Tuesday February 19

Topics for this Lecture:

Torque, center-of-mass, equilibrium.

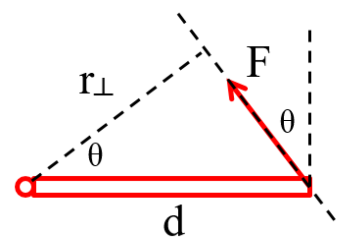
- Assignment 6 due next Friday
  Pre-class due 15min before class
  …like every class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: no Si this week
- Office Hours: 204 EAL, 3-4pm Thurs
  or by appointment (meisel@ohio.edu)

- Torque:
  - Referred to with Greek letter tau: τ
  - Force about an axis.
  - Clockwise: negative torque
  - Counter clockwise: positive torque
  - Directed perpendicular to Force & rotation

1. Draw a simplified version of your situation, including the forces in a free-body diagram
2. Select your axis of rotation
   - For static equilibrium, there is no rotation, so the rotation axis can be anywhere.
   - Choose your rotation axis so that a force you do not know provides zero torque.
3. Write expressions for the balanced forces & torques
   1. $\sum F_x = 0$
   2. $\sum F_y = 0$
   3. $\sum \tau = 0$
4. Work through the algebra & arithmetic

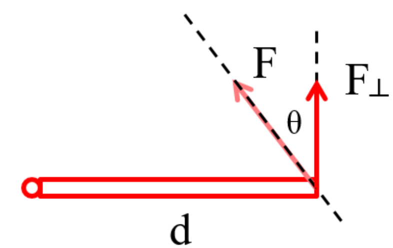
- Torque: $\tau = r \times F = |F||r| \sin(\theta)$
- Two ways to think about this:

(1) <u>Perpendicular distance</u>:



$\tau = F^* r_\perp = F^*[d^*\cos(\theta)]$

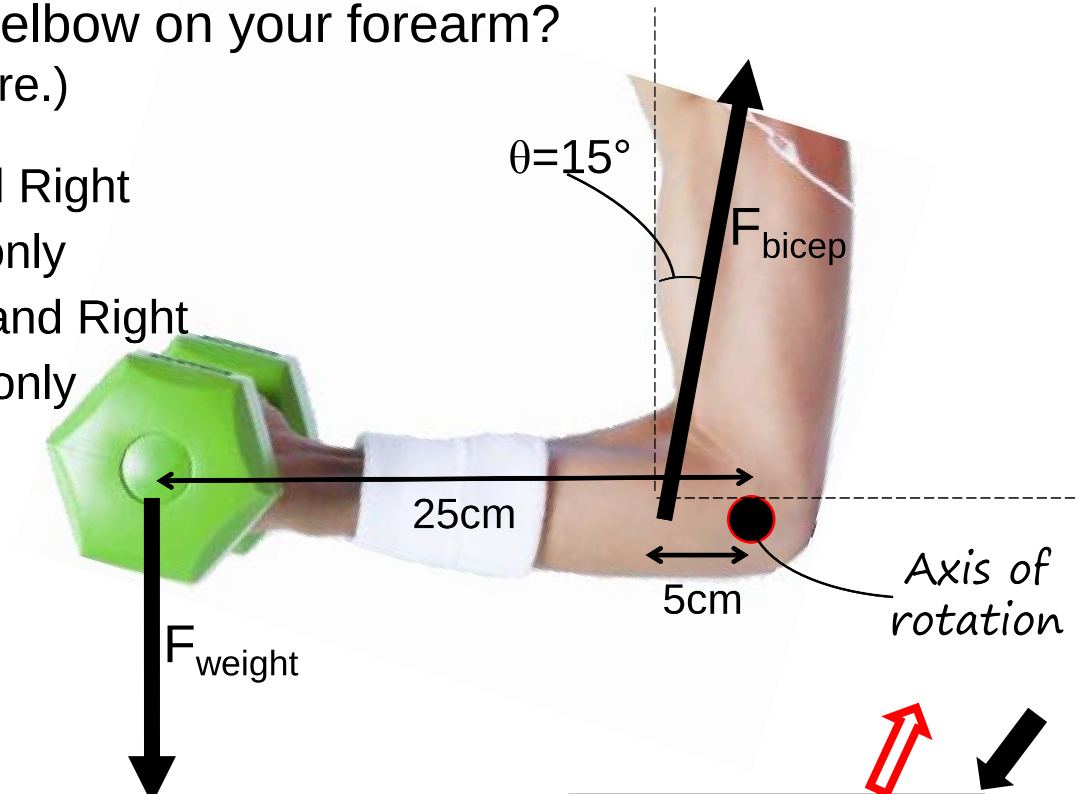
(2) <u>Perpendicular force</u>:



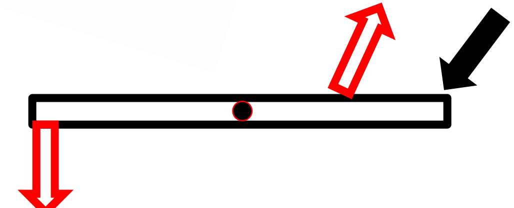
$\tau = F_\perp d = [F^*\cos(\theta)]^* d$

You’re getting swoll in the gym, holding a weight as shown.
Your elbow pushes on your forearm to keep it in static equilibrium
(i.e. to make sure your forearm doesn’t shift or rotate in the middle!).
What direction is the force of your elbow on your forearm?
(Hint: your elbow is <u>*not*</u> the fulcrum here.)

