Quick notes on

Simple Atmospheres

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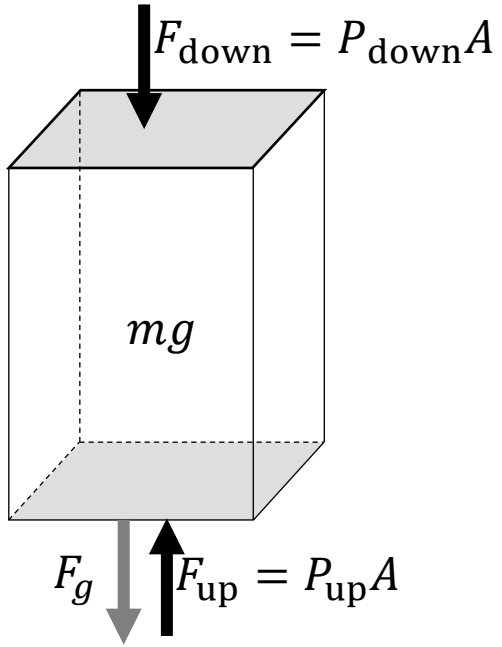
Ohio University - ASTR4201 - Fall 2020

the Isothermal Atmosphere



- Hydrostatic equilibrium:
  - Must satisfy $\vec{F}_{net} = m\vec{a} = 0$ for a static fluid element
  - Forces: Gravity downward & pressure inward from all sides
    - Non-vertical pressure forces cancel
    - Vertical forces cancel: $F_{\text{pressure,up}} = F_{\text{pressure,down}} + F_{\text{gravity}}$
      - $F_{\text{pressure}} = P \cdot A$
      - $A\left(P_{\text{up}} - P_{\text{down}}\right) = F_g = mg = \rho V g = \rho \Delta x A g$
      - $-\frac{\Delta P}{\Delta x} = \rho g \quad \longrightarrow \quad \boldsymbol{\frac{dP}{dr} = -\rho g(r)}$
- Equation of state:
  - $PV = Nk_BT \quad \longrightarrow \quad P = \frac{N}{V}k_BT = \frac{k_B}{\mu m_u}\rho T \quad \longrightarrow \quad \rho = \frac{\mu m_u}{k_B T}P$
- Substitute $\rho(P)$ into hydrostatic equilibrium equation and solve:
  - $P(r) = P_{\text{ref}}\exp(-r/H_P)$, where H_P is the pressure scale height
    This is the "barometric formula"

