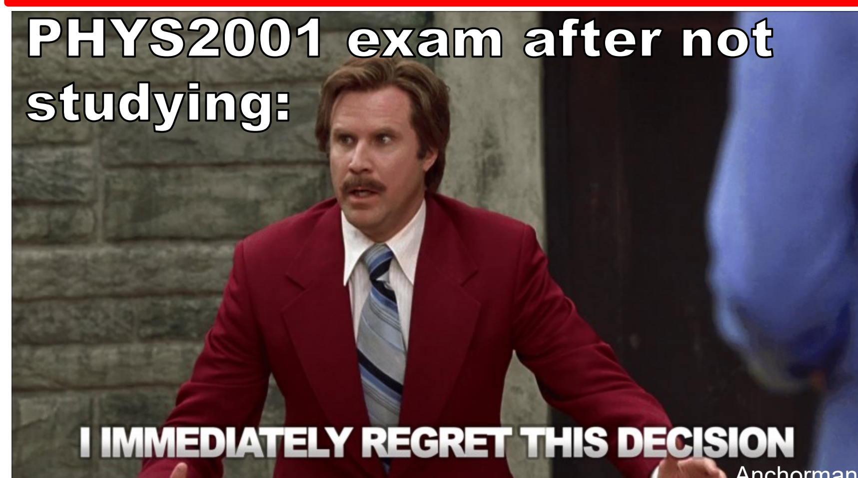
Thursday February 9

Topics for this Lecture:

Torque, rotation, & equilibrium.

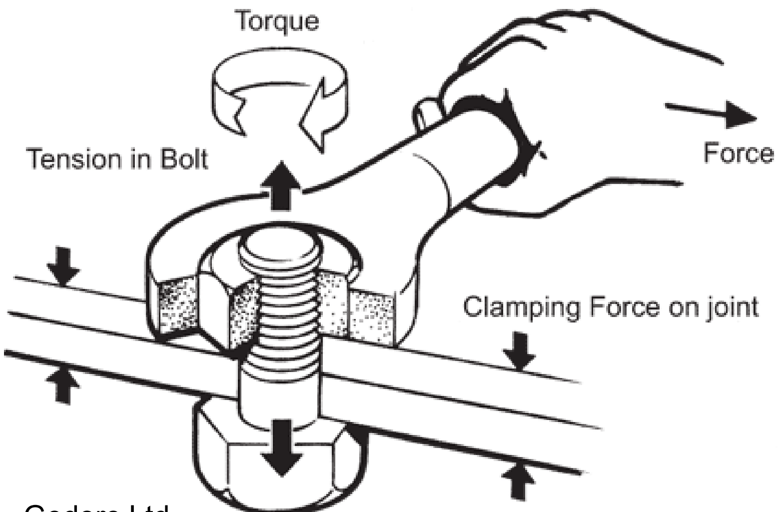
Torque is <u>not</u> on Exam 1.

- Torque:
  - Referred to with Greek letter tau: τ
  - Force about an axis.
  - Clockwise: negative torque
  - Counter clockwise: positive torque
  - Directed perpendicular to Force & rotation



- Assignment 5 due Friday
  …like almost every Friday
- Pre-class due 15min before class
  …like every class
- Help Room: Here, 6-9pm Wed/Thurs
- SI: Morton 326, M&W 7:15-8:45pm
- Office Hours: 204 EAL, 10-11am Wed
  or by appointment (meisel@ohio.edu)
- **Exam Monday February 13.**
  Morton 201 7:15-9:15PM
  - Email me ASAP if you have a class conflict or need special accommodations through accessibility services
  - ***Study!***

Torque:



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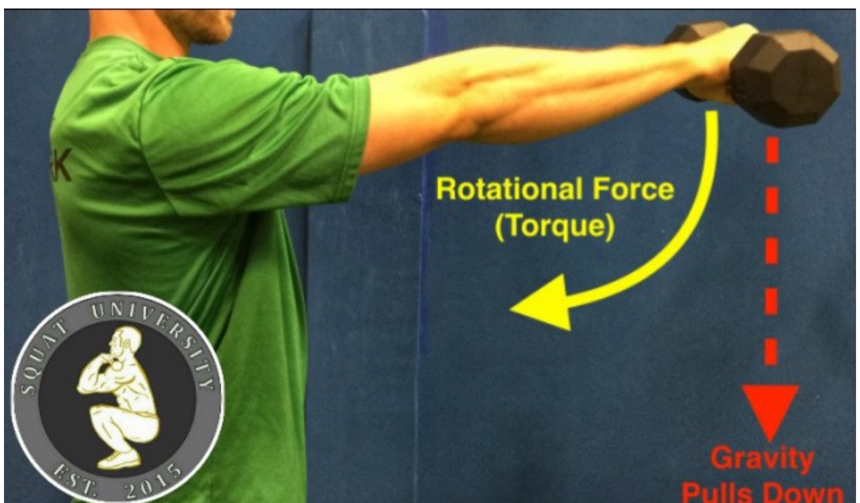


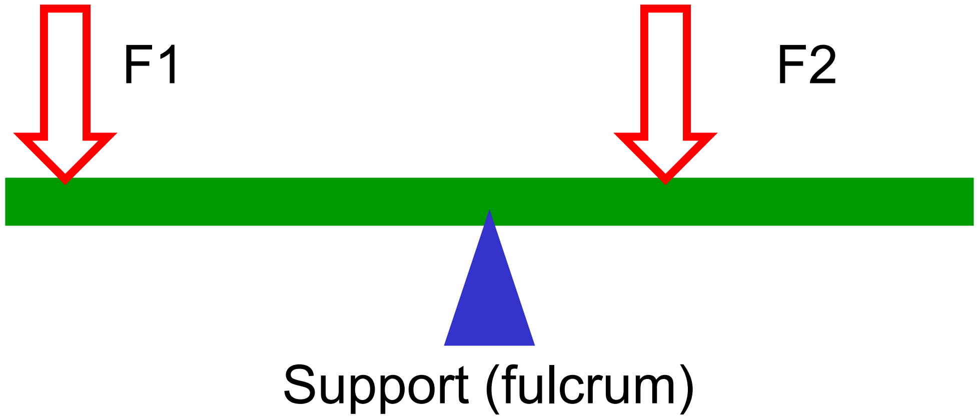
lagunaguiance

A board is placed on a pointed support.
Forces F1 and F2 push down on the board.
If the board stays stationary, how must the forces compare?

