## Tuesday February 28

Topics for this Lecture:

- Momentum

$$
\begin{gathered}
\sum \vec{F} \Delta t=\Delta \vec{p} \\
\text { Impulse }=\overrightarrow{\mathrm{J}}=\Delta \vec{p}
\end{gathered}
$$



- Assignment 8 due Friday after spring break
- Pre-class due 15 min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M\&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed or by appointment (meisel@ohio.edu)


A tennis ball cannon is mounted on sled in the middle of a frozen pond. The total mass of the unit (unloaded) is 50 kg .
The speed of each ball after being fired is $30 \mathrm{~m} / \mathrm{s}$ and the mass of each ball is 0.10 kg . How fast is the unit moving after it shoots 10 tennis balls to the right?
Presume it starts at rest.
(A) $0.06 \mathrm{~m} / \mathrm{s}$ left
(D) $0.6 \mathrm{~m} / \mathrm{s}$ right
(B) $0.06 \mathrm{~m} / \mathrm{s}$ right
(E) $6 \mathrm{~m} / \mathrm{s}$ left
(C) $0.6 \mathrm{~m} / \mathrm{s}$ left
(F) $6 \mathrm{~m} / \mathrm{s}$ right

1. $p_{\text {initial }}=p_{\text {final }}$
2. $p=m v \& v_{i}=0$..so $p_{i}=0$
3. $0=10^{*} p_{\text {ball,f }}+p_{\text {cannon }, f}$
4. $p_{\text {cannon }, f}=-10 p_{\text {ball,f }}$
5. $m_{c} v_{c, f}=-10 m_{b} v_{b, f}$
6. $v_{c, f}=-10 m_{b} v_{b, f} / m_{c}$
7. $v_{c, f}=\left(-10^{*} 0.1 \mathrm{~kg} * 30 \mathrm{~m} / \mathrm{s}\right) / 50 \mathrm{~kg}$
8. $\mathrm{v}_{\mathrm{c}, \mathrm{f}}=(-30 / 50) \mathrm{m} / \mathrm{s}=0.6 \mathrm{~m} / \mathrm{s}$ left

Three railroad cars of equal mass M are on a frictionless track. The one on the left is approaching at $6 \mathrm{~m} / \mathrm{s}$.
At the end, the two left cars are linked and traveling at $1 \mathrm{~m} / \mathrm{s}$, but the right-most car didn't connect properly and is free.
What is the speed of the separate un-linked car?

(A) $0 \mathrm{~m} / \mathrm{s}$
(B) $2 \mathrm{~m} / \mathrm{s}$
(C) $3 \mathrm{~m} / \mathrm{s}$
(D) $4 \mathrm{~m} / \mathrm{s}$
(E) $5 \mathrm{~m} / \mathrm{s}$
(F) $6 \mathrm{~m} / \mathrm{s}$
(G) $12 \mathrm{~m} / \mathrm{s}$
(H) $18 \mathrm{~m} / \mathrm{s}$

1. $p_{\text {initial }}=p_{\text {final }}$
2. $p_{1, i}+p_{2, i}+p_{3, i}=p_{1+2, f}+p_{3, f}$
3. $(\mathrm{m}) 6 \mathrm{~m} / \mathrm{s}+0+0=(2 \mathrm{~m})(1 \mathrm{~m} / \mathrm{s})+\mathrm{mv}_{3, \mathrm{f}}$
4. $(6 \mathrm{~m}) \mathrm{m} / \mathrm{s}=(2 \mathrm{~m}) \mathrm{m} / \mathrm{s}+\mathrm{mv}_{3, \mathrm{f}}$
5. $6 \mathrm{~m} / \mathrm{s}=2 \mathrm{~m} / \mathrm{s}+\mathrm{v}_{3, \mathrm{f}}$
6. $v_{3, f}=4 \mathrm{~m} / \mathrm{s}$