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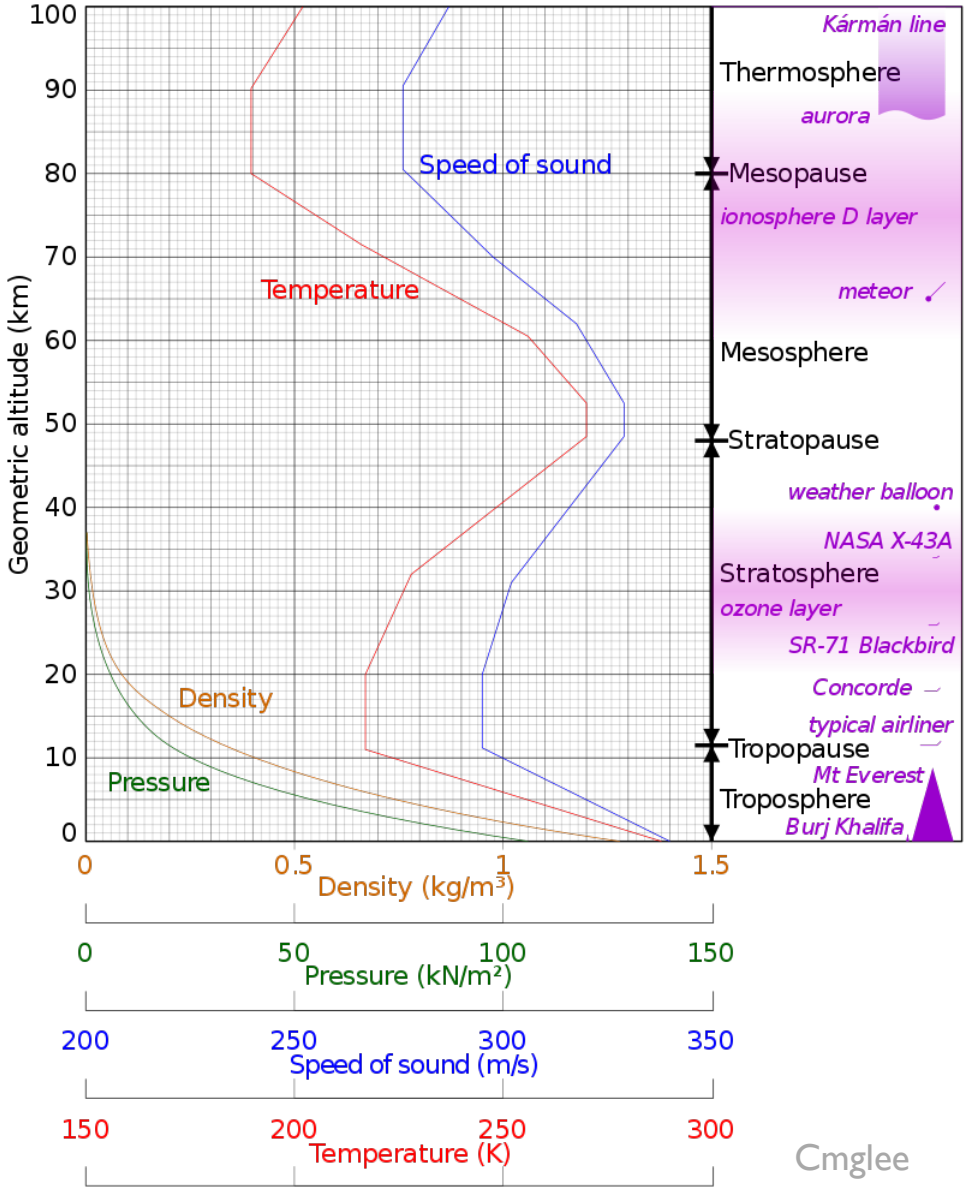
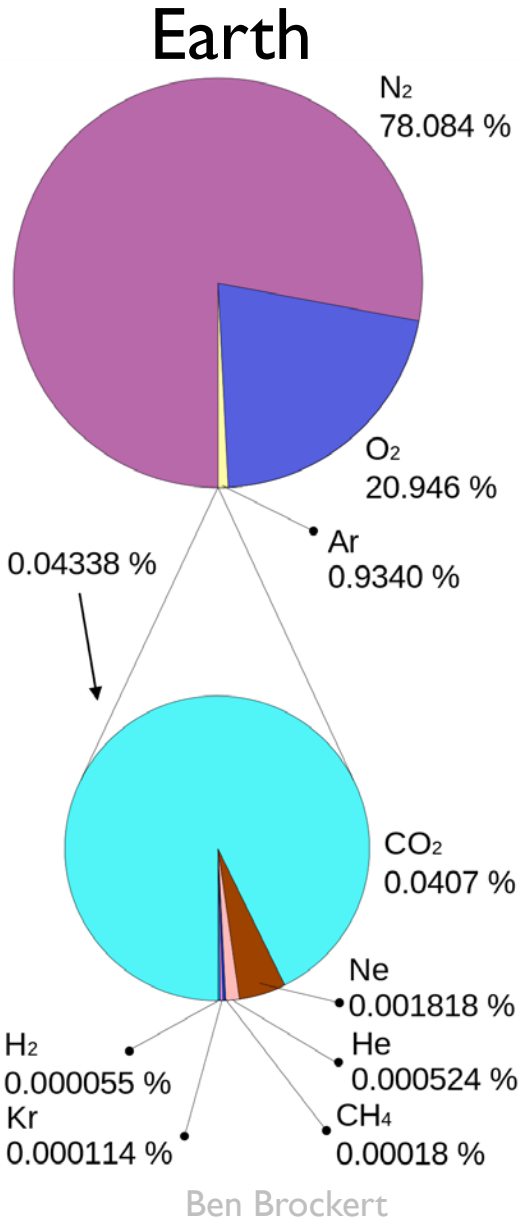
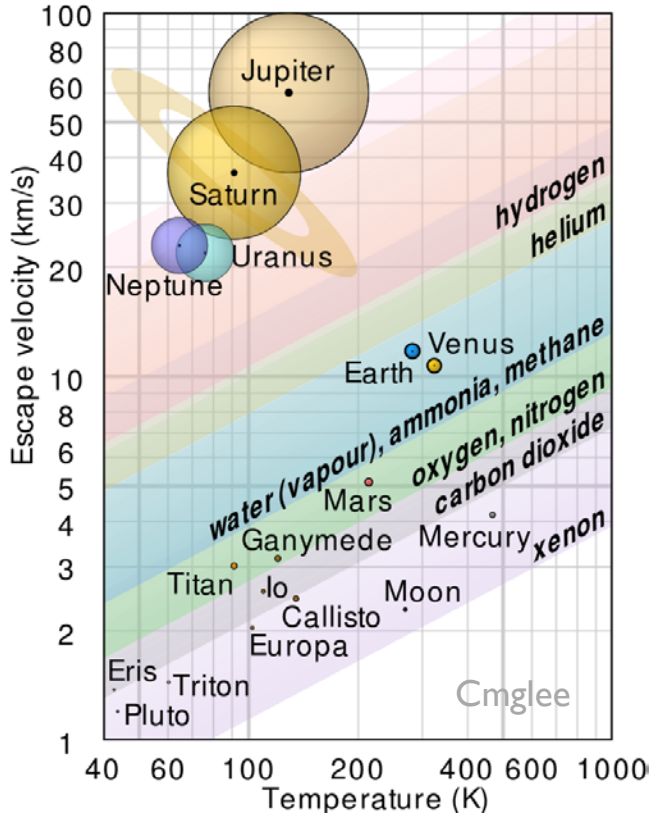


Atmosphere Composition (for μ)



- Venus: ~97% CO_2, ~3% N_2
- Mars: ~96% CO_2, ~2% Ar, ~2% N_2
- Jupiter: ~90% H_2, ~10% He
- Saturn: ~96% H_2, ~4% He
- Uranus: ~83% H_2, ~15% He, ~2% CH_4
- Neptune: ~80% H_2, ~20% He



Pro-solar abundances calculated from Lodders *ApJ* 2003