Quick notes on
Liquid Drop Model

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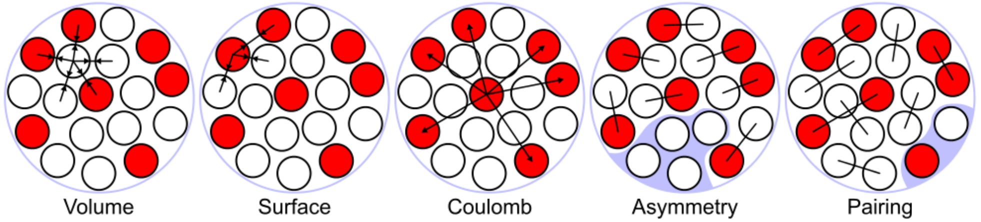
Ohio University - ASTR4201 - Fall 2020

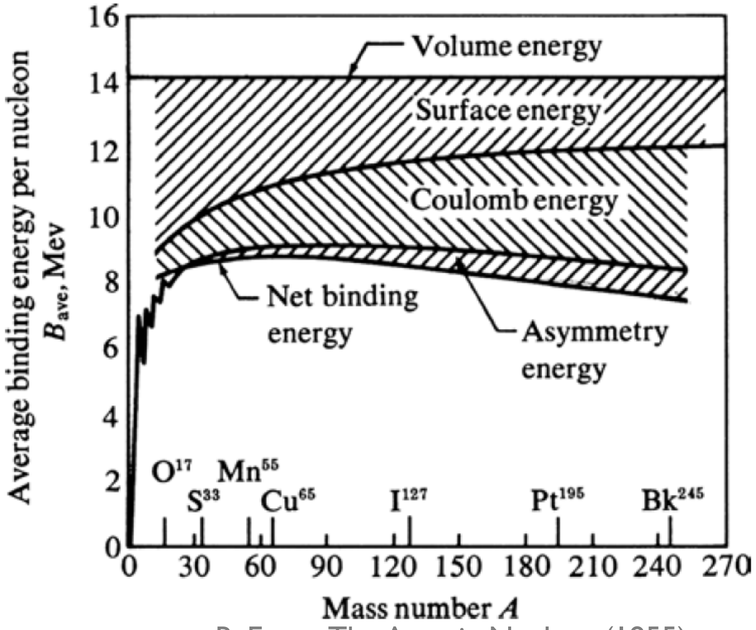
The Semi-Empirical Mass Formula



- BE(Z,A) = Volume - Surface - Coulomb - Asymmetry ± Pairing
- One mathematical parameterization* (*of many!*):

*from B. Martin, Nuclear and Particle Physics (2009)

