Thursday February 23

Topics for this Lecture:

- *Power (Energy expended over time)*
- *Momentum & Impulse*

- Assignment 7 due Friday
- Pre-class due 15min before class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed or by appointment (meisel@ohio.edu)

- Work:
  - Force applied to move an object over a given distance
- Impulse:
  - Force applied over a time-span, resulting in a change in momentum
  - A vector!
- Momentum:
  - Mass x velocity = momentum
  - m*v = p
  - A vector!
  - Conserved

$$\sum \vec{F}\Delta t = \Delta\vec{p}$$

$$\text{Impulse} = \vec{\mathrm{J}} = \Delta\vec{p}$$

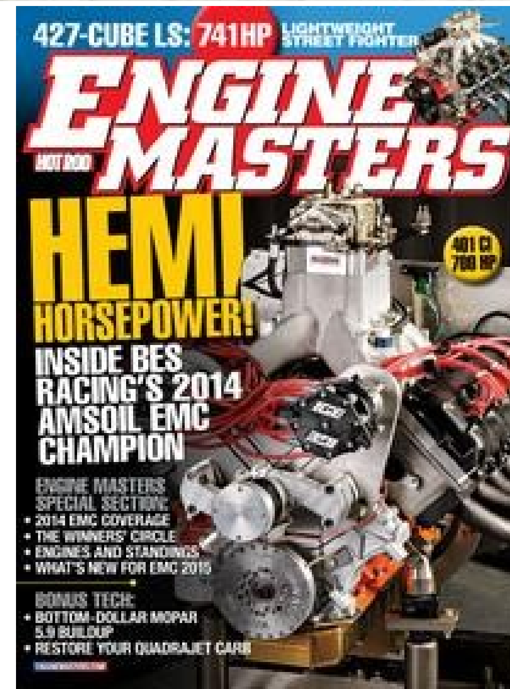
Power

- Rate at which work is done … which is the same as:
- Rate at which energy is expended
  - Units: J/s ≡ watt (W)
  - E.g. A 60W lightbulb uses 60J of energy every second
    *not the same as "watt equivalent"

$$P = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{\text{Change in Energy}}{\text{Time}}$$

- 1 Mechanical Horsepower $\approx$ 746W



A motor is capable of providing 500W of power.
How much energy can the motor provide when run for 20 s?

(A) 20 J (B) 25 J (C) 500 J (D) 10,000 J

1. $P = W/\Delta t$
2. $W = P*\Delta t$
3. $W = 500W*20s = (500J/s)*(20s) = 10,000J$

A person weighs 70kg.
What is the minimum amount of energy they need to expend in order to climb a 20m-tall cliff?
(They start and end at rest.)

(A) 14.3 J (B) 140 J (C) 196 J (D) 463 J
(E) 686 J (F) 13700 J

1. $KE_i + PE_i + W_{NC} = KE_f + PE_f$
2. Pick reference at base.
3. $0J + 0J + W_{NC} = 0\ J + mgh$
4. $W_{NC} = (70kg)\ (9.8m/s^2)(20m) = 13700J$

A person weighs 70kg. They climb a 20m-tall cliff in 3min.
What was their minimum power output?

(A) 76 W (B) 2380 W (C) 4570 W (D) 8290 W
(E) 12850 W (F) 41200 W

1. $P = W/\Delta t$
2. From previous slide, $W = 13700J$
3. $P = (13700J)/(180s) \approx 76$ W

