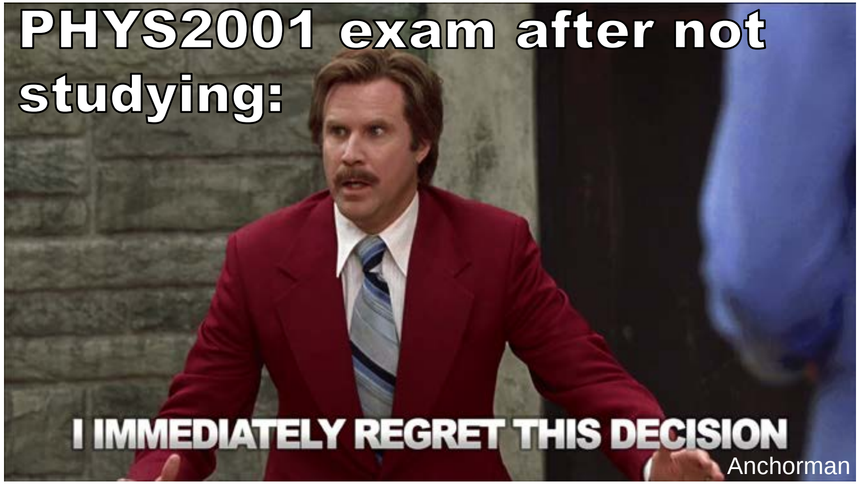
Thursday February 14

Topics for this Lecture:

Torque, rotation, & equilibrium.

Torque is <u>not</u> on Exam 1.

- Torque:
  - Referred to with Greek letter tau: τ
  - Force about an axis.
  - Clockwise: negative torque
  - Counter clockwise: positive torque
  - Directed perpendicular to Force & rotation



- Assignment 5 due Friday
  …like almost every Friday
- Pre-class due 15min before class
  …like every class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 226, M&Th 6:20-7:10pm
- Office Hours: 204 EAL, 3-4pm Thurs
  or by appointment (meisel@ohio.edu)
- **Exam Monday February 18.**
  Morton 201   7:15-9:15PM
  - Email me ASAP if you have a class conflict or need special accommodations through accessibility services
  - ***Study!***

Last time, on PHYS2001

- Exam 1 will cover:
  - Basics (e.g. scalar vs vector, graphs)
  - 1D kinematics (e.g. free-fall)
  - 2D kinematics (e.g. projectile motion)
  - Friction and tension

- You should bring:
  - one 8.5” x 11” sheet of paper with anything you like written and/or printed on either side
  - a calculator

So far we only have 2 concepts and 14 equations to deal with

Kinematics

2D Projectile motion eqns.

1. $v_x = v_{x,0}$
2. $x = x_0 + v_{x,0}t$
3. $v_y = v_{y,0} + a_y t$
4. $y = y_0 + v_{y,0}t + \frac{1}{2}a_y t^2$
5. $y = y_0 + \frac{1}{2}\left(v_{y,0} + v_y\right)t$
6. $v_y^2 = v_{y,0}^2 + 2a_y(y - y_0)$

Both

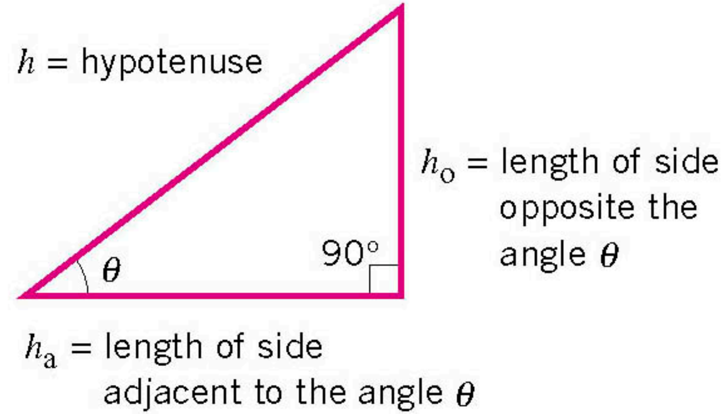
$$h = \sqrt{h_a^2 + h_o^2}$$

SOH-CAH-TOA

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



Dynamics

$$\vec{F}_{NET} = m\vec{a}$$

F_{grav} = mg   (g=9.8m/s^2)

$F_{\text{friction}} = \mu_{\textbf{kinetic}} F_{\text{normal}}$

$F_{\text{friction}} \leq \mu_{\textbf{static}} F_{\text{normal}}$

Scalars and Vectors

- *Scalars*: Have magnitude (roughly speaking, a "size")
- *Vectors*: Have magnitude ***and*** direction (relative to an origin)

Quick way to check: Does adding a geographical direction make sense?

- A force of 10 N to the East ? fine – vector
- A mass of 10kg to the West? nonsensical - scalar

Examples of *Scalars*:

- Time
- Mass
- Energy

Examples of *Vectors*:

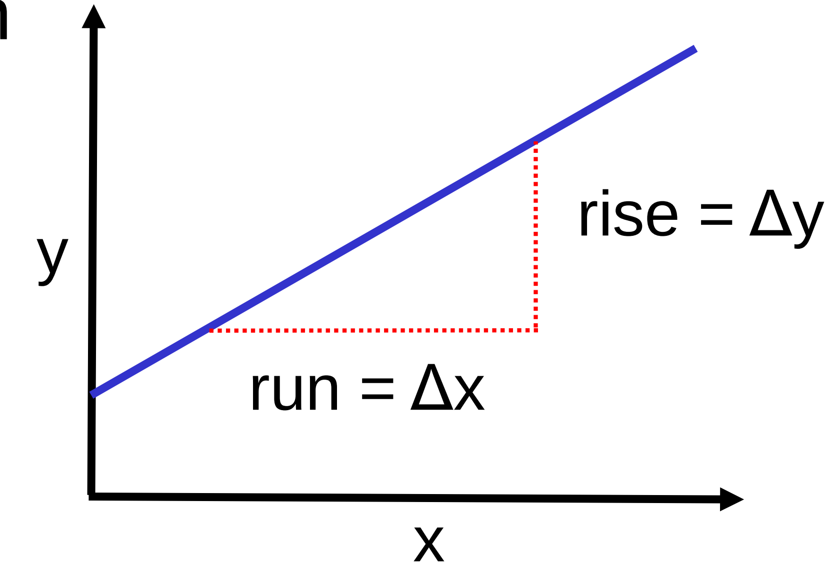
- Force
- Displacement
- Velocity

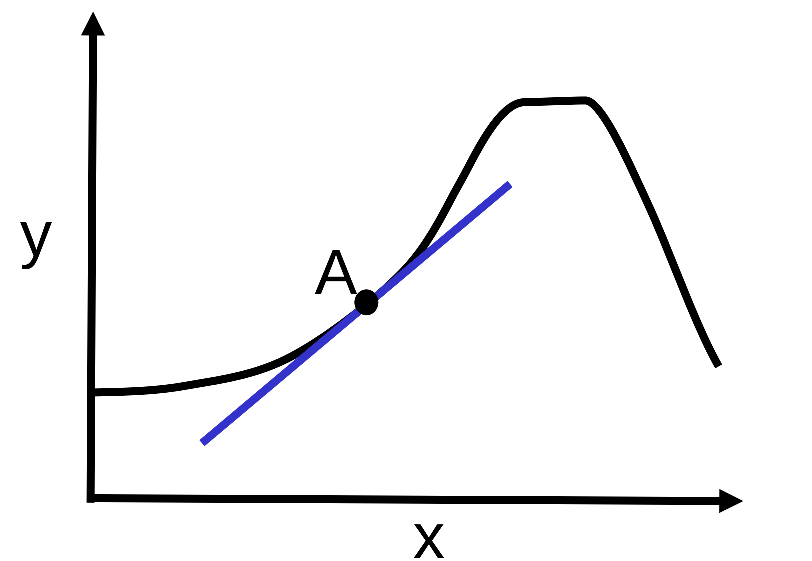
Slope of a Function on a Graph

- Slope = rise/run = Δy/Δx
- Up to the right is a positive slope.
- Down to the right is negative slope.

