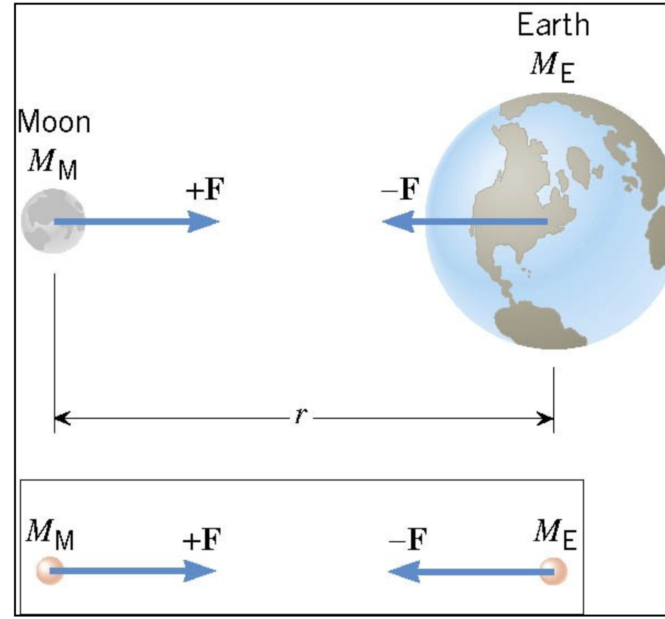
This is why hurricanes swirl opposite directions in N vs S hemisphere.



Earth rotation:
Looking "down" on N: CCW
Looking "up" at S: CW

Newton's Law of Universal Gravitation

📖 Sect 6.5



- All objects with mass attract all other objects with mass, where the attractive force is directed along a line between the two objects and the magnitude of the force is proportional to the magnitude of the masses & inversely proportional to the distance between the two objects.
- The force between objects of mass m_1 and m_2, whose centers of gravity are a distance r apart is: $\boxed{F = G\frac{m_1 m_2}{r^2}}$

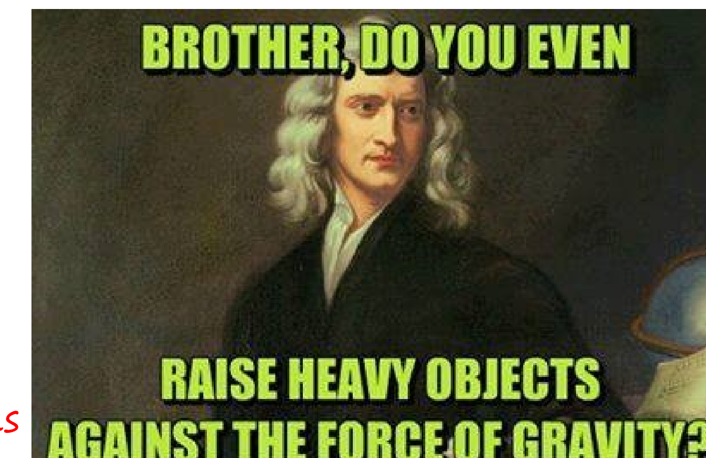
G = "big G"

- At the surface of the Earth,

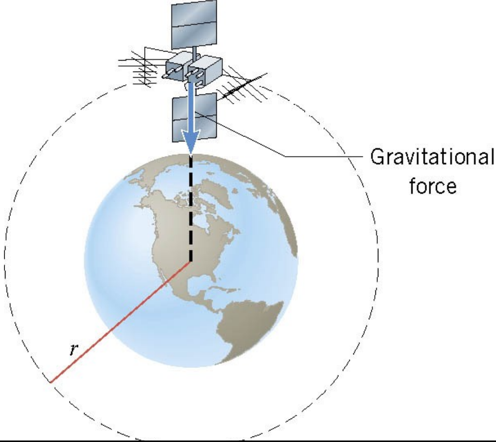
astronomical symbol for earth

  - $m_2 = M_{\oplus}$, $\mathrm{r} = R_{\oplus}$: $g = G\frac{M_{\oplus}}{R_{\oplus}^2}$
  - $F = mg$ = "weight"

This breakthrough linked celestial motions to motion on Earth.



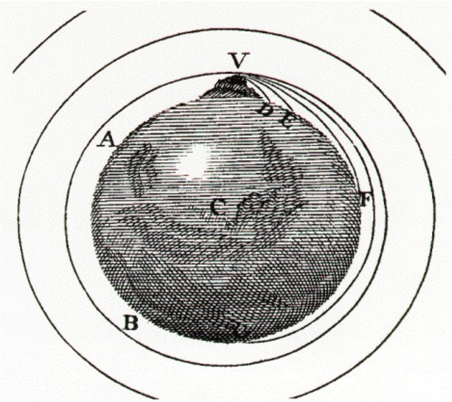
Gravity & Circular Motion: Orbit & Ballistics



- For objects orbiting the earth, gravity provides the centripetal force:
  - $F_c = m\frac{v_t^2}{r} = F_g = G\frac{mM_{Earth}}{r^2}$
  - $v_t^2 = G\frac{M_{Earth}}{r}$

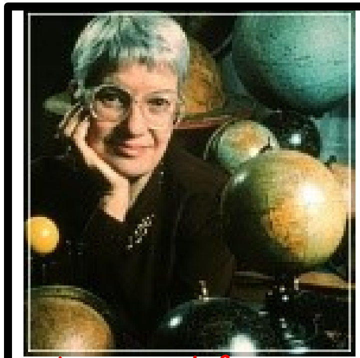
r is NOT the altitude. It is the distance from the center of the Earth.