(A) Up and Left
(B) Up and Right
(C) Left only
(D) Right only
(E) Down and Left
(F) Down and Right
(G) Up only
(H) Down only

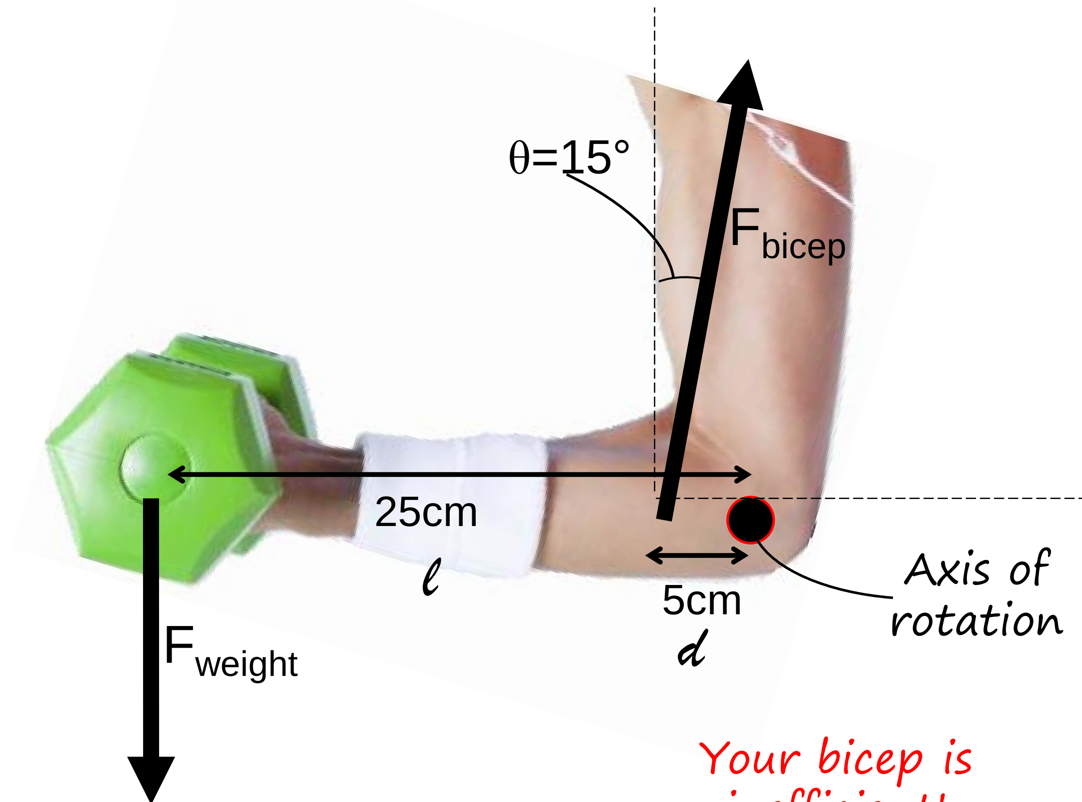


1. “Static Equilibrium” is key here.
   1. Balance Forces
   2. Balance Torque
2. To oppose the horizontal force from your bicep, your elbow must push left.
3. Your bicep & the weight are both producing positive torque. Your elbow must push down to counteract this with negative torque.



You're getting swoll in the gym, holding a weight as shown.
What force must your bicep exert to hold a 10kg weight like this?
(hint: your elbow is the fulcrum)

(A) 98 N
(B) 95 N
(C) 507N
(D) 1892 N
(E) 101 N
(F) 980 N



- Must balance torques:
  - $\sum\tau = 0 = \tau_{weight} - \tau_{bicep}$
  - $\tau_{weight} = F_{weight} * r_\perp = m*g*\ell$
  - $\tau_{bicep} = F_{b,\perp} * d = -F_b\cos(\theta)*d$
  - $m*g*\ell = F_b\cos(\theta)*d$
- Solve for F_b,
  - $F_b = (m*g*\ell)/(\cos(\theta)*d)$
  - $F_b = (10\text{kg}*9.8\text{m/s}^2*25\text{cm})/(5\text{cm}*\cos(15°)) = 507\text{N}$

Your bicep is inefficient!
You need way more force to lift a weight than the force of gravity.

Consider these three images of the Governor of California pumping iron.
For cases (A) and (B), Arnold's forearm is the same angle from horizontal, but his bicep is closer to vertical in case (B).
For cases (A) and (C), Arnold's bicep is the same angle from vertical, but his forearm is closer to horizontal in case (A).
If he has the same amount of weight in each hand for all cases, which case requires the greatest force from his bicep?

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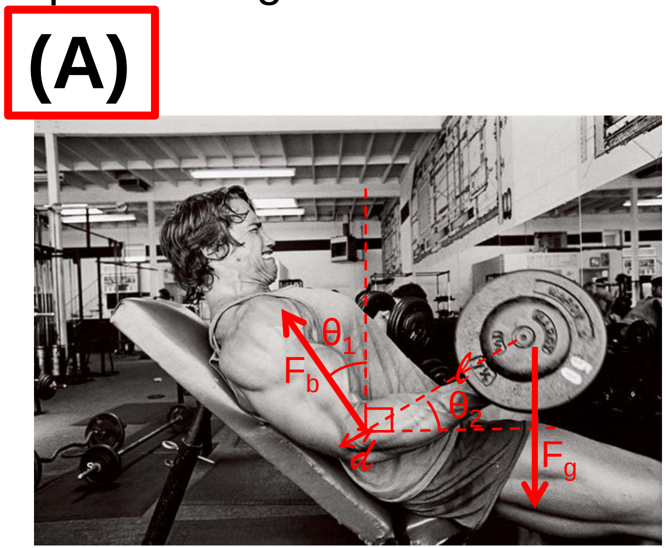
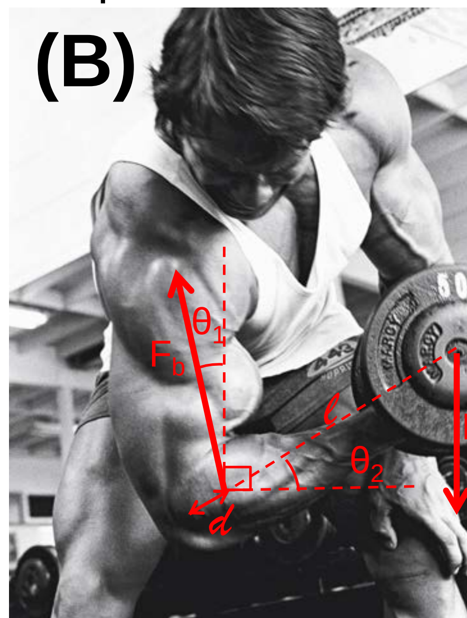
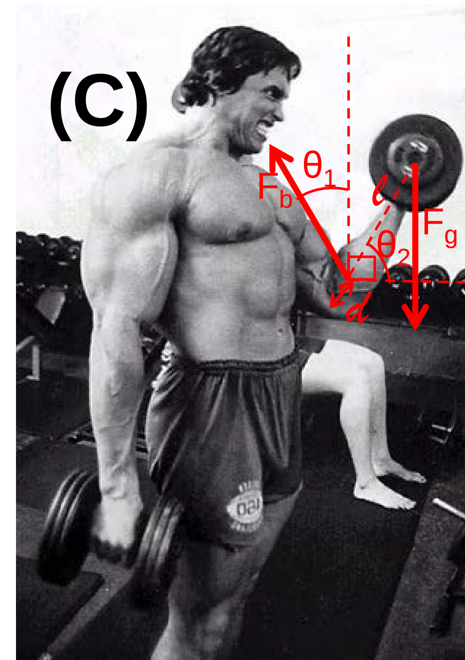
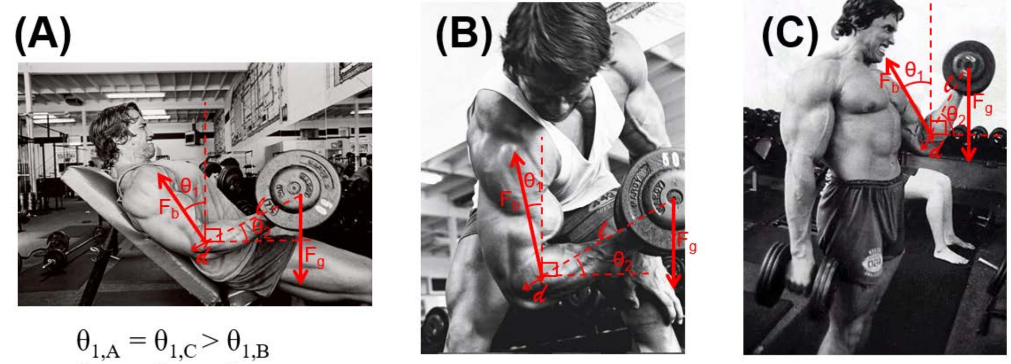