- Slope of a function at a point is the slope of tangent line.
- Slope of straight line same at any point.

- When finding slope off a graph, use the longest straight section possible in order to minimize your error.





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Problem Solving Example: Accelerating Car

A car accelerates uniformly from rest to a speed of 25 m/s in 8.0s.

Find the distance the car travels in this time and the constant acceleration of the car. Define the +x direction to be in the direction of motion of the car.

1D, *a*=constant Eqns:

1. $x = x_o + \bar{v}t$
2. $\bar{v} = \frac{v_0 + v}{2}$
3. $v = v_0 + at$
4. $x = x_0 + v_o t + (½)at^2$
5. $v^2 = v_0^2 + 2a(x - x_0)$

What do we know?

- "Accelerates uniformly": a = constant
- "from rest": $v_i = v(t=0) = 0$ m/s
- "to a speed of 25 m/s in 8 s": $v_f = v(t=8\text{s}) = 25$ m/s

What do we want to know?

- "Find the distance": $x_f - x_i$ = ? (say x_i=0, so x_f = ?)
- Find "the acceleration": a = ?

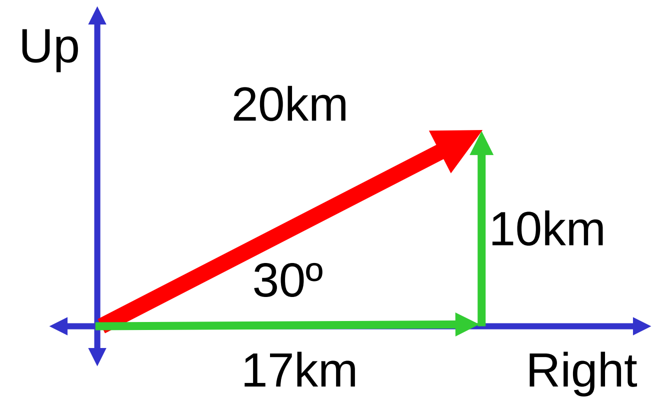
Which equations are useful?:

- Eqn. 1 gives distance …but need to combine with 2 to get average velocity
- Eqn. 3 gives the acceleration if you know the initial and final velocities

Solutions:

1. $x - x_0 = \bar{v}t = 0.5\left(0\frac{m}{s} + 25\frac{m}{s}\right)(8\ s) = 100\ m$
2. $a = \frac{(v - v_0)}{t} = \left(25\frac{m}{s} - 0\frac{m}{s}\right)\frac{1}{(8\ s)} = 3.1\ m/s^2$

Vector and Scalar Components



- Can refer to vector components
  - "x-component": 17km to the right
  - "y-component": 10km up
- Since we know the direction of a component (because it is along a single dimension) we can treat the **magnitude of a component as a scalar** and use the **sign** (+ or -) **to represent direction** (up/down or left/right)
  - x-component: +17km
  - y-component: +10km
- With these two components, our vector is fully defined

Projectile motion is like free-fall, but with horizontal uniform motion:

- For free-fall: acceleration 9.8 m/s^2 downward
- IF we choose y-axis vertical (+y up typically)
- Then: $a_x = 0$ m/s^2, $a_y = -9.8$m/s^2
- HTML5
- Our 2D equations of motion can be simplified:

$$\boxed{x = x_0 + \bar{v}_x t}$$

$$\boxed{\cancel{\bar{v}_x = \frac{1}{2}(v_{0x} + v_x)}}$$

$$\boxed{v_x = v_{0x} + \cancel{a_x t}}$$

$$\boxed{x = x_0 + v_{0x} t + \cancel{\frac{1}{2} a_x t^2}}$$

$$\boxed{\cancel{v_x^2 = v_{0x}^2 + 2a_x(x - x_0)}}$$

Uniform motion in x:

$$\boxed{v_x = v_{0x}}$$

$$\boxed{x = x_0 + v_{0x} t}$$

Free-fall in y:

$$\boxed{y = y_0 + \bar{v}_y t}$$

$$\boxed{\bar{v}_y = \frac{1}{2}(v_{0y} + v_y)}$$

$$\boxed{v_y = v_{0y} + a_y t}$$

$$\boxed{y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2}$$

$$\boxed{v_y^2 = v_{0y}^2 + 2a_y(y - y_0)}$$

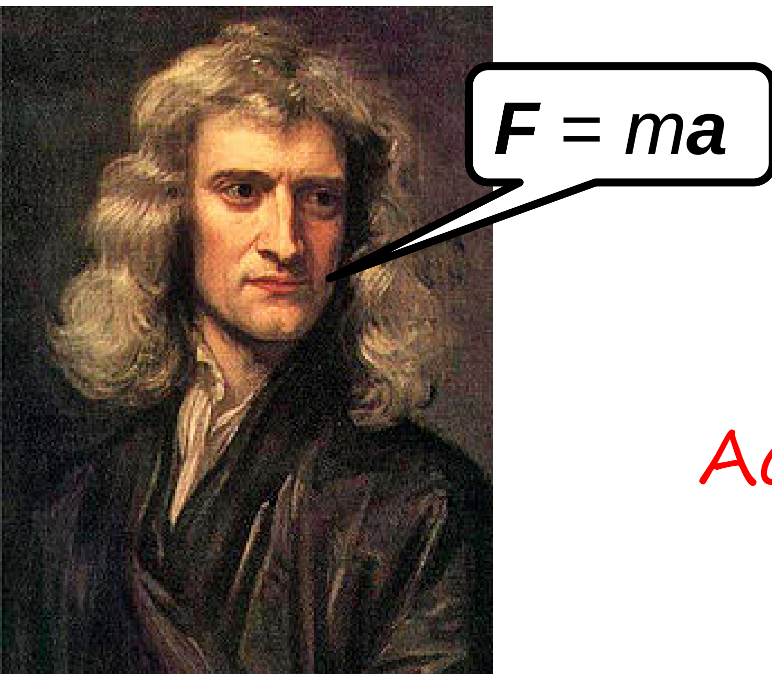
How to interpret a problem:

Often you will have to glean extra information from the problem to find out what you know, what you don't know, and what is asked. (This is almost always the case in real life)

- "Speed": magnitude of velocity
- "Max height": vertical velocity is zero
- "Just after __ leaves __": initial information ($t = 0$)
- "Just after __ hits __": final information ($t = t_{final}$)
- Look for clues about coordinates: "horizontal", "vertical"
- Note if there are any other forces other than gravity
    - If other forces are there, acceleration will not be 9.8m/s^2

A net force of 250N is exerted to the right on a large box of mass 50kg. What is the acceleration of the box? (1N= 1kg m/s^2)

(A) 0.2 m/s^2 to the right (B) 0.2 m/s^2 to the left (C) 1.25 m/s^2 to the right
(D) 1.25 m/s^2 to the left (E) 5.0 m/s^2 to the right (F) 5.0 m/s^2 to the left
(G) 12.5 m/s^2 to the right (H) 12.5 m/s^2 to the left



$$\vec{a} = \frac{\vec{F}_{NET}}{m} = \frac{250\text{N}}{50\text{kg}} = 5.0\text{m/s}^2$$

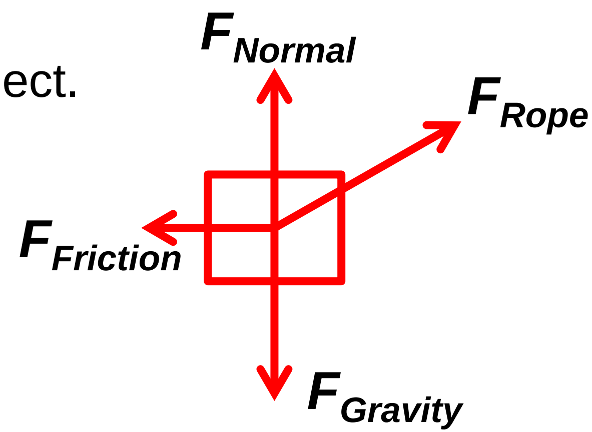
Acceleration same direction as the net force.

