Homework Assignment 2 **ASTR4201, Fall 2020**

Corresponds to Chapter 2 of "To Build a Star" (TBS) by E.F. Brown

1. *See below* Team: 1 Lead: Anthony
   The ocean's density only changes by 5% from surface to ocean floor. How much does gravity change? Assume the ocean floor is 2 miles below the surface.

2. TBS exercise 2.1 Team: 2 Lead: Michael

3. *See below* Team: 3 Lead: Ryan
   Calculate the weight of a column of air above a 1 m^2 area at sea level. Calculate the same above a 1 in^2 area. Finally, calculate the weight of earth's atmosphere.

4. TBS exercise 2.2 Team: 4 Lead: Jacob
   Hint: Recast density in terms of pressure. Note that you then have a first order homogeneous linear equation. Consult the solutions for common differential equations, e.g. in your math methods textbook.

5. *See below* Team: 1 Lead: Britt
   Calculate the average molecular mass of dry air, approximating the atmosphere as 78% nitrogen, 21% oxygen, 0.95% argon, and 0.05% carbon dioxide. Keep in mind that nitrogen and oxygen are diatomic gases in earth's atmosphere.

6. *See below* Team: 2 Lead: Sam
   Solve TBS exercise 2.3, as well as the following question.
   The sun is ~70% hydrogen, ~28% ~2% metals, which we denote by X=0.70, Y=0.28, Z=0.02.
   If we treat the metals as ^{14}N, what is he mean molecular weight of the solar composition?

7. TBS exercise 2.4 Team: 3 Lead: Josh

8. TBS exercise 2.5 Team: 4 Lead: Gula

9. TBS exercise 2.6 Team: 5 Lead: Justin

10. TBS exercise 2.7 Team: 1 Lead: Gavin

11. TBS exercise 2.8 Team: 2 Lead: Quinn

12. TBS exercise 2.9 Team: 3 Lead: Harshil

13. TBS exercise 2.10 Team: 5 Lead: Robert
    Hints: For part 4, "lowest order in $\delta R/R$" means to expand such that γ isn't in an exponent anymore. For part 5, $(1+\delta R/R)^2 \sim (1+2*\delta R/R)$ and $(1+\delta R/R)^{-2} \sim (1-2*\delta R/R)$. For part 6, consider the equation of motion for common systems (e.g. a spring).

Problem 1

HW prob 1

$$F = \frac{G\cancel{m_1}m_2}{r^2} = \cancel{m}g$$

$$\frac{Gm}{r^2} = g$$

Normal g

$$\frac{(6.67\times10^{-11})(5.972\times10^{24}\,kg)}{(6.371\times10^{6}\,m)^2} = g = 9.81\,m/s/s$$

New g

$r_{ocean} = r_{surface} - 3218.69\,m$

~~$r_{ocean} = 6.368\times10^{6}$~~ $r_{ocean} = 6.368\times10^{6}\,m$

$$\frac{Gm}{r^2} = g = \frac{(6.67\times10^{-11})(5.972\times10^{24}\,kg)}{(6.368\times10^{6}\,m)^2} = 9.823\,m/s/s$$

Problem 2

$$\frac{dP}{dr} = -\rho g(r) = \Delta P = P_s - P_a$$

water is incompressible, meaning its not pressure dependent, so $P_s = 0$

$$P_a = d\rho g(r)$$

$$1.013 \times 10^5 = d(10^3)(9.81)$$

$$\boxed{d = 10.3\,m}$$

Problem 3

Question 3

Calculate the weight of a column of air above a 1 m^2 area at sea level. Calculate the same above a 1 in^2 area. Finally, calculate the weight of earth's atmosphere.

$$P_0 - P(z) = gm(z)/\Delta A,$$

Weight above 1 m^2:

P_0 = 1atm = 101325 Pa

$P(z) = 0$

ΔA = 1 m^2

Solve for gm(z) (Weight = mg)

$gm(z) = P_0 \Delta A$

<u>gm(z) = 101325 N</u>

Weight above 1 in^2:

P_0 = 1atm = 14.7 psi

$P(z) = 0$

ΔA = 1 in^2

Solve for gm(z)

$gm(z) = P_0 \Delta A$

<u>gm(z) = 14.7 pounds = 65.4 N</u>

Weight of atmosphere:

Radius of Earth: 6.371×10^6 m

Surface Area = $4 \pi r^2 = 5.101 \times 10^{14}$ m^2

P_0 = 101325 Pa

$\Delta A = 5.101 \times 10^{14}$ m^2

<u>$gm(z) = P_0 \Delta A = 5.168 \times 10^{19}$ N</u>

Problem 4

$$\frac{dP}{dr} = -\rho g(r) \qquad P = \rho \frac{k_B T}{A m_u}$$

$$\frac{dP}{dr} = -\frac{A m_u g(r)}{k_B T} P$$

$$\int_{P_0}^{P} \frac{dP}{P} = \int_0^z -\frac{A m_u g}{k_B T} dr$$

$$\ln(P/P_0) = \frac{-A m_u g z}{k_B T}$$

$$P = P_0 \exp\left[-z \frac{A m_u g}{k_B T}\right]$$

$$P = P_0 e^{-z/H_p} \qquad H_p = \frac{k_B T}{A m_u g}$$

@ 288K

$$H_p = \frac{(1.38 \times 10^{-23}\ \frac{m^2\,\cancel{kg}}{s^2 \cancel{K}})(288\cancel{K})}{(28.97)(1.661 \times 10^{-27}\ \cancel{kg})(9.8\ m/s^2)} = \boxed{8.43 \times 10^3\ m}$$

$$z = H_p \Rightarrow P = P_0 e^{-1} = P_0/e \ \checkmark$$

Atmosphere is reasonably isothermal...

Simple Atmospheres lecture! Temperature ranges from ~190K to 290K (at absolute extremes)

Usually w/in ±50K temp range

Problem 5

want: avg. molecular mass dry air

given: atm. 78% nitrogen
21% oxygen
0.95% argon
0.05% carbon dioxide

nit. & oxy. diatomic gasses

from Ex. 2.2 in book, know ans.: $A = 28.97u$

atomic weights:

N: 14.0067 u
O: 15.999 u
Ar: 39.948 u
CO_2: 44.01 u

since diatomic, $N \to N_2$ and $O \to O_2$

so N_2: $14.0067u * 2 = 28.0134 u$
O_2: $15.999 u * 2 = 31.998 u$

multiply by percentages:

N_2: $28.0134u * 0.78 = 21.850452u$

O_2: $31.998u * 0.21 = 6.71958u$

Ar: $39.948u * 0.0095 = 0.379506u$

CO_2: $44.01u * 0.0005 = 0.022005u$

add these up:

$21.850452u + 6.71958u + 0.379506u$
$+ 0.022005u$

and get $A \approx 28.97u$

(same as given in book)

Problem 6

Samuel Fehringer
ASTR 4201 2.3

What is μ for a fully ionized ^4He gas ($A=4$, with 2 electrons per atom)?

$$\mu({}^4He + e^-) = \frac{4m_u \times n}{3nm_u} = \frac{4}{3}$$

The sun is ~70% hydrogen, ~28% helium, and ~2% metals, denoted by $x=0.70$, $y=0.28$, $z=0.02$
If we treat the metals as ^{14}N, what is the mean molecular weight of the solar composition?

$$\frac{1}{\mu_n} = \left(\frac{1}{1}\right)0.7 + \left(\frac{1}{4}\right)0.28 + \left(\frac{1}{14}\right)0.02 \rightarrow 1.296 = \mu_n$$

But since this is the sun...

$$\frac{1}{\mu_i} = \left(\frac{2}{1}\right)0.7 + \left(\frac{3}{4}\right) + \left(\frac{1}{2}\right)^{*}0.02 \longrightarrow 0.617 = \mu_i$$

* $\frac{1}{2}$ is used because heavier elements will have ~ the same number of protons and neutrons

Problem 7

ASTR4201 HW #2

Josh Olson

2.4 $m(r) = 4\pi \int_0^r \rho(r) r^2 dr$

Assuming constant density, $\rho(r) = \rho_c$.

$$m(r) = 4\pi \int_0^{\Delta r} \rho_c r^2 dr$$

$$m(\Delta r) = \frac{4}{3}\pi \rho_c (\Delta r)^3$$

$$\lim_{\Delta r \to 0} m(\Delta r) = \frac{4}{3}\pi \rho_c (0)^3 = 0$$

At the center $(r \to 0)$, $m(r) = 0$. This is because there is no mass interior to the center (no point can be "interior" of the center).