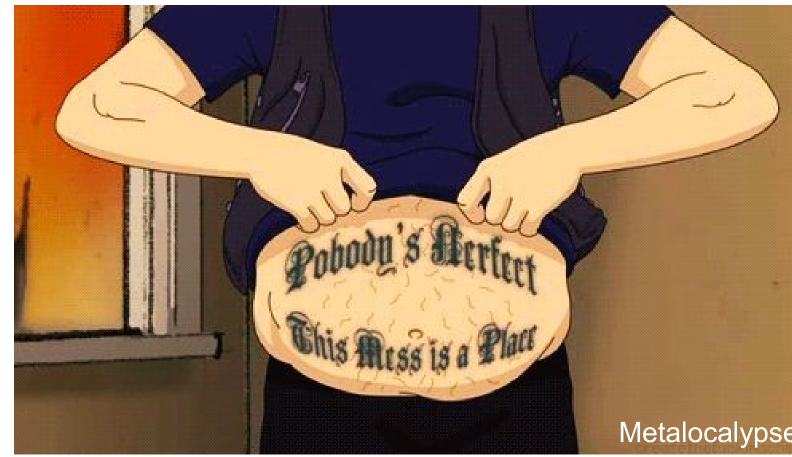
A person weighs 70kg. They climb a 20m-tall cliff.
Their body is only 20% efficient at converting food-energy into useful work.
How much food-energy must they consume to be able to make the climb?

(A) 2740 J (B) 10960 J (C) 13700 J (D) 17100 J

$\boxed{\text{(E) 68500 J}}$

1. (Useful Work) = (Total Work)*Efficiency
2. (Total Work) = (Useful Work)/(Efficiency)
3. From previous slide, W_{useful} = 13700J
4. W_{total} = (13700J)/(0.2) = 68,500J

…80% of the total energy spent would create non-useful work (e.g. body heat)

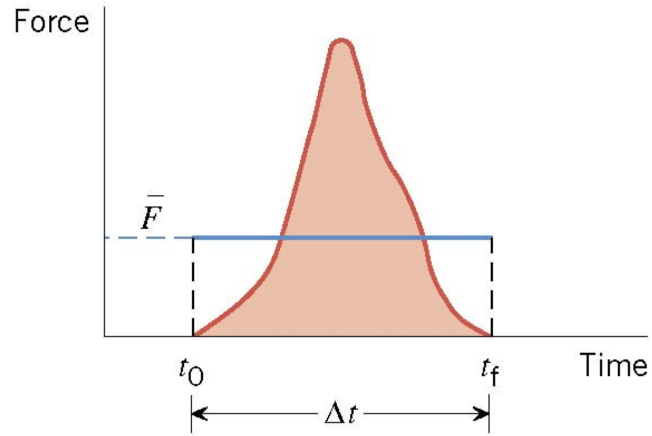


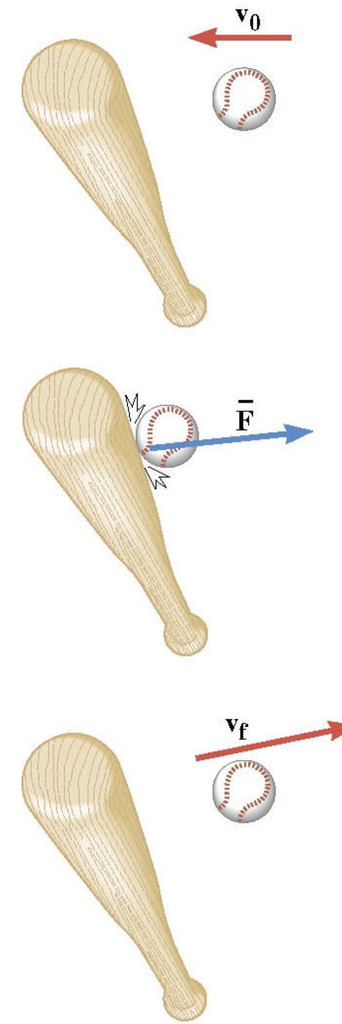
Efficiency factors are always necessary for real-life problems regarding energy generation & energy storage.

Fun fact:
1 Calorie = 1000 calories ≈ 4200 Joules

Force applied over time: "*Impulse*"

How do we describe the change in velocity of a baseball hit by a bat?