Forces: Learning Goals

- What is a force? How do we identify a force in a situation?
- What is the connection between force and motion?
- How do forces describe & relate to the interaction of two objects?
- Application:
  - Different forces: Contact & Non-contact forces
  - Different situations
- Representing forces with a "free-body diagram"
  1) Draw an object
  2) Draw & label an arrow for each force acting on that object.
     Often draw the arrows from the center of the object.

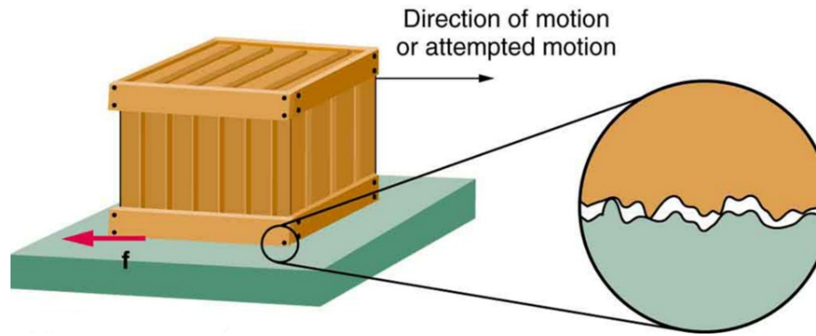
In this example, on the rock:

- The ground exerts a "normal force"
- The Earth's mass exerts a gravitational force
- The ground's unevenness exerts a frictional force
- The rope exerts a force



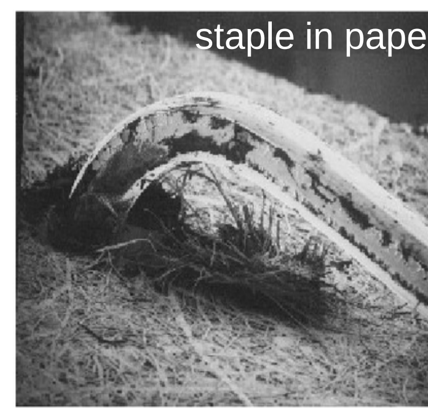
Friction: the force that opposes sliding

- Friction depends on the surface's
  - Normal Force: F_{normal}
  - Coefficient of Friction: μ Greek letter mu
  - $F_{friction} = \mu F_{normal}$

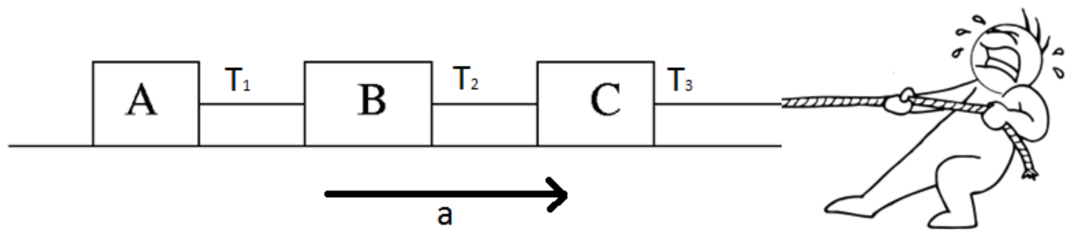


- The coefficient of friction depends motion & the surface
  - Motion:
    - If moving, "kinetic friction": $F_{friction} = \mu_{\mathbf{kinetic}} F_{normal}$
    - If stationary, "static friction" $F_{friction} \leq \mu_{\mathbf{static}} F_{normal}$

      …which means the static friction force only pushes as hard as it has to.
    - Once the force initiating sliding exceeds the static friction, you switch to kinetic friction.
  - The coefficient of friction depends on the material
    - Nothing is perfectly smooth. Typical values: ~0.01 to 1
    - See Table 5.1 of the textbook for some μ_k & μ_s



Three boxes are accelerating to the right at a rate of 2.0m/s^2.
All 3 have non-zero mass.
How to T_1, T_2, and T_3 relate?



(A) $T_1 = T_2 = T_3$

$\boxed{\text{(B) } T_1 < T_2 < T_3}$

(C) $T_3 < T_2 < T_1$

(D) $T_1 < T_2 = T_3$

(E) $T_3 = T_2 < T_1$

(F) $T_1 = T_2 < T_3$

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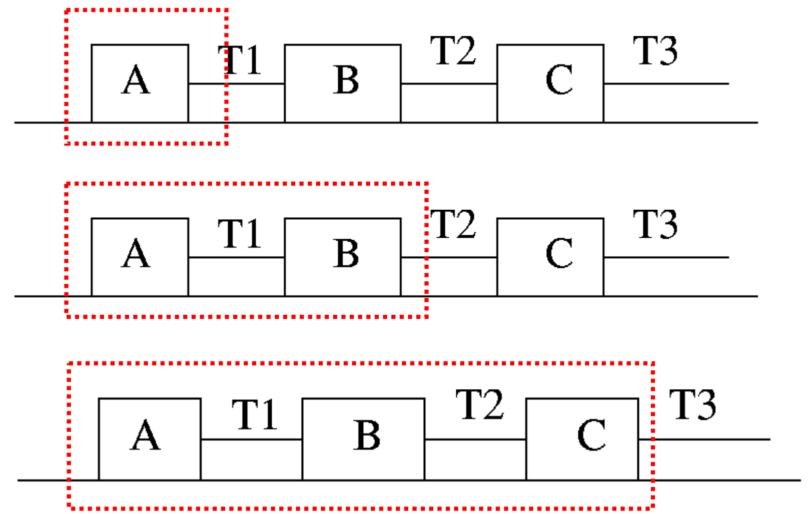
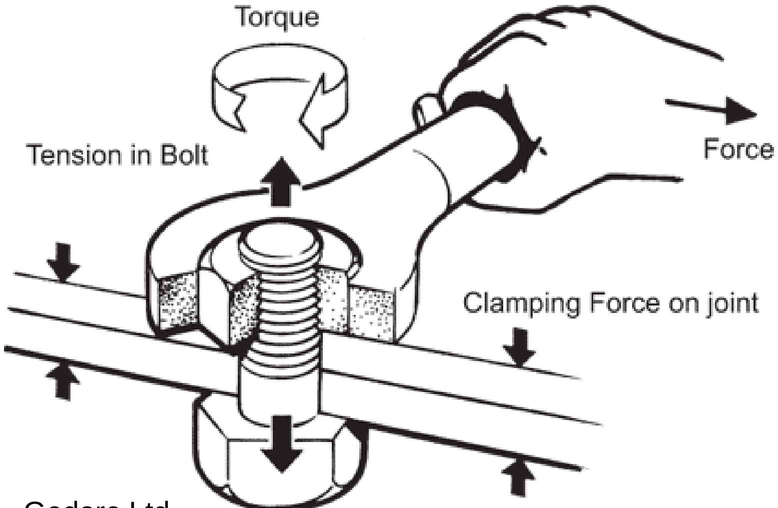
1. The boxes are connected via rope, so they have the same acceleration.
2. The tension of each rope section must provide the force to achieve this acceleration.
   - $F = ma = T$
3. However, each rope section is dragging a different amount of mass
   - $m_1 = m_A$
   - $m_2 = m_A + m_B$
   - $m_3 = m_A + m_B + m_C$
4. So, $T_3 > T_2 > T_1$



*Everything **after** this point is NOT on Exam 1*

Torque:



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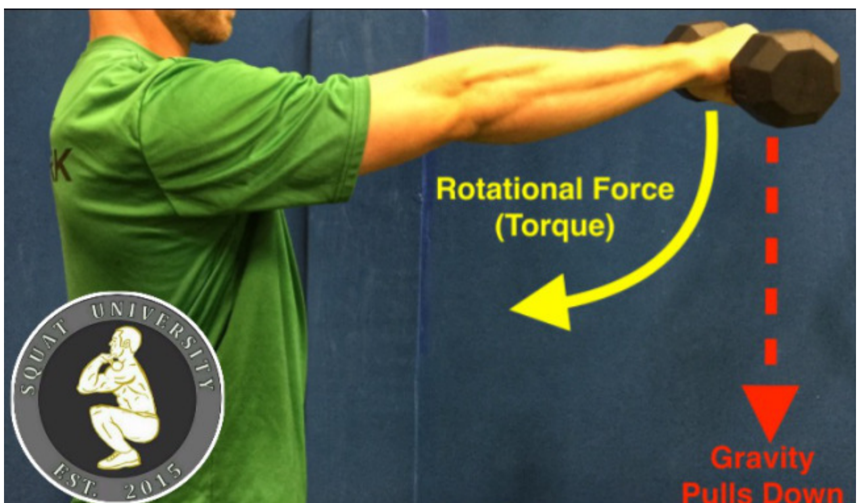

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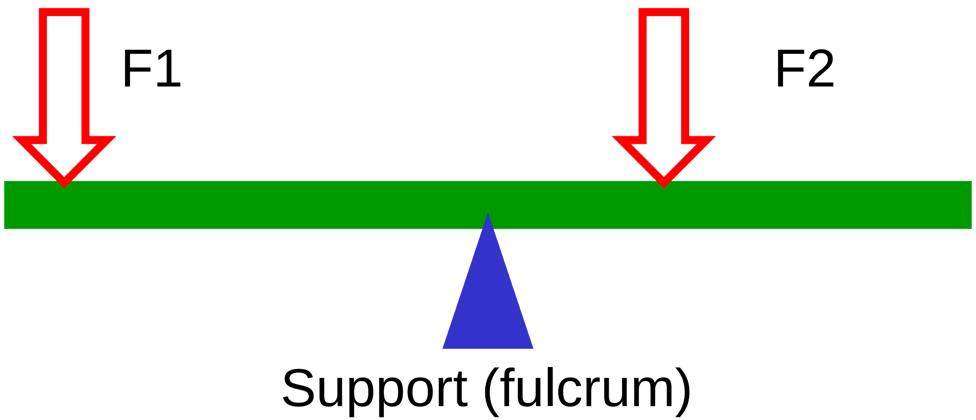
A board is placed on a pointed support.
Forces F1 and F2 push down on the board.
If the board stays stationary, how must the forces compare?

(A) F1 > F2

(B) F1 = F2

(C) F1 < F2



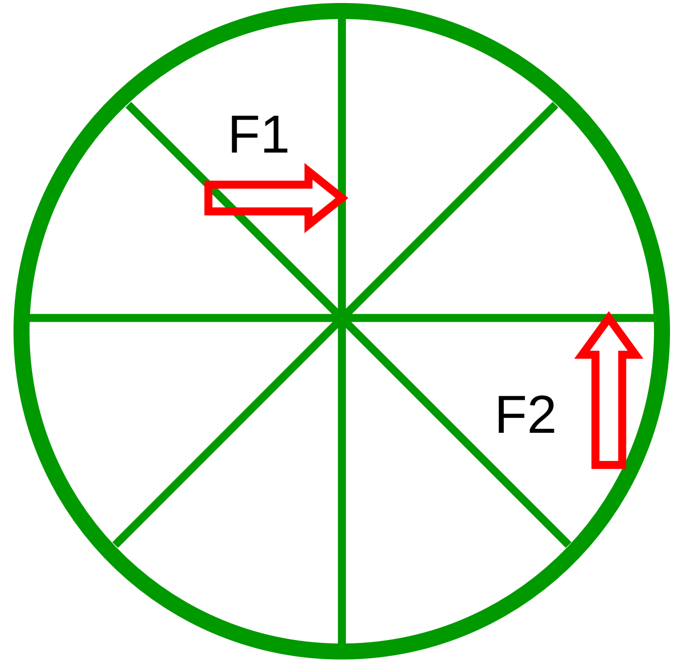
Force 2 has a much shorter “moment arm” (or “lever arm”) than Force 1.
Therefore, Force 2 must be greater to compensate if the board is going to stay stationary (i.e. in static equilibrium).

Two forces are exerted on a wheel as shown.
If the wheel is stationary, how do the two forces compare?

(A) F1 > F2

(B) F1 = F2

(C) F1 < F2



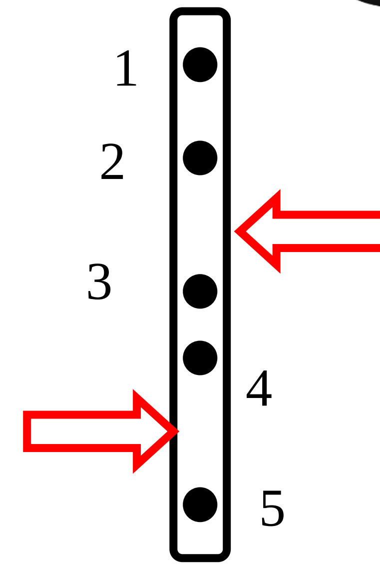
Force 1 is closer to the axis of rotation (a.k.a. the axle) than Force 2.
Therefore, to balance the torque, Force 1 must be greater than Force 2.

Rotation: The other type of motion

📖 Ch. 9

- To now we've only covered "translational" (a.k.a. linear) motion
- However, we also need "rotational" motion
  - i.e. motion about an axis (e.g. pivoting)
- All motion can be broken into translational & rotational parts
- For rotational motion, need to consider extended objects.
  i.e. where a force is applied matters.
- Key concepts: Torques, Static Equilibrium, Center of Mass (See Chapter 9)
  - Torque: Twisting force trying to cause rotation about an axis
  - Static equilibrium: acceleration & velocity are zero
    - "Velocity is zero" is a key point, because changing direction at a constant speed still corresponds to a change in velocity.
      Examples ***not*** in static equilibrium: Wheel rotating. Earth orbiting Sun.
    - Static equilibrium means forces ***and*** torques are balanced.
      - If horizontal forces are balanced, but off center, can twist!
        So not in static equilibrium.

Two forces are applied to a stick.
At which points could a third force be applied
(you can pick either direction and any force magnitude)
in order to create a situation with static equilibrium?



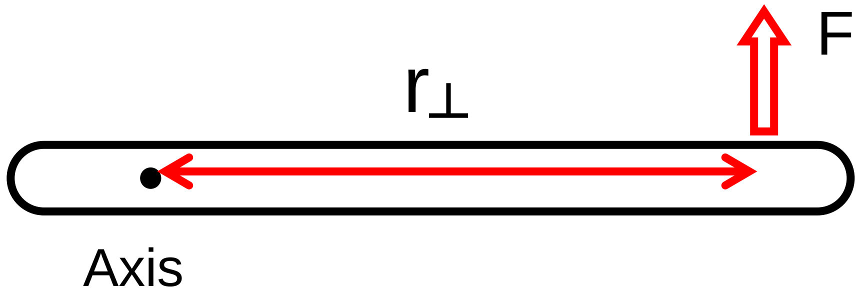
(A) 1 and 2 (B) 1 and 4

(C) 1, 2 and 5 (D) 1, 2, 4, and 5

- With only two forces, we clearly have some twist.
- Consider how each point individually could fix this:
  1. Force to right could balance with other rightward force about the leftward force (like a see-saw). ✓
  2. Same as (1). ✓
  3. Force to right will only add to twist. Force to left will only add to twist. ✗
  4. Same as (3). ✗
  5. Force to left could balance other leftward force about the rightward force (like a see-saw). ✓

Torque: Force applied to a lever some perpendicular distance from an axis

- Simplest case, Force perpendicular to a lever arm:



Greek letter 'tau'

- $\tau = F{*}r_\perp$ where $r_\perp$ is the distance from the 'axis of rotation'
  - Torque is always with respect to an axis of rotation (in fact, it's direction is along that axis of rotation). If it's a static problem, you can choose the axis of rotation.
  - Units: N*m [enter this way in LON-CAPA)