(A) F1 > F2

(B) F1 = F2

(C) F1 < F2



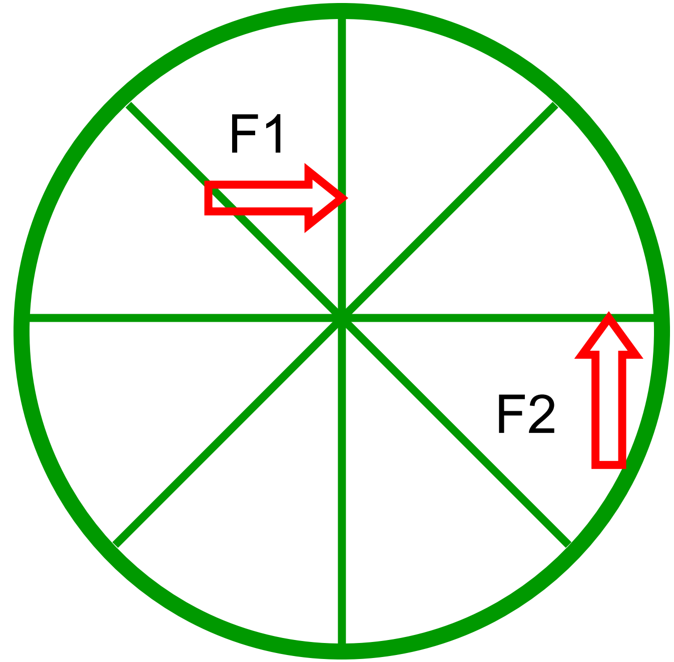
Force 2 has a much shorter “moment arm” (or “lever arm”) than Force 1. Therefore, Force 2 must be greater to compensate if the board is going to stay stationary (i.e. in static equilibrium).

Two forces are exerted on a wheel as shown.
If the wheel is stationary, how do the two forces compare?

(A) F1 > F2

(B) F1 = F2

(C) F1 < F2

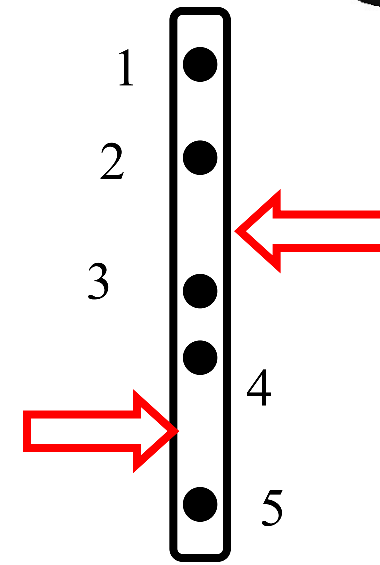


Force 1 is closer to the axis of rotation (a.k.a. the axle) than Force 2.
Therefore, to balance the torque, Force 1 must be greater than Force 2.

Rotation: The other type of motion

- To now we've only covered "translational" (a.k.a. linear) motion
- However, we also need "rotational" motion
  - i.e. motion about an axis (e.g. pivoting)
- All motion can be broken into translational & rotational parts
- For rotational motion, need to consider extended objects.
  i.e. where a force is applied matters.
- Key concepts: Torques, Static Equilibrium, Center of Mass (See Chapter 9)
  - Torque: Twisting force trying to cause rotation about an axis
  - Static equilibrium: acceleration & velocity are zero
    - "Velocity is zero" is a key point, because changing direction at a constant speed still corresponds to a change in velocity.
      Examples ***not*** in static equilibrium: Wheel rotating. Earth orbiting Sun.
    - Static equilibrium means forces ***and*** torques are balanced.
      - If horizontal forces are balanced, but off center, can twist!
        So not in static equilibrium.

Two forces are applied to a stick.
At which points could a third force be applied
(you can pick either direction and any force magnitude)
in order to create a situation with static equilibrium?



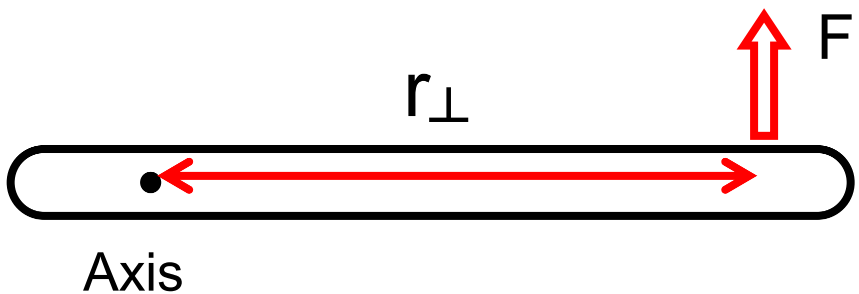
(A) 1 and 2 (B) 1 and 4

(C) 1, 2 and 5 (D) 1, 2, 4, and 5

- With only two forces, we clearly have some twist.
- Consider how each point individually could fix this:
    1. Force to right could balance with other rightward force about the leftward force (like a see-saw).
    2. Same as (1).
    3. Force to right will only add to twist. Force to left will only add to twist.
    4. Same as (3).
    5. Force to left could balance other leftward force about the rightward force (like a see-saw).