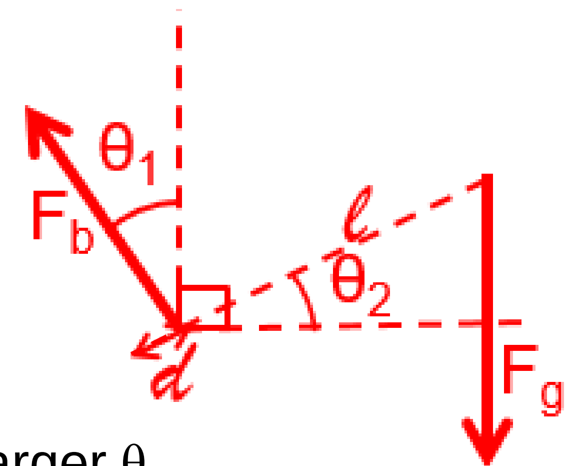


$$\theta_{1,A} = \theta_{1,C} > \theta_{1,B}$$
$$\theta_{2,A} = \theta_{2,B} < \theta_{2,C}$$
$$F_{g,1} = F_{g,2} = F_{g,\,3}$$

- Must balance torques:
  - $\sum\tau = 0 = \tau_{weight} - \tau_{bicep}$
  - $\tau_{weight} = F_{weight} * r_{\perp} = m*g*\ell*\cos(\theta_2)$
  - $\tau_{bicep} = F_{b,\perp} * d = F_b\cos(\theta_1) * d$
  - $m*g*\ell*\cos(\theta_2) = F_b\cos(\theta_1) * d$
- Solve for F_b,
  - $F_b = (m*g*\ell*\cos(\theta_2))/(\cos(\theta_1)*d)$
  - Keep in mind:
    - $\cos(0°) = 1$; $\cos(90°) = 0$
- Smaller θ_2 leads to a larger F_{bicep}
  - A more horizontal forearm requires more force
- Larger θ_1 leads to a larger F_{bicep}
  - A less vertical bicep requires more force
- Case A is tied for smallest θ_2 with case B, but has a larger θ_1.
- Case A is tied for largest θ_1 with case C, but has a smaller θ_2.
- Case A therefore requires the largest bicep force.



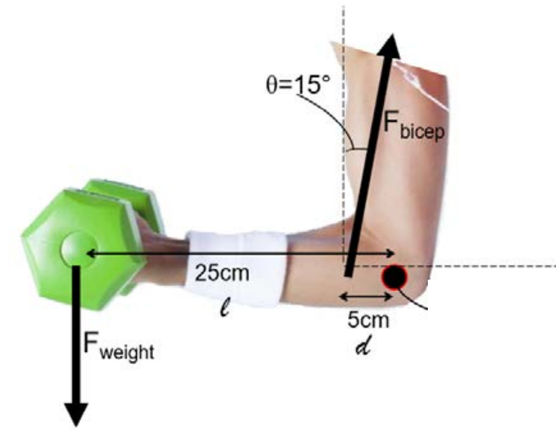
$\theta_{1,A} = \theta_{1,C} > \theta_{1,B}$
$\theta_{2,A} = \theta_{2,B} < \theta_{2,C}$
$F_{g,1} = F_{g,2} = F_{g,3}$

