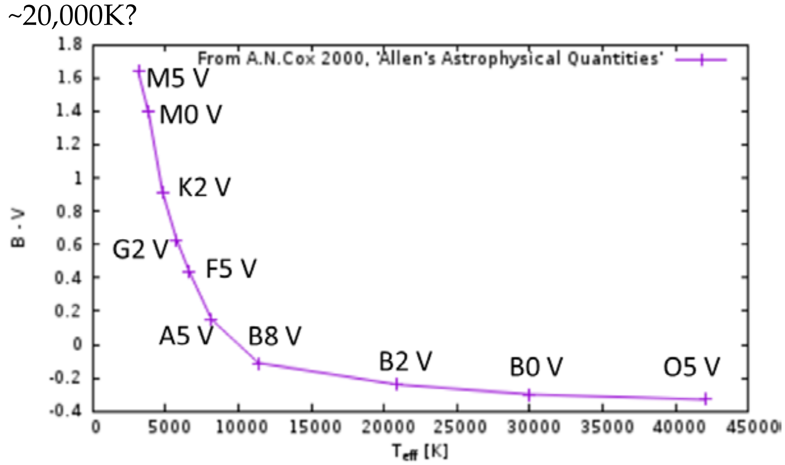
Homework Assignment 1 **ASTR4201, Fall 2020**

Corresponds to Chapter 1 of "To Build a Star" (TBS) by E.F. Brown

1. TBS exercise 1.1 Team: 1 Lead: Britt
2. TBS exercise 1.2 Team: 2 Lead: Sam
3. TBS exercise 1.3 Team: 4 Lead: Jacob
4. TBS exercise 1.4 Team: 3 Lead: Ryan
5. TBS exercise 1.5 Team: 1 Lead: Anthony
6. TBS exercise 1.6 Team: 2 Lead: Michael
   Hint: Recall the quotient rule
7. TBS exercise 1.7 Team: 3 Lead: Harshil
8. *See below* Team: 2 Lead: Quinn
   Suppose we have a K-type star that is 100 times more luminous than the sun. Given that the sun has a surface temperature of 5780K and a radius of 6.957×10^8m, what is the radius of our K-type star?
9. TBS exercise 1.8 Team: 3 Lead: Joshua
10. *See below* Team: 4,5 Lead: Gula, Justin
    Calculate B-V for the sun. Compare to the known value of 0.66.
    Hint: Assume the Flux in a band will correspond to the peak flux across the Full-width at half-maximum. Note that Vega is defined to have B-V =0.
11. TBS exercise 1.9 Team: 1 Lead: Gavin
12. *See below* Team: 5 Lead: Robert
    The trend for B-V versus spectral type is shown below. Why does B-V hardly change after ~20,000K?



$\frac{A}{4\pi d^2}$ is fraction of power

$$F = \frac{L}{4\pi d^2}$$

multiply by area for solar panel grid

$$F = \frac{LA}{4\pi d^2}$$

$$A = \frac{4\pi d^2 F}{L}$$

$$= \frac{4\pi(1.5 * 10^{11} m)^2 (70 * 10^6 W)}{3.86 * 10^{26} W}$$

$$\approx 5.1 * 10^4 \ m^2$$

ASTR 4201

Samuel Fehringer 8/26/2020

1.2 What would the flux be from a ~~star~~ with $L = 0.1 L_\odot$ at a distance of 10pc?

$$F = \frac{L}{4\pi r^2} \rightarrow \frac{3.86 \times 10^{25}\,W}{4\pi\,(100)} = 3.07 \times 10^{22}\ W/pc^2 \times \frac{1pc}{3.086 \times 10^{16}\,m} \times \frac{1pc}{3.086 \times 10^{16}}$$

$$= 3.22 \times 10^{-11}\ W/m^2$$

EXERCISE 1.3— The peak of the sun's spectrum is at a wavelength of approximately 500 nm. Estimate the number of photons from the sun striking $1\,m^2$ of Earth each second.

