Homework Assignment 7 **ASTR4201, Fall 2020**

Corresponds to Chapter 7 of "To Build a Star" (TBS) by E.F. Brown

1. *See below* Team: 1 Lead: Anthony
   A nucleus is held together by the strong force, which is mediated by virtual pions. In order for a conglomeration of nucleons to be considered a "state", the nucleons in a nucleus need to have time to have exchanged some pions. The virtual pions are created from the vacuum and exist for a lifetime limited by the uncertainty principle. This then sets the lower limit on what qualifies as a "state". Calculate this time given that the pion mass is ~140MeV/c^2.

2. TBS exercise 7.1 Team: 1 Lead: Brit

3. TBS exercise 7.2 Team: 2 Lead: Michael

4. *See below* Team: 3 Lead: Josh
   In the supergiant phase, a star that initially had 15 $M_\odot$ ZAMS will be have ~10 $M_\odot$ mass and ~50 $R_\odot$ radius. In table 7.1, you'll notice that the surface luminosity hardly changes beyond the carbon burning phase. Show why.

5. *See below* Team: 1 Lead: Gavin
   What would the maximum mass of an H-rich white dwarf be? How can a hydrogen white dwarf be ruled-out observationally?

6. *See below* Team: 3 Lead: Ryan
   How much heat does it cost to have electron capture on ^{56}Fe? BE(^{56}Fe)= 492.26 MeV and BE(^{56}Mn)=489.35 MeV.

7. TBS exercise 7.3 Team: 3 Lead: Harshil

8. *See below* Team: 2 Lead: Sam
   Suppose you're standing on the surface of a white dwarf. What is the difference in force between your upper 10 kg and your lower 10 kg? What about on a neutron star? Compare to the typical tensile strength of a human tendon of ~1,000N.

9. *See below* Team: 2 Lead: Quinn
   A black hole is "black" because light can't escape. Find the radius within which a given amount of mass will have an escape velocity equal to the speed of light. This is the Schwarzschild radius. For a solar mass, what would the average density of this object be?

10. TBS exercise 7.4 Team: 5 Lead: Robert

11. *See below* Team: 4 Lead: Jacob
Consider the core of a massive star. Prior to collapse, suppose it has roughly a solar mass, white dwarf radius, and is uniformly rotating at the solar surface rotation frequency of 4×10^{-7} Hz. Assuming no angular momentum is lost and both objects are uniform spheres, what will the rotation rate be when this core collapses to the size of a neutron star?

12. TBS exercise 7.5 Team: 5 Lead: Justin

13. *See below* Team: 4 Lead: Gula
Follow TBS 7.5, but assume this is an accreting white dwarf, where the radius is 6,500 km and the accretion rate is 10^{18} g/s.

Problem 7.1

7.)

Look for shortest possible limit based on Heisenberg

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$x = 1 \times 10^{-15}\ m$

$m = 140 MeV/c^2 = 2.50 \times 10^{-28}\ Kg$

$$m \Delta x \Delta v$$

$$m \Delta x \frac{\Delta x}{\Delta t} \geq \frac{\hbar}{2}$$

$$m \frac{\Delta x^2}{\Delta t} \approx \frac{\hbar}{2}$$

$$\frac{2m\Delta x^2}{\hbar} \approx \Delta t$$

$$\frac{2(2.50 \times 10^{-28}\ kg)(1 \times 10^{-15}\ m)^2}{1.05 \times 10^{-34}\ Js} = \boxed{4.76 \times 10^{-24}\ s}$$

Or

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta t \approx \frac{\hbar}{2\Delta E}$$

$$\Delta t \approx \frac{6.58 \times 10^{-16}\ eV \cdot s}{2 \cdot 140 \times 10^{6}\ eV}$$

$$\boxed{\Delta t \approx 2.74 \times 10^{-24}\ s}$$

ASTR HW7 Brit Kenady

Problem 7.2

7.1

reaction:

$$^4He + {}^4He + {}^4He \rightarrow {}^{12}C$$

binding energy of ^4He: 28.296 MeV
binding energy of ^{12}C: 92.162 MeV

Mass of proton: 938.28 MeV/c^2
mass of neutron: 939.57 MeV/c^2
mass of electron: 0.5110 MeV/c^2