You're driving home from a bug-spray convention, moving along the highway at $30 \mathrm{~m} / \mathrm{s}$ in your 1000kg wagon.
An army of mosquitos ( $m_{\text {mosquito }}=5$ milligrams) has decided enough is enough and they ram your vehicle, ultimately sticking themselves to your windshield.
If the mosquito army charges your car at $2 \mathrm{~m} / \mathrm{s}$, how many mosquitos would it take to slow you down by $0.001 \mathrm{~m} / \mathrm{s}$ ?
Note that the army will not significantly increase the mass of the wagon.
(A) $10^{3}$
(B) $10^{4}$
(C) $10^{5}$
(D) $10^{6}$
(E) $10^{7}$
(F) $10^{8}$

1. $p_{\text {initial }}=p_{\text {final }}$
2. $p_{\text {wagon }, i}+p_{\text {army }, \mathrm{i}}=p_{\text {wagon+army }, f}$
3. $m_{w} v_{w, i}+N_{m} m_{m} v_{m, i}=\left(m_{w}+N_{m} m_{m}\right) v_{f}$
4. "army will not significantly increase the mass of the wagon": $m_{w}+N_{m} m_{m} \rightarrow m_{w}$
5. $m_{w} v_{w, i}+N_{m} m_{m} v_{m, i}=m_{w} v_{f}$
6. $N_{m}=\left(m_{w} v_{f}-m_{w} v_{w, i}\right) /\left(m_{m} v_{m, i}\right)$
7. "slow you down by $1 \mathrm{~m} / \mathrm{s}$ ": $\mathrm{v}_{\mathrm{f}}-\mathrm{v}_{\mathrm{w}, \mathrm{i}}=-0.001 \mathrm{~m} / \mathrm{s}$
8. $\mathrm{N}_{\mathrm{m}}=1000 \mathrm{~kg}(-0.001 \mathrm{~m} / \mathrm{s}) /\left(5 \mathrm{e}-6 \mathrm{~kg}^{*}(-2 \mathrm{~m} / \mathrm{s})\right)$
9. $\mathrm{N}_{\mathrm{m}}=(1 \mathrm{kgm} / \mathrm{s}) /(1 \mathrm{e}-5 \mathrm{kgm} / \mathrm{s})=100,000=10^{5}$


Learning his lesson, a lone mosquito survivor with a mass of 5 mg is retreating at $2 \mathrm{~m} / \mathrm{s}$. Your wagon, which has a mass of 1000 kg , hits him at $30 \mathrm{~m} / \mathrm{s}$, sticking the single mosquito to your windshield and slightly increasing the mass of your vehicle.
By how much does your wagon slow down?
(A) $10^{-3} \mathrm{~m} / \mathrm{s}$
(B) $10^{-4} \mathrm{~m} / \mathrm{s}$
(C) $10^{-5} \mathrm{~m} / \mathrm{s}$
(D) $10^{-6} \mathrm{~m} / \mathrm{s}$
(E) $10^{-7} \mathrm{~m} / \mathrm{s}$
(F) $10^{-8} \mathrm{~m} / \mathrm{s}$

1. $p_{\text {initial }}=p_{\text {final }}$
2. $p_{\text {wagon,i }}+p_{\text {mosquito, } i}=p_{\text {wagon+mosquito,f }}$

3. $m_{w} v_{w, i}+m_{m} v_{m, i}=\left(m_{w}+m_{m}\right) v_{f}$
4. $v_{f}=\left\{m_{w} v_{w, i}+m_{m} v_{m, i}\right\} /\left\{m_{w}+m_{m}\right\}$
5. $\mathrm{v}_{\mathrm{f}}=\left\{1000 \mathrm{~kg}^{*} 30 \mathrm{~m} / \mathrm{s}+\left(5 \times 10^{-6} \mathrm{~kg}\right) 2 \mathrm{~m} / \mathrm{s}\right\} /\left\{1000 \mathrm{~kg}+5 \times 10^{-6} \mathrm{~kg}\right\}$
6. $v_{f}=29.99999986 \mathrm{~m} / \mathrm{s}$
7. $\mathrm{v}_{\mathrm{i}}-\mathrm{v}_{\mathrm{f}}=1.4 \times 10^{-7} \mathrm{~m} / \mathrm{s} \sim 100 \mathrm{~nm} / \mathrm{s}$