Torque: Force applied to a lever some perpendicular distance from an axis

- Simplest case, Force perpendicular to a lever arm:



Greek letter 'tau'

- $\tau = F{*}r_{\perp}$ where $r_{\perp}$ is the distance from the 'axis of rotation'
  - Torque is always with respect to an axis of rotation (in fact, it's direction is along that axis of rotation). If it's a static problem, you can choose the axis of rotation.
  - Units: N*m [enter this way in LON-CAPA)

Sec. 9.2

Torque: Force applied to a lever some ***perpendicular distance*** from an axis

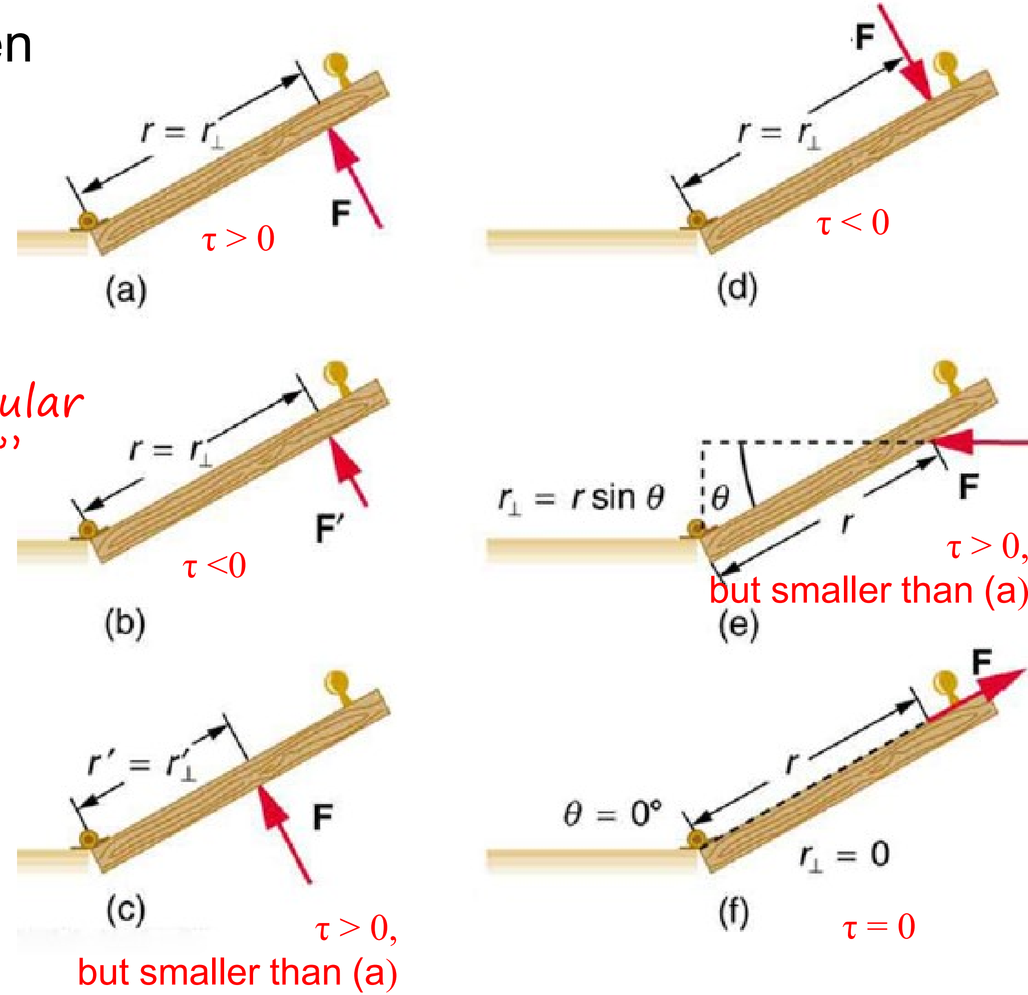
- Torque is a "cross product" between the lever arm length ***r*** and the force applied ***F***
- Mathematically speaking,
  - $\tau = r \times F$
  - Here, this means,

$$\tau = |F|\boxed{|r|\sin(\theta)}$$

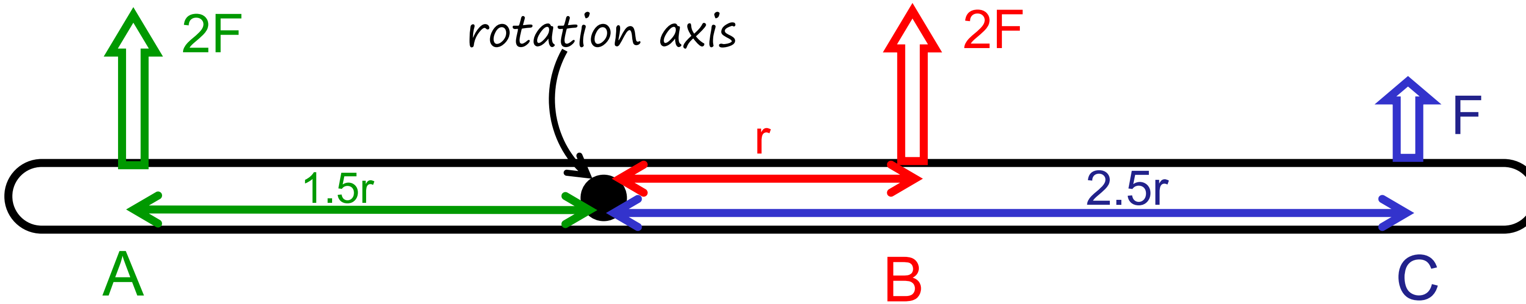
"perpendicular distance"

- Often need trig to find the perpendicular distance.

- The sign convention is (due to the cross-product):
  - \+ for counter-clockwise
  - \- for clockwise



Which of the following forces provides the largest magnitude for torque about the axis (indicated by the solid circle)?



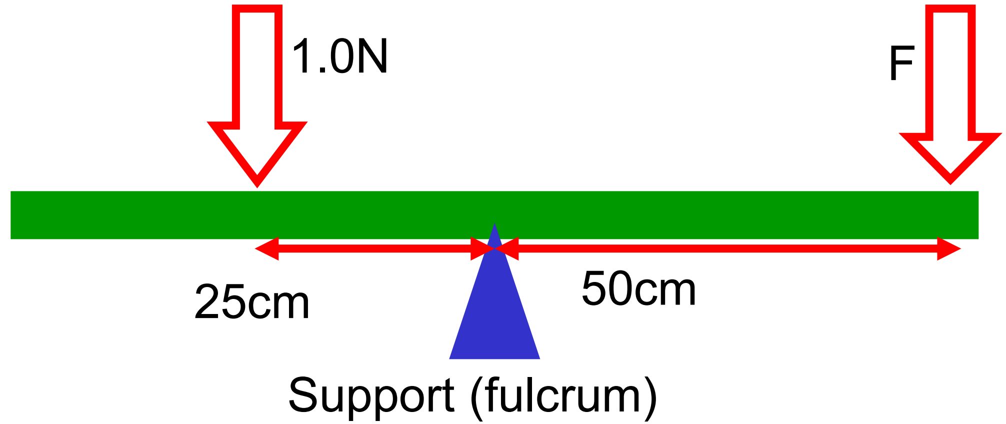
(A) A   (B) B   (C) C   (D) All same

- All forces are perpendicular to lever arm, so
  - $\tau = r \times F = |F||r|\sin(\theta) = |F||r|\sin(90°) = |F||r|$
- Just need to compare product of lever arm length & force:
  - $|\tau_A|$ = 2F*1.5r = 3F*r τ_A : Clockwise (CW): (-)
  - $|\tau_B|$ = 2F*r = 2F*r τ_B : Counterclockwise (CCW): (+)
  - $|\tau_C|$ = F*2.5r = 2.5F*r τ_C : CCW: (+)

What force, F, is required to balance this lever on the fulcrum?