- Impulse: Integral of F(t), i.e. area under the F vs t graph
  - $\vec{J} = \int F(t)dt = \sum \vec{F}\Delta t$
- Impulse describes the change in direction and is therefore a vector quantity
- Impulse: Change in momentum
  - $\vec{J} = \vec{F}\Delta t = m\vec{a}\Delta t = m\frac{\Delta\vec{v}}{\Delta t}\Delta t = m\Delta v = m\vec{v}_f - m\vec{v}_i = \Delta\vec{p}$
  - *Impulse in a given direction only affects momentum in that direction

The same impulse is applied to a toy car, a bowling ball, and a truck, all of which are initially at rest.
Which object is traveling fastest after the impulse is applied?

(A) toy car (B) bowling ball (C) truck (D) all have the same final speed

1. $\vec{J} = \Delta\vec{p}$
2. $\vec{J} = \Delta\vec{p} = \Delta(m\vec{v})$
3. Impulse is just a push, so it won't change mass…
4. $\vec{J} = \Delta\vec{p} = \Delta(m\vec{v}) = m(\Delta\vec{v})$
5. $\Delta\vec{v} = \frac{\vec{J}}{m}$
6. Smallest mass = largest change in velocity

Two balls of equal mass are dropped on the table from the same height.
Ball A bounces back up to roughly the same height from which it was dropped.
Ball B does not bounce back up at all.
Which ball receives the greater impulse from the table?

A. Ball A

B. Ball B

C. Both feel the same impulse

Ball A stopped & then reversed direction …this means there was a greater change in momentum than for Ball B, which just stopped.

For example, if the initial velocity is -5m/s and final velocity is + 5m/s (presume +y is up):

A.

$$\sum \vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0$$

$$\sum \vec{F}\Delta t = m(+5) - m(-5)$$

$$\sum \vec{F}\Delta t = m(+5 - (-5))$$

$$\sum \vec{F}\Delta t = 10m$$

B.