A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
The mass of the first block is 1150 g , and its velocity is $+0.550 \mathrm{~m} / \mathrm{s}$ after the bullet passes through it. The mass of the second block is 1530 g .
What is the initial momentum of the blocks+bullet system?
(A) $1.42 \mathrm{kgm} / \mathrm{s}$
(B) $1420 \mathrm{kgm} / \mathrm{s}$
(C) $-1.42 \mathrm{kgm} / \mathrm{s}$
(D) $-1420 \mathrm{kgm} / \mathrm{s}$
(E) $142 \mathrm{kgm} / \mathrm{s}$
(F) $-142 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$
(a) Before collision

$m_{\text {block } 1}=1150 \mathrm{~g}$

$$
\begin{aligned}
& m_{\text {block } 2}=1530 \mathrm{~g} \\
& m_{\text {bullet }}=4.00 \mathrm{~g}
\end{aligned}
$$

(b) After collision

1. $p_{\text {initial }}=p_{\text {bullet }, \mathrm{i}}+\mathrm{p}_{\text {block } 1, \mathrm{i}}+\mathrm{p}_{\text {block } 2, \mathrm{i}}$
2. $p=m v$
3. only the bullet has non-zero initial velocity
4. $p_{\text {initial }}=p_{\text {bullet, }, i}=m_{\text {bullet }} v_{\text {bullet, }, i}$
5. $p_{\text {initial }}=\left(4 \times 10^{-3} \mathrm{~kg}\right)(355 \mathrm{~m} / \mathrm{s})$
6. $p_{\text {initial }}=1.42 \mathrm{kgm} / \mathrm{s}$

A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
The mass of the first block is 1150 g , and its velocity is $+0.550 \mathrm{~m} / \mathrm{s}$ after the bullet passes through it. The mass of the second block is 1530 g .
What is the final momentum of the blocks+bullet system?
(A) $0 \mathrm{kgm} / \mathrm{s}$
(B) $0.142 \mathrm{kgm} / \mathrm{s}$
(C) $1.42 \mathrm{kgm} / \mathrm{s}$
(D) $-1.42 \mathrm{kgm} / \mathrm{s}$
(E) $0.613 \mathrm{kgm} / \mathrm{s}$
(F) $2.03 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$

(a) Before collision

(b) After collision

1. Momentum is conserved [no external forces]
2. $p_{\text {final }}=p_{\text {initial }}=p_{\text {bullet }, \mathrm{i}}=\left(4 \times 10^{-3} \mathrm{~kg}\right)(355 \mathrm{~m} / \mathrm{s})=1.42 \mathrm{kgm} / \mathrm{s}$

A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
The mass of the first block is 1150 g , and its velocity is $+0.550 \mathrm{~m} / \mathrm{s}$ after the bullet passes through it. The mass of the second block is 1530 g .
What is the final velocity of the second block?
(A) $0.00 \mathrm{~m} / \mathrm{s}$
(B) $0.550 \mathrm{~m} / \mathrm{s}$
(C) $1.42 \mathrm{~m} / \mathrm{s}$
(E) $0.920 \mathrm{~m} / \mathrm{s}$
(D) $355 \mathrm{~m} / \mathrm{s}$
(F) $0.513 \mathrm{~m} / \mathrm{s}$

(a) Before collision

$m_{\text {block } 1}=1150 \mathrm{~g}$

$m_{\text {bullet }}=4.00 \mathrm{~g}$

1. $p_{\text {initial }}=p_{\text {final }}$
2. $p_{\text {bullet, }, i}=p_{\text {bullet }, f}+p_{\text {block } 2, f}+p_{\text {block } 1, f}$
3. $m_{\text {bullet }} v_{\text {bullet }, \mathrm{i}}=\left(m_{\text {bullet }}+m_{\text {block } 2}\right) v_{f}+m_{\text {block } 1} v_{\text {block } 1, f}$
4. $1.42 \mathrm{kgm} / \mathrm{s}=(1.534 \mathrm{~kg}) \mathrm{v}_{\mathrm{f}}+0.632 \mathrm{kgm} / \mathrm{s}$
5. $\mathrm{v}_{\mathrm{f}}=(1.42 \mathrm{kgm} / \mathrm{s}-0.632 \mathrm{kgm} / \mathrm{s}) /(1.534 \mathrm{~kg})$
6. $v_{f}=0.513 \mathrm{~m} / \mathrm{s}$