$$\lim_{r \to 0} g(\Delta r) = \frac{G \lim_{r \to 0} m(\Delta r)}{(\Delta r)^2}$$

$$= \frac{G(0)}{(\Delta r)^2} = 0$$

At the center, $g(r) = 0$ as well.

$$\frac{dP}{dr} = -\rho \frac{G \lim_{r \to 0} m(r)}{r^2}$$

$$= -\rho \frac{G(0)}{r^2}$$

$\boxed{\frac{dP}{dr} = 0}$ at the center.

Problem 8

1. $\rho(r) = \rho_c$

Mass function can be obtained using eq (2.6)

$$m(r) = 4\pi \int_0^r \rho(r)\, r^2\, dr$$

$$= 4\pi \rho_c \int_0^r r^2\, dr$$

$$= 4\pi \rho_c \left.\frac{r^3}{3}\right|_0^r$$

$$= \frac{4\pi}{3} r^3 \rho_c \quad \text{----} \; (1)$$

The volume is a sphere

Since the density is constant we can express eq (1) through the global stellar parameters M and R.

$$\rho_c = \frac{M}{V} = \frac{M}{\frac{4\pi}{3} R^3}$$

2.

$$m(r) = 4\pi \int_0^r \rho(r)\, r^2\, dr$$

$$m(r) = \cancel{4\pi} \int_0^r \frac{M}{\frac{\cancel{4\pi}}{3} R^3} \cdot r^2\, dr$$

$$= \frac{M}{\frac{1}{\cancel{3}} R^3} \frac{r^3}{\cancel{3}}$$

$$m(r) = \frac{M r^3}{R^3} \quad \text{---} \; 2$$

3. Pressure at surface, $P(R) = 0$

$$dP = -\rho \frac{G . m(r)}{r^2} dr$$

$$\int_{P_c(r=0)}^{P(R)} dP = -\frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{M}{R^3} \int_0^R \frac{r^3}{r^2} \cdot dr$$

$$P(R) - P_c(r=0) = -\frac{M}{\frac{4}{3}\pi R^{\cancel{6}4}} \frac{\cancel{R^2}}{2}$$

$$\cancel{-}P_c(r=0) = \cancel{-}\frac{3}{8\pi} \cdot \frac{M^2}{R^4} G$$

$$P_c(r=0) = \frac{3}{8\pi} \frac{M^2}{R^4} G$$

4.

$$\rho_c = \frac{M}{\frac{4\pi}{3}R^3} \text{---} 1$$

$$P_c = \frac{3}{8\pi}\frac{M^2}{R^4}G \dots 3$$

$$P_c = \rho_c\left(\frac{k_B}{\mu m_u}\right)T_c \dots 2.5$$

$$P_c = \frac{3}{4\pi \cdot 2} \cdot \frac{M}{R^3}\frac{M}{R^2}G$$

$$= \frac{M}{\frac{4\pi}{3}R^3} \cdot \frac{M}{2R^2}G$$

$$= \frac{GM}{2R}\rho_c$$

$$P_c = \rho_c \frac{k_B}{\mu m_u} T_c$$

$$\frac{GM}{2R}\cancel{\rho_c} = \cancel{\rho_c}\frac{k_B}{\mu m_u}T_c$$

$$\overline{T_c} = \frac{G \mu m_u}{2 k_B} \frac{M}{R} \quad \text{---} \cdot 4$$

for $M = M_\odot$, $R = R_\odot$, $\mu = 0.6$

$M_\odot = 1.99 * 10^{30}$ kg $\quad$ μ = mean moleculer weight.

$R_\odot = 6.96 * 10^{8}$ m

m_u = hass mass of one atomic mass unit

$1u = 1.661 * 10^{-27}$ kg.

$k_B = 1.380649 * 10^{-23}$ J.K^{-1}

$G = 6.67 * 10^{-11}$ N kg^{-2} m^{2}

Putting the constants in eq (4) we get.

$\overline{T_c} = 7.6 * 10^{6}$ K

The actual value is

$\overline{T_c} = 14.4 * 10^{6}$ K

constant density model give the Lower Limit.