$$\sum \vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0$$

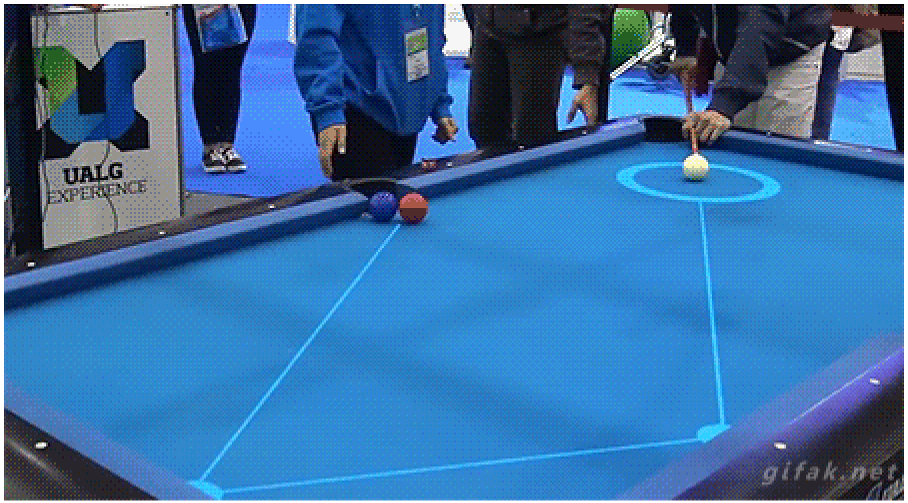
$$\sum \vec{F}\Delta t = m(0) - m(-5)$$

$$\sum \vec{F}\Delta t = m(0 - (-5))$$

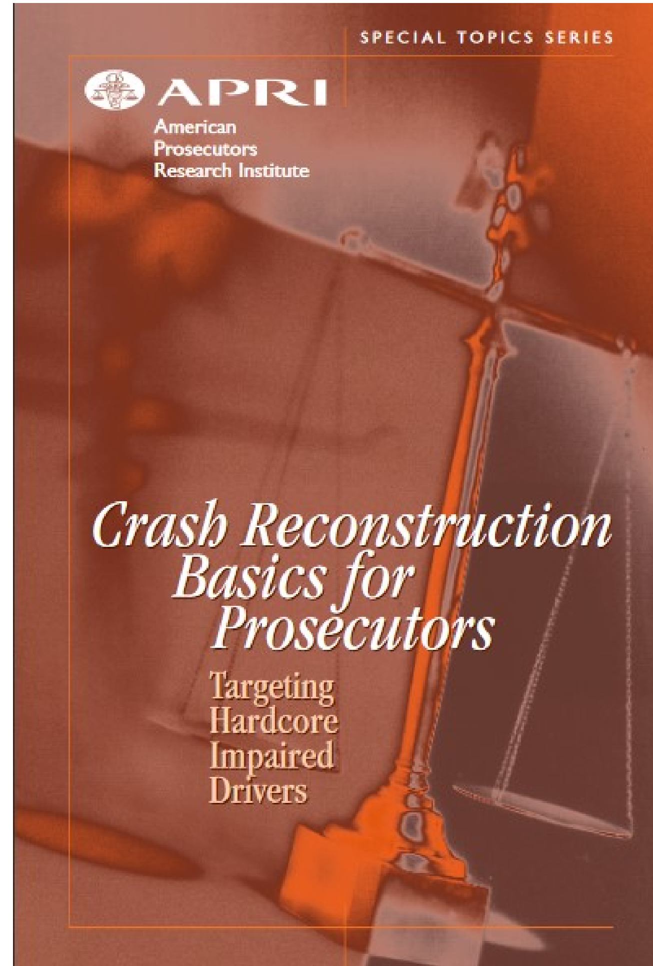
$$\sum \vec{F}\Delta t = 5m$$

Collisions

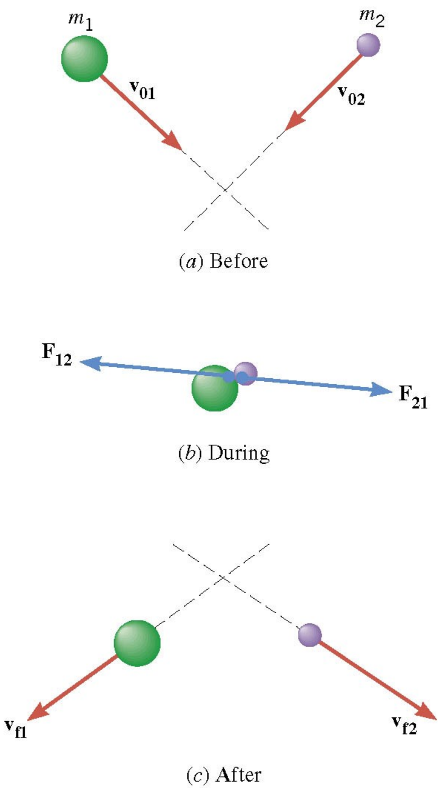
How must we strike the cue-ball to make this shot?



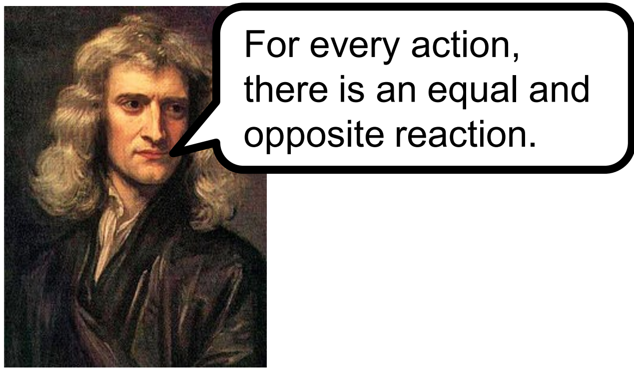
How can we reconstruct a car accident?