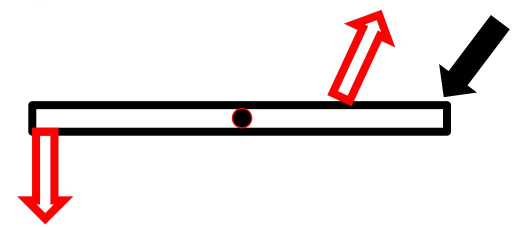


Tips for solving static equilibrium problems

no translational or rotational motion

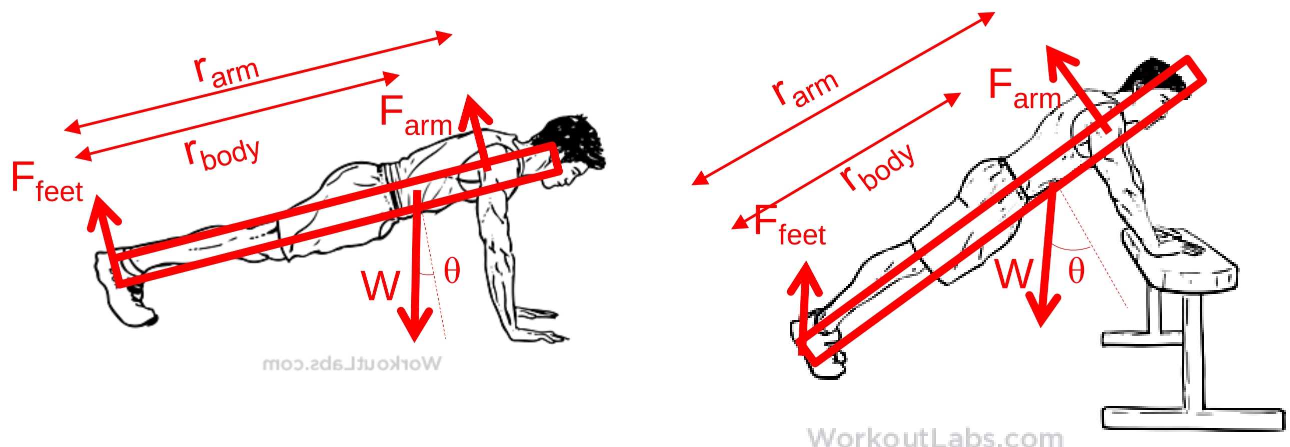
1. Draw a simplified version of your situation, including the forces in a free-body diagram
2. Select your axis of rotation
   - For static equilibrium, there is no rotation, so the rotation axis can be anywhere.
   - Choose your rotation axis so that a force you do not know provides zero torque.
3. Write expressions for the balanced forces & torques
   1. $\sum F_x = 0$
   2. $\sum F_y = 0$
   3. $\sum \tau = 0$
4. Work through the algebra & arithmetic





Static Equilibrium example: Push-ups

- Why are incline push-ups easier than push-ups on the ground?



How did we choose r_{body}? Center-of-gravity

1. Draw free-body diagrams for both situations
2. Choose the feet as the axis of rotation
   because we don’t care about the force of the feet on the ground
3. Write expressions for sum of forces & torques:
   - $\sum F_x = 0$; $\sum F_y = 0$; $\sum \tau = 0$
   - Here just need to consider torques:
     - $\sum \tau = \tau_{arm} - \tau_{weight}$ …. So: $\tau_{arm} = \tau_{weight}$ … i.e. $F_{arm} r_{arm} = W\cos(\theta) r_{body}$
     - $F_{arm} = W\cos(\theta) r_{body} / r_{arm}$ (note: $\cos(\theta) \to 0$ as 90°.)

   Because arm is perpendicular to body

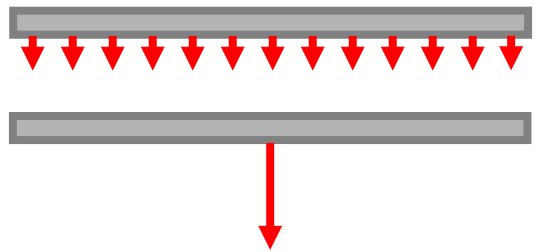
4. For both cases, arm is perpendicular to body. But, θ is larger for incline push-ups.

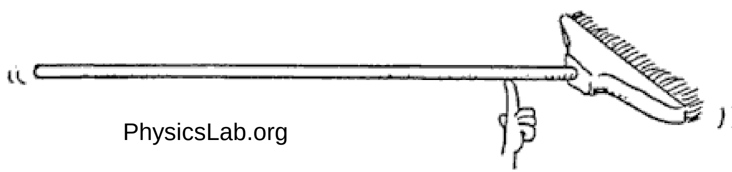
Center of Gravity (a.k.a. Center of Mass)

Sec. 9.3

- For extended objects, it is preferable to draw gravity acting on that object at a single location
  - In reality, gravity acts on several parts of an object.
  - However, the average interaction is as if all of the object's mass was concentrated at center of the object's mass distribution
- This location is known as the "center of gravity" (or "center of mass")
- For a symmetric object, actual center
- For irregular object, will be offset
- Think about where you would have to hang an object from for it to hang balanced & not tilted

Youtube:@CrazyRussianHacker

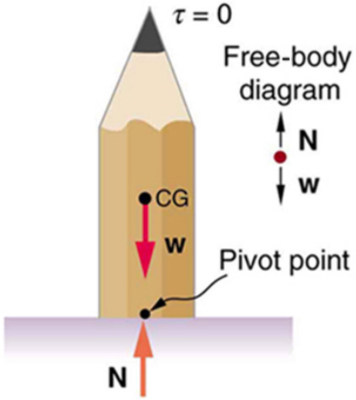




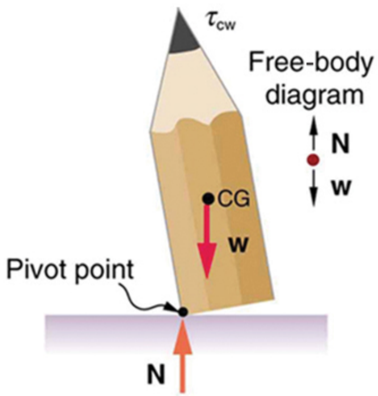
PhysicsLab.org

Stability & Equilibrium

If center of gravity (CG)
over or under point(s) of support, then stable.

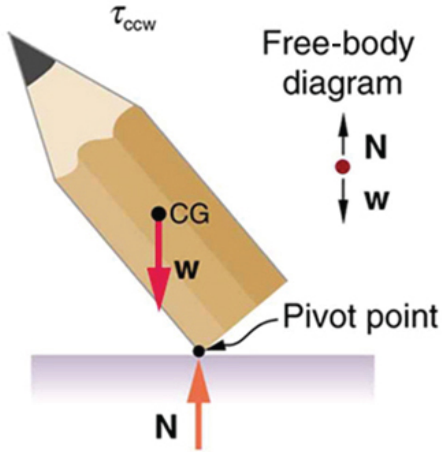


***Stable* Equilibrium**:



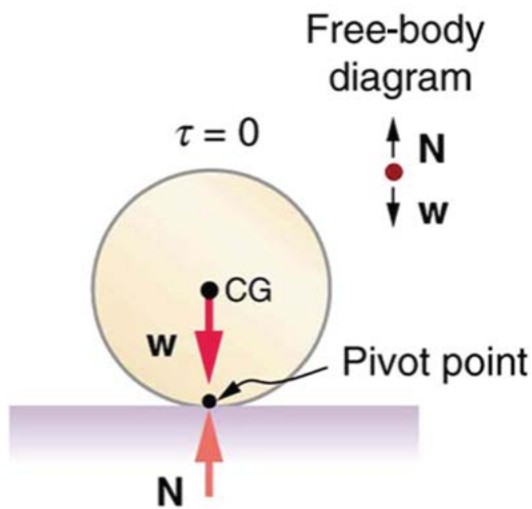
displace from equilibrium
and it returns

