Quick notes on

Eddington Gray Atmosphere

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Assumptions

- "Gray" (frequency-independent) opacity
  - …though we know there is a frequency dependent (especially bound-bound)
- Local thermodynamic equilibrium: thermalized environment
  - …though at ~ 1 optical depth, photons aren't thermalizing
- Eddington approximation: Intensity is linear in $\cos\theta$, so doesn't have high-order anisotropy
  - …though at ~1 optical depth, photons won't have lots of scattering

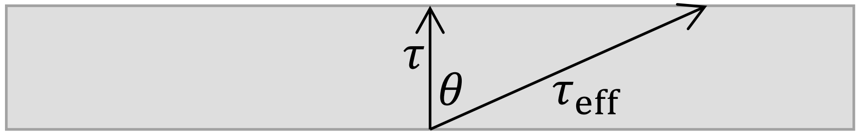
This doesn't sound very realistic, how does it do?

Radial Temperature Dependence

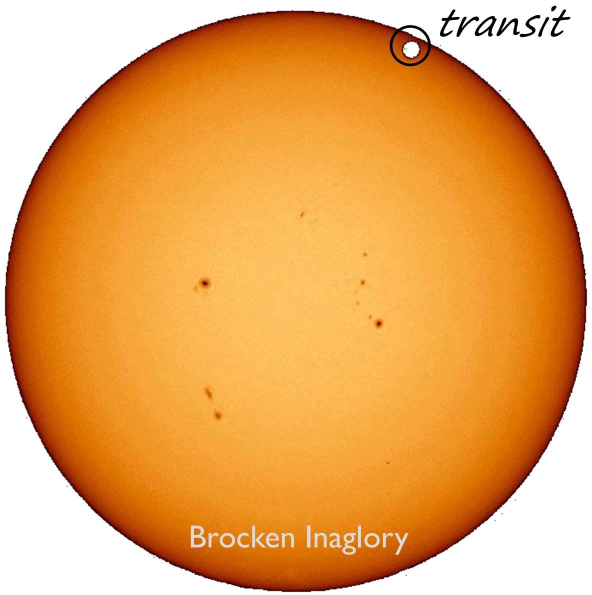
- Eddington approximation yields $T^4 = \frac{3}{4} T_{\text{eff}}^4 \left(\tau + \frac{2}{3}\right)$
- What would the core temperature be?
  - See quick notes on Photon Diffusion for $\bar{l} \approx 0.1 \text{ cm}$
  - So, $\tau_{\text{core}} \approx \frac{R_{\odot}}{\bar{l}} \approx \frac{7e8\,m}{1e{-}3m} \approx 7e11$
  - Noting $T_{\text{eff}} \approx 5800 \text{ K},\ T_{\text{core}} \approx \sqrt[4]{\frac{3}{4} 5800^4 (7e11)} \approx 5 \text{ MK}$
  - …to be compared to the real value of 15 MK
    and the constant density estimate of 6 MK

- At $\tau = 0, \frac{T}{T_{\text{eff}}} = \frac{1}{\sqrt[4]{2}} = 0.841$, while the non-Eddington gray result is $\sqrt[4]{\frac{\sqrt{3}}{4}} = 0.811$
- The weak τ-dependence of T justifies our ~constant T argument made for the vertical motion of the Hayashi track in the Quick Notes on Star formation

Limb Darkening



- Consider right near the surface (~1 optical depth)
  We can approximate this as a plane-parallel atmosphere.
- If we view the plane from an angle, the effective optical depth τ_{eff} is modified from the optical depth you would have viewing the plane head-on τ
  - We see $\cos\theta = \frac{\tau}{\tau_{\text{eff}}}$, so when we view the photosphere from angle θ, we're viewing light coming from a depth $\tau = \tau_{\text{eff}} \cos\theta$
  - By definition, we see light from $\tau_{\text{eff}} \sim 1$, so $\tau \approx \cos\theta \equiv \mu$



- Then, $I(\mu,\tau) = \frac{3}{4\pi}\sigma_{\text{SB}} T_{\text{eff}}^4 \left(\frac{2}{3} + \tau\right)$

$$= I(\mu,\mu) = \frac{3}{4\pi}\sigma_{\text{SB}} T_{\text{eff}}^4 \left(\frac{2}{3} + \mu\right)$$

- $\frac{I(\mu)}{I(1)} = \frac{3}{5}\left(\frac{2}{3} + \mu\right)$ … 1 at $\mu = 1$ and 0.4 at $\mu = 0$