5

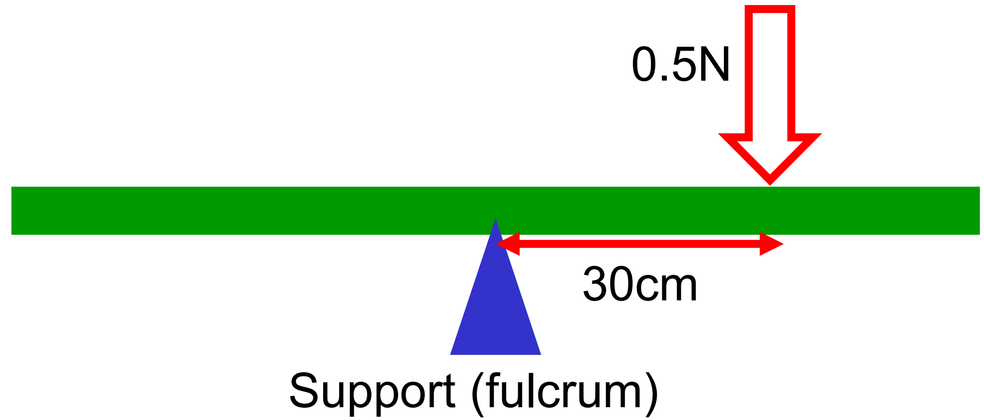
(A) 0.5N
(B) 1.0N
(C) 2.0N



1. Choose axis of rotation:
   - Fulcrum is a natural choice, since then the force due to the fulcrum doesn't need to be considered
   - *If you chose a different axis, you would need to consider the perpendicular distance from the fulcrum to that axis & the force the fulcrum would apply to the lever
2. Sum the torques about the chosen axis to cancel (because want "balance")
   1. $\sum\tau = 0 = \tau_{left} - \tau_{right} = (r_{left}*F_{left}) - (r_{right}*F_{right})$
   2. $r_{left}*F_{left} = r_{right}*F_{right}$
   3. $F_{right} = (r_{left}*F_{left})/r_{right} = (1.0N)(25cm)/50cm = 0.5N$

A force of 0.5N is applied downward, 30cm rightward from a fulcrum. Where, with respect to the fulcrum, would a downward force of 0.7N need to be applied in order to balance the lever?

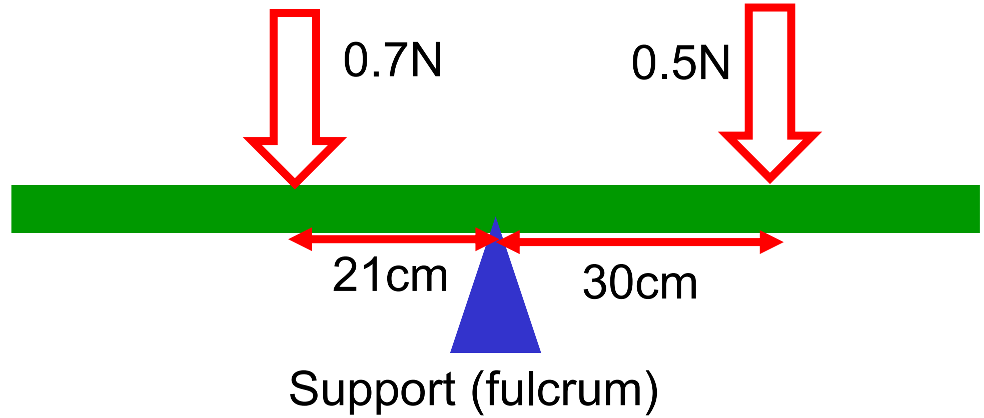
A. 21cm to the right
B. 21cm to the left
C. 30cm to the left
D. 70cm to the left



1. Choose axis of rotation:
   - Fulcrum is a natural choice, since then can ignore the force due to the fulcrum
2. Sum the torques about the chosen axis to cancel (because want “balance”)
   1. Can tell that a downward force on the left will be needed to compensate for the downward force on the right. (because need $+\tau$ to compensate for $-\tau$)
   2. $\sum\tau = 0 = \tau_{left} - \tau_{right} = (r_{left} * F_{left}) - (r_{right} * F_{right})$
   3. $r_{left} * F_{left} = r_{right} * F_{right}$
   4. $r_{left} = (r_{right} * F_{right})/F_{left} = (30cm)(0.5N)/(0.7N) \approx 21cm$ (to the left)

A force of 0.5N is applied downward, 30cm rightward from a fulcrum.
A force of 0.7N is applied downward, 21cm leftward from a fulcrum.
What is the force of the fulcrum on the lever?