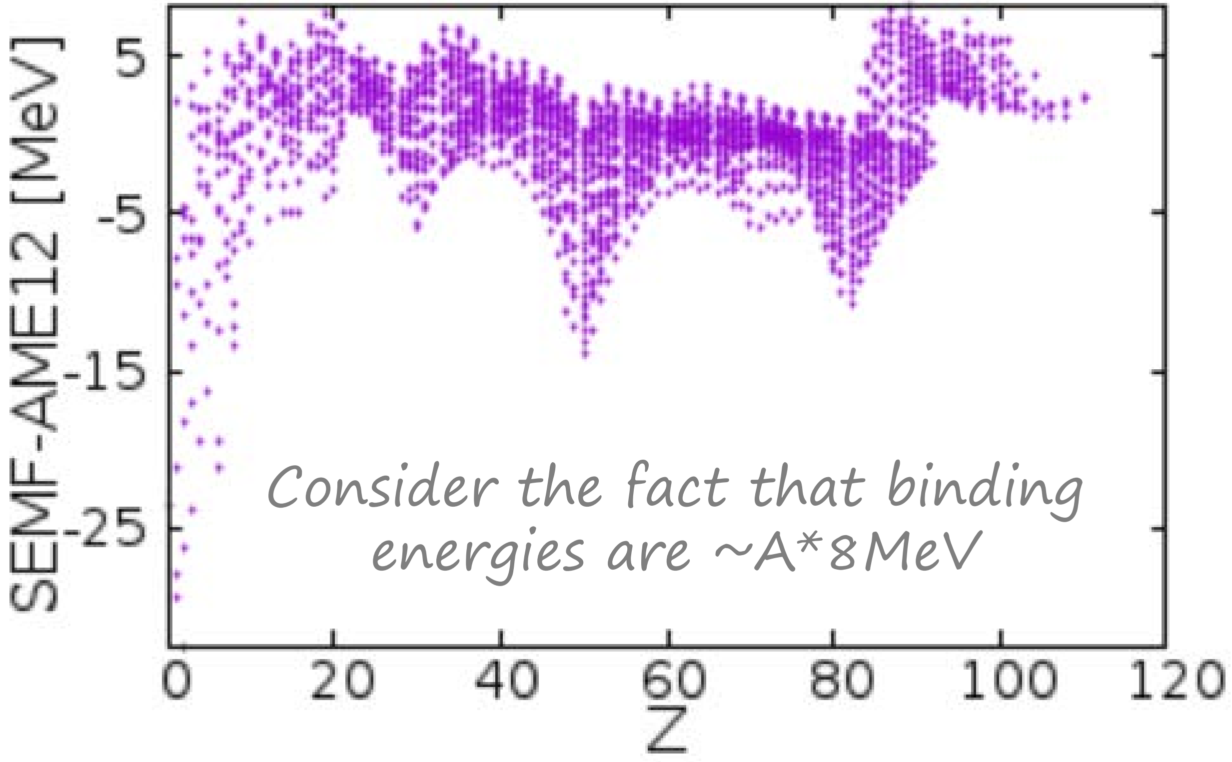
  - $BE(Z,A) = a_v f_v(A) - a_s f_s(A) - a_c f_c(Z,A) - a_a f_a(Z,A) + i a_p f_p(A)$
    - **Volume**: Nucleons have some self-binding, so: $f_v(A) = A$
    - **Surface**: Since radius goes as $R \propto A^{1/3}$ and surface area goes as $SA \propto R^2$, $f_s(A) = A^{2/3}$
    - **Coulomb**: Energy for a charged sphere goes as $\frac{q^2}{R}$ and $R \propto A^{1/3}$, so $f_c(Z,A) = \frac{Z(Z-1)}{A^{1/3}}$
    - **Asymmetry**: Z=N favored (want Z=A/2) but lesser problem for large A, so $f_a(Z,A) = \frac{\left(Z-\frac{A}{2}\right)^2}{A}$
    - **Pairing**: Favor spin-0 nucleon pairs & disfavor unpaired nucleons, empirically $f_p(A) = \left(\sqrt{A}\right)^{-1}$ Even-Z, Even-N: $i = +1$
      - Odd-Z, Odd-N: $i = -1$
      - Even-Odd: $i = 0$
    - a_i are fit to data



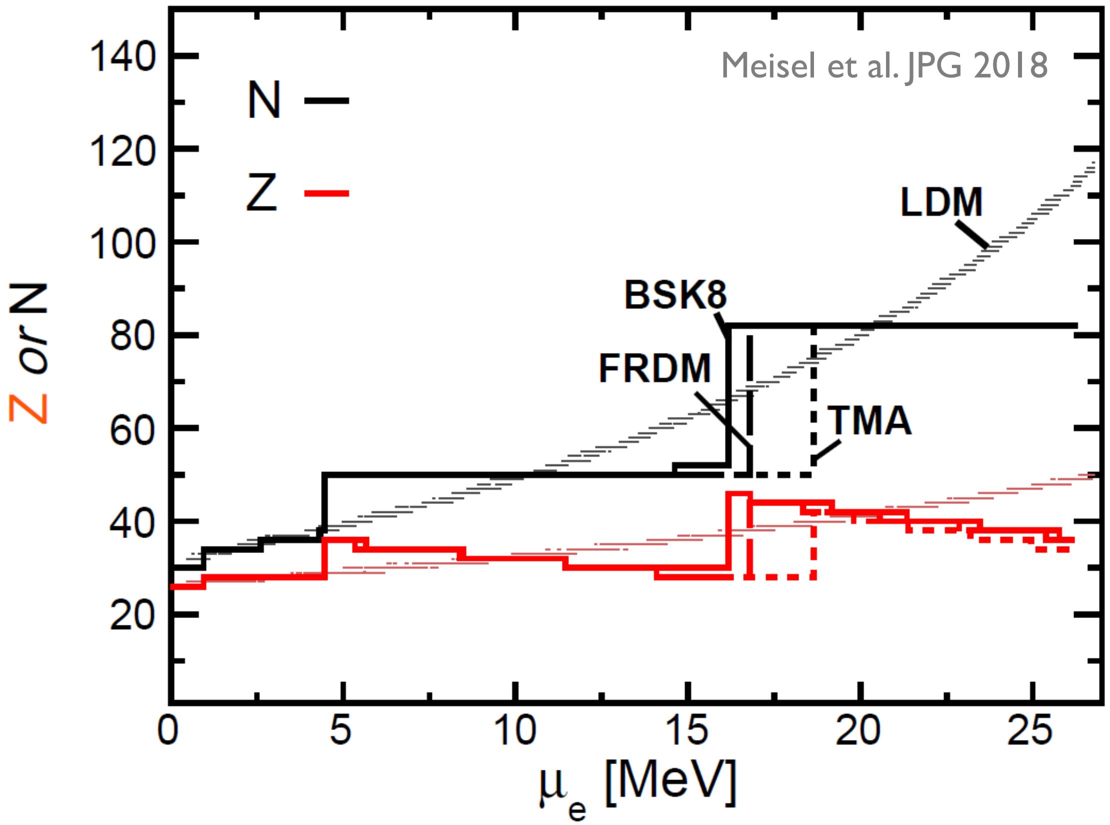
R. Evans, The Atomic Nucleus (1955)