***Unstable* equilibrium**:
displaced but does not
return to equilibrium



Neutral Equilibrium: The most boring kind of equilibrium

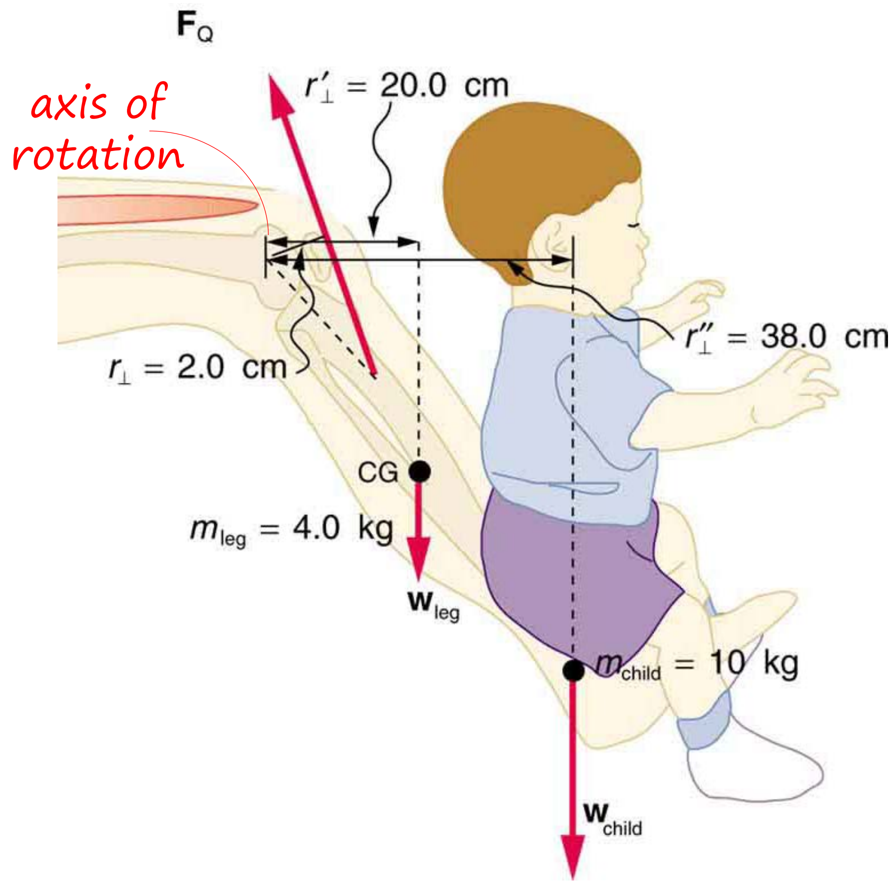
***Neutral* equilibrium**: displace & object winds up in a new stable state
(e.g. rolling around a sphere)



Neutral
(a)

A child is sitting on an adult's leg. Given the measurements shown, what is the torque due to the child if we use the knee-cap as the axis of rotation?

(A) +0.388 Nm
(B) – 0.388 Nm
(C) + 3.80 Nm
(D) – 3.80 Nm
(E) + 3.72 Nm
(F) – 3.72 Nm
(G) + 37.2 Nm
(H) – 37.2 Nm



$\tau = F * r_\perp$

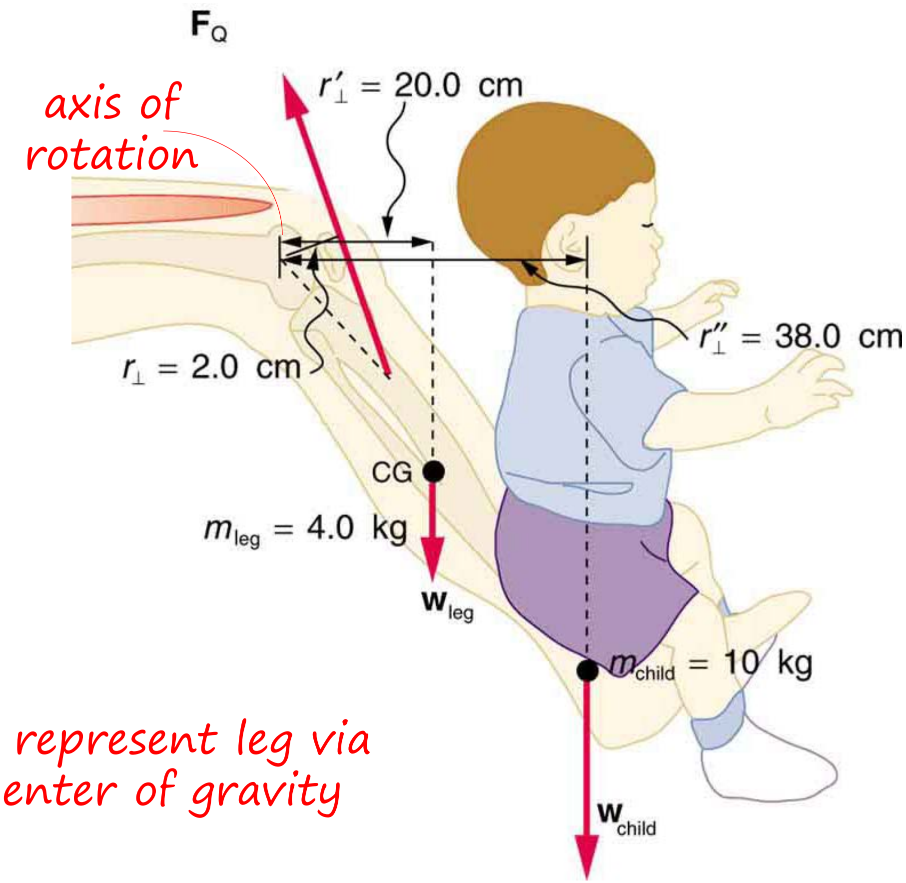
$= -m_{child} g * r_\perp''$

$= -(10kg)(9.8m/s^2)(0.38m)$

$= -37.2$ N*m

A child is sitting on an adult's leg. Given the measurements shown, what is the torque due to the leg if we use the knee-cap as the axis of rotation?