^4He has 2 protons, 2 electrons, 2 neutrons
^{12}C has 6 protons, 6 electrons, 6 neutrons

change in mass:

$$3[(2*938.28)+(2*0.5110)+(2*939.57)-28.296]-[(6*938.28)+(6*0.5110)+(6*939.57)-92.162]$$

$$= 7.274\ MeV/c^2$$

so, ~7.3 MeV

Given:

luminosity → $30\ L_\odot$

mass → $0.45\ M_\odot$

Want:

lifetime

convert:

MeV → J ⇒ $(7.274\ \text{MeV})(1.60218E{-13}\ \text{J/MeV})$

$\approx 1.165E{-12}$ J

$M_\odot$ → kg ⇒ $(0.45 M_\odot)(1.989E30\ \frac{\text{kg}}{M_\odot})$

$\approx 8.951E29$ kg

$L_\odot$ → J/s ⇒ $(30 L_\odot)(3.826E26\ \frac{\text{J/s}}{L_\odot})$

$\approx 1.148E28$ J/s

u → kg ⇒ $(12u)(1.66054E{-27}\ \frac{\text{kg}}{u})$

$\approx 1.993E{-26}$ kg

(from "mass of ^4He" * 3
[because have 3 atoms])

need form $\frac{E}{M}$:

$$\frac{1.165E{-}12\ J}{1.993E{-}26\ kg} \approx 5.845\ E13\ J/kg$$

get energy: $8.951E29\ kg * 5.845E13\ J/kg$

$\approx 5.232\ E43\ J$

plug in for time:

$$T = \frac{5.489\ E43\ J}{1.148E28\ J/s} \approx 4.557E15\ s$$

Ans. /60/60/24/365 $\approx$ 1.445 E 8 yrs.

or lifetime $\approx$ 145 mega-years

7.2)
$$\frac{1}{\sqrt{G\bar{\rho}}}$$

$$= \frac{1}{\sqrt{(6.674\times10^{-11})(3.6\times10^{9})}}$$

$$= 2.04\ \mathrm{s}$$

Problem 7.4

$$t_{KH} = \frac{GM^2}{RL} = \frac{G(10\,M_\odot)^2}{(50\,R_\odot)(83\times10^3\,L_\odot)}$$

$$= \frac{\left(6.67\times10^{-11}\ m^3/kg\cdot s^2\right)\left(10\cdot\left(1.989\times10^{30}\ kg\right)\right)^2}{\left(50\cdot\left(696\times10^6\ m\right)\right)\left(83\times10^3\cdot\left(3.85\times10^{26}\ \frac{m^2\cdot kg}{s^3}\right)\right)}$$

$$\frac{\frac{\cancel{m^3\cdot kg}}{\cancel{s^2}}}{\frac{\cancel{m^3\cdot kg}}{\cancel{s^3}}} = s$$

$$= 2.37\times10^{10}\ s = 2.75\times10^5\ \text{days} = \boxed{752\ yr}$$

$$\left.\begin{array}{l} O_2:\ \tau = 2.58\ yr \\ Si:\ \tau = 18.3\ d \end{array}\right\} < 752\ yr$$

take-away: nuclear timescale is shorter than Kelvin-Hemholtz timescale, so core details aren't communicated to surface

Problem 7.5

Homework 7.5

- What would the maximum mass of an H-rich white dwarf be? How can a hydrogen white dwarf be ruled-out observationally?

