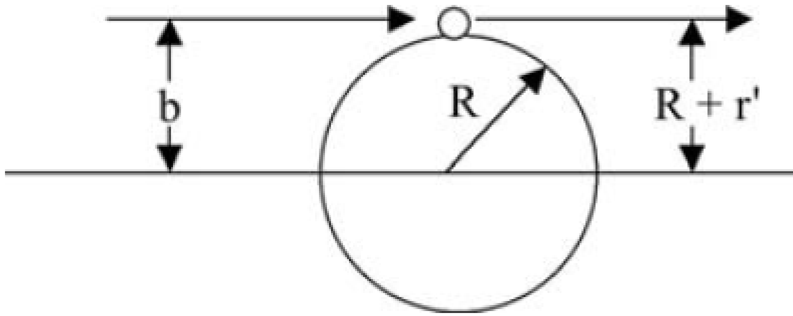
# *Quick notes on* Cross Sections

Zach Meisel

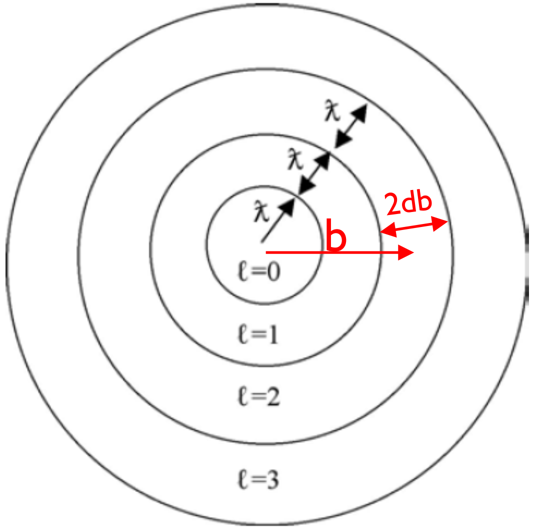
Ohio University - ASTR4201 - Fall 2020

# Geometric Cross Section




Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

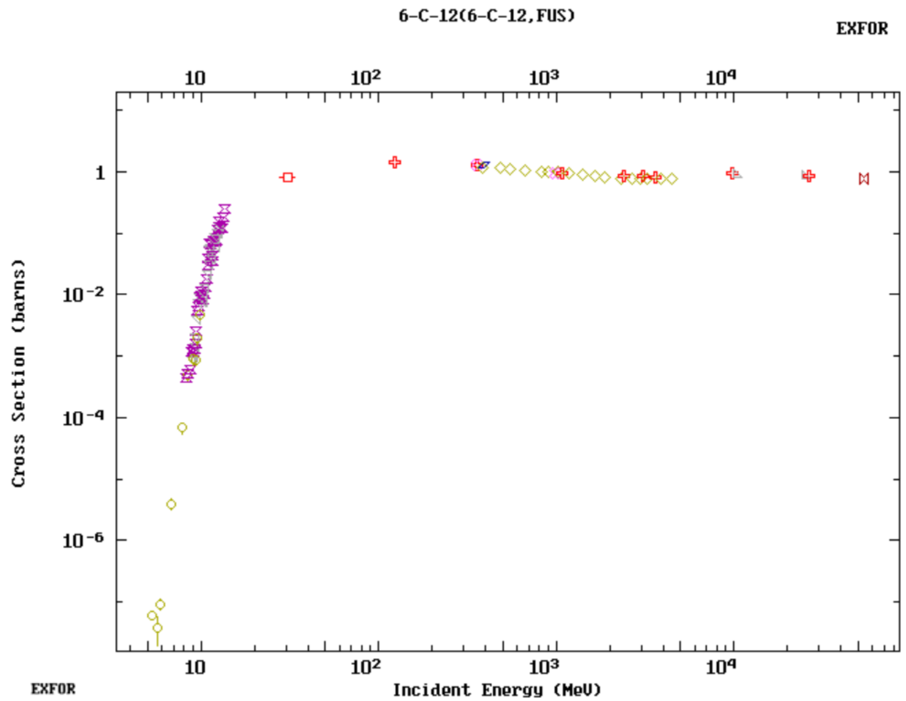
- For a grazing collision with impact parameter $b = R + r'$, relative orbital angular momentum is $\left|\vec{l}\right| = l\hbar = |\vec{r} \times \vec{p}| = pb$
- For our wave-like particle, $p = \frac{h}{\lambda} = 2\pi\hbar/\lambda$ …. and so $l\hbar = 2\pi b\frac{\hbar}{\lambda}$ …i.e. $b = \frac{l}{2\pi}\lambda$
- Collisions at a given impact-parameter slice $b \pm db$ correspond to a given $l$
- The geometric cross section for one slice is:
  - $\sigma_l = \sigma_{b\pm db} = \sigma_{b+db} - \sigma_{b-db} = \pi(b+db)^2 - \pi(b-db)^2$
    $= \pi(l+1)^2\left(\frac{\lambda}{2\pi}\right)^2 - \pi l^2\left(\frac{\lambda}{2\pi}\right)^2 = \pi\left(\frac{\lambda}{2\pi}\right)^2(2l+1)$
- So the total cross section is $\sigma_{total} = \sum_l \pi\left(\frac{\lambda}{2\pi}\right)^2(2l+1)$
- Including quantum mechanics, the probability of tunneling through a barrier corresponding to $l$ is the transmission coefficient $T_l$, so $\sigma_{tot} = \pi\left(\frac{\lambda}{2\pi}\right)^2 \sum_l (2l+1)\ T_l$ where $0 \leq T_l \leq 1$
- For relatively high energy (e.g. a ~100MeV heavy-ion beam), $T_l = \begin{matrix} 1\ \ for\ l < l_{max} \\ 0\ \ for\ l > l_{max} \end{matrix}$, sharp-cutoff limit
  where $l_{max}$ takes $b = R + r'$, so $l_{max} = \frac{2\pi(R+r')}{\lambda}$ and so $\sigma_{tot} \approx \pi\left(\frac{\lambda}{2\pi}\right)^2 l_{max}^2$