Collisions



1. Consider two spheres colliding.
2. When they impact (i.e. are touching) the force of sphere 1 on sphere 2 is equal & opposite to the force of sphere 2 on sphere 1
   - $F_{12} = -F_{21}$

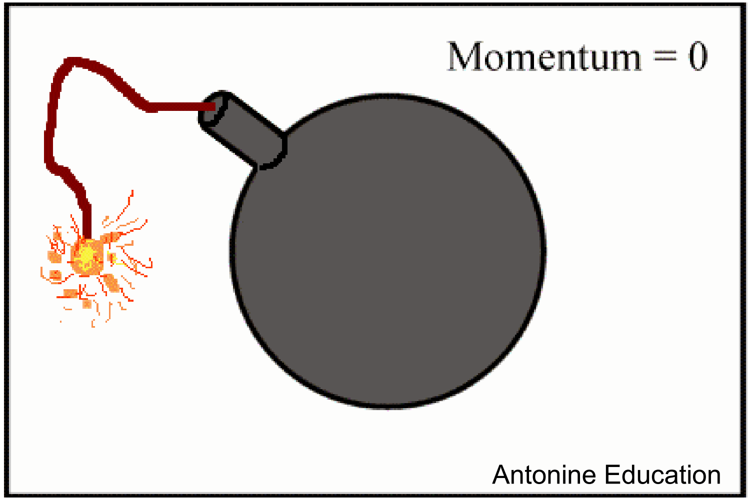


3. Since the contact time is the same,
   - $|\vec{J}_{12}| = |\vec{F}_{12}|\Delta t = |\vec{F}_{21}|\Delta t = |\vec{J}_{21}|$
   - The impulse on each is the same magnitude, but in opposite directions
4. Therefore, each sphere has an equal change in momentum, but in the opposite direction
   - $\vec{p}_{1,f} = \vec{p}_{1,i} + \Delta\vec{p}$
   - $\vec{p}_{2,f} = \vec{p}_{2,i} + \Delta\vec{p}$
   - Because of directionality, $\Delta\vec{p}$ will increase $\vec{p}_{1,f}$ or $\vec{p}_{2,f}$ and decrease the other …so the total momentum will be unchanged!
5. Therefore, **momentum is conserved** [i.e. Total momentum stays the same]

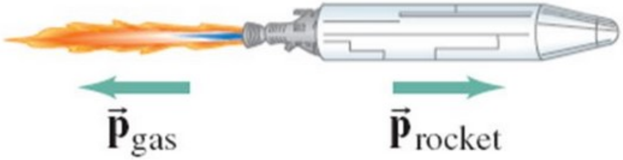
Explosions & Recoils (from perspective of momentum conservation)

- Total momentum is conserved (individually along each axis).
  - $p_{x,f} = p_{x,i}$, $p_{y,f} = p_{y,i}$, $p_{z,f} = p_{z,i}$
- A group of objects with zero initial momentum will remain with zero total momentum [Unless acted on by an outside force]
- Each individual object can still gain momentum individually!
- ….it just has to cancel-out with the momentum of another object or objects

Example: *Explosions*



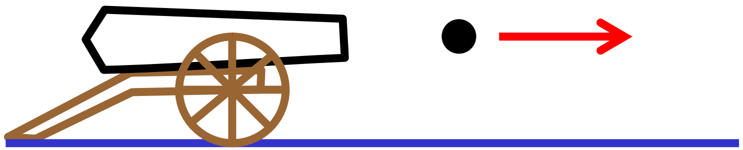
Example: *Rockets*



Example: *Recoils*

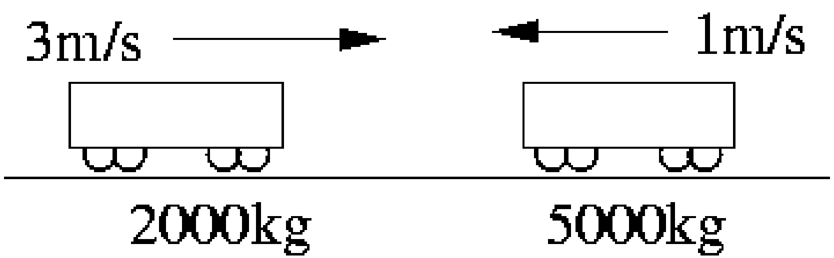
An old cannon with a mass of 600kg is sitting on a frozen lake. The cannon fires a 5kg cannon ball at a speed of 300m/s to the right, where the cannon starts at rest.
What is the velocity of the cannon after it has been fired?