Flux [Watts/meter²] = [$\frac{\text{Joules}}{\text{sec}} \cdot \frac{1}{\text{meter}^2}$]

$F = \frac{L}{4\pi d^2} = \frac{3.86}{4\pi (1.5)^2} \times 10^{(26-22)} \frac{W}{m^2} = 1.37 \times 10^3\ J/m^2 s$

Luminosity of Sun

$L_\odot = 3.86 \times 10^{26}\ W$

Distance from Sun to Earth

$d = 1AU = 1.5 \times 10^{11}\ m$

Energy per photon

$E = \frac{hc}{\lambda} \quad \frac{6.63 \cdot 3}{500} \times 10^{(-34+8+9)}\ J/\gamma = 3.98 \times 10^{-19}\ J/\gamma$ (Joules per photon)

Planck's Constant

$h = 6.63 \times 10^{-34}\ J \cdot s$

Speed of Light

$c = 3 \times 10^8\ m/s$

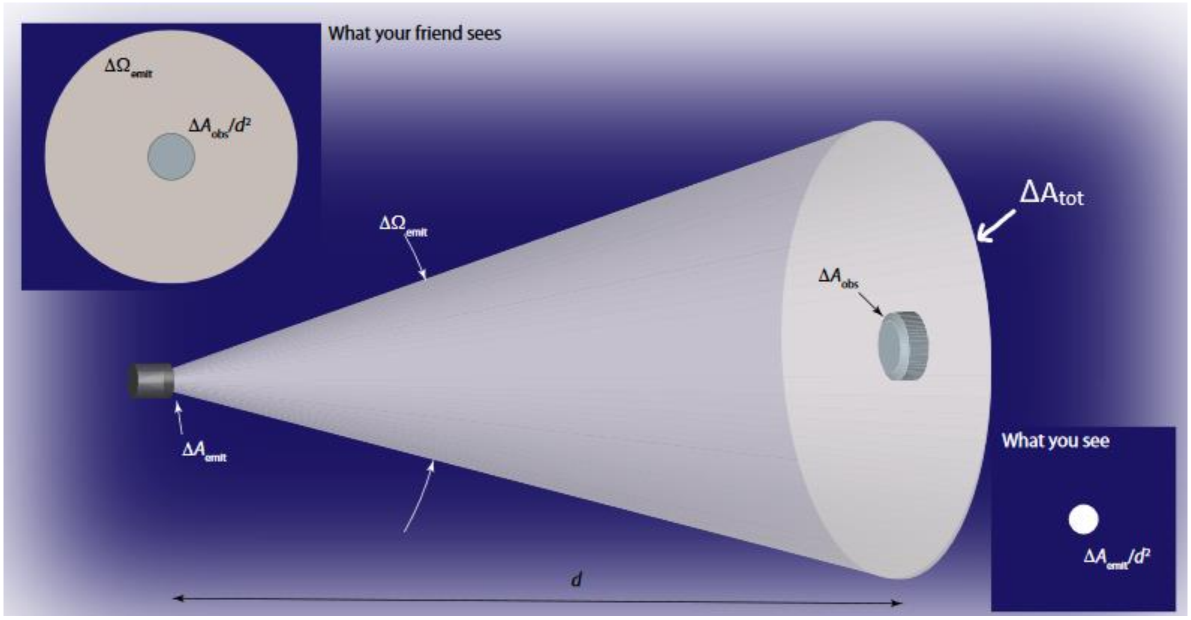
Peak Sun Spectrum Light Wavelength

$\lambda = 500 \times 10^{-9}\ m$

Flux / Energy per photon = Photons striking $1m^2$ per second

$\frac{F}{E} = \frac{1.37}{3.98} \times 10^{3+19} \frac{J \cdot \gamma}{m^2 \cdot s \cdot J} = \boxed{3.44 \times 10^{22}\ \gamma/m^2 \cdot s}$

Exercise 1.4



$$I_{\lambda,\text{emit}} = \frac{\Delta E_{\text{emit}}}{\Delta t\ \Delta A_{\text{emit}}\ \Delta\lambda\ \Delta\Omega_{\text{emit}}}.$$

