Quick notes on
Random Walk

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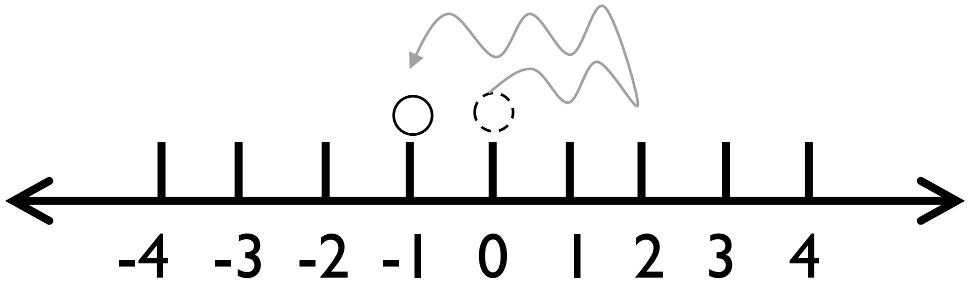
Ohio University - ASTR4201 - Fall 2020

One-dimensional Random Walk

- Consider a hopping circle on a number line that can only jump by one space at a time, where $\Delta x = \pm 1$, and the direction of each hop ($+$ or $-$) is random



- The average distance traveled after N hops is $\bar{d} = \overline{\sum_{i=1}^{N} \Delta x_i} = \sum_{i=1}^{N} \overline{\Delta x_i}$
  - For all steps, the average is of +1 and -1, which is zero …so $\bar{d} = 0$
- Instead, consider a measure of the magnitude of distance traveled, d^2
  - $\overline{d^2} = \overline{\left(\sum_{i=1}^{N} \Delta x_i\right)^2} = \overline{\left(\sum_{i=1}^{N} \Delta x_i\right)\left(\sum_{i=1}^{N} \Delta x_i\right)} = \sum_{i=1}^{N} \overline{\Delta x_i^2} + 2\sum_{i=1}^{N}\sum_{j=1}^{N} \overline{\Delta x_i \Delta x_j}$
  - For each Δx_i^2, you either have $(-1)^2$ or 1^2, which are both 1
  - For each $\Delta x_i \Delta x_j$, the Δx are independent, so the average is over +1 and -1, which is zero
  - So $\overline{d^2} = \sum_{i=1}^{N} \overline{\Delta x_i^2} = N$
- So, we consider the average positive distance traveled from the origin to be $\sqrt{\overline{d^2}} = \sqrt{N}$ for step size 1. For average step size l, then distance $\sim l\sqrt{N}$
- For 3D and unequal step sizes, distance traveled: $\sqrt{\overline{r^2}} = \bar{l}\sqrt{⅓N}$ (Rev. Mod Phys. 1943)

One-dimensional Random Walk …but fancier

- For the same scenario, of our N steps, say that m have been to the right and $N - m$ have been to the left, where the probability of any given step going right is p_r and going left is $1 - p_r$.
- Probability of a sequence of steps is the probability of having m right steps and $N - m$ left steps: $p_r^m (1 - p_r)^{N-m}$
- But we could have taken many different sequences to wind up at m steps right, where the total number of sequences is the "N choose m" problem of combinatorics, where the answer is the binomial coefficient $\binom{N}{m} = \frac{N!}{m!(N-m)!}$
- So, $p_N(m; p_r) = \frac{N!}{m!(N-m)!} p_r^m (1 - p_r)^{N-m}$, the binomial distribution
  - $\bar{m} = N p_r$
  - $\sigma_m^2 = \overline{m^2} - \bar{m}^2 = N p_r (1 - p_r)$