Eq. 7.2 → $M_{ch} = 1.456 \left(\frac{2}{\mu_e}\right)^2 M_\odot$

for hydrogen → $\mu_e = 1$

$M_{ch} = 1.456 (2)^2 M_\odot$

$M = 5.824 M_\odot$ is maximum mass

A hydrogen white dwarf can be ruled-out observationally because a hydrogen white dwarf would have a radius much larger than that of a helium or carbon white dwarf. With such a differenke in radius size for stars of the same mass, that has never been seen.

Problem 7.6

HW7 Q6

$^{56}Fe \rightarrow {}^{56}Mn$

$BE_{Fe} = 492.26$ MeV

$BE_{Mn} = 489.35$ MeV

$m_p = 938.3$ MeV/c^2

$m_n = 939.6$ MeV/c^2

$m_e = .511$ MeV/c^2

$(26m_p + 30m_n + 26m_e – BE_{Fe}/c^2) - (25m_p + 31m_n + 25m_e – BE_{Mn}/c^2)$

<u>3.8 MeV</u>

Problem 7.7

~~P~~: $n = \frac{N}{V}$

$N = nV$
$= (0.16\ fm^{-3})\left(\frac{4}{3}\pi R_0^3\right)$

$\simeq 1.16$

Total mass $= 1.67 \times 10^{-27} \times 1.16$
$= 1.94 \times 10^{-27}$ kg

$\rho_m = \frac{m}{V}$

$= \frac{1.94 \times 10^{-27}\ kg}{\frac{4}{3}\pi R_0^3} = 2.72 \times 10^{17}\ kg/m^3$

$U_g = -\frac{3}{5} \frac{G m^2}{R}$ $\qquad \rho \propto \frac{M}{R^3}$

$= -\frac{3}{5} G m^2 \left(\frac{\rho}{m}\right)^{1/3}$ $\qquad R \propto \left(\frac{M}{\rho}\right)^{1/3}$

$= -\frac{3}{5}(6.67 \times 10^{-11})(2.8 \times 10^{30})^2 \left(\frac{2.72 \times 10^{17}}{2.8 \times 10^{30}}\right)^{1/3}$

$= -1.358 \times 10^{46}$ J

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Problem 7.8

Samuel Fehringer 19 November 2020
ASTR 4201 Homework 7.8

Suppose you're standing on the surface of a white dwarf. What is the difference in force between your upper 10 kg and your lower 10 kg? What about on a neutron star? Compare to the tensile strength of a human tendon of ~1,000 N.

White Dwarf

$$F = G(m_1 m_2 / r^2)$$

$$= (6.67408 \times 10^{-11} \frac{m^3}{kg\, s^2})\left(\frac{1.197 \times 10^{30} kg \quad 10 kg}{8584884.63 m^2}\right)$$

$$= 10,839,684.26 \text{ N}$$

If we take $r + 1.88$ m ...

$$F = 10,839,679.51 \text{ N}$$

A difference of 4.75 N, the rough equivalent of wearing 1/2 lb ankle weights on each ankle.

Note: I used Procyon B as my white dwarf

Neutron Star

$$F = G(m_1 m_2 / r^2)$$

$$= (6.67408 \times 10^{-11} \frac{m^3}{kg\, s^2})\left(\frac{2.784 \times 10^{30} kg \quad 10 kg}{10,000 m^2}\right)$$

$$= 1.858063872 \times 10^{13} \text{ N}$$

If we take $r + 1.88$ m ...

$$F = 1.857365437 \times 10^{13} \text{ N}$$

A difference of 6.98435×10^9 N ...