A. 0.5 m/s to the left
B. 0.5 m/s to the right
C. 2.5 m/s to the left
D. 2.5 m/s to the right
E. 300 m/s to the left
F. 5.0 m/s to the left



1. Momentum is conserved: $p_{final} = p_{initial}$
2. Initially, everything is at rest: $p_{initial} = 0$
3. $p_{final} = 0 = p_{c},f + p_{b,f}$
4. $p_{c,f} = -p_{b,f}$
5. $m_c v_{c,f} = -m_b v_{b,f}$
6. $v_{c,f} = -(5kg*300m/s)/600kg = -2.5m/s$

A 2000kg railroad car is traveling at 3m/s along a track.
A 5000kg railroad car is travelling on the same track,
but going in the opposite direction at 1m/s.
The two railroad cars collide and stick together.
What is the speed and direction of the smashed-together railroad cars?

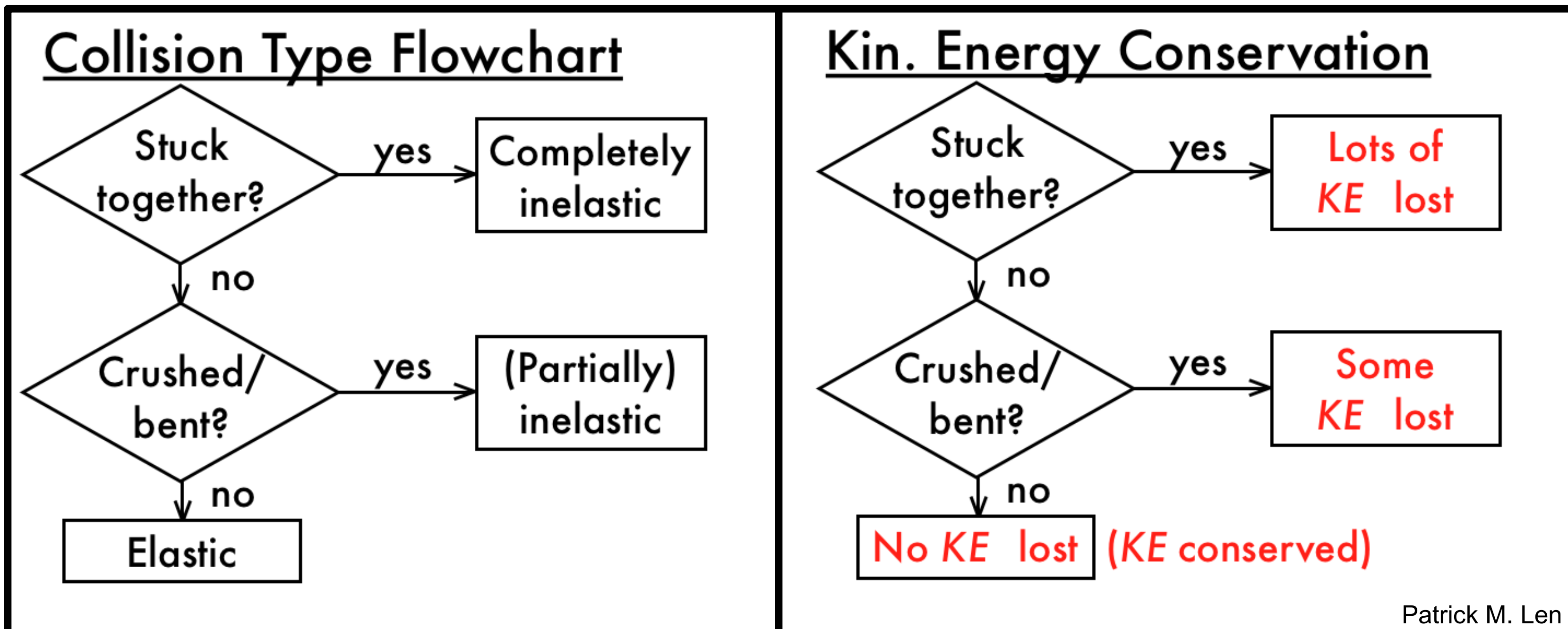


A. 0.14m/s to the right
B. 0.14m/s to the left
C. 1.6m/s to the right
D. 1.6m/s to the left
E. 7.0m/s to the right
F. 7.0m/s to the left

