An introduction to

Stellar Equilibrium

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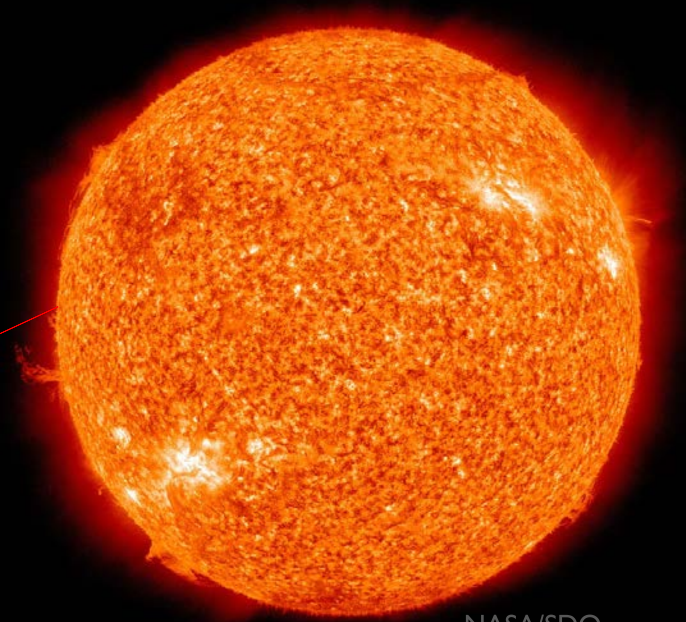
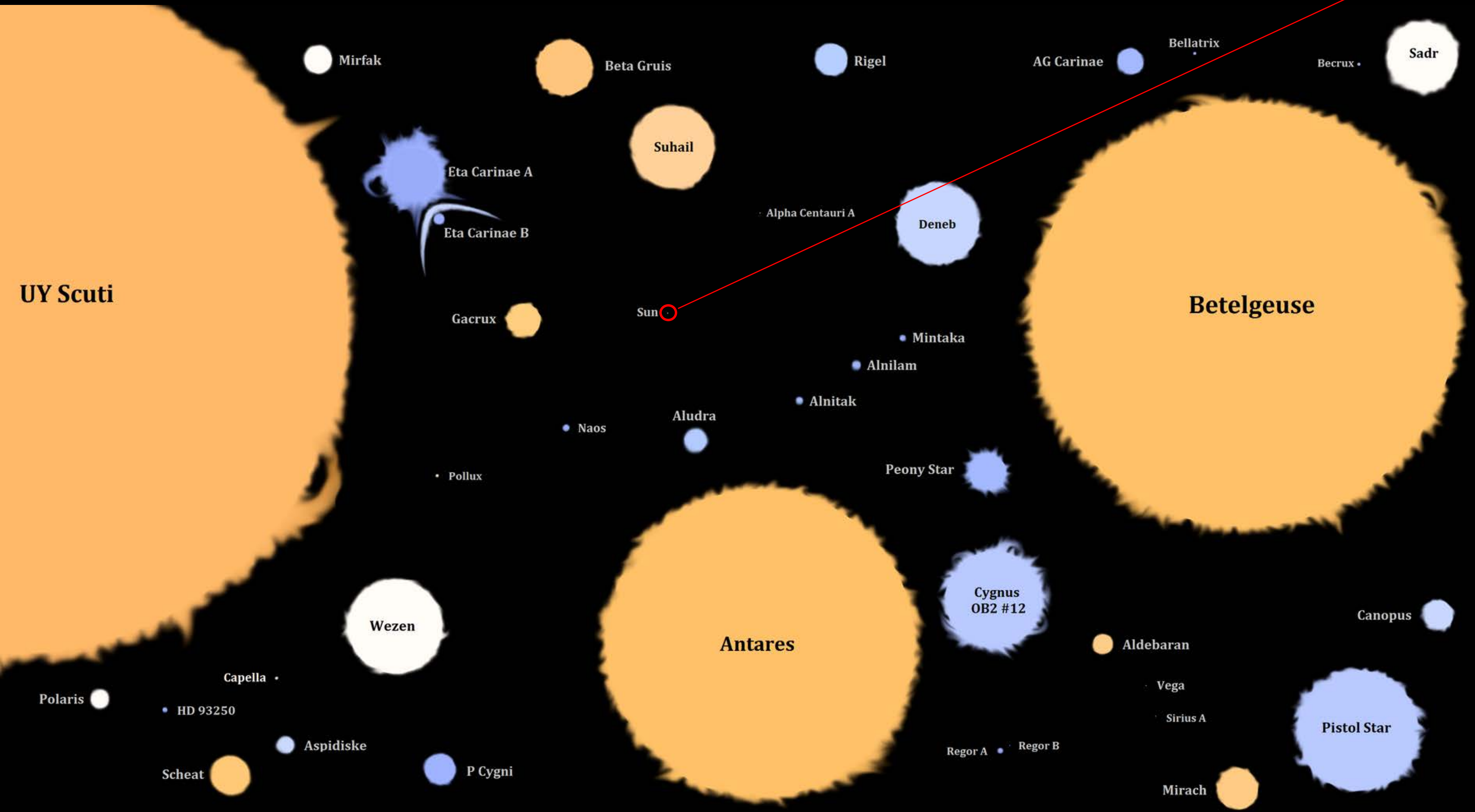
What is a star?