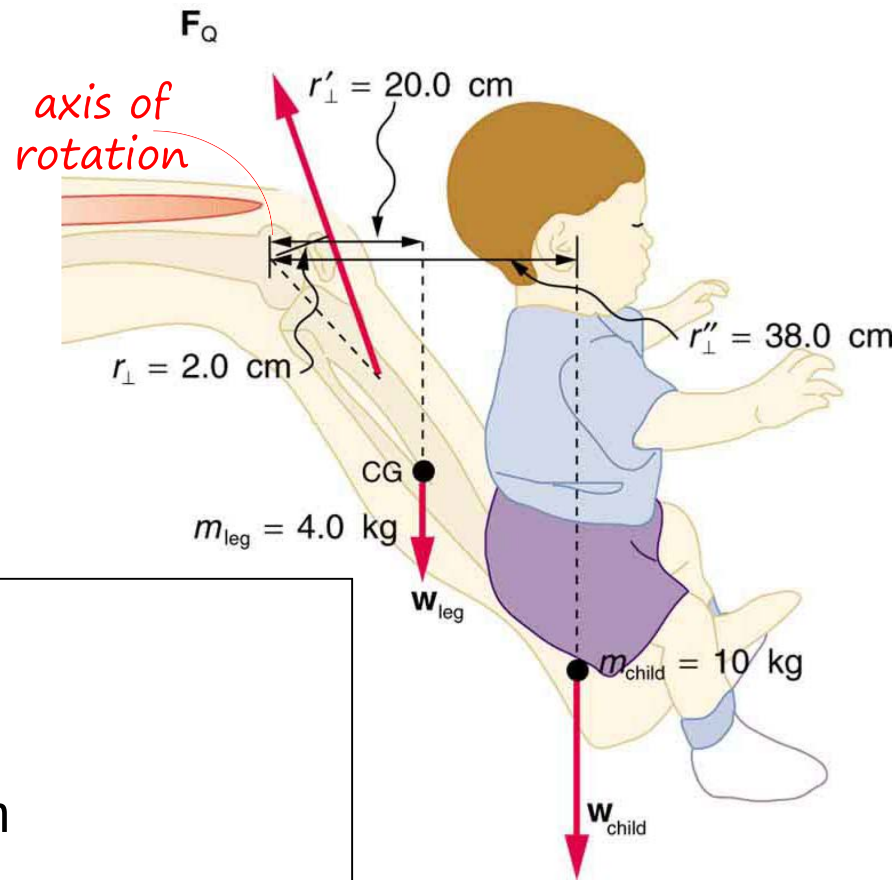
(1) +0.8 Nm
(2) – 0.8 Nm
(3) + 3.20 Nm
(4) – 3.20 Nm
(5) + 7.84 Nm
(6) – 7.84Nm
(7) + 15.8 Nm
(8) – 15.8 Nm



$$\tau = F^*r_\perp$$
$$= -m_{leg}g^*r_\perp'$$
$$= -(4kg)(9.8m/s^2)(0.2m)$$
$$= -7.84\ N^*m$$

Can represent leg via its center of gravity

A child is sitting on an adult's leg, as shown. What is the tension in the upper leg muscle (pink arrow pointing to the upper-left)?



(1) 0.0 N

(2) 225 N

(3) 2252 N

(4) 45 N

(5) 0.9 N

(6) 5600 N

Balance torques:

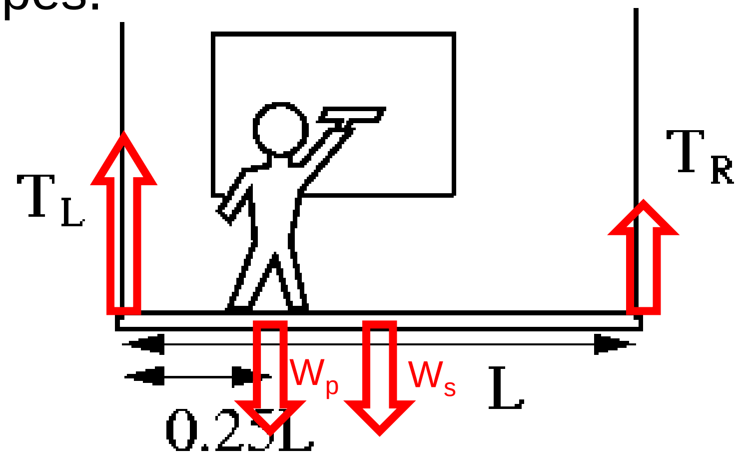
1. $\sum\tau = \tau_{muscle} - \tau_{leg} - \tau_{child} = 0$
2. $\tau_{muscle} = m_{leg}g*r_\perp' + m_{child}g*r_\perp''$
3. $= 7.84Nm + 37.2Nm = 45.04Nm$
4. $\tau_{muscle} = F_{muscle}*r_\perp$
5. $F_{muscle} = \tau_{muscle}/r_\perp = (45.04Nm)/0.02m = 2252N$

A 800N painter stands on a 400N scaffold (a uniform rectangular board) with one rope supporting each end.
The painter stands one-quarter of the length of the scaffold from the left side.
Compare the tension in the two ropes.

(A) $T_L > T_R$

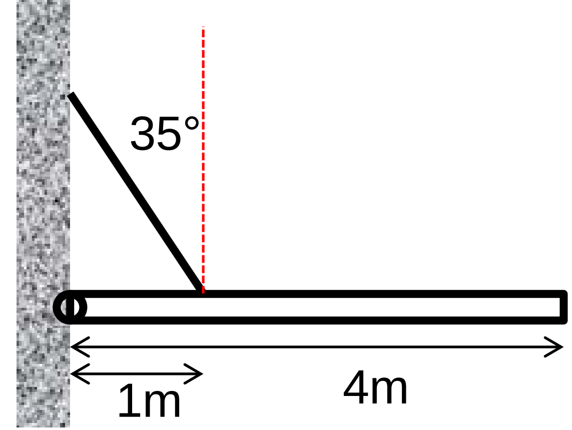
(B) $T_L < T_R$

(C) $T_L = T_R$



- Consider torque axes where ropes connect
  - Left rope: $\sum\tau = \tau_{rope,r} - \tau_{painter,1} - \tau_{scaffold} - = T_R{*}L - W_p(0.25{*}L) - W_{scaffold}(0.5{*}L) = 0$
  - Right rope: $\sum\tau = -\tau_{rope,l} + \tau_{painter,2} + \tau_{scaffold} = (-T_L{*}L) + W_p(0.75{*}L) + W_{scaffold}(0.5{*}L) = 0$
- From the left rope:
  - $T_R = (W_p{*}0.25)+(W_s{*}0.5)$
- From the right rope:
  - $T_L = (W_p{*}0.75)+(W_s{*}0.5)$

A 1500N beam is attached to a wall by a hinge.
The beam is 4.0m long, and the mass of the beam is distributed evenly along the beam.
The beam is supported by a cable that is attached 1.0m to the beam from the wall and makes an angle of 35° from the vertical.
We want to find the tension in the cable.