- Objects orbiting Earth are really just constantly falling towards Earth, but they're moving fast enough to maintain a constant radius.

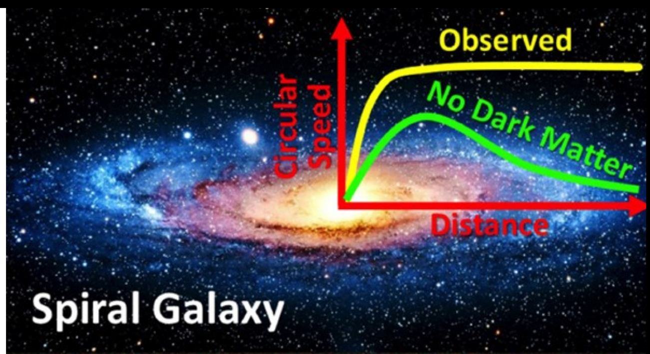


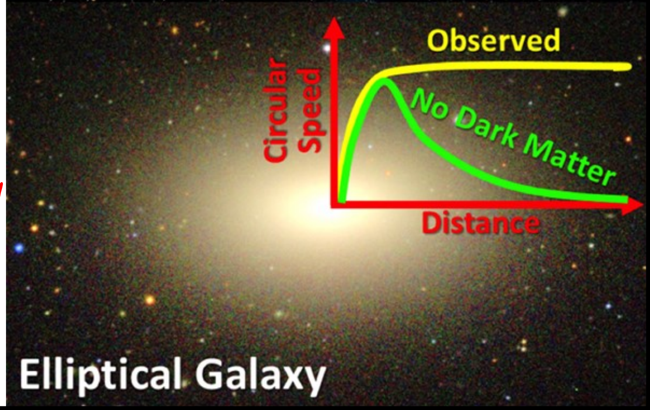
~8km/s is required to orbit Earth.

http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/NewtMtn/NewtMtn.html



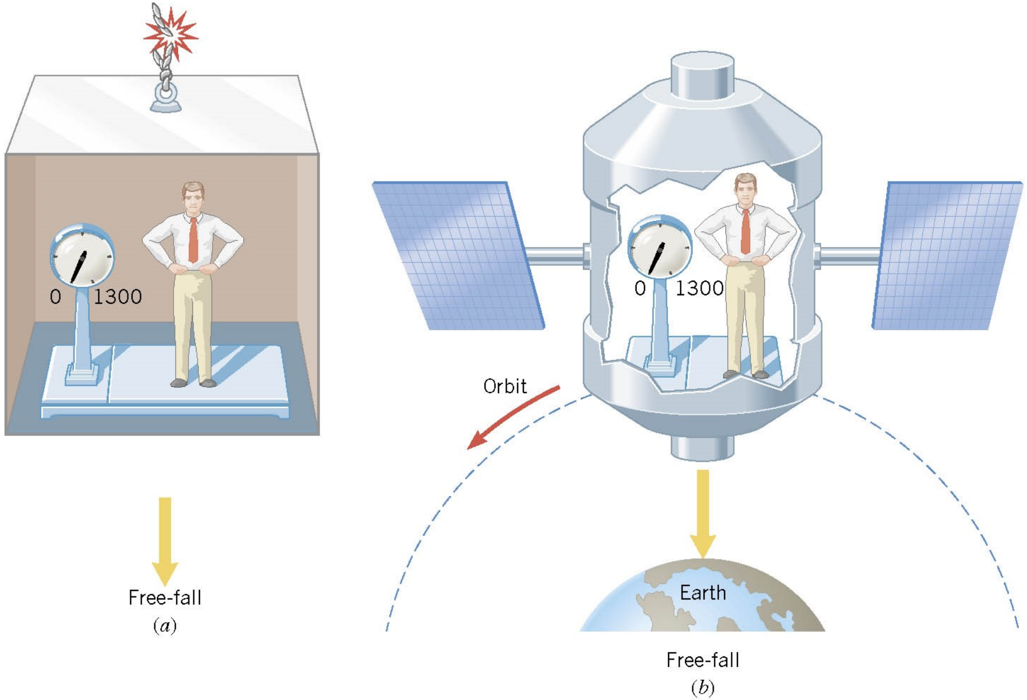
The need for Dark Matter was first realized by Vera Rubin because the orbital velocities of stars in galaxies didn't follow the $v_t \sim r^{-1/2}$ trend as expected.