1. Momentum is conserved: $p_{final} = p_{initial}$
2. $p_{initial} = m_1v_1 + m_2v_2$ = (2000kg)(3m/s) - (5000kg)(1m/s) = 1,000kg*m/s
3. They stick together after the collision: m_{final} = 2000kg + 5000kg = 7000kg
4. $p_f = m_{final}v_{final} = p_{initial}$ = 1,000kg*m/s
5. $v_{final} = p_{final}/m_{final}$ = (1,000kg*m/s)/(7000kg) = 0.14m/s to the right

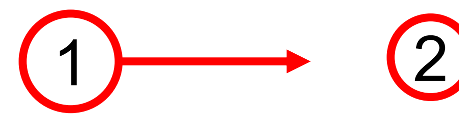
Collision Classification

- ***All*** collisions conserve momentum.
- Collisions that conserve kinetic energy KE are called: *"elastic"* e.g. billiards
- Collisions that ***do not*** conserve KE are called: *"inelastic"* e.g. car crash
- When colliding objects stick together, their collision is: "perfectly inelastic"
  e.g. mosquito on windshield



Elastic collisions between 2 objects

- "Collision": Momentum is conserved
  - $p_{1,i} + p_{2,i} = p_{1,f} + p_{2,f}$
  - $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$
- "Elastic": Kinetic energy is conserved
  - $KE_i = KE_f$
  - $\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$



- Say initially object 1 is moving and object 2 is at rest.

i.e. $m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$ and $\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$

…using these two equations and some algebra: $v_{1,f} = \frac{1}{2}\left(1 - \frac{m_2}{m_1}\right) v_{2,f}$

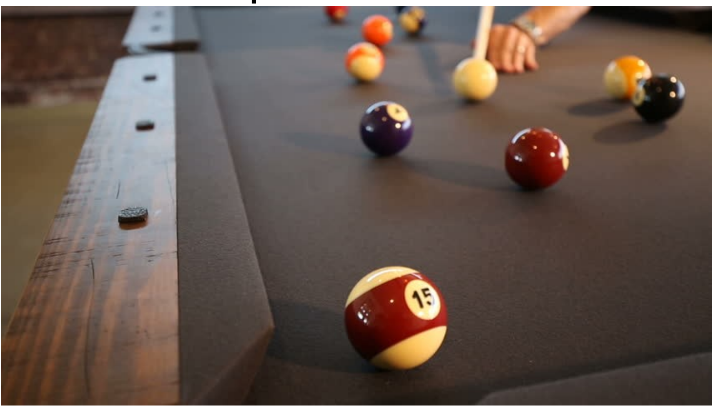
If:

A. $m_1 > m_2$: $\frac{m_2}{m_1} < 1$ …meaning: $v_{1,f} > 0$ …so both 1 and 2 move forward

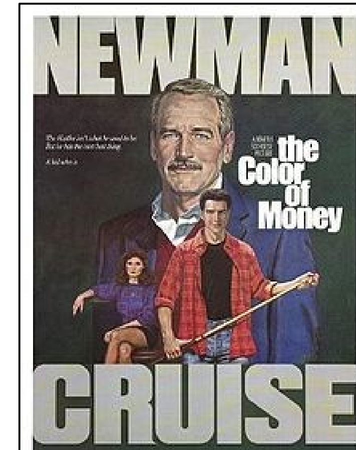
B. $m_1 < m_2$: $\frac{m_2}{m_1} > 1$ …meaning $v_{1,f} < 0$ …so 1 bounces back & 2 moves slowly forward