Problem 9

2 Homework 2

Question 9: Exercise 2.6 —The central temperature T_c is a measure of the average kinetic energy of a particle at the stellar center. Use the central temperature that you found for the constant density star in exercise 2.5 and estimate the time that such a particle would take to cross a distance R. How does this time compare to the orbital period of a satellite orbiting just outside the stellar surface?

Solving (2.5)

$$P = \frac{M^2}{R^4}\frac{3G}{8\pi} \tag{10}$$

$$P = \frac{\rho k_B}{\mu m_u}T \tag{11}$$

$$T = \frac{\mu m_u}{\rho k_B}\frac{M^2}{R^4}\frac{3G}{8\pi} = \frac{1}{2}\frac{\mu m_u}{k_B}\frac{GM}{R} \tag{12}$$

nets $T = 6.9 \times 10^6$ K for the central temperature using $R = R_\odot = 696 \times 10^6$ m and $M = M_\odot = 2 \times 10^{30}$ kg. Using the equation for the average kinetic energy of a particle in an ideal gas,

$$T_{AV} = \frac{3}{2}k_B T \tag{13}$$

and obtaining the velocity,

$$\begin{aligned} v_{AV} = \sqrt{\frac{2T_{AV}}{m_u}} &= \sqrt{\frac{3k_B T}{m_u}} && (14)\\ &= \sqrt{\frac{3G\mu \cancel{m_u}\cancel{k_B}M_\odot}{2\cancel{m_u}\cancel{k_B}R_\odot}} && (15)\\ &= \sqrt{\frac{3\mu G M_\odot}{2R_\odot}} && (16) \end{aligned}$$

nets $v_{AV} = 415.5$ km/s. As t_{AV} can be calculated from velocity using

$$\begin{aligned} t_{AV} &= \frac{R_\odot}{v_{AV}} && (17)\\ &= \frac{R_\odot}{\sqrt{\frac{3\mu G M_\odot}{2R_\odot}}}\frac{2R_\odot}{2R_\odot} && (18)\\ &= \frac{2R_\odot^2}{\sqrt{6\mu G M_\odot R_\odot}} = \sqrt{\frac{2R_\odot^3}{3\mu G M_\odot}} && (19)\\ &= \sqrt{\frac{2}{3\mu}}\sqrt{\frac{R_\odot^3}{GM_\odot}} && (20) \end{aligned}$$

This crosses $R_\odot$ in 1675 seconds. Using Kepler's third law to determine the orbital period for a particle at the surface of the sun

$$t_o = 2\pi\sqrt{\frac{R_\odot^3}{GM_\odot}} \tag{21}$$

nets an orbital period of $\sim 10^4$ seconds. The ratio of the t_{AV} and t_0 is calculated as

$$\phi = \frac{t_{AV}}{t_0} = \frac{\sqrt{\frac{2}{3\mu}}\cancel{\sqrt{\frac{R_\odot^3}{GM_\odot}}}}{2\pi\cancel{\sqrt{\frac{R_\odot^3}{GM_\odot}}}} = \sqrt{\frac{1}{6\pi^2\mu}} \tag{22}$$

which nets a ratio of $\phi \approx 0.1677$.

Exercise 2.7

Problem 10

	B2	B8	F0	F5	G5	M0	M7
$M/M_\odot$	9.8	3.8	1.6	1.3	0.92	0.51	0.12
$R/R_\odot$	5.6	3.0	1.5	1.3	0.92	0.60	0.18
$L/L_\odot$	5800.0	180.0	6.5	3.2	0.79	0.08	0.003

$$\rho = \frac{M}{4/3\pi R^3} \qquad T_c = \frac{1}{2}\left\{\frac{GM}{R}\frac{\mu m_u}{k_B}\right\} \qquad P_c = \frac{3}{8\pi}\left\{\frac{G(M)^2}{R^4}\right\}$$

$$\rho_\odot = \frac{M_\odot}{4/3\pi R_\odot^3} \qquad T_{c\odot} = \frac{1}{2}\left\{\frac{GM_\odot}{R_\odot}\frac{\mu m_u}{k_B}\right\} \qquad P_{c\odot} = \frac{3}{8\pi}\left\{\frac{G(M_\odot)^2}{R_\odot^4}\right\}$$

