Quick notes on Convection

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Convection and Composition Gradients

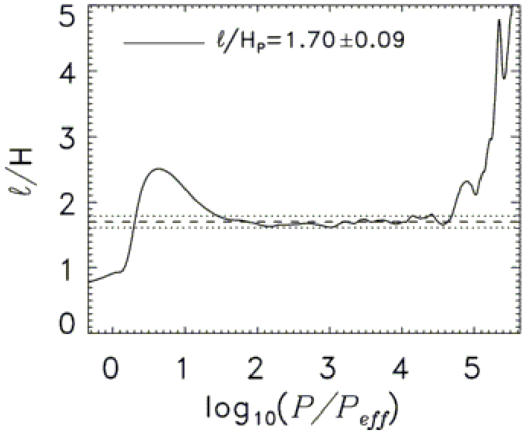
- In the presence of a steep (defined by the Schwarzschild criterion) temperature gradient, convection occurs
- What if there is also a composition gradient?
  - Convection sets in when entropy decreases with increasing radius
  - In deriving the Schwarzschild criterion, in TBS we use the fundamental thermodynamic relation $dU = TdS - PdV$
    …but where there is a non-uniform composition,
    this becomes $dU = TdS - PdV + \mu dN$,
    meaning energy can change with changes in composition
  - So we generalize to get the *Ledoux criterion*,
    $\frac{d\ln(T)}{d\ln(P)} < \frac{\gamma-1}{\gamma} - \frac{d\ln(\mu)}{d\ln(P)}$
- A negative composition gradient (light elements on top of heavy), increases the stability to convection, meaning higher Temperature-gradients can be supported

C. Malone et al. ApJ 2011

Convection Length Scale

- Convection is complicated and really requires a 3D treatment to capture. Nonetheless, back-of-the-envelope estimates give us some insight
- The *mixing length,* l_m, describes the distance the "blob" in our convection scenario will travel until dissolving into the surrounding environment.
  - $l_m = \alpha H_P$, where H_P is the pressure-scale height we encountered in the barometric formula for the isothermal atmosphere (See TBS Exercise 2.2)
    - This obviously is a poor approximation when there is a large temperature gradient
  - $\alpha \approx 1$ and $H_P = \frac{k_B T}{\mu m_u g}$
    - …for the solar photosphere: $l_m \sim 10^5$ m (compare to $R_\odot \sim 10^9$ m)
    - …for $R_\odot/2$: $l_m \sim 10^8$ m
    - …for the solar core: infinite!
  - In practice, l_m calibrated to (1) the standard solar model, (2) 3D calculations

Trampedach & Stein ApJ 2011

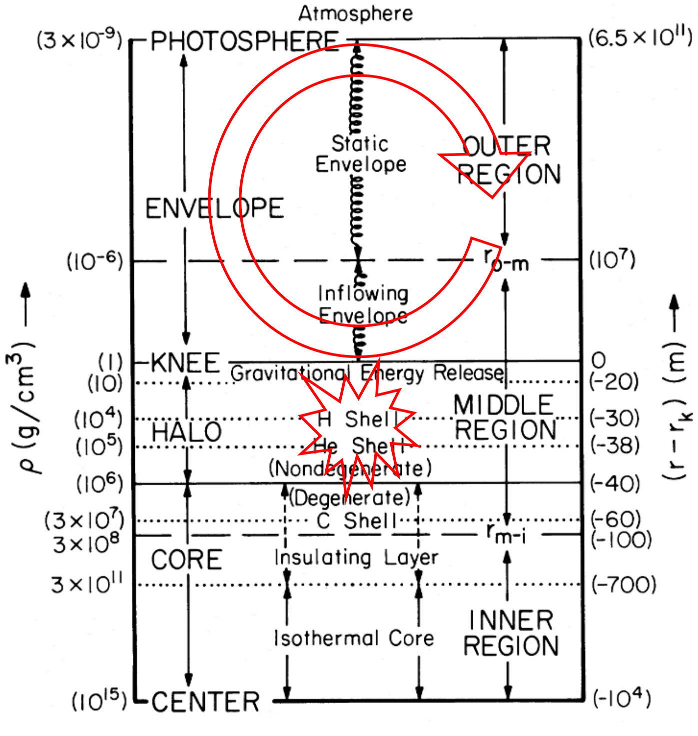


Convection Velocity

- Consider the amount of energy that is transported.
  This will be the difference between an upward blob & a downward blob:
  $L = \frac{dE}{dt}|_\mathrm{u} - \frac{dE}{dt}|_\mathrm{d} = \left(\frac{3}{2}Nk_BT/dt\right)_\mathrm{u} - \left(\frac{3}{2}Nk_BT/dt\right)_\mathrm{d} = \frac{3}{2}Avnk_B(T_\mathrm{u} - T_\mathrm{d})$
  $= \frac{3}{2}(4\pi r^2)vnk_B\Delta T = 6\pi r^2\frac{\rho}{\mu m_u}k_B v\Delta T$
    - For $L_\odot, r = R_\odot/2 \ldots v\Delta T \approx 100\,\frac{\mathrm{m}}{\mathrm{s}}\mathrm{K}$
- Now, consider the blob as a buoyant cylinder that achieves terminal velocity when the buoyant force + gravity equals the drag force
    - $F_\mathrm{u} = F_\mathrm{buoy} - F_\mathrm{grav} = gV\Delta\rho = \frac{GM}{R^2}Al\Delta\rho$
    - $F_d = F_\mathrm{drag} = \frac{\mathrm{Work}}{\mathrm{Distance}} = \frac{\Delta KE}{\mathrm{Distance}} = \frac{1}{2}\rho v^2 A$
    - $v = \sqrt{2\frac{GM}{R^2}l\frac{\Delta\rho}{\rho}}$ …for an ideal gas where $P_\mathrm{blob} = P_\mathrm{env}, v = \sqrt{2\frac{GM}{R^2}l\frac{\Delta T}{T}}$
- Solving both for $R_\odot/2, v{\sim}100\ \mathrm{m/s}$ and $\Delta T{\sim}1\ \mathrm{K}$!

Convection Timescale

- For $R_{\odot}/2$: $v \sim 100$ m/s and $l_m \sim 10^8$ m
- So the convection timescale is $t \sim \frac{l_m}{v} \sim 10^6$ s ~week
- Therefore, $t_{\text{convection}} \gg t_{\text{dynamical}}$, meaning hydrostatic equilibrium applies even to convective regions of a star
- Consider the hypothetical Thorne-Zytkow Object (TZO): a neutron star inside a red giant



Thorne & Zytkow ApJ 1977

- L and g are much larger, so $v \sim 10^5 \frac{\text{m}}{\text{s}}$ (still $\ll v_{\text{sound}}$) and $l_m \sim 10^3$ m which means $t_c \sim 0.01s$
- This is comparable to β-decay times for the proton-rich nuclides produced near the NS surface that are convected upward (and often back)
- Leads to the "interrupted" rapid proton-capture (irp) process

R. Cannon MNRAS 1993