A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
The mass of the first block is 1150 g , and its velocity is $+0.550 \mathrm{~m} / \mathrm{s}$ after the bullet passes through it. The mass of the second block is 1530 g . What is the ratio of the final kinetic energy to the initial kinetic energy $\left(\mathrm{KE}_{\mathrm{f}} / \mathrm{KE}_{\mathrm{i}}\right)$ ?
(A) 0.0
(C) 0.38
(E) $5.0 \times 10^{-2}$
(B) $1.0 \times 10^{-3}$
(D) $1.5 \times 10^{-3}$
(F) 1.0

(a) Before collision

(b) After collision

1. $\mathrm{KE}_{\mathrm{i}}=1 / 2 \mathrm{~m}_{\text {bullet }} \mathrm{V}^{2}{ }_{\text {bullet }, \mathrm{i}}=252 \mathrm{~J}$
2. $\mathrm{KE}_{\mathrm{f}}=1 / 2 \mathrm{~m}_{\text {block } 1} \mathrm{v}^{2}$ block $1, \mathrm{f}+1 / 2\left(\mathrm{~m}_{\text {block2 }}+\mathrm{m}_{\text {bullet }}\right) \mathrm{v}_{\mathrm{f}}^{2}=0.376 \mathrm{~J}$
3. $\mathrm{KE}_{\mathrm{f}} / \mathrm{KE}_{\mathrm{i}}=1.5 \times 10^{-3}$

Recall, Collision Classification

- All collisions conserve momentum.
- Collisions that conserve kinetic energy KE are called: "elastic" e.g. billiards
- Collisions that do not conserve KE are called: "inelastic" e.g. car crash
- When colliding objects stick together, their collision is: "perfectly inelastic" e.g. mosquito on windshield


A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
The mass of the first block is 1150 g , and its velocity is $+0.550 \mathrm{~m} / \mathrm{s}$ after the bullet passes through it. The mass of the second block is 1530 g .
What is the speed of the bullet after going through block 1 , but before going into block 2 ?
(A) $355 \mathrm{~m} / \mathrm{s}$
(B) $197 \mathrm{~m} / \mathrm{s}$
(C) $354 \mathrm{~m} / \mathrm{s}$
(D) $0 \mathrm{~m} / \mathrm{s}$
(E) $142 \mathrm{~m} / \mathrm{s}$
(F) $19.7 \mathrm{~m} / \mathrm{s}$


Block 1
Block 2
(a) Before collision

$m_{\text {block } 1}=1150 \mathrm{~g}$


1. $p_{\text {initial }}=p_{\text {final }}$
(b) After collision
2. $p_{\text {bullet }, \mathrm{i}}=p_{\text {bullet }, \mathrm{f}}+p_{\text {block } 2, f}+p_{\text {block } 1, f}$
3. $\mathrm{m}_{\text {bullet }} \mathrm{v}_{\text {bullet }, \mathrm{i}}=\mathrm{m}_{\text {bullet }} \mathrm{V}_{\text {bullet }, \mathrm{f}}+\mathrm{m}_{\text {block2 }}(0)+m_{\text {block1 }} v_{\text {block1,f }}$
4. $1.42 \mathrm{kgm} / \mathrm{s}=(0.004 \mathrm{~kg}) \mathrm{v}_{\mathrm{f}}+0.632 \mathrm{kgm} / \mathrm{s}$
5. $\mathrm{v}_{\mathrm{f}}=(1.42 \mathrm{kgm} / \mathrm{s}-0.632 \mathrm{kgm} / \mathrm{s}) /(0.004 \mathrm{~kg})=197 \mathrm{~m} / \mathrm{s}$