A luminous, massive object where the inward force of gravity is balanced by outward pressures.



NASA/SDO

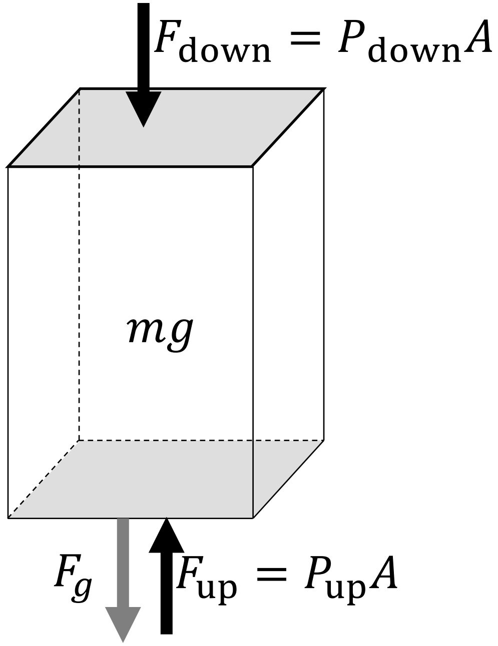


GiovanniMartin16

Beyond this, the details can vary quite a bit.

Hydrostatic equilibrium: *balancing pressure and gravity*

- Newton's second law says that for any object, the sum of all forces $\vec{F}_{net} = m\vec{a}$, where m is the object's mass and $\vec{a}$ is the net acceleration
- For a chunk of star that is "static" (not accelerating), $\vec{F}_{net} = 0$
- The two forces acting on our chunk of star are:
  - Gravity pulling inward ("down")
  - Pressure pushing on the chunk from all sides
    - …but horizontal pressure forces cancel-out
- Vertical forces must be equal to statisfy $\vec{F}_{net} = 0$
  - $F_{\text{pressure,up}} = F_{\text{pressure,down}} + F_{\text{gravity}}$
    - $F_{\text{pressure}} = P \cdot A$
    - $F_{\text{gravity}} = mg$
  - $A\left(P_{\text{up}} - P_{\text{down}}\right) = F_g = mg = \rho V g = \rho \Delta x A g$
    - $-\frac{\Delta P}{\Delta x} = \rho g \quad \longrightarrow \quad \boldsymbol{\frac{dP}{dr} = -\rho g(r)}$

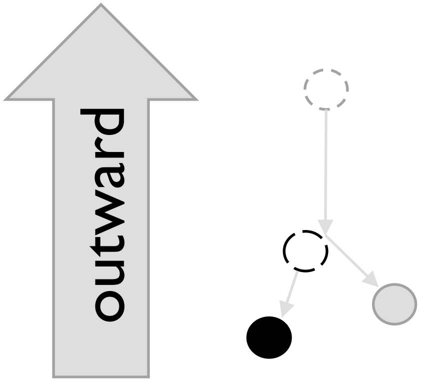


The pressure difference on the top and bottom of a chunk of star must be equal to the gravitational force on that chunk of star.