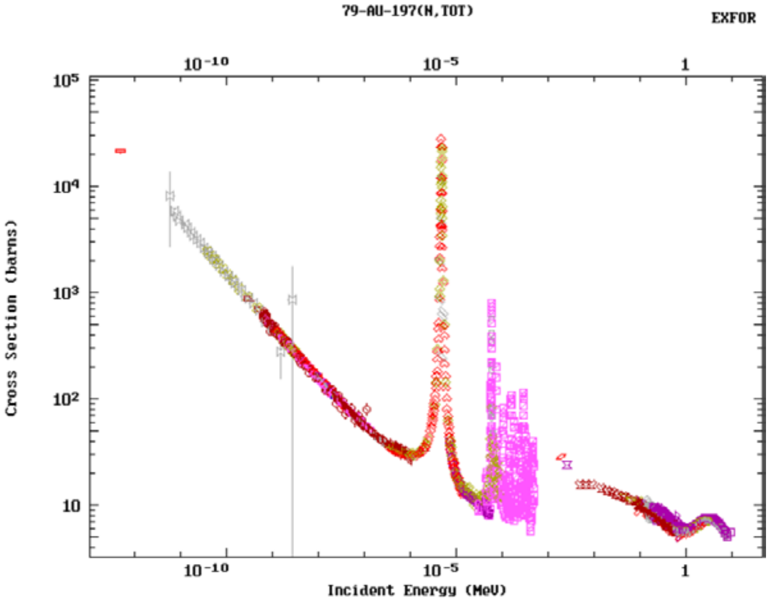

Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

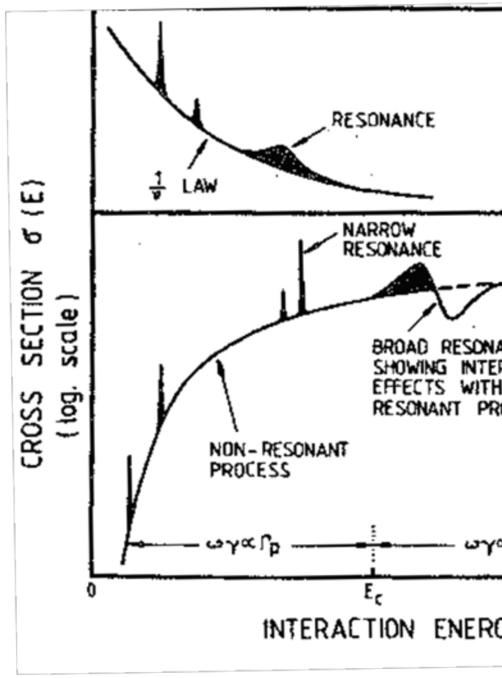
# Geometric Cross Section vs Data

- Consider the example of $^{12}C+^{12}C$
  - For 1-100MeV, geometric cross section ~1b
  - $E_{pk} = 0.122\left(Z_1^2 Z_2^2 \frac{A_1 A_2}{A_1+A_2} T_9^2\right)^{1/3}$ MeV center-of-mass
  - For explosive carbon burning (~2GK): 3.8MeV
    - $E_{lab} = E_{CM}\frac{A_{beam}+A_{target}}{A_{target}} = 7.6$ MeV
    - $\sigma_{expt} \sim 10^{-5}$ b




- Consider $^{197}$Au+n
  - Geometric for 10MeV ~2b
  - $E \sim k_B T = \frac{T_9}{11.6045}$ MeV C-of-M
  - For s-process ~25keV, $\sigma_{expt} \sim 13$ b
  - For neutrons, $T_l \propto \sqrt{E}$ for $l = 0$ and ~0 otherwise
  - $\sigma_{tot} \propto \pi\left(\frac{\lambda}{2\pi}\right)^2 \sqrt{E} \propto \left(\frac{1}{\sqrt{E}}\right)^2 \sqrt{E}$
  - $\propto \left(\frac{1}{v^2}\right)\sqrt{v^2} \propto \frac{1}{v}$




Rolfs & Rodney, Cauldrons in the Cosmos (1988)