Quick notes on
Saha Equation

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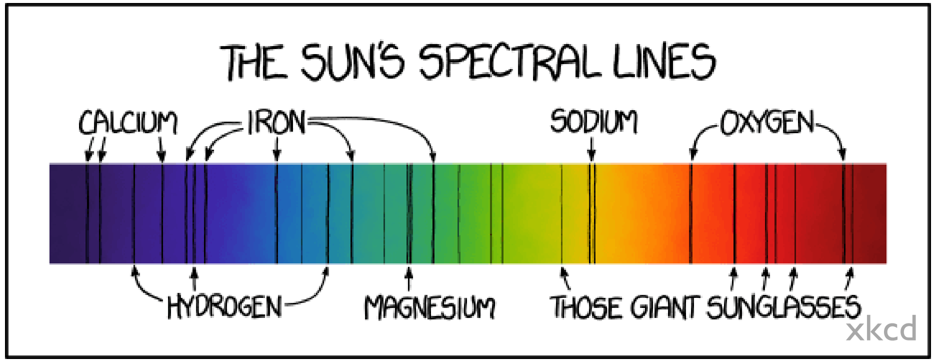
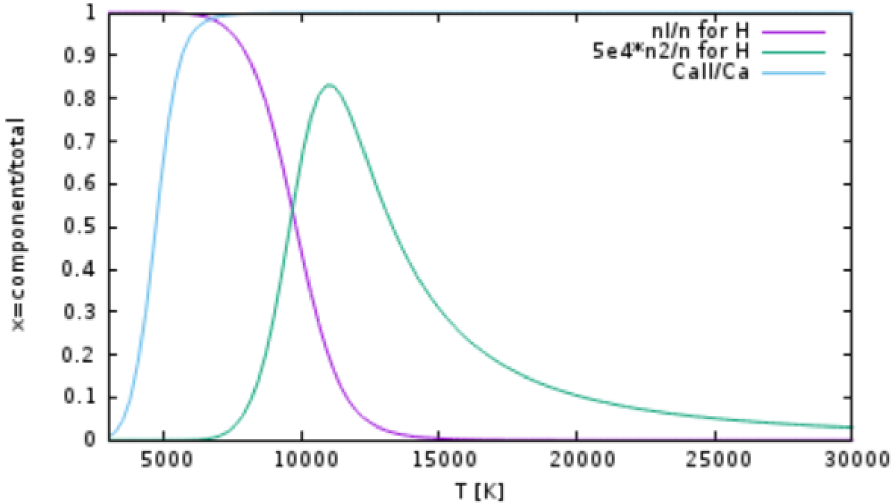
When does the Saha equation apply?

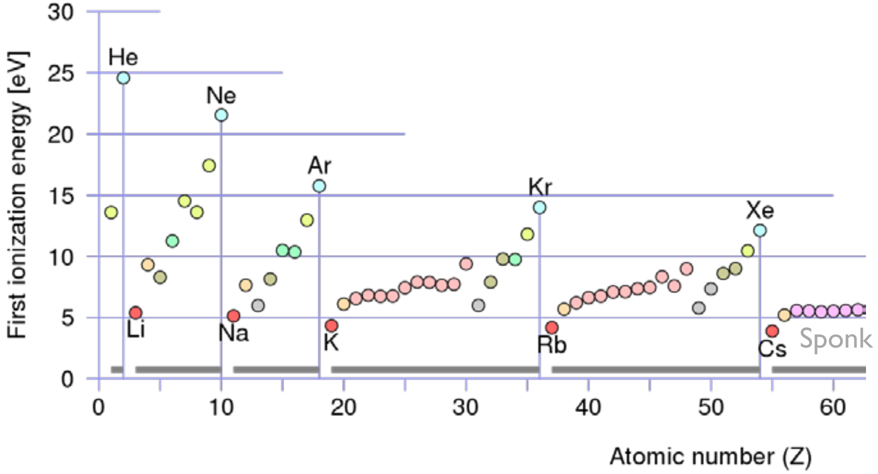
- $\frac{N_{i+1}}{N_i} \approx 2\frac{1}{n_e}\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2}\frac{g_{i+1,gs}}{g_{i,gs}}\exp\left(-\left(\frac{E_{i+1,gs}-E_{i,gs}}{k_B T}\right)\right)$ *"≈" because assuming in ground-states*
- The latter term assumes Boltzmann statistics are valid: $P \sim \exp\left(-{}^{E}/_{k_B T}\right)$
- This is only true if $n \ll n_Q$, the quantum concentration, which is the $()^{3/2}$ stuff
- For the solar photosphere, $\frac{n}{n_Q} \sim 10^{-6}$
- For the solar core, $\frac{n}{n_Q} \sim 10^{2}$
  - Obviously Saha equation is no good here.
  - The solar core is partially supported by electron degeneracy pressure!
  - This does not mean the solar core isn't ionized. It means the photons (emitted due to the high temperature) aren't doing the ionizing. It's the pressure.

Stellar Abundance Ratios



- Suppose we observe absorption lines in a stellar atmosphere, from which we infer (from the line widths) that there are 400 more Ca II absorbers than Hα absorbers
- Does the photosphere have 400x more Ca than H?
  - *No! Need to know fraction of H in n=2 and singly ionized Ca*
- Use Saha equation for $\frac{n_{H,n=2}}{n_H}$ and $\frac{n_{Ca\ II}}{n_{Ca}}$ and find $\left(\frac{n_{H,n=2}}{n_H}\right) \Big/ \left(\frac{n_{Ca\ II}}{n_{Ca}}\right) = \frac{5.5e{-9}}{0.92} \sim 6 \times 10^{-9}$
- $\frac{n_{Ca}}{n_H} = \frac{n_{Ca\ II}}{n_{H,n=2}} \left(\frac{n_{Ca}}{n_{Ca\ II}}\right) \left(\frac{n_{H,n=2}}{n_H}\right) \sim 2 \times 10^{-6}$ which agrees with what is shown in the Quick Notes on Simple Atmospheres





Saha Equation: Not just for atoms!

- The Saha equation $\frac{N_{i+1}}{N_i} \approx 2\,\frac{1}{n_e}\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} \frac{g_{i+1,gs}}{g_{i,gs}} \exp\left(\frac{-E_{ion}}{k_B T}\right)$ refers to thermal equilibrium for ionization & recombination reactions
- For nuclear physics, one can work out almost the same relation for radiative capture and photodisintegration reactions
- E.g. for $(p,\gamma) - (\gamma,p)$ equilibrium

$$\frac{N_{Z,A}}{N_{Z+1,A+1}} \approx \frac{2}{n_p}\left(\frac{\mu_{\text{red}} k_B T}{2\pi\hbar^2}\right)^{3/2} \frac{g_{Z,A}}{g_{Z+1,A+1}} \exp\left(\frac{-Q_{p,\gamma}}{k_B T}\right)$$

- This explains the reaction network path in explosive burning:
  - *rp*-process in XRBs follows $Q_{p,\gamma} \sim 1\ \mathrm{MeV}$
  - *r*-process in NS mergers follows $Q_{n,\gamma} \sim 2\ \mathrm{MeV}$