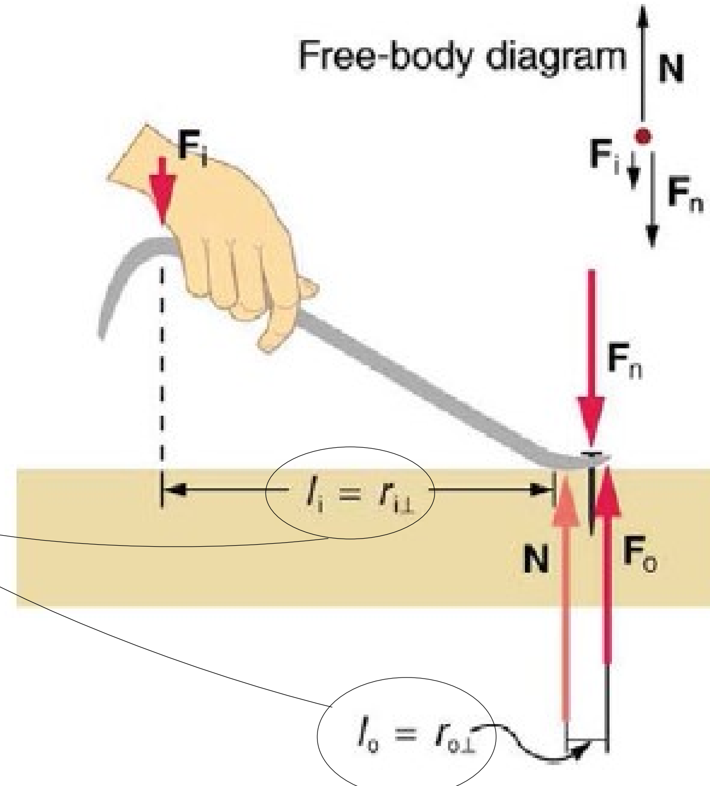
A. 0N
B. 1.2N down
C. 1.2N up
D. 0.2N up
E. 0.2N down
F. 0.35N up



1. $\sum F_y = ma_y = 0$
2. Therefore, $F_{down} = F_{up}$
3. $F_{up} = F_{fulcrum} = 0.7N + 0.5N = 1.2N$ (up)

Levers: Mechanical advantage

- Levers can act as a force multiplier
- Mechanical advantage "MA" quantifies the force multiplication:
  - MA = F_{output}/F_{input}
- Levers work because of torque:
  - $\tau = |F||r|\sin(\theta)$
  - A small force at a large distance
    is equivalent to
    a large force at a short distance
- **"Distance" is the <u>perpendicular distance</u> between the force & axis**

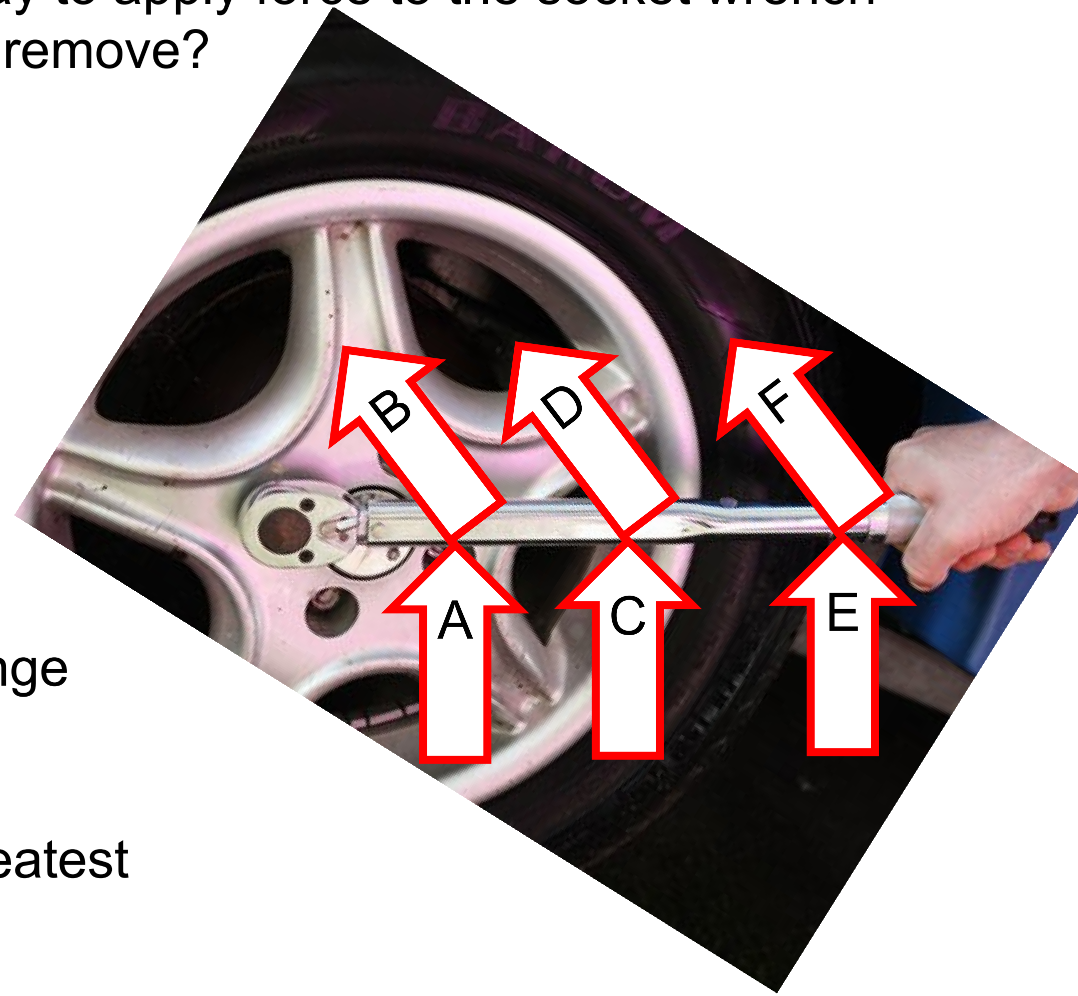


You just got some sweet new rims for your car and so you need to take off the old ones.

Which arrow indicates the best way to apply force to the socket wrench so that the lug nuts are easiest to remove?

(A) A
(B) B
(C) C
(D) D
$\boxed{\text{(E) E}}$
(F) F

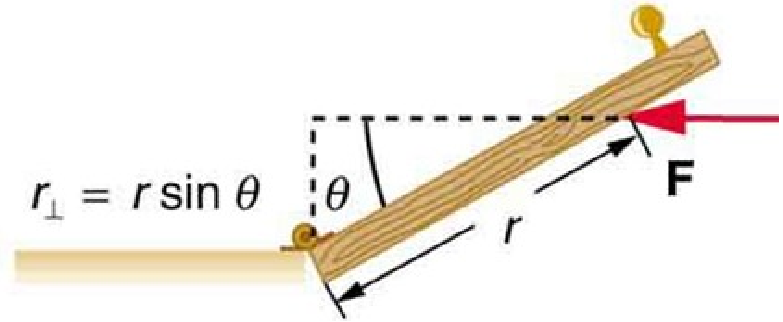
(G) A & B
(H) C & D
(I) E & F



- Want to *maximize* **<u>perpendicular distance</u>** from hinge (in this case the lug nut).
- "E" has the largest perpendicular distance, so the torque will be greatest for a force applied there.