$$\frac{\rho}{\rho_\odot} = \frac{\frac{M}{4/3\pi R^3}}{\frac{M_\odot}{4/3\pi R_\odot^3}} \rightarrow \frac{\frac{3M}{4\pi R^3}}{\frac{3M_\odot}{4\pi R_\odot^3}} \rightarrow \frac{\cancel{3}M}{\cancel{4\pi}R^3}\cdot\frac{\cancel{4\pi}R_\odot^3}{\cancel{3}M_\odot} = \frac{MR_\odot^3}{R^3M_\odot}$$

$$\frac{T_c}{T_{c\odot}} = \frac{\frac{1}{2}\left\{\frac{GM}{R}\frac{\mu m_u}{k_B}\right\}}{\frac{1}{2}\left\{\frac{GM_\odot}{R_\odot}\frac{\mu m_u}{k_B}\right\}} \rightarrow \frac{\frac{GM\mu m_u}{2Rk_B}}{\frac{GM_\odot\mu m_u}{2R_\odot k_B}} \rightarrow \frac{\cancel{G}M\cancel{\mu m_u}}{\cancel{2}R\cancel{k_B}}\cdot\frac{\cancel{2}R_\odot\cancel{k_B}}{\cancel{G}M_\odot\cancel{\mu m_u}} = \frac{MR_\odot}{RM_\odot}$$

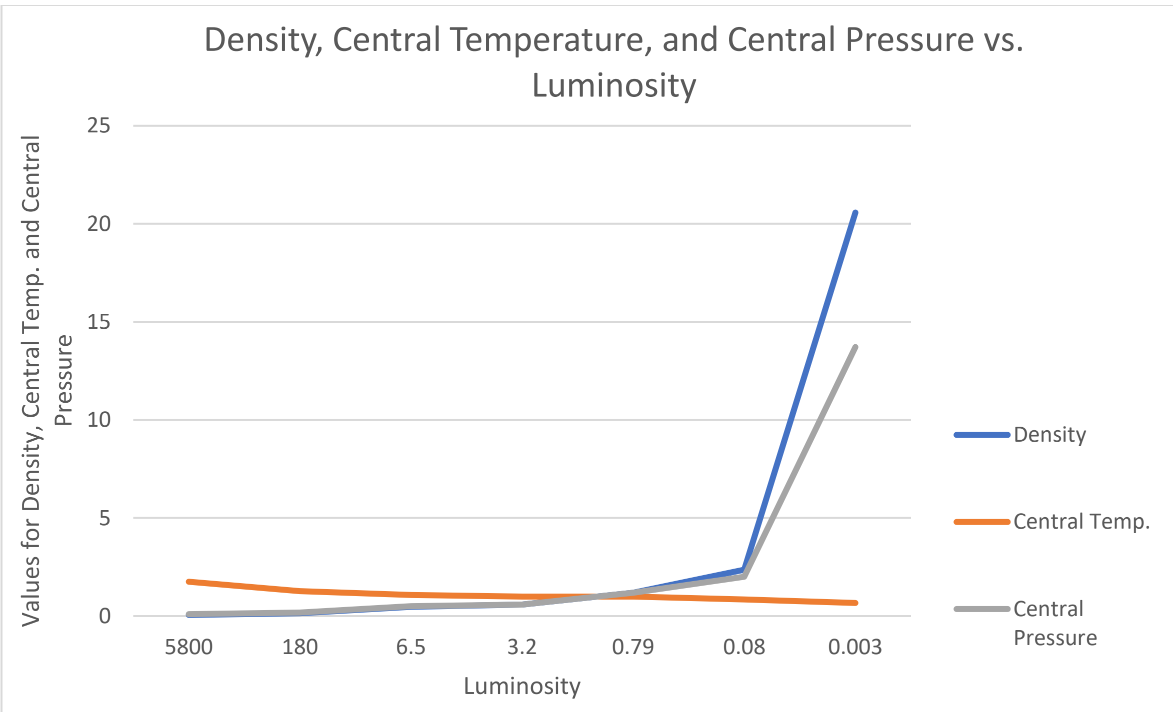
$$\frac{P_c}{P_{c\odot}} = \frac{\frac{3}{8\pi}\left(\frac{GM^2}{R^4}\right)}{\frac{3}{8\pi}\left(\frac{GM_\odot^2}{R_\odot^4}\right)} = \frac{\frac{3GM^2}{8\pi R^4}}{\frac{3GM_\odot^2}{8\pi R_\odot^4}} \rightarrow \frac{\cancel{3G}M^2}{\cancel{8\pi}R^4}\cdot\frac{\cancel{8\pi}R_\odot^4}{\cancel{3G}M_\odot^2} = \frac{M^2R_\odot^4}{R^4M_\odot^2}$$