“Weightlessness”

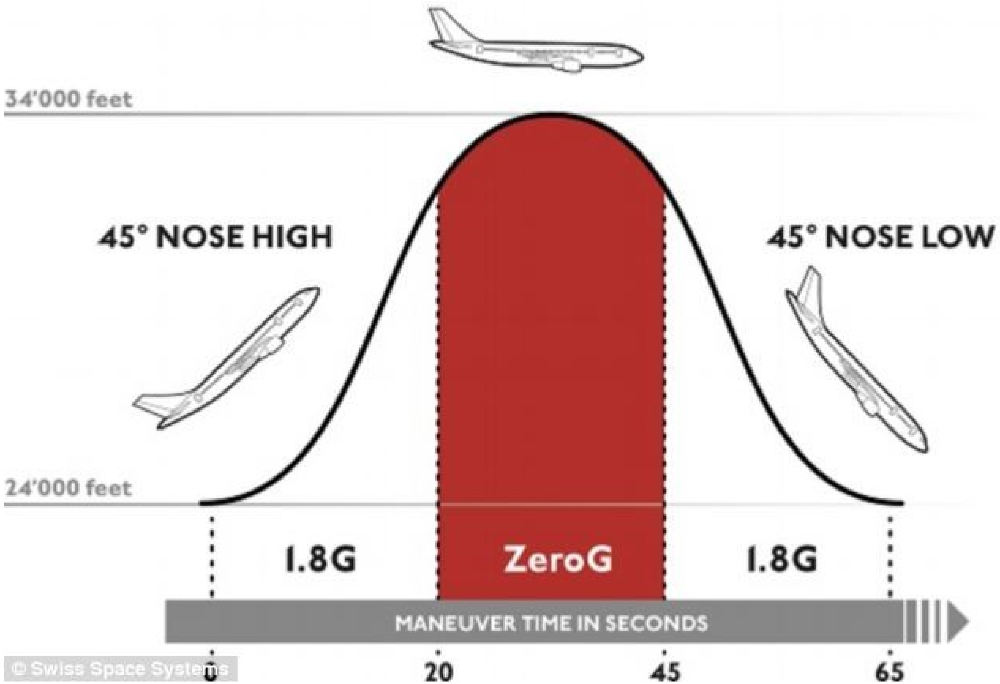
- Being “weightless” just means no normal force is required to support an object.
- Gravity is not “off” and you don’t have to be infinitely far from a massive object
- On the International Space Station, astronauts are falling toward earth at the same rate as the station, so they have no apparent weight.



https://www.youtube.com/watch?v=WMK36dpHIkg

“Weightlessness”: The Vomit Comet

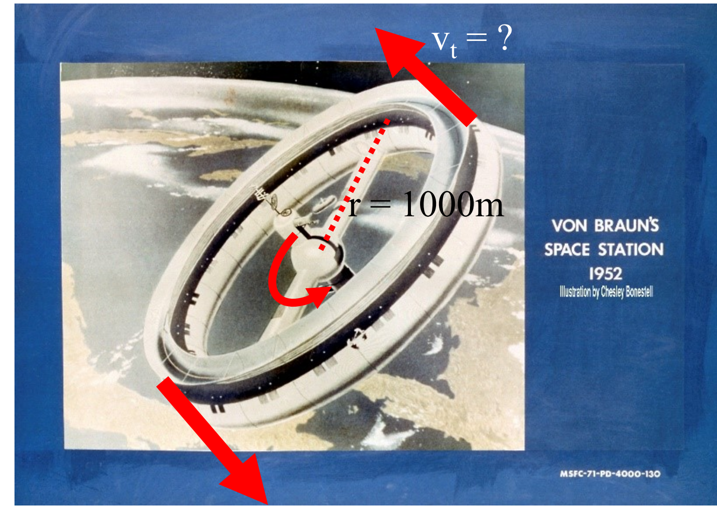
Can experience weightlessness (‘zero g’) using parabolic flight paths:



https://www.youtube.com/watch?v=BTkFIE_-kL8&sns=em

12

A space station designer wants to mimic Earth's gravity by employing a rotating-ring design. Astronauts will inhabit the outer-ring layer, at a radius of 1000m, which rotates about the center at some velocity.
How fast must the outer-ring layer be moving?



(A) 8000m/s (B) 99m/s (C) 9.8m/s
(D) 1000m/s (E) 31m/s (F) 9800m/s

1. On Earth, F=mg. On the station, $F=ma_c$
2. So, need to have: $a_c = g = 9.8 m/s^2$.
3. $a_c = v_t^2/r = g$
4. $v_t = \sqrt{gr} = \sqrt{(9.8\frac{m}{s^2})(1000m)} \approx 99m/s$ …~220mph!

13

What is the corresponding angular velocity, ω?

(A) 100rad/s (B) 10rad/s (C) 1rad/s (D) 0.1rad/s

$v_t = r\omega$ … $\omega = v_t/r = (99m/s)/(1000m) \approx 0.1 rad/s$ …~1rpm

Google: "Stanford torus"