Sec. 9.2

Torque: Force applied to a lever some ***perpendicular distance*** from an axis

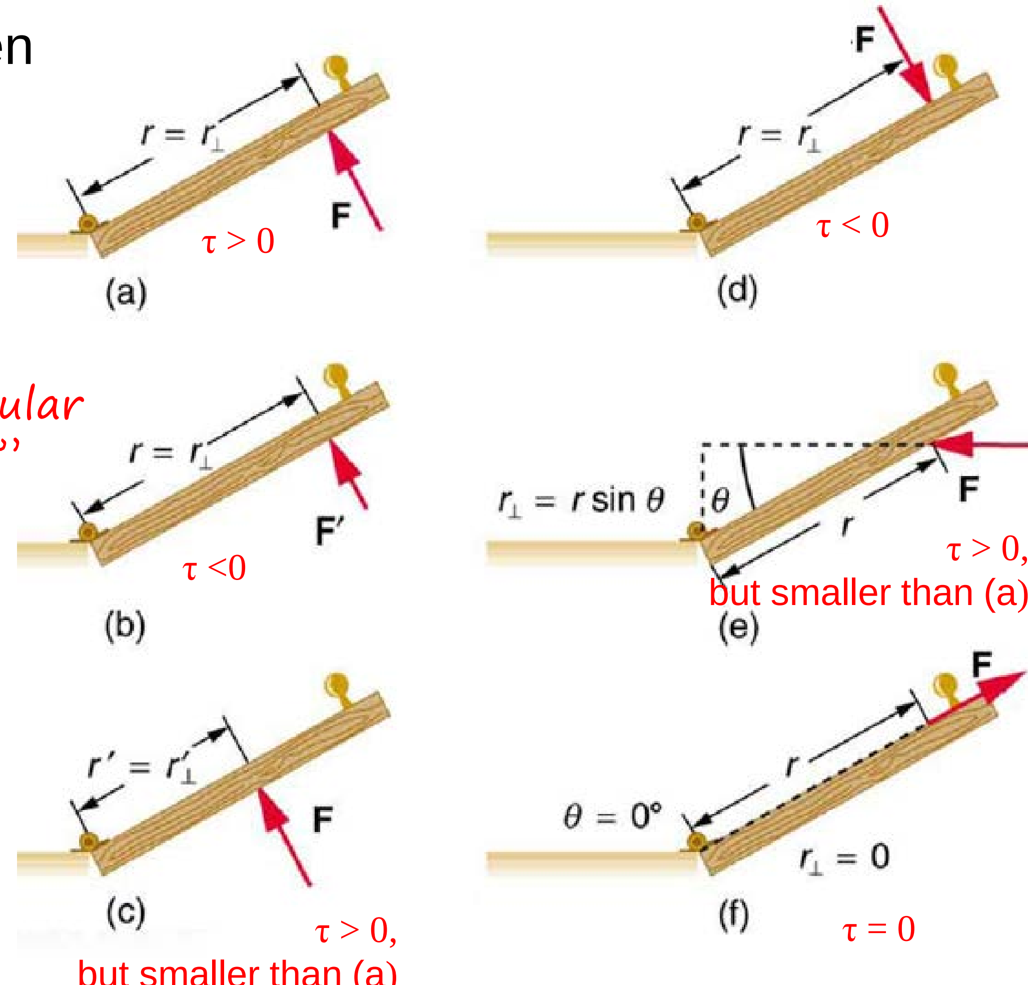
- Torque is a “cross product” between the lever arm length ***r*** and the force applied ***F***
- Mathematically speaking,
  - $\tau = r \times F$
  - Here, this means,

$$\tau = |F|\boxed{|r|\sin(\theta)}$$

“perpendicular distance”

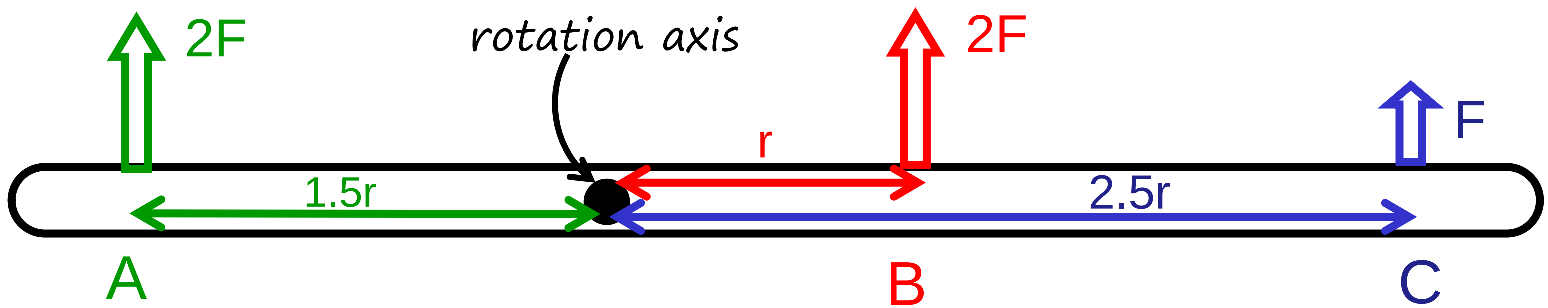
- Often need trig to find the perpendicular distance.

- The sign convention is
  (due to the cross-product):
  - + for counter-clockwise
  - - for clockwise



Sec. 9.2

Which of the following forces provides the largest magnitude for torque about the axis (indicated by the solid circle)?

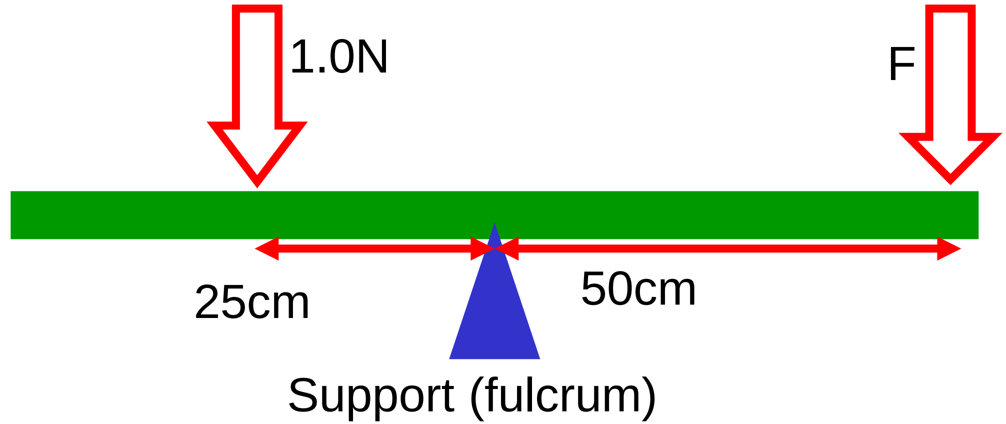


(A) A　　(B) B　　(C) C　　(D) All same

- All forces are perpendicular to lever arm, so
  - $\tau = r \times F = |F||r|\sin(\theta) = |F||r|\sin(90°) = |F||r|$
- Just need to compare product of lever arm length & force:
  - $|\tau_A|$ = 2F*1.5r = 3F*r　　τ_A : Clockwise (CW): (-)
  - $|\tau_B|$ = 2F*r = 2F*r　　τ_B : Counterclockwise (CCW): (+)
  - $|\tau_C|$ = F*2.5r = 2.5F*r　　τ_C : CCW: (+)

What force, F, is required to balance this lever on the fulcrum?