$$\boxed{\frac{\rho}{\rho_\odot} = \frac{MR_\odot^3}{R^3M_\odot}, \quad \frac{T_c}{T_{c\odot}} = \frac{MR_\odot}{RM_\odot}, \quad \frac{P_c}{P_{c\odot}} = \frac{M^2R_\odot^4}{R^4M_\odot^2}}$$

↳ Relationships that are followed by ratios

- See that none of ratios have a dependence on Luminosity
- Next step is to see how different stellar types act

Luminosity	Density	Central Temp.	Central Pressure	Stellar Type
5800	0.0558	1.75	0.0976	B2
180	0.1407	1.266	0.1783	B0
6.5	0.4741	1.0667	0.5057	F0
3.2	0.5917	1	0.5917	F5
0.79	1.181	1	1.181	G5
0.08	2.361	0.85	2.006	M0
0.003	20.576	0.6667	13.717	M7



The higher luminosity shows a higher temperature but lower density and pressure. As the luminosity decreases, so does the temperature, while the density and pressure increase.

ASTRO HW

Problem 11

2.8) When a protostar contracts:

Pressure	rises
Density	rises
Temperature	rises
radius	decreases
energy	decreases

$\underbrace{T_c = \frac{1}{2}\left[\frac{GM}{R}\frac{\mu m_u}{k_B}\right]}_{(1)}$; $\underbrace{P_c = \frac{3}{8\pi}\left[\frac{GM^2}{R^4}\right]}_{(2)}$; $\underbrace{E = -\frac{GM^2}{R}}_{(3)}$

$R\downarrow$ • Contracting implies Radius is Decreasing.

$T\uparrow$ • In equation (1), when you divide by a decreasing R, the temperature will rise.

$P\uparrow$ • In equation (2), you will get the same result as T, but with Pressure rising.

$\rho\uparrow$ • The density will rise because the star is contracting and also not expelling any of its matter.

$E\downarrow$ • Total Energy decreases because the gravitational energy of the star is converted into heat and light which radiates away into space.

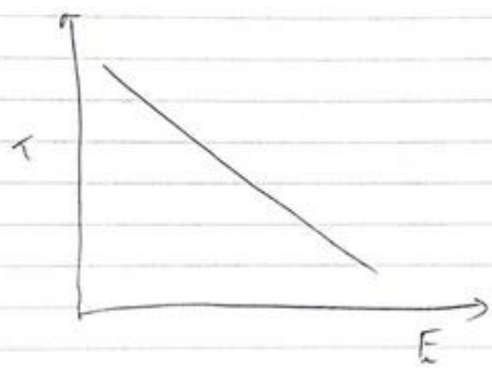
Problem 12

2.9 $E = K + \Omega = \frac{\Omega}{2}$

$$= -\frac{3}{2} N k_B \bar{T} \Rightarrow \frac{-2E}{3Nk_B} = \bar{T}$$

$$\frac{\Delta \bar{T}}{\Delta E} \Rightarrow \frac{d\bar{T}}{dE} = \frac{-2}{3Nk_B} \approx -\frac{4.8286 \times 10^{22}}{N}$$

$$\left(N = \frac{M}{\mu m_0} \right)$$



If additional heat is provided the central temperature would increase for some time but would eventually revert to its equilibrium state.

[NOT CORRECT. Adding energy decreases the temperature. See the figure.]

Problem 13

Tuesday, April 14, 2020 7:11 PM

Note that it is also true that $(1+x)^a$ expands to $1+ax$, which is used later on, e.g. in Part 4.