Where does the pressure come from?

- Re-phrased, what is providing the energy to power a star?

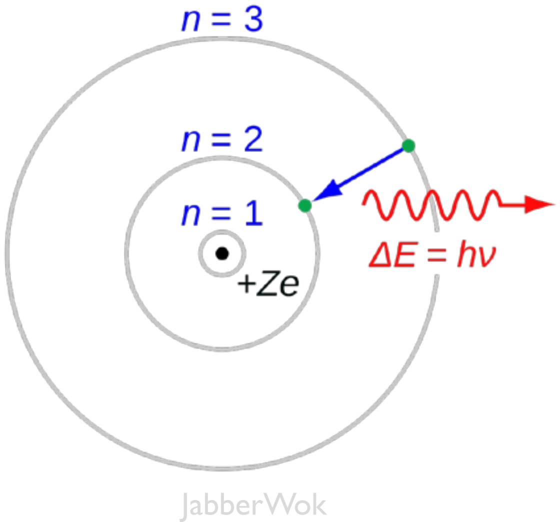
Gravity?



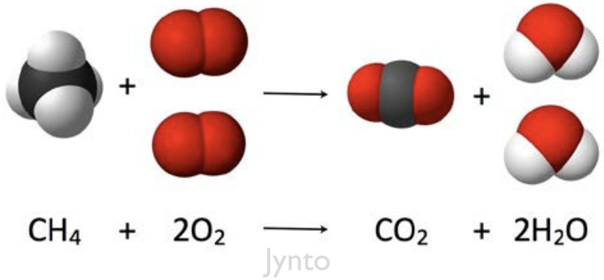
kinetic

$\langle v \rangle \propto T$

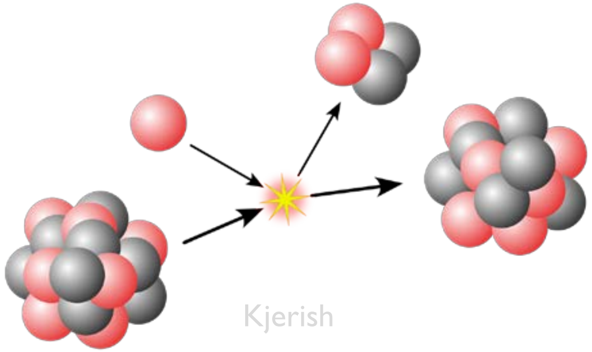
atomic



Combustion?

