Homework Assignment 4 **ASTR4201, Fall 2020**

Corresponds to Chapter 4 of "To Build a Star" (TBS) by E.F. Brown

1. *See below* Team: 1 Lead: Gavin
   Using semiclassical arguments and the proposal from Bohr that angular momentum is quantized ($L = n\hbar$), derive the -13.6eV Rydberg energy for the most-bound state of hydrogen that is listed in equation 4.1.
   Hint: The virial theorem is for more than just gravity.

2. *See below* Team: 2 Lead: Michael
   Figure 4.3 shows that there would be relatively wide gaps in the solar spectrum from absorption lines from various series. Calculate the wavelength at which the relatively wide gaps in the solar spectrum would appear for the Lyman, Balmer, and Paschen series. Do any of these gaps appear in Figure 4.1?

3. TBS exercise 4.1 Team: 3 Lead: Ryan

4. *See below* Team: 3 Lead: Josh
   Using semiclassical arguments and assuming angular momentum is quantized ($L = n\hbar$), derive the Bohr radius.

5. *See below* Team: 4 Lead: Gula
   Box 4.1 uses the estimate that the volume of all atoms has to be less than half of that available in the gas in order for atoms to not overlap. If we assume the atoms are spheres, is this reasonable? How much volume could the spheres take up?
   Hint: We are essentially talking about the atomic packing factor.

6. *See below* Team: 1 Lead: Anthony
   A nuclear form of the Saha equation exists, where we use nucleon density instead of electron density, nucleon mass instead of electron mass, and a nuclear reaction energy release as opposed to ionization energy release. Assuming the astrophysical *r*-process happens with a neutron density of 10^{20} neutrons/cm^3 and a temperature of 1 GK, calculate the neutron separation energy (Energy release from an (n,γ) reaction) that results in (n,γ)-(γ,n) equilibrium. Assume the neutron-capture parent and daughter have the same degeneracy.

7. *See below* Team: 2 Lead: Sam
   The hydrogen in the photosphere of the sun is mostly neutral. Would fluorine be similar or more ionized? Why?

8. *See below* Team: 2 Lead: Quinn
   If an electron is confined to a volume defined by its de Broglie wavelength and is non-relativistic, what is the number density in terms of the temperature? Do you recognize this quantity?

9. *See below* Team: 5 Lead: Justin
   The term in equation 4.6 that is raised to the 3/2 power is known as the quantum concentration. When an environment approaches this density, the matter is degenerate (i.e. Boltzmann statistics are no longer good). Compare this number density for electrons and nucleons at 1 GK. Then, convert to mass density, assuming that 1 g/mol is close enough, and compare to the average density of a white dwarf and of a neutron star.

10. TBS exercise 4.2 Team: 4 Lead: Jacob
    Hint: Consider the quantity $(1\text{-}x)^2/x$

11. *See below* Team: 3 Lead: Harshil
    Verify that the general solution for the driven harmonic oscillator (equation at the top of Box 4.2 on page 51) is correct.

12. *See below* Team: 1 Lead: Brit
    Show that $(\omega_0^2 - \omega^2) \approx 2\omega_0(\omega_0 - \omega)$ for $\omega \approx \omega_0$.

13. *See below* Team: 5 Lead: Robert
    How much narrower would you expect a spectral line in an A1 supergiant star to be relative to the same spectral line in an A1 main sequence star? Compare to the inset of Figure 4.7 and comment on the possible causes for any discrepancies. Assume the contribution to the line width from pressure depends linearly on the pressure and assume the A1 supergiant has the same mass as the A1 main sequence star.