(1) $f(x) = f(a) + f'(a)(x-a) + \dots$

near x_0:

$f(x) = f(x_0) + f'(x_0)(x - x_0)$

$= A x_0^a + A a x_0^{a-1}(x - x_0)$

$= A x_0^a + A a x_0^a (x - x_0)/x_0$

$f(x) = f_0 (1 + a(x - x_0)/x_0)$

$f(x_0 + \partial x) = f_0 (1 + a\,\delta x / x_0)$

---

(2) from 2.7 $\frac{dP}{dr} = \frac{-\rho G m(r)}{r^2} \Rightarrow \frac{\Delta P}{\Delta r} = \frac{-\rho G m(r)}{r^2}$

$P(R + \Delta r) = 0$ from problem description

$P(R) = \frac{\rho G M \Delta r}{R^2}$ $\qquad \frac{m}{4\pi R^2} = \rho \Delta r$

$P(R) = \frac{G M m}{4\pi R^4}$

$4\pi R^2 P(r) - \frac{G M m}{4\pi R^2} = 0$

---

(3) $\rho = \frac{M}{V} = \frac{M}{R^3} = M R^{-3}$

$\rho' = \rho\left(1 - 3\frac{\delta R}{R}\right)$

---

(4) $P' V'^{\gamma} = P V^{\gamma}$ $\qquad V = m/\rho$

$P' m^{\gamma'} \rho^{-\gamma'} = P m^{\gamma} \rho^{-\gamma}$ $\qquad m^{\gamma'} = m^{\gamma}$

$P' \rho^{-\gamma'} = P \rho^{-\gamma}$

$P' = P\left(\frac{\rho}{\rho'}\right)^{-\gamma}$ $\qquad \rho' = \rho\left(1 - 3\frac{\delta R}{R}\right)$

$= P\left(\frac{\rho}{\rho(1 - 3\delta R/R)}\right)^{-\gamma}$

$= P(1 - 3\delta R/R)^{\gamma}$

$\approx P(1 - 3\gamma\delta R/R)$

5

$$4\pi R^2 P(R) - \frac{GMm}{R^2} \Rightarrow 4\pi(R+\delta R)^2 P\left(1 - 3\gamma\frac{\delta R}{R}\right) - \frac{GMm}{(R+\delta R)^2} = F$$

$$4\pi R^2\left(1+\frac{\delta R}{R}\right)^2 P\left(1 - 3\gamma\frac{\delta R}{R}\right) - \frac{GMm}{R^2(1+\delta R/R)^2} = F$$

$$4\pi R^2\left(1+2\frac{\delta R}{R}\right) P\left(1 - 3\gamma\frac{\delta R}{R}\right) - \frac{GMm\,(1-2\delta R/R)}{R^2(1+2\delta R/R)(1-2\delta R/R)} = F$$

$\approx 1 - 4\,\delta R^2/R^2 \Rightarrow 1$

$$4\pi R^2 P\left(1 + (2-3\gamma)\frac{\delta R}{R}\right) - \frac{GMm}{R^2}\left(1 - 2\frac{\delta R}{R}\right) = F$$

$$\cancel{4\pi R^2 P - \frac{GMm}{R^2}} + 4\pi R^2 P(2-3\gamma)\frac{\delta R}{R} + \frac{2GMm\,\delta R}{R^3} = F$$

$$\frac{GMm}{R^2}(2-3\gamma)\frac{\delta R}{R} + \frac{2GMm}{R^2}\frac{\delta R}{R} = F$$

$$\frac{GMm\,\delta R}{R^3}(4-3\gamma) = F$$

6

$$m\,\delta\ddot{R} = \frac{GMm\,\delta R}{R^3}(4-3\gamma) = C\,\delta R \qquad C = \frac{GM(4-3\gamma)}{R^3} = \frac{4\pi\rho G(4-3\gamma)}{3}$$

$$\frac{\rho 4\pi}{3} = \frac{M}{R}$$

$$\delta R = A e^{i(-C)t}$$

$$\omega^2 = -C \Rightarrow \frac{2\pi}{\sqrt{-C}} = T$$

$$= \frac{2\pi\sqrt{3}}{\sqrt{-4\pi\rho G(4-3\gamma)}}$$

$$= \sqrt{\frac{3\pi}{-\rho G(4-3\gamma)}}$$

if $\gamma = \frac{5}{3}$, $T = \sqrt{\frac{3\pi}{\rho G}}$