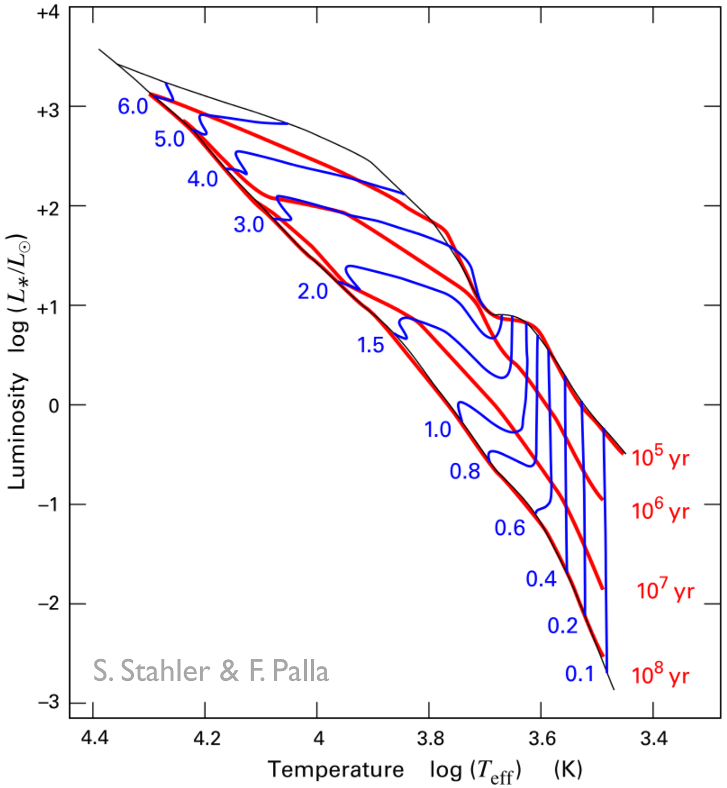


Nuclear power?



Gravitational Power: *the Kelvin-Helmholtz time*

- A star has a gravitational potential energy $E \sim \frac{GM}{R^2}$, which can be released by contraction
- If gravitational energy were powering the sun, then the Sun's luminosity L would be the rate at which this energy is being consumed
- The time it would take for all energy to be exhausted is $\tau_{KH} = \frac{E}{L} \sim \frac{GM^2}{RL}$
- Using the Sun's mass, radius, and luminosity, we get $\tau_{KH,sun} \sim 30\ Myr$
- This is obviously way too short based on radiological and geologic evidence
- The Kelvin-Helmholtz timescale is still interesting, as this is the time it takes for a proto-star to transition into a full-fledged star

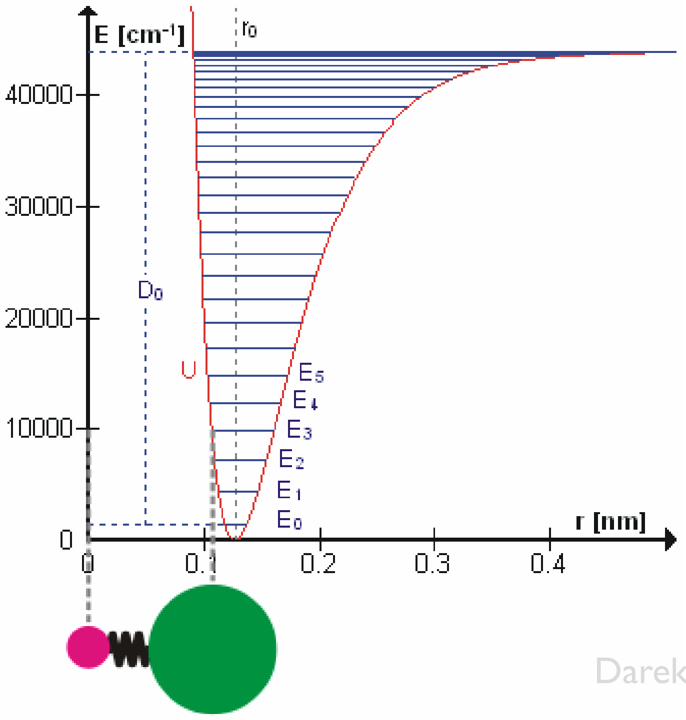


Chemical power: *Breaking atomic bonds*

- Chemical reactions, like combustion, are good enough to power cars and even whole cities
- The energy released is from breaking bonds between atoms
- This isn't all that much energy (~1eV ~ 1.6×10^{-19} J)
- The sun's luminosity is ~10^{34} W = 10^{34} J/s ~10^{45} eV/s
- The sun has ~10^{57} atomic nuclei
- If all nuclei in the sun were atoms with atomic bonds, breaking all of these would be enough energy to power the sun for:

$$\tau_{Chem} = \frac{N_{atoms} E_{atomic-bond}}{L_\odot} \sim 10^{12}\ s \sim 32\ kyr$$