1. Calculate the incident energy that falls on your camera aperture ΔE_{obs}.

$$\Delta E_{obs} = \frac{\Delta A_{obs}\Delta E_{em}}{\Delta A_{tot}}$$

$$\Delta\Omega_e = \frac{\Delta A_t}{d^2} \rightarrow \Delta A_t = \Delta\Omega_e d^2$$

$$\Delta E_o = \frac{\Delta A_o \Delta E_e}{\Delta\Omega_e d^2}$$

2. What solid angle $\Delta\Omega_{obs}$ is subtended by the rays entering the aperture?

$$\Delta\Omega_o = \frac{\Delta A_e}{d^2}$$

3. Now compute your intensity

$$I_{\lambda,\text{obs}} = \frac{\Delta E_{\text{obs}}}{\Delta t\ \Delta A_{\text{obs}}\ \Delta\lambda\ \Delta\Omega_{\text{obs}}}.$$

$$I_o = \frac{\Delta A_o \Delta E_e}{\Delta\Omega_e d^2}\frac{1}{\Delta t\Delta A_o \Delta\lambda}\frac{d^2}{\Delta A_e} \rightarrow I_o = \frac{\Delta A_o \Delta E_e d^2}{\Delta t \Delta A_o \Delta\lambda\Delta\Omega_e d^2 \Delta A_e}$$

$$I_o = \frac{\Delta E_e}{\Delta t \Delta A_e \Delta\lambda \Delta\Omega_e} = I_e$$

Anthony D'Alessandro

1.5

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

$a = 2hc^2$

$b = \frac{hc}{k_B T}$

$$= \frac{a\lambda^{-5}}{e^{b/\lambda} - 1}$$

$$\frac{d}{dB} = \frac{\left(e^{b/\lambda} - 1\right)\left(-5a\lambda^{-6}\right) - \left(a\lambda^{-5}\right)\left(-b\lambda^{-2} e^{b/\lambda}\right)}{\left(e^{b/\lambda} - 1\right)^2} = 0$$

$$\left(e^{b/\lambda} - 1\right)\left(-5a\lambda^{-6}\right) - \left(a\lambda^{-5}\right)\left(-b\lambda^{-2} e^{b/\lambda}\right) = 0$$

$$\left(\lambda^6\right)\left(-5\cancel{a}\lambda^{-6} e^{b/\lambda} + 5\cancel{a}\lambda^{-6}\right) = \left(-\cancel{a}b\lambda^{-7} e^{b/\lambda}\right)\left(\lambda^6\right)$$

$$-5\lambda e^{b/\lambda} + 5\lambda = -b\lambda^{-1} e^{b/\lambda}$$

$$b e^{b/\lambda} = 5\lambda e^{b/\lambda} - 5\lambda$$

$$\frac{b e^{b/\lambda}}{5\left(e^{b/\lambda} - 1\right)} = \lambda$$

$$\frac{1}{5}\left(\frac{b}{\lambda}\right) \frac{e^{b/\lambda}}{e^{b/\lambda} - 1} = 1$$

Find graphical interception

CMB

$$\lambda_{pkCMB} = 290_{nm}\left(\frac{10000\,K}{2.73\,K}\right)$$

$$\lambda_{pkcmb} = 1.06 \times 10^6\,nm$$

$$\lambda_{pk} = 1.06 \times 10^{-3}\,m$$

$$\lambda_{pk} = 4.965$$

$$B = \frac{hc}{k_B T}$$

$$\lambda_{pk} = \frac{1}{4.965} \frac{hc}{k_B T}$$

Sun

$$\lambda_{pk\,sun} = 290_{nm}\left(\frac{10000\,K}{5780\,K}\right)$$

$$\lambda_{pk} = 502_{nm}$$

Part 2

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left[e^{h\nu/kT}-1\right]^{-1} = \frac{2h\nu^3}{c^2 e^{h\nu/kT}-c^2}$$

$F(T) = 2h\nu^3 \quad G(T) = c^2 e^{h\nu/kT} - c^2$

$F'(T) = 6h\nu^2 \quad G'(T) = (0)(e^{h\nu/kT}) + (c^2)\left(\frac{h}{kT}e^{h\nu/kT}\right) - 0$

$$0 = \frac{F'(T)G(T) - G'(T)F(T)}{[G(T)]^2}$$

$$F'(T)G(T) - G'(T)F(T) = 0$$

$$F'(T)G(T) = G'(T)F(T)$$

$$(6h\nu^2)(c^2e^{h\nu/kT} - c^2) = (2h\nu^3)\left(\frac{c^2h}{kT}e^{h\nu/kT}\right)$$

$$6h\nu^2c^2e^{h\nu/kT} - 6h\nu^2c^2 = \frac{2h\nu^3c^2h}{kT}e^{h\nu/kT}$$

$$3\,\cancel{6h\nu^2c^2}(e^{h\nu/kT}-1) = \frac{\cancel{2h}\nu^{\cancel{3}}\cancel{c^2}h}{kT}e^{h\nu/kT}$$