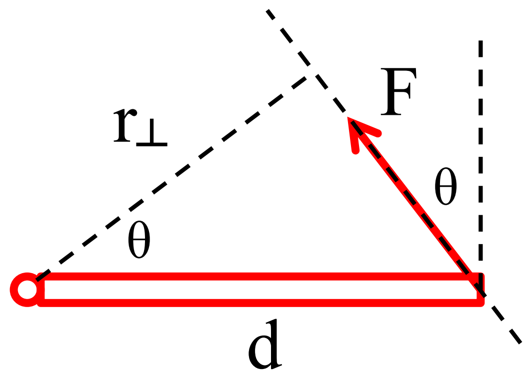
Torque: "Perpendicular distance" & "Perpendicular force"

- Torque: $\tau = r \times F = |F||r|\sin(\theta)$
- The angle between the force & the lever arm matters.



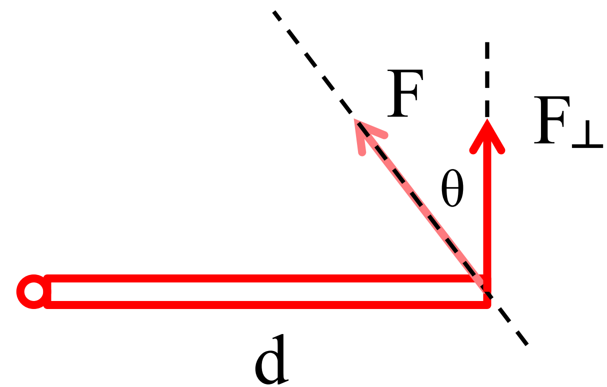
- Two ways to think about this:

(1) Perpendicular distance:



$\tau = F^*r_\perp = F^*[d^*\cos(\theta)]$

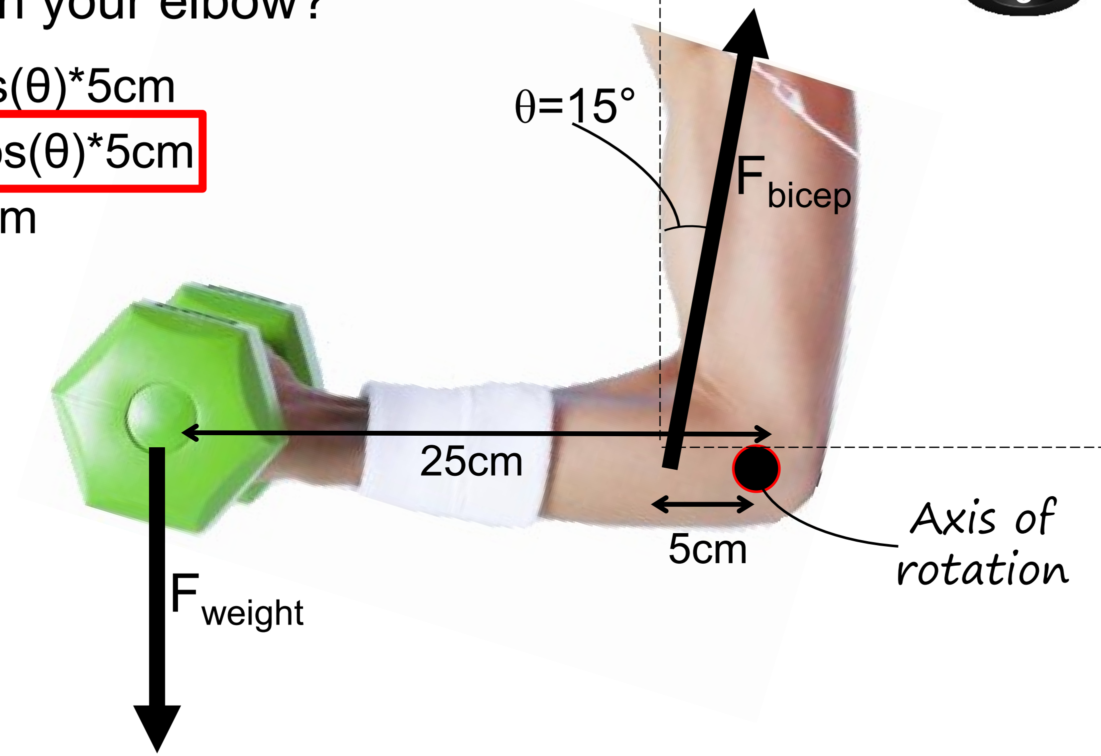
(2) Perpendicular force:



$\tau = F_\perp d = [F^*\cos(\theta)]^*d$

You're getting swoll in the gym, curling a weight as shown. What is the torque of your bicep on your elbow?

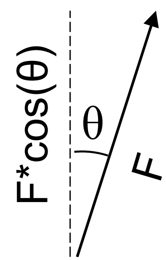
(A) $F_b*\sin(\theta)*5cm$
(B) $F_b*\cos(\theta)*5cm$
(C) $-F_b*\sin(\theta)*5cm$
(D) $\boxed{-F_b*\cos(\theta)*5cm}$
(E) F_b*5cm
(F) $-F_b*5cm$
(G) None of these



Method 1:

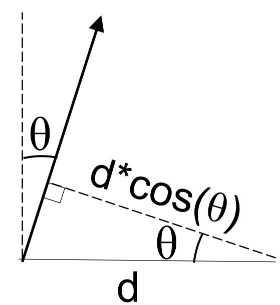
1. Torque is distance multiplied by perpendicular force
2. $\tau = F_\perp d = [F*\cos(\theta)]*d$
3. $\tau = -F_{bicep}*\cos(\theta)*d$