- …obviously not long enough



Darekk2

This skull is ~6Myr old

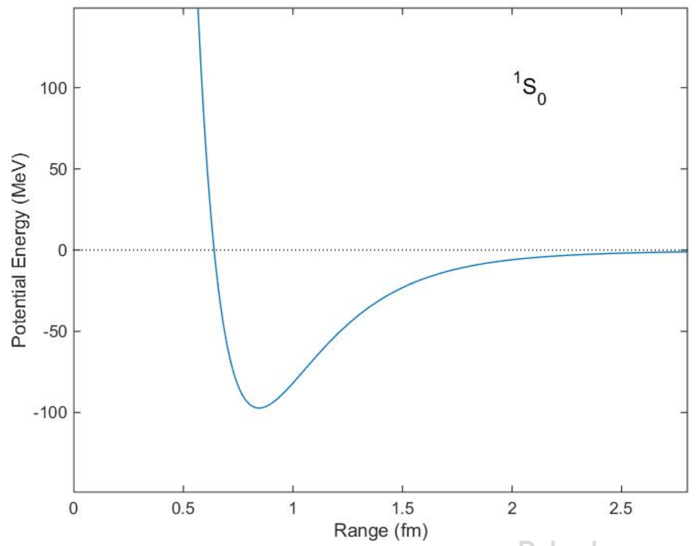
Didier Descouens & Brunet et al. Nature 2002

Nuclear power: *Breaking nuclear bonds*

- Nuclear power is in a league of its own for energy production
- The energy released is from reconfiguring nucleons, which may be breaking or forming nuclear bonds
- This is a lot more energy (~10^6eV ~ 1.6×10^{-13} J)
- The sun's luminosity is ~10^{34} W = 10^{34} J/s ~10^{45} eV/s
- The sun has ~10^{57} atomic nuclei
- If all nuclei in the sun underwent nuclear reactions:

$$\tau_{Nuc} = \frac{N_{atoms} E_{nuclear-bond}}{L_\odot} \sim 10^{18}\ s \sim 32\ Gyr$$

- …which gets the job done



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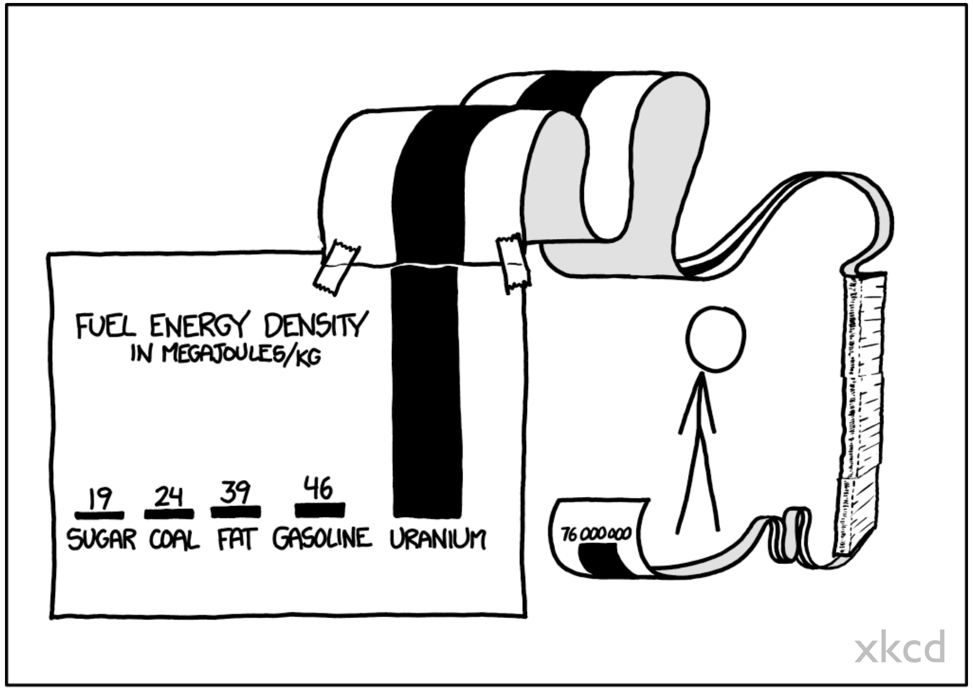

xkcd

SCIENCE TIP: LOG SCALES ARE FOR QUITTERS WHO CAN'T FIND ENOUGH PAPER TO MAKE THEIR POINT *PROPERLY.*