$$3(e^{h\nu/kT}-1) = \frac{\nu h}{kT}e^{h\nu/kT}$$

$$3 = \frac{\frac{\nu h}{kT}e^{h\nu/kT}}{e^{h\nu/kT}-1}$$

let $y = \frac{h}{kT}$

$$\frac{\nu y e^{\nu y}}{e^{\nu y}-1} = 3$$

y = constant, so graph $\frac{xe^x}{e^x-1} =$

crosses at 2.82

$$\nu_{pk} = \frac{2.82(1.381\times10^{-23})}{6.626\times10^{-34}} = \boxed{5.87\times10^{10}\ \text{Hz}\cdot T}$$

Part 1

$f = \frac{c}{\lambda_{pk}} \quad \lambda_{pk} = 290\,\text{nm}\left(\frac{10000}{T}\right)$

$$f = \frac{3\times10^8}{(2.9\times10^{-7})\left(\frac{10000K}{T}\right)}$$

$$f = \frac{1.03\times10^{15}}{10000} = \boxed{1.03\times10^{11}\ \text{Hz}\cdot\frac{K}{T}}$$

1.7) The amount of energy absorbed is equal to amount of energy with which the photon is emitted and therefore there is no net flux for thermal emission. 1.7

1

$T_{e\odot} = 5700 \quad K$

Wien's Law $\rightarrow$ $\lambda_{max} T = 2.89 \times 10^{-3}\ mK$

$$\lambda_{max}(5700K) = 2.89 \times 10^{-3}\ mK$$

$$\lambda_{max} \simeq 500\ nm \rightarrow \text{V-filter}$$

2

$$\lambda_{max}(8000\ K) = 2.89 \times 10^{-3}\ mK$$

$$\lambda_{max} = 362\ nm \rightarrow \text{U-filter}$$

ASTRO HW #1

8.) $L_\odot = 3.86 \times 10^{26}\,W$ $L_K = 100 L_\odot$ $\sigma_{SB} = 5.7 \times 10^{-8}\ W/m^2 \cdot K^4$

$T_\odot = 5780\,K$ $R_K = ?$

$R_\odot = 6.957 \times 10^{8}\,m$ $T_K = 5{,}300\,K$ found from HR Diagram

$$L_K = 4\pi \sigma_{SB} R_K^2 T_K^4$$

$$R_K = \sqrt{\frac{L_K}{4\pi \sigma_{SB} T_K^4}}$$

$$R_K = \sqrt{\frac{100(3.86 \times 10^{26})}{4\pi (5.7 \times 10^{-8})(5{,}300)^4}}$$

$$R_K = 8.264 \times 10^{9}\,m$$

ASTR 5201 HW

Justin Warren

September 1, 2020

1 Homework 1

Question 10: Calculate B-V for the sun. Compare to the known value of 0.66. Hint: Assume the Flux in a band will correspond to the peak flux across the Full-width at half-maximum. Note that Vega is defined to have B-V = 0

Using (1.4)

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left[exp\left(\frac{h\nu}{k_BT}\right) - 1\right]^{-1} \tag{1}$$

and

$$\lambda = \frac{c}{\nu} \Rightarrow \nu = \frac{c}{\lambda} \tag{2}$$

then

$$B_\nu(T) = \frac{2hc}{\lambda^3}\left[exp\left(\frac{hc}{k_BT\lambda}\right) - 1\right]^{-1} \tag{3}$$

with

$$F_{band} = \int F_\lambda T(\lambda) d\lambda = \int \frac{2hc}{\lambda^3}\left[exp\left(\frac{hc}{k_BT\lambda}\right) - 1\right]^{-1} T(\lambda)d\lambda \tag{4}$$

and finally

$$B - V = -2.5\log\left[\frac{\int_{B-band} \frac{2hc}{\lambda^3}\left[exp\left(\frac{hc}{k_BT\lambda}\right) - 1\right]^{-1} T(\lambda)d\lambda}{\int_{V-band} \frac{2hc}{\lambda^3}\left[exp\left(\frac{hc}{k_BT\lambda}\right) - 1\right]^{-1} T(\lambda)d\lambda}\right] \tag{5}$$

making the substitution

$$u = \frac{hc}{k_B T \lambda} \Rightarrow \lambda = \frac{hc}{k_B T u} \tag{6}$$

$$-\frac{k_B T}{hc} du = \frac{d\lambda}{\lambda^2} \tag{7}$$

then

$$B - V = -2.5 \log \left[\frac{\int_{B-band} u \left[e^u - 1\right]^{-1} T(\frac{1}{u}) du}{\int_{V-band} u \left[e^u - 1\right]^{-1} T(\frac{1}{u}) du} \right] \tag{8}$$