Good luck, you'll need it

Note: I used the crab pulsar as my neutron star

9.) $KE = PE$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$m =$ mass of photon (Cancels so I assume it doesn't matter that photons are massless)

$V = c = 3\times10^8$ m/s

$G = 6.674\times10^{-11}$ $m^3/Kg\cdot s^2$

$M = M_\odot = 1.989\times10^{30}$ Kg

$r = r_{schwarzschild} = r_s$

$$\frac{1}{2}\cancel{m}c^2 = \frac{GM_\odot \cancel{m}}{r_s}$$

$$\frac{1}{2}c^2 = \frac{GM_\odot}{r_s}$$

$$\boxed{r_s = \frac{2GM_\odot}{c^2}}$$

$$V = \frac{4}{3}\pi r_s^3$$

plug in r_s:

$$= \frac{4}{3}\pi\left(\frac{2GM_\odot}{c^2}\right)^3$$

$$= \frac{4}{3}\pi \cdot \frac{8G^3M_\odot^3}{c^6}$$

$$V = \frac{32\pi G^3 M_\odot^3}{3c^6}$$

$$\rho = \frac{M_\odot}{V}$$

Plug in V:

$$= \frac{M_\odot}{\left(\frac{32\pi G^3 M_\odot^3}{3c^6}\right)}$$

$$\rho = \frac{M_\odot 3c^6}{32\pi G^3 M_\odot^3} = \frac{3c^6}{32\pi G^3 M_\odot^2}$$

$$\rho = \frac{3(3\times10^8)^6}{32\pi(6.674\times10^{-11})^3(1.989\times10^{30})^2}$$

$\rho = 1.853\times10^{19}$ Kg/m³

$$\frac{GM_\odot}{r^2} = a = \frac{v^2}{r}$$

$$\frac{GM_\odot}{r^2} = a = \frac{(2\pi r f)^2}{r}$$

$$\left(\frac{\sqrt{GM_\odot}}{2\pi f}\right)^{\frac{2}{3}} = r$$

$$\left(\frac{\sqrt{6.674 \times 10^{-11} \times 1.98847 \times 10^{30}}}{\pi \times 33 \times 2}\right)^{2/3}$$

Result:

$1.45604\ldots \times 10^5$

If the radius is more than ~150 km matter at the equator is not bound to the star, this is too small to be a normal star.

and too small to be a white dwarf, which is what the question is asking

Problem 7.11

Before	After

$M = M_\odot$ $\quad R_i = 7000\text{ km} \rightarrow R_f = 10\text{ km}$

$\omega_i = 4\times10^{-7}\text{ Hz} \rightarrow \omega_f = ?$

$L = I\omega$ $\qquad$ sphere $\Rightarrow$ $I = \frac{2}{5}MR^2$

(Before & after)

$L_i = L_f$

$I_i\omega_i = I_f\omega_f$

$\cancel{\tfrac{2}{5}}\cancel{M_\odot}R_i^2\,\omega_i = \cancel{\tfrac{2}{5}}\cancel{M_\odot}R_f^2\,\omega_f$

$700^2\omega_i = \omega_f$ $\qquad \Leftarrow R_i = 700\,R_f$

$$\boxed{\omega_f = 0.196\text{ Hz}}$$

Question 12: EXERCISE 7.5 - Let's estimate the luminosity and surface temperature of an accreting neutron star. Assume a mass of 1.4 $M_{\odot}$ and a radius of 10 km. How much gravitational energy (in MeV) is released when a proton falls onto the surface (use a Newtonian approximation for the gravitational potential). How does this compare to the energy released (per proton) from the fusion of hydrogen into helium? Now suppose the neutron star is accreting at 10^{17} g/s, which is a typical rate for many observed systems. What would be the luminosity generated by this accretion? Suppose the luminosity were emitted thermally from the surface of the neutron star. What would be the surface effective temperature? In what band (e.g., visible, IR, UV, X-ray) would you want to observe this system?

Given a mass $M = 1.4M_{\odot}$ and $R_{NS} = 10^4$ m, integrating the law of universal gravitation from $R_{NS} \leq R \leq \infty$ and using the mass of the proton in kilograms

$$V_{NS} = \int_{R_{NS}}^{\infty} \frac{GMm}{r^2} dr = -\left. \frac{GMm}{r} \right|_{R_{NS}}^{\infty} = \frac{GMm}{R_{NS}} \tag{70}$$

and converting to MeV, the energy released for a proton impacting the surface of the neutron star would be 195.10 MeV, approximately 29.2 times more than the 6.68 MeV of energy per hydrogen nuclei released by the p-p I branch.