C. $m_1 = m_2$: $v_{1,f} = 0$ …meaning $v_{2,f} = v_{1,i}$ …so they swap velocities!

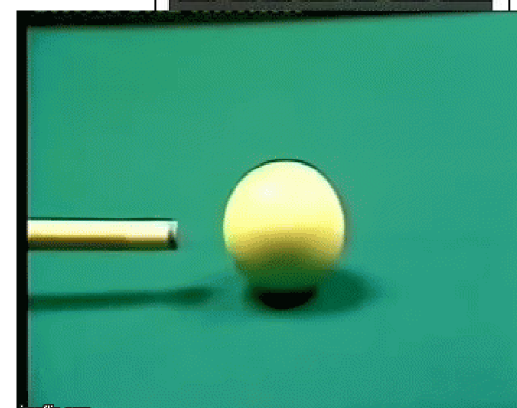
You fancy yourself a young Tom Cruise in the Color of Money.
You want to sink a numbered ball (m_n=0.16kg) in a corner pocket, which the numbered ball is sitting directly in front of.
However, you need to avoid the cue ball falling into the pocket as well (a “scratch”).
Should you hit the cue ball (m_c=0.17kg) directly at the numbered-ball, right toward the corner pocket?



A. Yes
B. Only if you strike it soft
C. Only if you strike it hard
D. No, you’ll scratch.

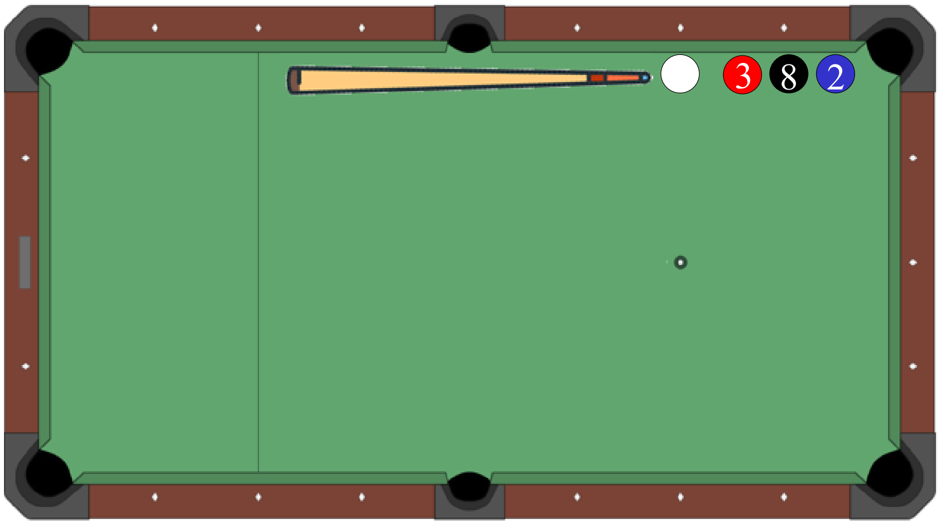


1. Pool balls don’t easily compress, so this is an elastic collision.
2. The initially moving mass (m_c) is greater than the initially stationary mass (m_n).
3. Therefore, both objects will be moving forward after the collision.
4. So the cue ball will probably roll into the corner pocket. You’ll scratch!



https://www.youtube.com/watch?v=GZHJaLMnI1o

You're playing a game of eight-ball and you're solids. You've recovered from an embarrassing scratch, but you need to avoid losing. You want to sink the 2-ball in the corner pocket, but you need to avoid having the 8-ball fall in too. The balls are lined-up as shown. Can you strike the cue ball (m_c=0.17kg) along the line of numbered balls (m_n=0.16kg) without losing?



A. Yes
B. Only if you strike it soft
C. Only if you strike it hard
D. No, you'll lose

1. Pool balls don't easily compress, so this is an elastic collision.
2. The cue-ball will transfer its momentum on to the 3-ball, and roll slightly forward with it.
3. But, the 3-ball will transfer all of its momentum to the 8-ball and come to a rest (blocking the cue-ball's roll afterward).
4. The 8 will then strike the 2, coming to a rest and sending the 2 into the corner pocket.