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At what point does it make the most sense to place the axis of rotation?

(A) Hinge

(B) Point Cable attaches

(C) Center of beam

(D) Right End

- The forces involved are: force of hinge on the board, the tension of the cable pulling on the board, and gravity pulling the board down.
- If we place the axis of rotation at the hinge, we can ignore the force of the wall on the board.
- Then, by equating our torques, the torque from the weight of the board can be related to the torque of the cable on the board.

A 1500N beam is attached to a wall by a hinge.
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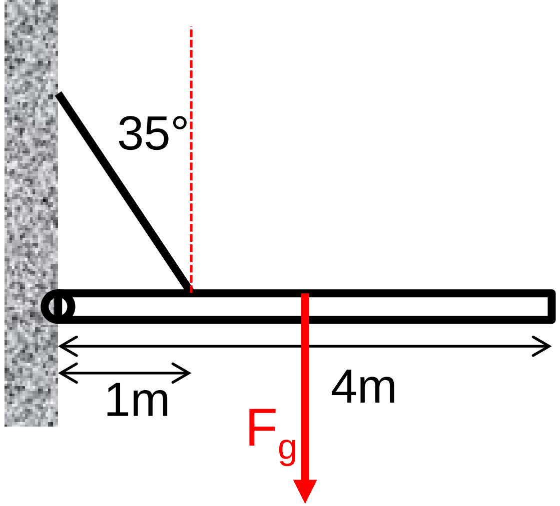


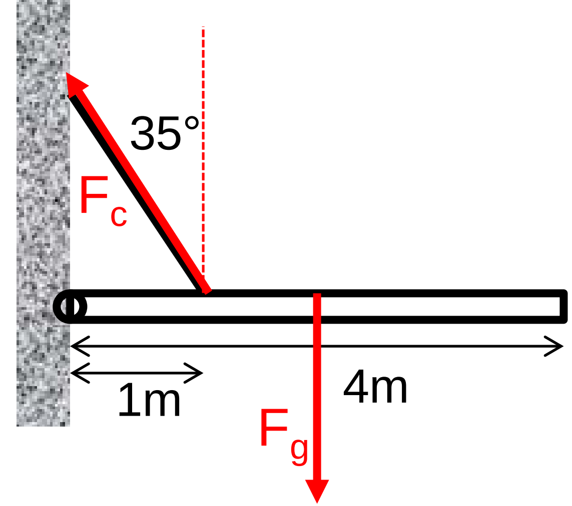
What is the lever-arm for the force due to gravity, where the hinge is the axis?

(A) 1m (B) 2m (C) 3m (D) 4m

- The force of gravity will act as if it is pulling on the center-off mass of the board.
- The board is a uniform object, so the center-of-mass will be at the physical center.

A 1500N beam is attached to a wall by a hinge.
The beam is 4.0m long, and the mass of the beam is distributed evenly along the beam.
The beam is supported by a cable that is attached 1.0m to the beam from the wall and makes an angle of 35° from the vertical.

What is the tension in the cable?



(A) 1500N (B) 2460N (C) 3000 N **(D) 3660 N** (E) 5230 N (F) 6000N

1. Choose the hinge as the axis of rotation, so that we can ignore its force, and then balance the torques about there.
2. $\sum \tau = \tau_{cable} - \tau_{gravity} = 0$
3. $\tau_{cable} = \tau_{gravity} = m_{board} g * (L_{board}/2) = (1500N)(2m) = 3000Nm$
4. $\tau_{cable} = F_{cable,\perp} r_{cable} = F_{cable} \cos(\theta) r_{cable}$
5. $F_{cable} = \tau_{cable}/[r_{cable}\cos(\theta)] = (3000Nm)/(1m*\cos(35°)) \approx 3660N$

A 1500N beam is attached to a wall by a hinge.
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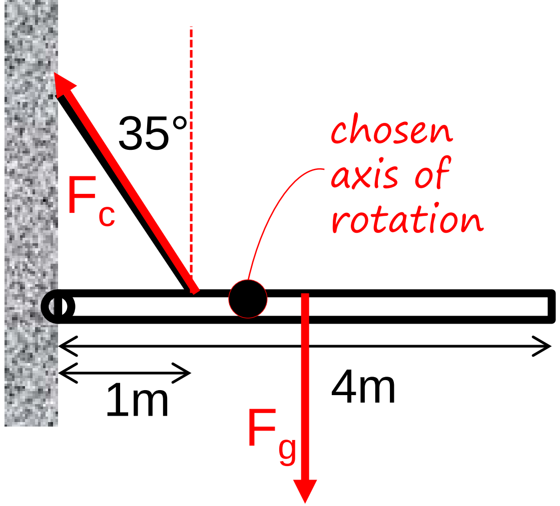
The beam is supported by a cable that is attached 1.0m to the beam from the wall and makes an angle of 35° from the vertical.



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What direction is the force of the hinge on the board?

(A) Up & Right (B) Directly Right (C) Down & Right

1. Want to choose a different axis of rotation, so we can consider what torque the hinge exerts on the board.
   We can do this because the board is in static equilibrium, i.e. not rotating about any point.
2. We can choose the point in between where the cable connects and the board's center of mass.
   - (If we chose for instance, the right end of the board, this would be a valid choice, but it wouldn't help us solve for the wall's force).
3. Want to balance the torques.
   1. Weight & cable provide negative torques.
   2. So wall must provide positive torque by pushing down.