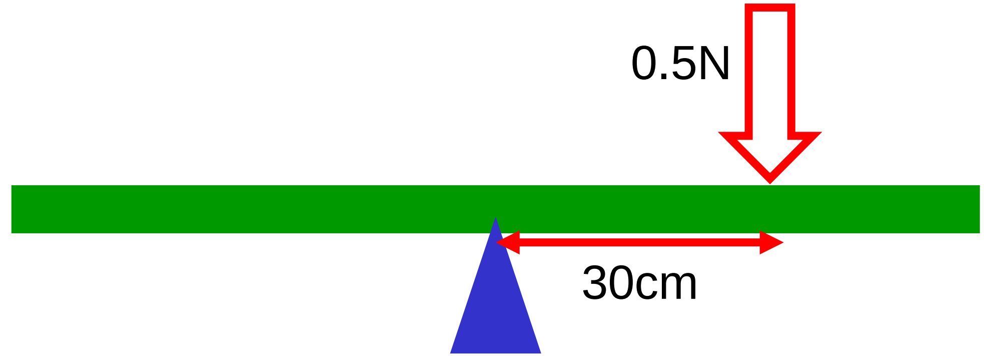
(A) 0.5N
(B) 1.0N
(C) 2.0N



1. Choose axis of rotation:
   - Fulcrum is a natural choice, since then the force due to the fulcrum doesn’t need to be considered
   - *If you chose a different axis, you would need to consider the perpendicular distance from the fulcrum to that axis & the force the fulcrum would apply to the lever
2. Sum the torques about the chosen axis to cancel (because want “balance”)
   1. $\sum\tau = 0 = \tau_{left} - \tau_{right} = (r_{left}*F_{left}) - (r_{right}*F_{right})$
   2. $r_{left}*F_{left} = r_{right}*F_{right}$
   3. $F_{right} = (r_{left}*F_{left})/r_{right} = (1.0N)(25cm)/50cm = 0.5N$

A force of 0.5N is applied downward, 30cm rightward from a fulcrum. Where, with respect to the fulcrum, would a downward force of 0.7N need to be applied in order to balance the lever?

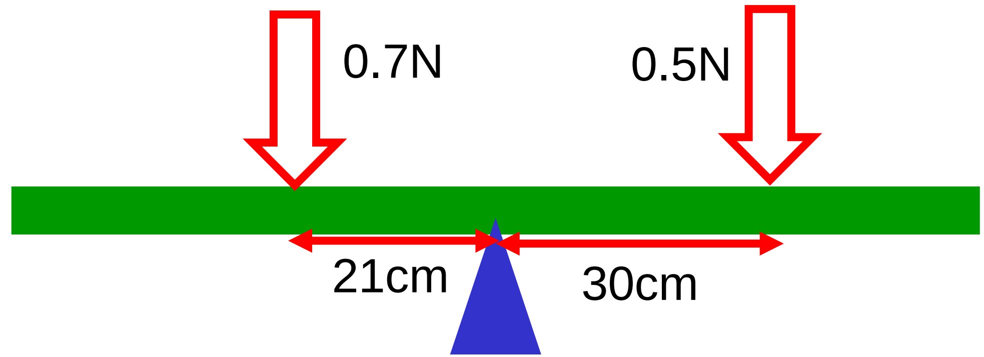
A. 21cm to the right
B. 21cm to the left
C. 30cm to the left
D. 70cm to the left



1. Choose axis of rotation:
   - Fulcrum is a natural choice, since then can ignore the force due to the fulcrum
2. Sum the torques about the chosen axis to cancel (because want "balance")
   1. Can tell that a downward force on the left will be needed to compensate for the downward force on the right. (because need $+\tau$ to compensate for $-\tau$)
   2. $\sum\tau = 0 = \tau_{left} - \tau_{right} = (r_{left}*F_{left}) - (r_{right}*F_{right})$
   3. $r_{left}*F_{left} = r_{right}*F_{right}$
   4. $r_{left} = (r_{right}*F_{right})/F_{left} = (30cm)(0.5N)/(0.7N) \approx 21cm$ (to the left)

A force of 0.5N is applied downward, 30cm rightward from a fulcrum.
A force of 0.7N is applied downward, 21cm leftward from a fulcrum.
What is the force of the fulcrum on the lever?

A. 0N
B. 1.2N down
C. 1.2N up
D. 0.2N up
E. 0.2N down
F. 0.35N up

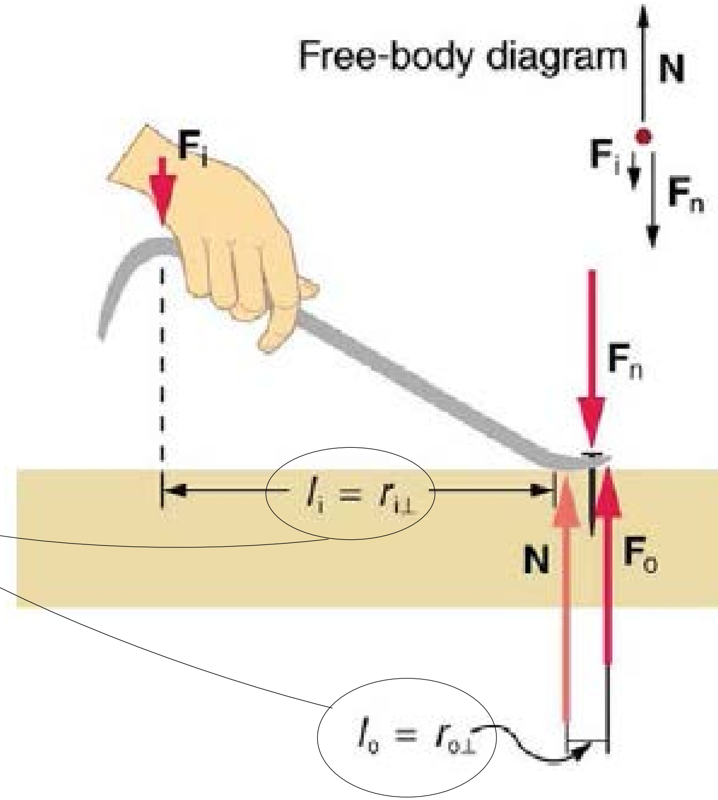


1. $\sum F_y = ma_y = 0$
2. Therefore, $F_{down} = F_{up}$
3. $F_{up} = F_{fulcrum} = 0.7N + 0.5N = 1.2N$ (up)

Levers: Mechanical advantage

Sec. 9.5

- Levers can act as a force multiplier
- Mechanical advantage "MA" quantifies the force multiplication:
  - MA = F_{output}/F_{input}
- Levers work because of torque:
  - $\tau = |F||r|\sin(\theta)$
  - A small force at a large distance is equivalent to a large force at a short distance
- **"Distance" is the <u>perpendicular distance</u> between the force & axis**



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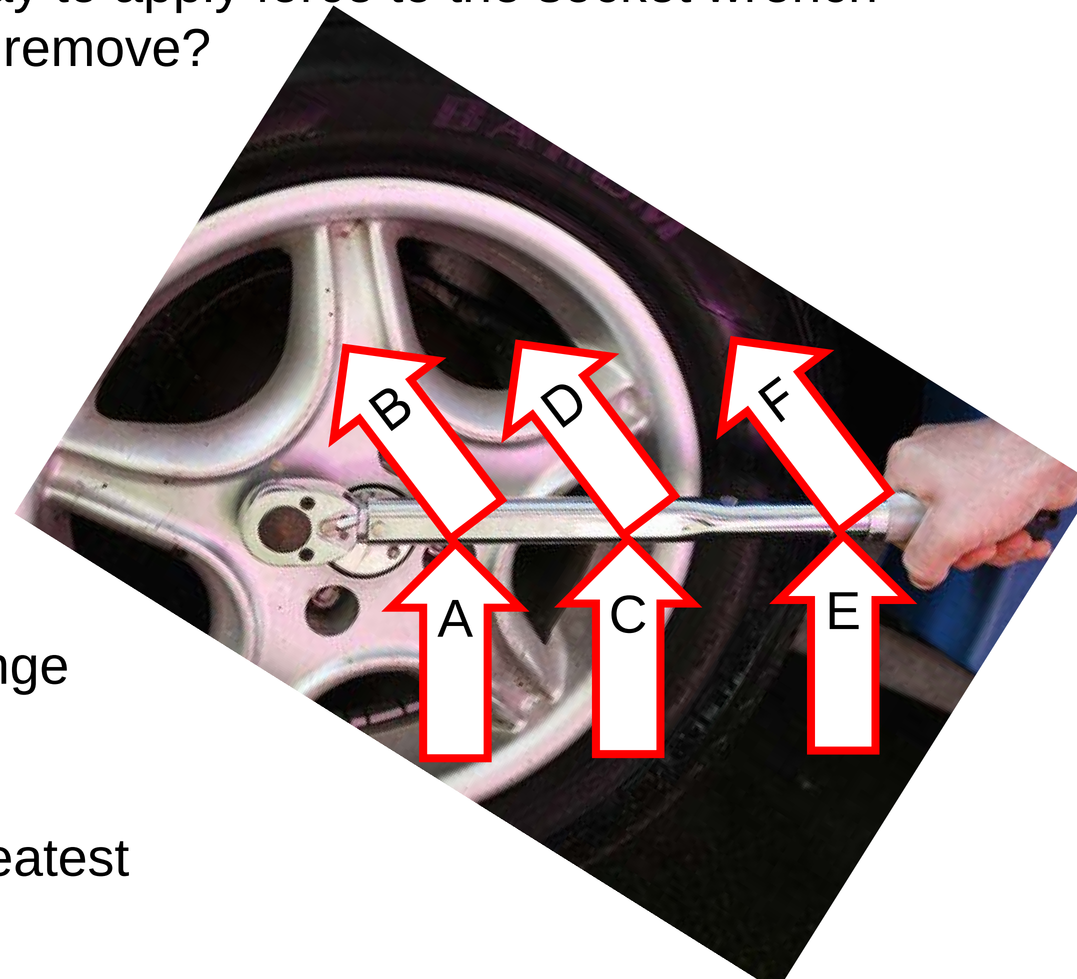
You just got some sweet new rims for your car and so you need to take off the old ones.
Which arrow indicates the best way to apply force to the socket wrench so that the lug nuts are easiest to remove?