Using the FWHM as the flux value for the B and V filters, 89 nm and 84 nm respectively, and integrating utilizing Gaussian quadrature using the Python 3.8 code:

```
import numpy as np
import scipy.integrate as integrate

def wein(X,FWHM):
    return (X *FWHM)/(np.exp(X) - 1)

def usub(LAM,T):
    return (1239.841984)/(8.617333262e-05 * T * LAM)

B = np.arange(360e-9,560e-9,0.1e-9)/1e-9
V = np.arange(470e-9,700.1e-9,0.1e-9)/1e-9

Bint = integrate.quad(wein,usub(350,5600),usub(580,5600),args=(1/usub(94,5600)))
Vint = integrate.quad(wein,usub(470,5600),usub(680,5600),args=(1/usub(89,5600)))
Bint_VEGA = integrate.quad(wein,usub(350,10000),usub(580,10000),args=(1/usub(94,10000)))
Vint_VEGA = integrate.quad(wein,usub(470,10000),usub(680,10000),args=(1/usub(89,10000)))

B_V = -2.5*np.log10(Bint[0]/Vint[0])
B_V_VEGA = -2.5*np.log10(Bint_VEGA[0]/Vint_VEGA[0])

B_V_Sun = B_V - B_V_VEGA
```

Running this code gets a B-V of 0.30 for the Sun, and -0.14 for Vega. Vega is defined as the "zero" of the B-V magnitude. Adjusting using the equation below:

$$(B - V)_{Sun,Actual} = (B - V)_{Sun} - (B - V)_{Vega} \tag{9}$$

Nets a B-V for the sun of 0.44, less than the B-V from literature value of 0.66.

The color of a star is defined as the ratio between the flux in two wavelength bands, expressed as the difference in magnitude

B-V = measures the ratio of fluxes in the B and V band for a particular star

$$B\text{-}V = m_B - m_V = -2.5 \log\left[\frac{\int_B F_\lambda d\lambda}{\int_V F_\lambda d\lambda}\right]$$

$$= -2.5 \log\left[\frac{(F_\lambda \cdot \Delta\lambda)_{B\text{-band}}}{(F_\lambda \cdot \Delta\lambda)_{V\text{-band}}}\right]$$

$\Delta\lambda_V$ = width at the half maximum for V-band filter is created at 551nm
= 88 nm

$\Delta\lambda_B$ = 94 nm

λ_V = 551 nm   T = 5778

λ_B = 445 nm

$$F_\lambda = \kappa B_\lambda$$

$$F_\lambda = \kappa \frac{2hc^2}{\lambda^5}\left[\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right]^{-1}$$

$$F_{\lambda_B} = \kappa \frac{2hc^2}{\lambda_B^5}\left[\exp\left(\frac{hc}{\lambda_B k_B T}\right) - 1\right]^{-1}$$

$$F_{\lambda_V} = \kappa \frac{2hc^2}{\lambda_V^5}\left[\exp\left(\frac{hc}{\lambda_V k_B T}\right) - 1\right]^{-1}$$

$$B\text{-}V = -2.5 \log\left[\frac{F_{\lambda_B} \cdot \Delta\lambda_B}{F_{\lambda_V} \cdot \Delta\lambda_V}\right]$$

