An introduction to

Math Fundamentals

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Algebra: "*solving for x*"

- Example 1:
  - $y = a*x$
  - We can move a variable that is in the numerator (top part of the fraction) on one side of the equation to the denominator on the other side of the equation.
  - So "$y = a*x$" becomes "$y/a = x$".
- Example 2:
  - $y = a/x$
  - This becomes, $y*x = a$
  - Which becomes $x = a/y$
- Example 3:
  - $y = a*x^2$
  - First we can apply the same manipulation as above, so our equation becomes: $y/a = x^2$
  - Now we need to "undo" the exponent, which is done by taking a "root". When the exponent is 2, we use a square root: $x = \sqrt{y/a}$

Working with exponents

- The meaning of positive and negative exponents:
  - A positive exponent means that number is multiplied by itself some number of times, where the number of times is the exponent:
    - Example 1: $x^2 = x*x$
    - Example 2: $10^2 = 10*10 = 100$
  - A negative exponent is almost the same thing, but the numbers are actually in the denominator.
    - Example 1: $x^{-2} = 1/(x*x)$
    - Example 2: $10^{-1} = 1/(10*10) = 1/100 = 0.01$
- When multiplying numbers, add exponents:
  - Example 1: $10^a*10^b = 10^{a+b}$
  - Example 2: $(2x10^2)*(3x10^{-1}) = (2*3)x10^{(2-1)} = 6x10^1 = 6*10 = 60$
- When dividing numbers, subtract exponents:
  - Example 1: $10^a/10^b = 10^{a-b}$
  - Example 2: $(2x10^2)/(3x10^{-1}) = (2/3)x10^{(2-(-1))} = 0.667x10^3 = 0.667*10*10*10 = 667$

A factor of 10 is an "order of magnitude". So 100 is two orders of magnitude larger than 1.

Units as a tool to check your answer

- Making sure that you have the right unit is one way to check that you got the algebra right (though it is of course no guarantee!).
  - Example: A car takes 2 hours to travel 60 miles. How fast was its average speed?
    - Speed = (Distance Traveled)/(Elapsed Time)
    - Speed = (60 mile)/(2 hour)
    - Speed = (60/2)*(mile/hour)
    - Speed = 30 mile/hour = 30 miles per hour ...which is the unit you know & love, and is also a reasonable speed for a car to travel (if a bit slow).
    - If you had accidentally used Speed = (Distance Traveled)*(Elapsed Time), you would get 120 mile*hour.
    - This is a bit fast for a car, but not necessarily unreasonable. However, mile*hour is not a proper unit for speed, so you know a mistake happened.