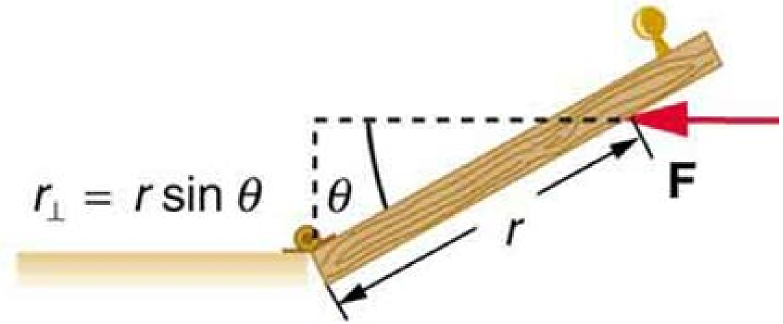
(A) A
(B) B
(C) C
(D) D
(E) E
(F) F
(G) A & B
(H) C & D
(I) E & F



- Want to *maximize* **perpendicular distance** from hinge (in this case the lug nut).
- "E" has the largest perpendicular distance, so the torque will be greatest for a force applied there.

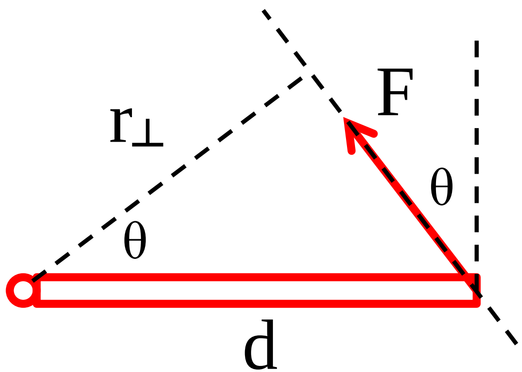
Torque: "Perpendicular distance" & "Perpendicular force"

- Torque: $\tau = r \times F = |F||r|\sin(\theta)$
- The angle between the force & the lever arm matters.



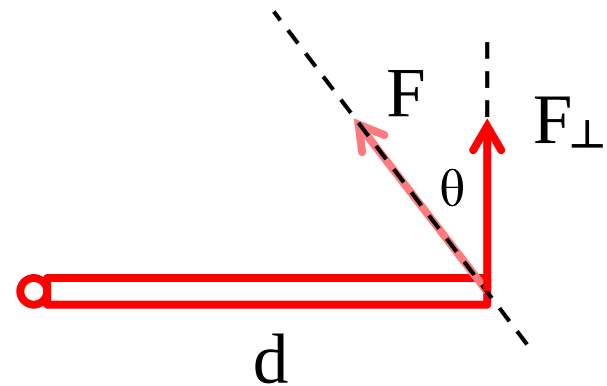
- Two ways to think about this:

(1) <u>Perpendicular distance</u>:



$\tau = F{*}r_\perp = F{*}[d{*}\cos(\theta)]$

(2) <u>Perpendicular force</u>:



$\tau = F_\perp d = [F{*}\cos(\theta)]{*}d$

You're getting swoll in the gym, curling a weight as shown. What is the torque of your bicep on your elbow?

(A) F_b*sin(θ)*5cm

(B) F_b*cos(θ)*5cm

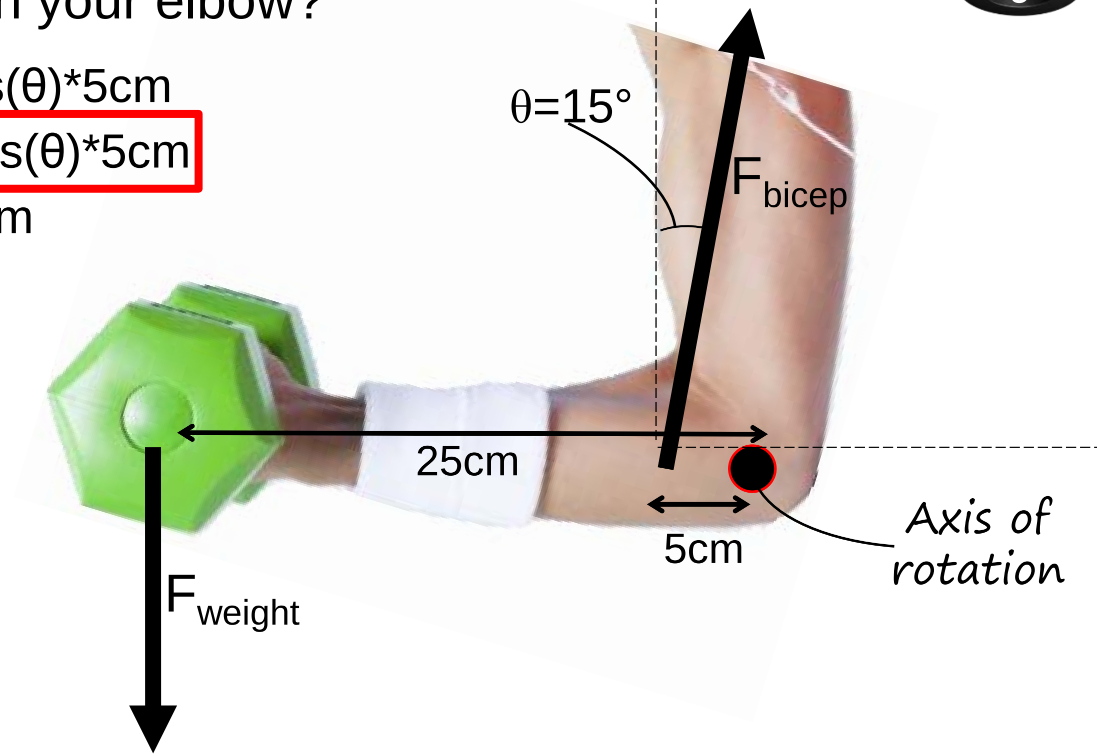
(C) -F_b*sin(θ)*5cm

(D) -F_b*cos(θ)*5cm

(E) F_b*5cm

(F) -F_b*5cm

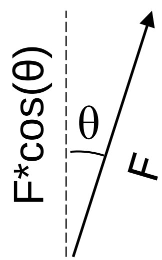
(G) None of these



Method 1:

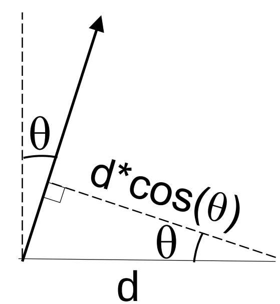
1. Torque is distance multiplied by perpendicular force
2. $\tau = F_{\perp}d = [F^*\cos(\theta)]^*d$
3. $\tau = -F_{bicep}{}^*\cos(\theta)^*d$