For sun
T = 5600 K

For Vega
T = 10 000 K

```
void BV(){
   string star[2] = {"Sun","Vega"};
   Double_t KB = 1.38064852E-23; //Boltzmann constant m2 kg s-2 K-1 //
   Double_t h = 6.62607004E-34; // plank constant kg m2 s-1
   Double_t c = 3E+08; // speed of light m/s
   Double_t T[2]= {5600,10000}; // sun, Vega temperature K
   Double_t lambda[2] = { 435E-9 , 551E-9 }; // wave length peak for B and V band at
lambda in meter
   Double_t dellambda[2] = { 78E-9 , 99E-9 };
   Double_t B_lambda [2];
   Double_t B_V[2] ;
   for (int j =0 ; j<2 ;j++){
      for (int i = 0 ; i<2 ; i++){
      B_lambda [i] = ( (2 * h * TMath::Power(c,2)) / TMath::Power(lambda[i],5) ) *
TMath::Power( TMath::Exp( h*c / (lambda[i]*KB*T[j]) )  - 1 ,-1 );
      }
      B_V[j] = -2.5 * TMath::Log10( ( B_lambda[0] * dellambda[0] ) / ( B_lambda[1] *
dellambda[1] ) );
      cout <<"B-V_"<<  star [j]<< "="<<  B_V[j]<<endl;
   }
  cout <<"B-V ="<< "  "<<  B_V[0]-B_V[1]<<endl;
}


//root [0] .L BV.C
//root [1] BV()
//B-V_Sun=0.333864
//B-V_Vega=-0.22561
//B-V =  0.559475


//  NOTE:
//       W = kg m2 s-3
//       unit of Intensity W sr-1 m-2 nm-1
//       To get this : 2*h*c/lambda ->  2 * kg m2 s-1 * m2 s-2 / nm5 -> kg m2 s-3   m2
nm-5  -> W sr-1 m2 nm-5 ->   W sr-1 m-3
```

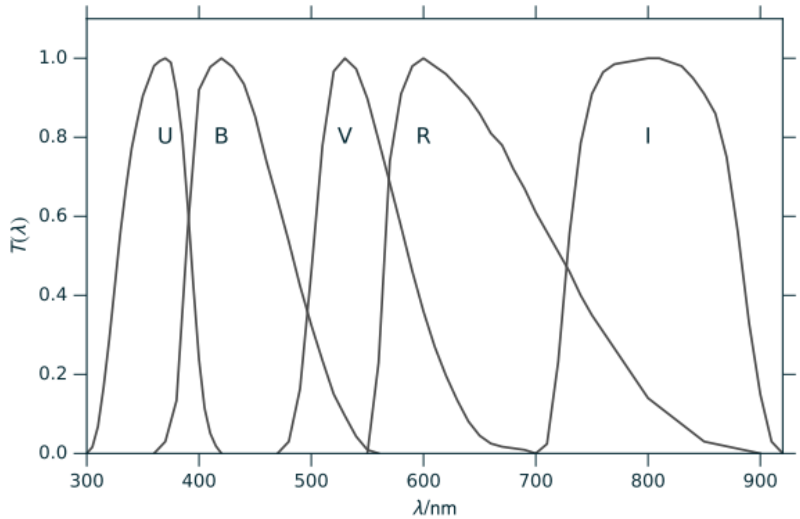
How would the B-V index of the sun compare to that of a hotter star, e.g., one with T_{eff}=8,000 K?

- Since the B-V index is found by looking at the ratio of fluxes in the B and V band, we must look at the spectrum of the stars to try and compare. Due to the temperature of the hotter star, when we look at that temperature on a spectrum, the star will be bluer than the sun. The higher temperature from the spectrum also shows that the wavelength will be shorter in the B band for the hotter star (due to the relation of Wien's displacement law). Due to the sun's T_{eff}, it will have a longer wavelength in the V band as opposed to the B band. Also, while looking at the Hertzsprung Russel Diagram, the hotter star will have a B-V closer to zero, while the sun is around the 0.5 range.