A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
The mass of the first block is 1150 g , and its velocity is $+0.550 \mathrm{~m} / \mathrm{s}$ after the bullet passes through it. The mass of the second block is 1530 g .
If the bullet takes 0.4 ms to go through block 1 , what is the force of the bullet on the block?
(A) 0.158 N
(B) 158 N
(C) 1580 N
(D) 0 N
(E) -1580 N
(F) -158 N

(a) Before collision
$m_{\text {block } 1}=1150 \mathrm{~g}$

(b) After collision
$m_{\text {block 2 }}=1530 \mathrm{~g}$
$m_{\text {bullet }}=4.00 \mathrm{~g}$

1. $\mathrm{F}_{\text {on block1 }} \Delta \mathrm{t}=\Delta \mathrm{p}_{\text {block1 }}$
2. $\mathrm{F}_{\text {on block } 1}\left(4 \times 10^{-4} \mathrm{~s}\right)=\mathrm{p}_{\text {block } 1, \mathrm{f}}-\mathrm{p}_{\text {block } 1, \mathrm{i}}$
3. $F\left(4 \times 10^{-4} s\right)=m_{\text {block } 1} v_{\text {block } 1, f}-0$
4. $F=\left(1.150 \mathrm{~kg}^{*} 0.550 \mathrm{~m} / \mathrm{s}\right) /\left(4 \times 10^{-4} \mathrm{~s}\right)=1580 \mathrm{kgm} / \mathrm{s}^{2}=1580 \mathrm{~N}$

A 4.00-g bullet is moving horizontally with a velocity of $+355 \mathrm{~m} / \mathrm{s}$, where the + sign indicates that it is moving to the right.
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If the bullet takes 0.4 ms to go through block 1 , what is the force of the block on the bullet?
(A) 0.158 N
(B) 158 N
(C) 1580 N
(D) 0 N
(E) -1580 N
(F) -158 N

(a) Before collision

(b) After collision

1. An action force will have an equal \& opposite reaction force.
2. $F_{\text {bullet-on-block }}=-F_{\text {block-on-bullet }}=1580 \mathrm{~N}$
3. $F_{\text {block-on-bullet }}=-1580 \mathrm{~N}$

Recall, Force applied over time: "Impulse"

(a) Before

(b) During

(c) After

- Impulse: Integral of $F(t)$, i.e. area under the $F$ vs $t$ graph
- Impulse: Change in momentum
- $\vec{J}=\vec{F} \Delta t=m \vec{a} \Delta t=m \frac{\Delta \vec{v}}{\Delta t} \Delta t=m \Delta v=m \vec{v}_{f}-m \vec{v}_{i}=\Delta \vec{p}$
"Ballistic Pendulum" (lab after ‘wooo, spring break')

-Perfectly inelastic collision: Ball launched via spring mechanism into pendulum
- Will determine change in KE
-Will get final KE using conservation of energy (measuring final PE) \& conservation of momentum
-Will get initial KE by measuring initial velocity (using 1D kinematics)

A ball is attached to a wire, held horizontally, and dropped. It strikes a block that is sitting on a horizontal, frictionless surface.
Air resistance is negligible and the collision is elastic.
The block is more massive than the ball.
Which of the following are conserved as the ball swings down?
(A) Ball's Kinetic Energy
(B) Ball's Momentum
(C) Ball's Total Mechanical Energy
(D) $A \& B$
(E) A \& C (F) B \& C

(G) A, B, \& C

1. Initial kinetic energy is zero (at rest) ...but clearly moving at the bottom of the swing.

So KE clearly not conserved (Nor need it be!)
2. Similarly, momentum clearly zero (at rest) but moving at bottom of swing.

Why is momentum not conserved? Ball acted-on by external forces! (tension+gravity)
3. Even if the pendulum were initially swinging, velocity would be downward at first, then horizontal. Tension is an outside force acting on the ball, converting $p_{y}$ to $p_{x}$.
4. Energy is always conserved. Here PE is converted into KE.