Minus because CW



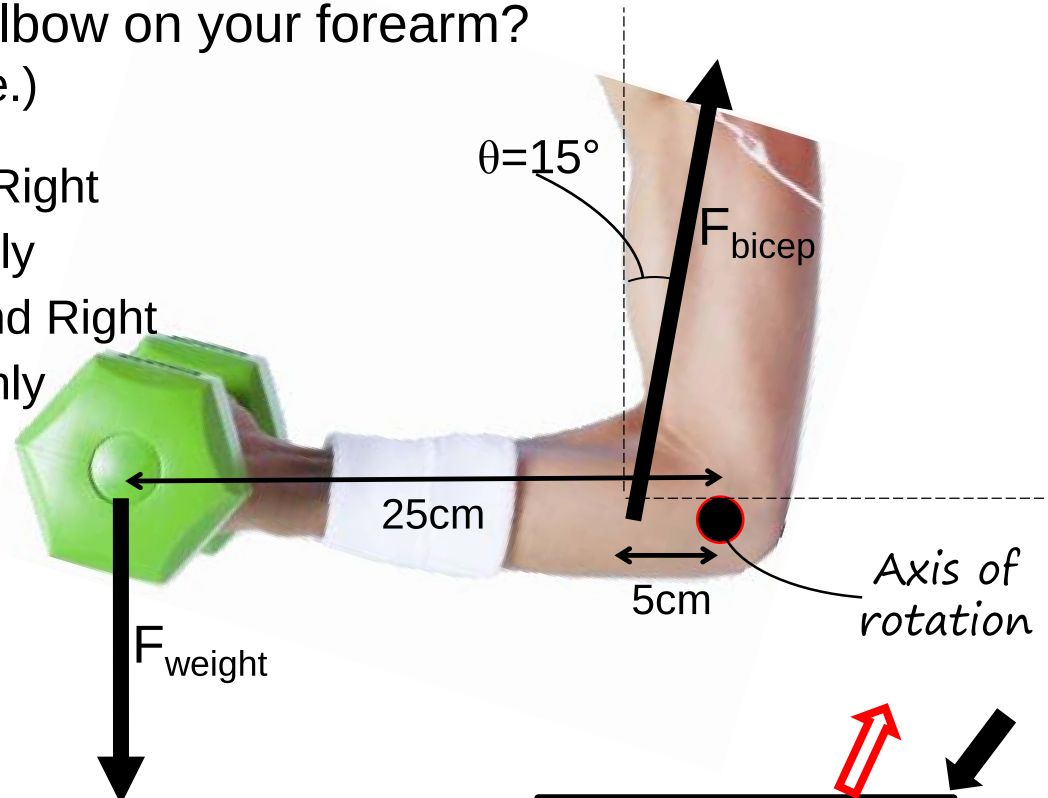
Method 2:

1. Torque is force multiplied by perpendicular distance
2. $\tau = Fr_{\perp} = F^*[d^*\cos(\theta)]$
3. $\tau = -F_{bicep}{}^*d^*\cos(\theta)$

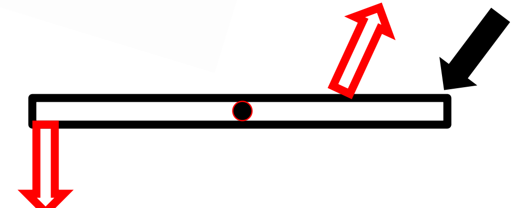


You're getting swoll in the gym, holding a weight as shown.
Your elbow pushes on your forearm to keep it in static equilibrium (i.e. to make sure your forearm doesn't shift or rotate in the middle!).
What direction is the force of your elbow on your forearm?
(Hint: your elbow is ***not*** the fulcrum here.)

(A) Up and Left
(B) Up and Right
(C) Left only
(D) Right only
(E) Down and Left
(F) Down and Right
(G) Up only
(H) Down only

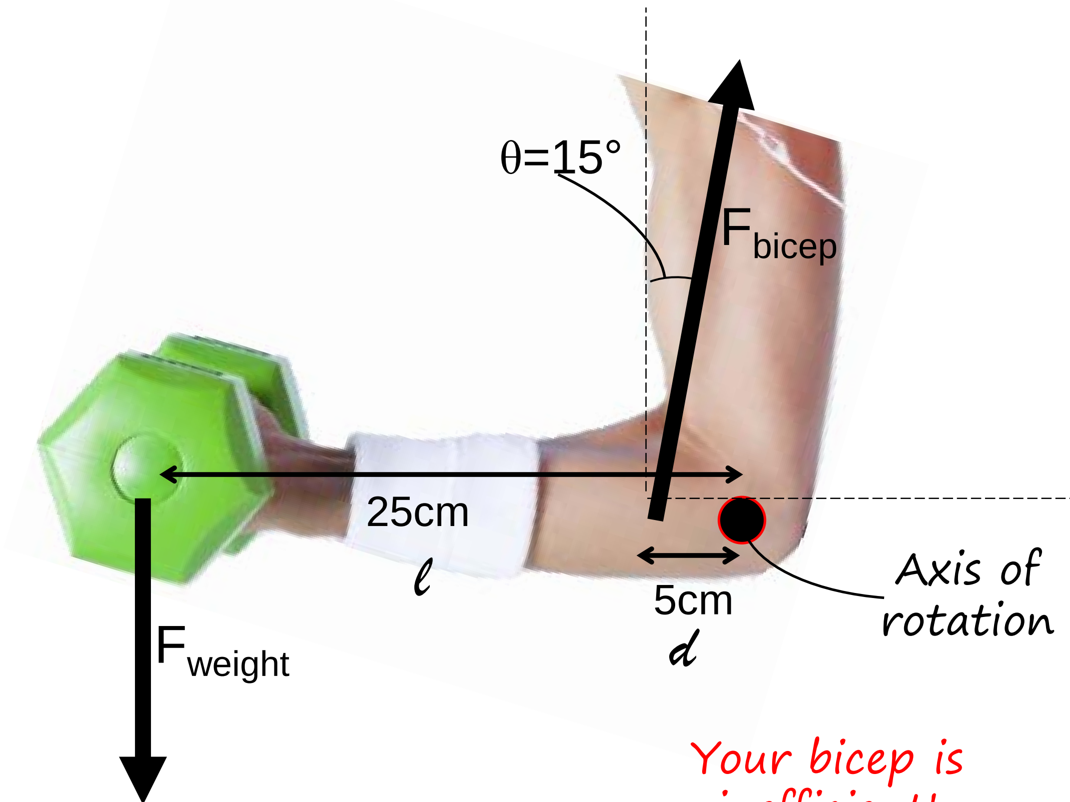


1. "Static Equilibrium" is key here.
   1. Balance Forces
   2. Balance Torque
2. To oppose the horizontal force from your bicep, your elbow must push left.
3. Your bicep & the weight are both producing positive torque. Your elbow must push down to counteract this with negative torque.



You're getting swoll in the gym, holding a weight as shown.
What force must your bicep exert to hold a 10kg weight like this?
(hint: your elbow is the fulcrum)

(A) 98 N
(B) 95 N
(C) 507N
(D) 1892 N
(E) 101 N
(F) 980 N



- Must balance torques:
  - $\sum\tau = 0 = \tau_{weight} - \tau_{bicep}$
  - $\tau_{weight} = F_{weight} * r_{\perp} = m*g*\ell$
  - $\tau_{bicep} = F_{b,\perp} * d = -F_b \cos(\theta) * d$
  - $m*g*\ell = F_b \cos(\theta) * d$
- Solve for F_b,
  - $F_b = (m*g*\ell)/(\cos(\theta)*d)$
  - $F_b = (10kg*9.8m/s^2*25cm)/(5cm*\cos(15°)) = 507N$

Your bicep is inefficient!
You need way more force to lift a weight than the force of gravity.