Minus because CW



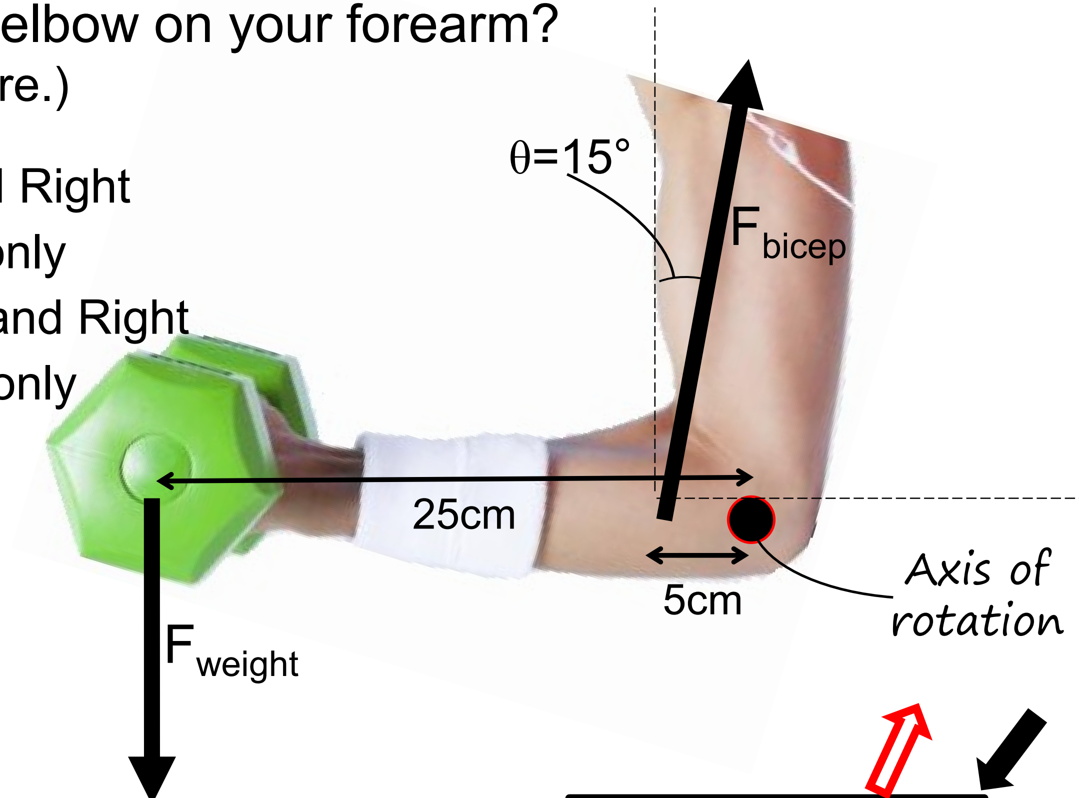
Method 2:

1. Torque is force multiplied by perpendicular distance
2. $\tau = Fr_\perp = F*[d*\cos(\theta)]$
3. $\tau = -F_{bicep}*d*\cos(\theta)$

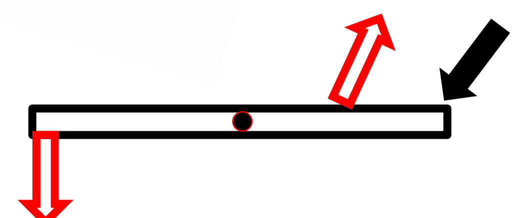


You're getting swoll in the gym, holding a weight as shown.
Your elbow pushes on your forearm to keep it in static equilibrium
(i.e. to make sure your forearm doesn't shift or rotate in the middle!).
What direction is the force of your elbow on your forearm?
(Hint: your elbow is _*not*_ the fulcrum here.)

(A) Up and Left
(B) Up and Right
(C) Left only
(D) Right only
(E) Down and Left
(F) Down and Right
(G) Up only
(H) Down only

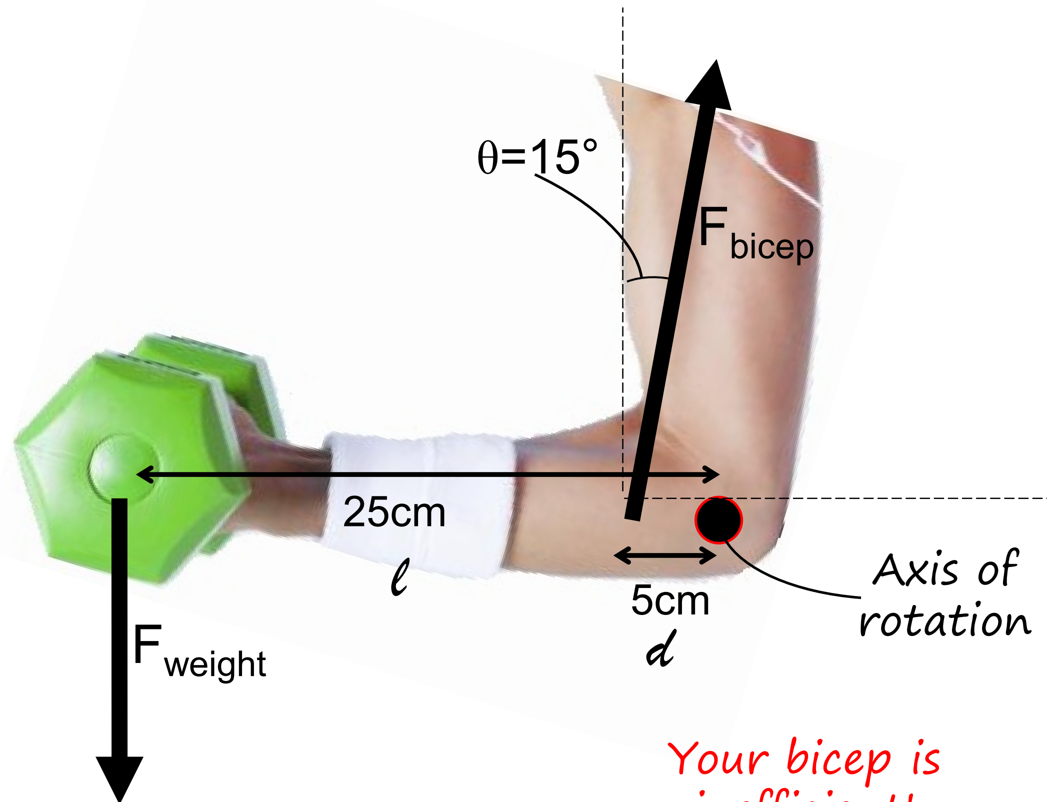


1. "Static Equilibrium" is key here.
   1. Balance Forces
   2. Balance Torque
2. To oppose the horizontal force from your bicep, your elbow must push left.
3. Your bicep & the weight are both producing positive torque. Your elbow must push down to counteract this with negative torque.