Converting the mass accretion rate to nuclei/s $\dot{H}$

$$\dot{H} = \frac{\dot{M} N_A}{m_{^1\mathrm{H}}} \tag{71}$$

yields a nuclei accretion rate of 5.98×10^{40}nuclei/s. Knowing this, calculating the luminosity in Joules/s

$$L = \dot{H} V_{NS} \times \left(1.602 \times 10^{-19} J\right) \tag{72}$$

yields a luminosity of 1.86×10^{30} W. Converting to the effective surface temperature

$$T_{eff} = \left[\frac{L}{4\pi R^2 \sigma_{SB}}\right]^{\frac{1}{4}} \tag{73}$$

yields an effective surface temperature of 12.71×10^6 K. Using Wien's law to get the peak of emission for the surface of the neutron star

$$\lambda_{pk} = 290\,\mathrm{nm} \left[\frac{10000\,\mathrm{K}}{T_{eff}}\right] \tag{74}$$

yields a peak surface wavelength of 0.228 nm, directly in highest energy band of X-ray emission. This system would therefore best be observed with an X-ray telescope.

Problem 7.13

13. $1.4 M_{\odot}$ is the theoretical upper limit to the mass a white dwarf can have.

$M = 1.4 M_{\odot} = 1.4 * 1.99 * 10^{30}$ kg $= 2.786 * 10^{30}$ Kg

$R = 6500$ Km $= 6500000$ m

$m_p = 1.6726219 * 10^{-27}$ kg.

$G = 6.67408 * 10^{-11}\ m^3 kg^{-1} s^{-2}$

Using newtonian approximation for gravitational potential

1. $V = \dfrac{GMm}{R}$

$= \dfrac{6.67408 * 10^{-11} * 2.786 * 10^{30} * 1.6726219 * 10^{-27}}{6500\,000}$

$= 4.7847 * 10^{-14}$ J

$= 0.2986$ MeV

2. energy released by p-p I branch $= 26.73$ MeV

Since in p-p I branch 4 protons turn into 1 helium nucleus we devide the energy released by 4.

$(26.73\ \text{MeV} / 4) = 6.6825$ MeV

energy released is Less than that by 22.3 time

3. $\dot{n} = \dot{m} \dfrac{N_A}{\mu^1{}_H}$

$= 10^{18}\ g/s \cdot \dfrac{6.0221409 \times 10^{23}}{1.00782503224}$

$= 5.9753833 \times 10^{41}$ nuclie/sec.

4. Luminosity = ?

$L = \dot{n} \times V \times (1.602 \times 10^{-19} J)$

$= 5.97 \times 10^{41} \times 0.2896\ MeV \times 1.602 \times 10^{-19} J$

$= 2.85 \times 10^{28}\ W$

5. The surface effective temperature

$T_{eff} = \left[\dfrac{L}{4\pi R^2 \sigma_{SB}} \right]^{1/4}$

$= \left[\dfrac{2.85 \times 10^{28}\ \cancel{W}}{4\pi \times (65 \times 10^5\ \cancel{m})^2 \times 5.7 \times 10^{-8}\ \cancel{W} \cdot \cancel{m^{-2}}\ K^{-4}} \right]^{1/4}$

$= 175307.97\ K$

$= 1.7530797 \times 10^5\ K$

Type of radiation emitted is UV.

6. Using Wien's Law to get the peak emission for the surface.

$$\lambda_{pk} = 290 \text{ nm} \left[\frac{10000}{T_{eff}} \right]$$

$$= 290 \text{ nm} \left[\frac{10000 \text{ K}}{175307.97} \right]$$

$$= 16.54 \text{ nm}$$

peak surface wavelength $\lambda = 16.54$ nm

This system would be observed with a UV telescope.