The SEMF is often close enough

Typically within ~1 percent of right BE



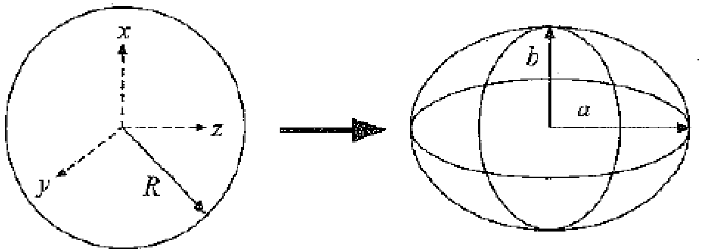
Sometimes used for neutron star crusts (though often with a shell correction)



Here, a_v = 15.302, a_s =16.518, a_c = 0.687, a_a = 88.974, and a_p = 5.898, with the functional form on the previous slide

Nuclear Fission: Splitting a Liquid Drop

- Consider deforming a nucleus: volume and number of nucleon pairs are conserved, but the surface gets larger and the charges get spaced further apart.
  - i.e. The Coulomb penalty of the SEMF decreases, but the surface penalty increases
  - The change in energy: $\Delta E = BE_{final} - BE_{initial} = (E_c' + E_s{}') - (E_c + E_s)$
- Parameterize the nuclear shape as an ellipsoid, $R(\theta) = R_0[1 + \alpha_2 P_2(\cos\theta)]$, where $a = R_0(1 + \alpha_2)$, $b = R_0(1 + \alpha_2)^{-1/2}$
- Expanding, $E_c' \approx a_c \frac{Z^2}{A^{1/3}}\left(1 - \frac{1}{5}\alpha_2^2\right)$ and $E_s' \approx a_s A^{2/3}\left(1 + \frac{2}{5}\alpha_2^2\right)$, so is $\Delta E = \frac{\alpha_2^2}{5}\left(2a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}}\right)$
- The drop will split when $E_c \geq 2E_s$