You're getting swoll in the gym, holding a weight as shown.
What force must your bicep exert to hold a 10kg weight like this?
(hint: your elbow is the fulcrum)

(A) 98 N
(B) 95 N
(C) 507N
(D) 1892 N
(E) 101 N
(F) 980 N



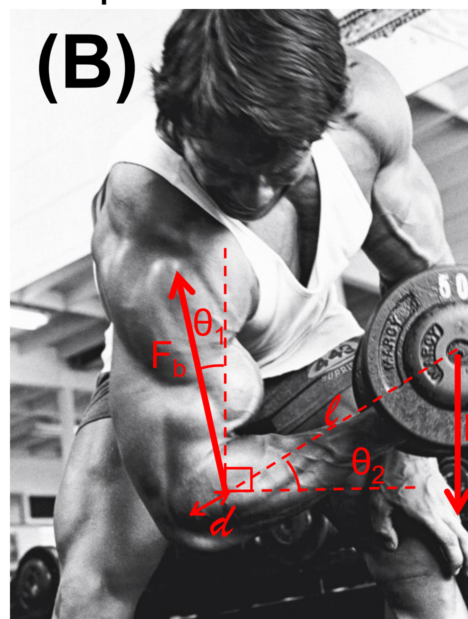
- Must balance torques:
  - $\sum\tau = 0 = \tau_{weight} - \tau_{bicep}$
  - $\tau_{weight} = F_{weight} * r_{\perp} = m*g*\ell$
  - $\tau_{bicep} = F_{b,\perp} * d = -F_b \cos(\theta) * d$
  - $m*g*\ell = F_b \cos(\theta) * d$
- Solve for F_b,
  - $F_b = (m*g*\ell)/(\cos(\theta)*d)$
  - $F_b = (10kg*9.8m/s^2*25cm)/(5cm*\cos(15°)) = 507N$

Your bicep is inefficient!
You need way more force to lift a weight than the force of gravity.

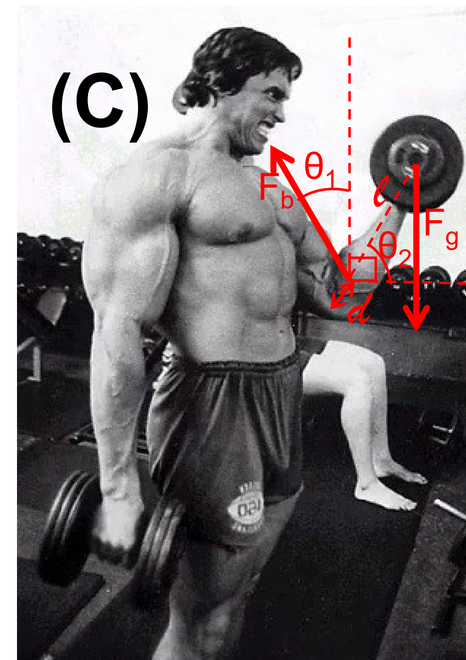
Consider these three images of the Governor of California pumping iron.
For cases (A) and (B), Arnold’s forearm is the same angle from horizontal, but his bicep is closer to vertical in case (B).
For cases (A) and (C), Arnold’s bicep is the same angle from vertical, but his forearm is closer to horizontal in case (A).
If he has the same amount of weight in each hand for all cases, which case requires the greatest force from his bicep?

(A)

(B)



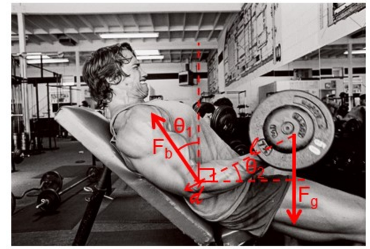
(C)



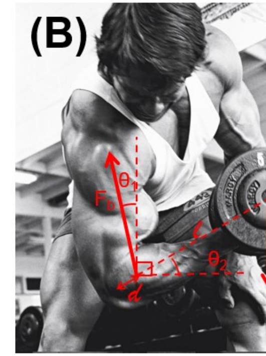
$$\theta_{1,A} = \theta_{1,C} > \theta_{1,B}$$
$$\theta_{2,A} = \theta_{2,B} < \theta_{2,C}$$
$$F_{g,1} = F_{g,2} = F_{g,3}$$

- Must balance torques:
  - $\sum\tau = 0 = \tau_{weight} - \tau_{bicep}$
  - $\tau_{weight} = F_{weight} * r_{\perp} = m*g*\ell*\cos(\theta_2)$
  - $\tau_{bicep} = F_{b,\perp} * d = F_b \cos(\theta_1) * d$
  - $m*g*\ell*\cos(\theta_2) = F_b \cos(\theta_1) * d$
- Solve for F_b,
  - $F_b = (m*g*\ell*\cos(\theta_2))/(\cos(\theta_1)*d)$
  - Keep in mind:
    - $\cos(0°) = 1$; $\cos(90°) = 0$
- Smaller θ_2 leads to a larger F_{bicep}
  - A more horizontal forearm requires more force
- Larger θ_1 leads to a larger F_{bicep}
  - A less vertical bicep requires more force
- Case A is tied for smallest θ_2 with case B, but has a larger θ_1.
- Case A is tied for largest θ_1 with case C, but has a smaller θ_2.
- Case A therefore requires the largest bicep force.

(A)



(B)



(C)

$\theta_{1,A} = \theta_{1,C} > \theta_{1,B}$
$\theta_{2,A} = \theta_{2,B} < \theta_{2,C}$
$F_{g,1} = F_{g,2} = F_{g,3}$