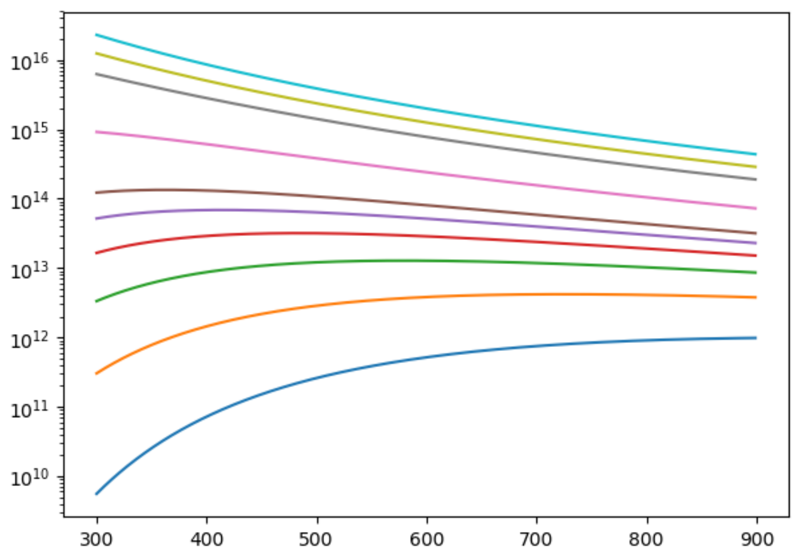
1.12

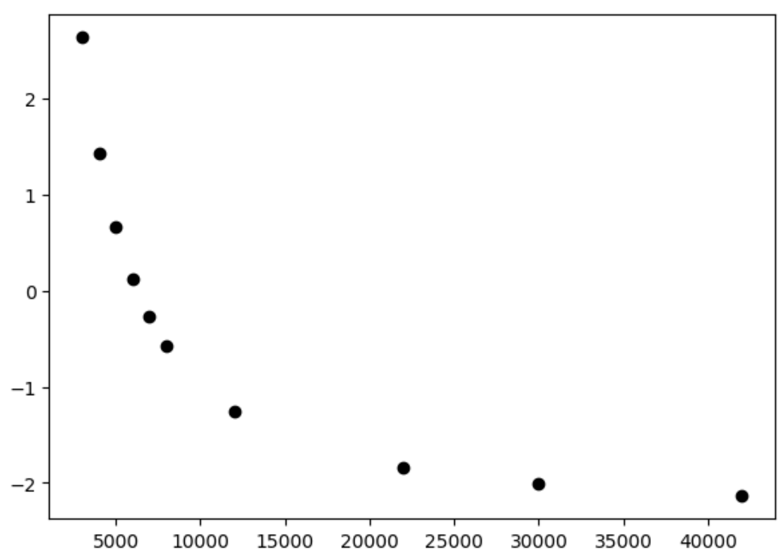
As the effective temperature increases, the shape of the black body spectrum in the range of the B and V filters also changes. Since the black body spectrum's peak condenses toward smaller and smaller wavelengths as the temperature increases, the B and V filters are both sampling an area of the spectrum that is nearly flat, causing the difference to be nearly zero.

The python code attempts to recreate the plot given in the problem, though rather than using transmission functions for the filters, all wavelength within a given range around the peak of the b and v filters is used.



U
B
V
R
I
$T(\lambda)$
λ/nm





```python
# -*- coding: utf-8 -*-
"""
Created on Tue Sep  1 06:28:38 2020

"""

import numpy
import pylab

#constants
h = 6.626*10.0**-34.0
c = 3.0*10.0**8.0
kb = 1.381*10.0**-23.0

#ranges
lam_min = 300.0
lam_max = 900.0
lam_mat = numpy.arange(lam_min, lam_max, 1)
blam_mat = numpy.arange(lam_min, lam_max, 1)
temps = [3000.0, 4000.0, 5000.0, 6000.0, 7000.0, 8000.0, 12000.0, 22000.0, 30000.0, 42000.0]
BV = numpy.arange(0, 10, 1, float)

#black body equation
def Blam(T,lam):
    return ((2.0*h*c**2.0)/((lam*10.0**-9.0)**5.0))/(numpy.exp(h*c/((lam*10.0**-9.0)*kb*T)) - 1.0)

fig = pylab.figure(figsize=(7,5), dpi=100)

k = 0
for Temp in temps:
    j = 0
    B = 0.0
```

```
    V = 0.0
    for i in lam_mat:
        blam_mat[j] = Blam(Temp, i)

        #estimated range for B filter
        if (i > 350 and i < 560):
            B+=blam_mat[j]

        #estimated range for V filter
        if (i > 470 and i < 700):
            V+=blam_mat[j]

        j+=1

    print("B-V at "+str(Temp)+"K = "+str(-2.5*numpy.log(B/V)))
    BV[k] = -2.5*numpy.log(B/V)

    pylab.plot(lam_mat, blam_mat)

    k+=1

pylab.yscale('log')
pylab.show()

fig = pylab.figure(figsize=(7,5), dpi=100)
pylab.plot(temps, BV, 'ko')
pylab.show()
```