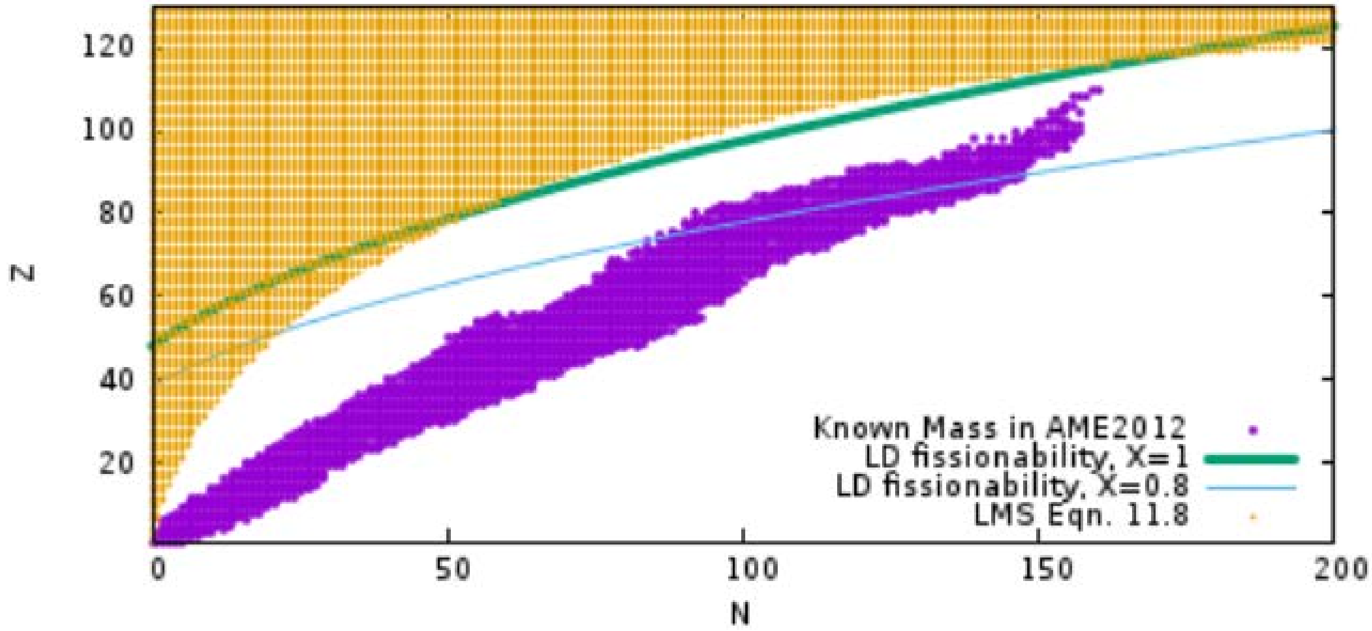
B.R. Martin, Nuclear and Particle Physics (2009)

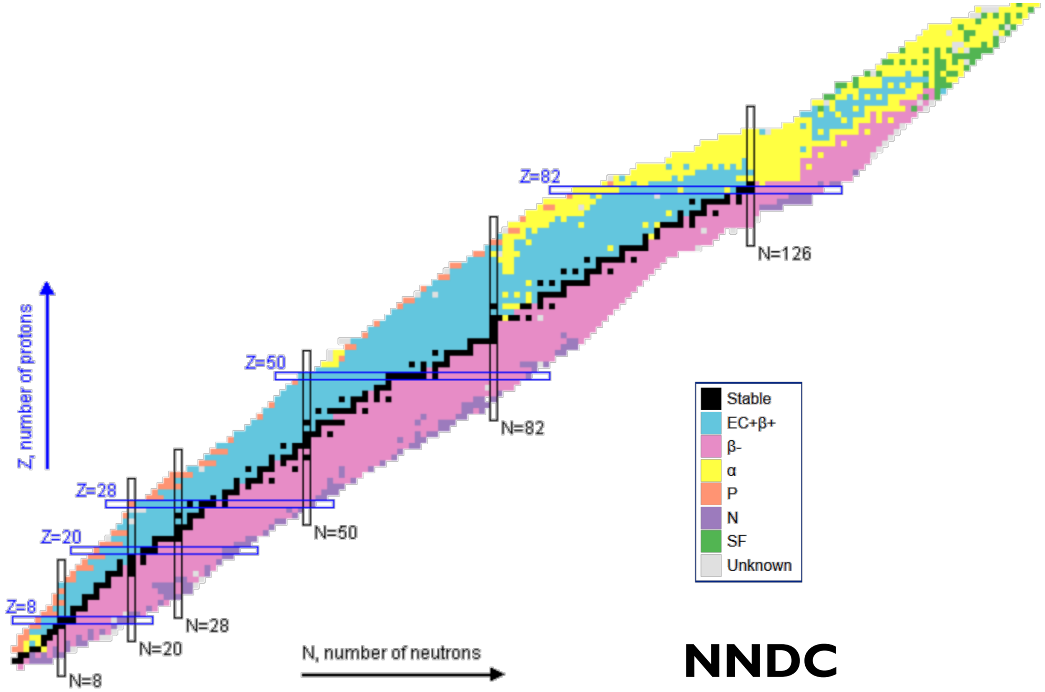
Nuclear Fission: Splitting a Liquid Drop

- Fissionability of a nucleus (in this naïve picture) is: $x = \frac{E_C}{2E_S} \equiv \frac{Z^2/A}{\left(Z^2/A\right)_{critical}}$
- In practice, (Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry)

$$\left(Z^2/A\right)_{critical} = 50.8333\left[1 - 1.7826\left(\frac{N-Z}{A}\right)^2\right]$$

  - Note larger fissionability (i.e. closer to $\left(Z^2/A\right)_{critical}$) means a nucleus is more prone to fission





NNDC