A 1500N beam is attached to a wall by a hinge.
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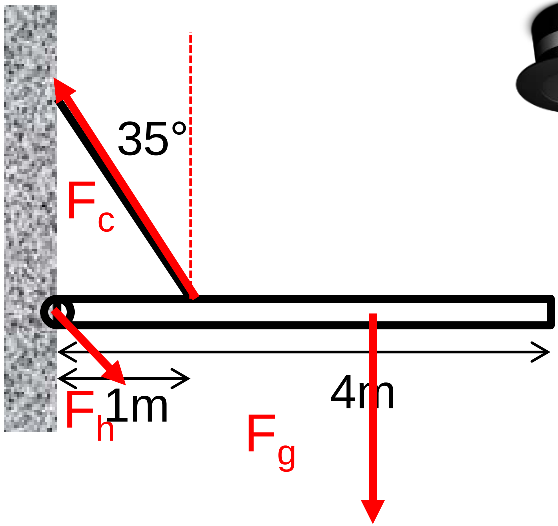
The beam is supported by a cable that is attached 1.0m to the beam from the wall and makes an angle of 35° from the vertical.



What is the magnitude of the force of the hinge on the board?

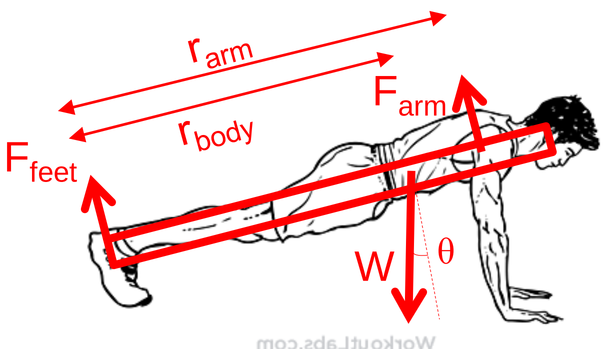
(A) 1498 N (B) 3660 N **(C) 2580 N** (D) 2100 N

1. Just use F=ma
2. $\sum F_y = F_{c,y} - F_{g,y} - F_{h,y} = 0$
3. $F_{h,y} = F_{c,y} - F_{g,y}$
4. $F_{h,y} = F_c\cos(\theta) - F_g = (3660N)*\cos(35°) - 1500N = 1498N$
5. $\sum F_x = F_{h,x} - F_{c,x} = 0$
6. $F_{h,x} = F_c\sin(\theta) = (3660N)*\sin(35°) = 2100N$
7. $F_h = \sqrt{F_{h,x}^2 + F_{h,y}^2} = 2580$ N

Static Equilibrium example: Push-ups & Center of Gravity

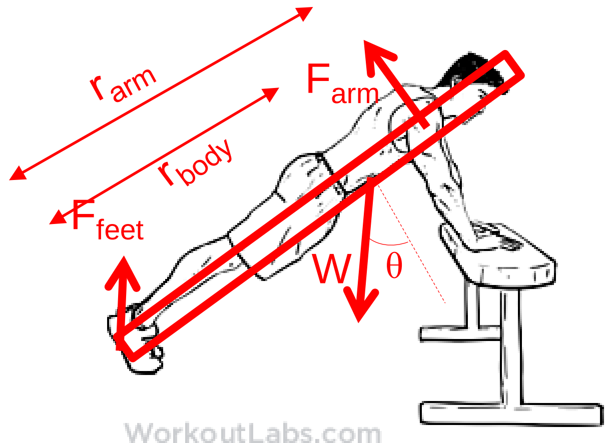
- How else can we make push-ups easier?

Regular:



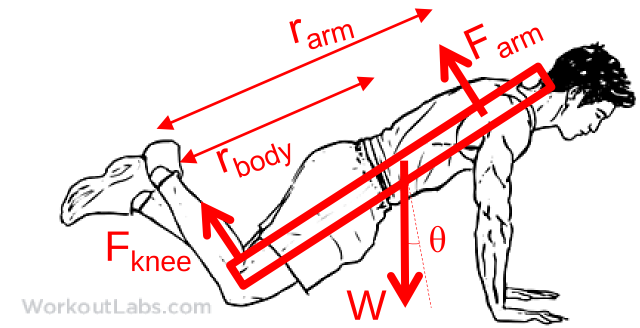
$$F_{arm} r_{arm} = W\cos(\theta) r_{body}$$

Incline:



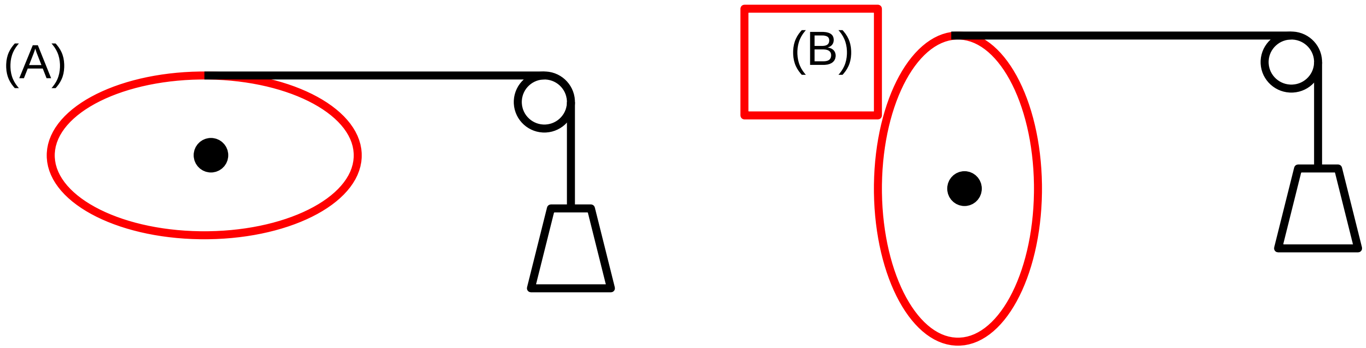
- Reduce perpendicular force of gravity
- So need less arm torque to counteract body torque

"Modified":



- Reduce weight.
- Reduce perpendicular distance for arms, but reduce perpendicular distance more for weight (because CoG is almost in same spot in body)

A cable is attached to an elliptical rim and attached via a pulley to a weight.
The black dot represents the axis.
In which configuration is the torque on the ellipse the greatest?



- Longer lever arm for B, so greater torque.
- Fun fact:
  - Exercise machines take advantage of elliptical gears to adjust the force at different points of motion.
  - Your lever-arm is fixed, but the lever to the connected load varies.